



CredGuild

STUDY NOTES

CFA® Program Level I

Derivatives

2027 curriculum · 10 readings · 32 practice questions

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ABOUT THIS EDITION

CredGuild · CFA® Program Level I · Derivatives

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USING THIS BOOK

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How to use this book

These are the CredGuild notes for the 2027 CFA® Program Level I curriculum, Derivatives: 10 readings. They are the same notes as on credguild.com, set out for reading on paper or offline.

What each reading contains

- **What you'll learn:** the learning outcome statements the reading covers.
- **The notes, module by module:** worked examples, key concepts, common exam traps, exam shortcuts and a bottom line.
- **A quick check after each module:** one question with its answer and the full explanation.
- **Practice questions at the end, with the answers.** The full explanation for each one is free on the reading's page at credguild.com; the link is under the answers.

The coloured boxes

KEY CONCEPT

Definitions and formulas to know.

Example. A worked problem, step by step. Try it before you read the working.

COMMON EXAM TRAPS

Mistakes that cost marks.

EXAM SHORTCUTS

Faster ways through a question.

BOTTOM LINE

What to remember from the module.

Using it with the website

Paper is good for reading. Practice works better on the site: it scores each answer as you go, explains every option, keeps a list of the questions you got wrong and spreads the readings across the weeks before your exam. If you are not sure where to start, the free diagnostic test (credguild.com/cfa/level-1/diagnostic/) gives you a score in each topic.

If this book helps you, pass it on to someone else who is studying.

CFA® PROGRAM LEVEL I · TOPIC

Derivatives

Exam weight 6–9% · 10 readings

What forwards, futures, swaps and options are and why issuers and investors use them, how no-arbitrage pricing and replication set their values, put-call parity and the one-period binomial model.

Reading 70	Derivative Instrument and Derivative Market Features
Reading 71	Forward vs Futures
Reading 72	Derivative Benefits, Risks, and Issuer and Investor Uses
Reading 73	Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives
Reading 74	Pricing and Valuation of Forward Contracts and for an Underlying With Varying Maturities
Reading 75	Pricing and Valuation of Futures Contracts
Reading 76	Interest Rate Swap
Reading 77	Pricing and Valuation of Options
Reading 78	Put Call Parity
Reading 79	Binomial Option Pricing Model

DERIVATIVES · READING 70

Derivative Instrument and Derivative Market Features

What you'll learn

LOS 70.a Define a derivative, identify its underlying, contract price, size and settlement date, and describe long/short exposure, cash vs physical settlement, and hedging vs speculation.

LOS 70.b Describe derivative markets and contrast exchange-traded (standardized, centrally cleared) with OTC (customized, dealer) derivatives, including the central clearing mandate.

MODULE 70.1

Derivatives Markets

This reading defines derivatives, their basic features and their uses, and compares exchange-traded derivatives with those traded over the counter. A candidate must be able to compute the payoff of a forward to each side, distinguish hedging from speculation and deliverable from cash-settled contracts, and contrast exchange-traded and OTC markets, including the role of central clearing.

LOS 70.a — What a derivative is and its basic features

A **derivative** is a contract whose value comes from (is *derived* from) the value of something else at a specified future date. That something else is the **underlying**: most often a stock or bond price, a stock or bond index, or an interest rate, but also a currency, a commodity, credit, or even the weather, a cryptocurrency or longevity.

Every derivative contract spells out four basic features:

KEY CONCEPT

Feature	Meaning	Example (the forward below)
Underlying	Asset or variable that drives the contract's value	Shares of a listed company
Contract price (e.g., forward price)	Price at which the future trade will take place	\$62 per share
Contract size	Quantity of the underlying covered	300 shares
Settlement date (maturity date)	Date on which the exchange (or payment) occurs	Three months from today

The forward price is normally chosen so that the contract is worth zero to both parties at initiation, and neither side pays anything to enter it.

Long and short exposure

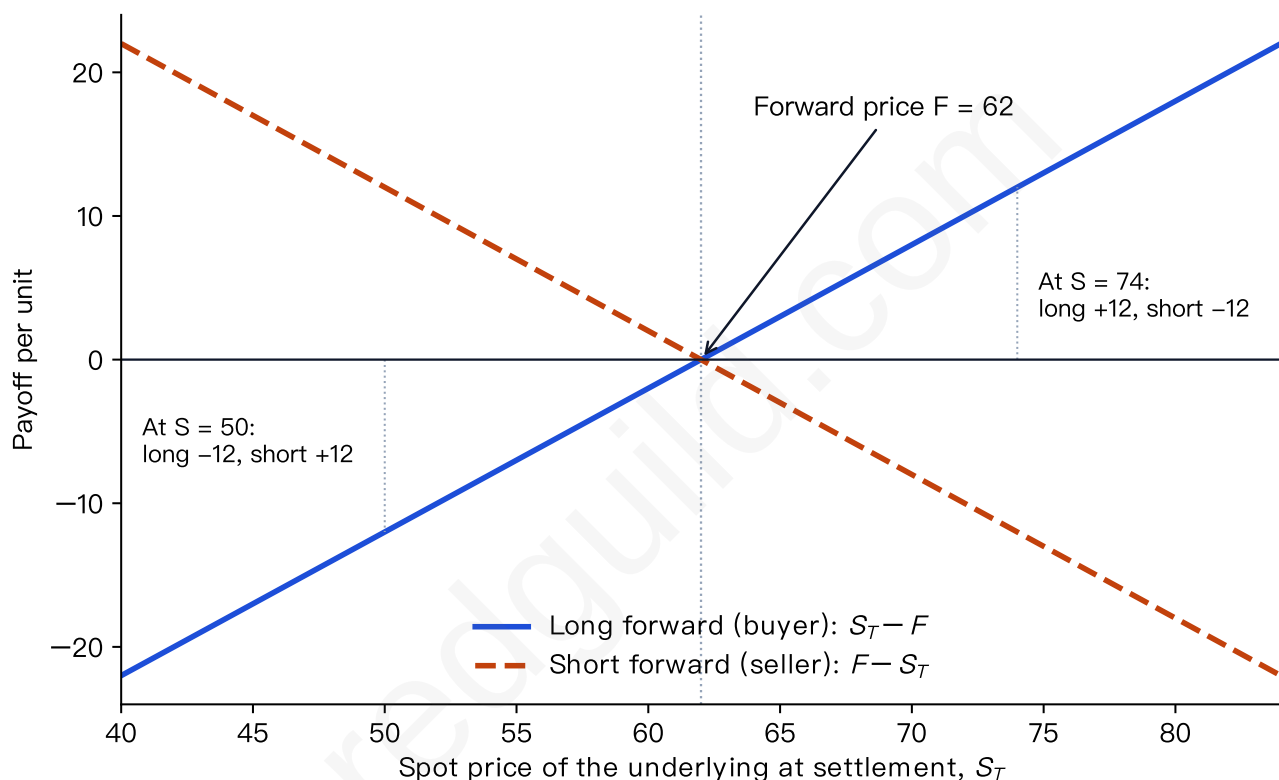
- The buyer of a forward agrees to buy the underlying. She has **long exposure** and gains when the spot price at settlement is above the forward price.

- The seller agrees to deliver the underlying. He has **short exposure** and gains when the spot price at settlement is below the forward price.
- One side's gain equals the other side's loss (a zero-sum outcome before costs).

KEY CONCEPT

$$\text{Payoff to long at settlement} = S_T - F \quad \text{Payoff to short} = F - S_T$$

Because neither side pays anything to enter the contract, this payoff is also each party's profit or loss on the forward.



Payoff at settlement of a long and a short forward with a forward price of 62

Example. A forward commits one party to buy 300 shares at \$62 in three months.

- If the spot price at settlement is \$74, the buyer pays $300 \times 62 = \$18,600$ for shares worth $300 \times 74 = \$22,200$ and gains \$3,600. The seller loses \$3,600.
- If the spot price at settlement is \$50, the buyer loses $300 \times (62 - 50) = \$3,600$ and the seller gains the same amount.

Deliverable versus cash-settled

- A **deliverable contract** requires the underlying to be exchanged for the contract price at settlement.
- A **cash-settled contract** exchanges only the net gain or loss: the losing side pays $|S_T - F| \times \text{size}$ to the winning side. With a settlement price of \$50 in the example above, the buyer pays the seller \$3,600 in cash instead of paying \$18,600 for shares worth \$15,000. Ignoring trading costs, the two methods are

economically the same.

Hedging versus speculating

- **Hedging:** a party with an existing risk uses a derivative to offset it, fully or partially. A holder of shares who sells them forward locks in the sale price.
- **Speculating:** a party with no existing exposure takes one on through the derivative, which increases its risk.
- A useful rule for hedgers: take the derivative position now that matches the trade the party must make later in the cash market. A firm that will receive foreign currency will have to sell it, so it sells the currency forward. A firm that must pay foreign currency buys it forward.

Why use derivatives instead of a cash market trade?

- Exposure can be obtained with little cash, so the position is highly leveraged.
- Transaction costs can be much lower than for the equivalent cash trade.
- Entering the position may move the price of the underlying less than an equivalent cash trade would.

Underlyings and the main derivative types

A higher interest rate benefits the buyer of an interest rate forward but hurts the buyer of a bond forward, because bond prices fall when rates rise. Commodities may be hard (mined or extracted, such as gold and oil) or soft (grown, such as coffee and cattle). Currency forwards are quoted as price currency per unit of base currency: USD/EUR 1.10 means one euro costs 1.10 US dollars. The long side agrees to buy the base currency (here, euros) at the forward rate and pays the price currency; the short side sells the base currency.

The main derivative types are forwards, futures, options and swaps. A **call option** lets its buyer purchase the underlying on a later date at a price set today, with no obligation to do so; a **put option** gives its buyer the same kind of right to sell. In a plain **interest rate swap**, the two sides exchange periodic payments on the same notional amount: one pays a fixed rate, and the other pays a floating rate set from a market reference rate (MRR). The fixed-rate payer is in effect borrowing at a fixed rate to hold a floating-rate bond. In a **credit default swap**, one party makes fixed periodic payments, and the other pays only if the issuer of the underlying credit instrument defaults.

LOS 70.b — Derivative markets: exchange-traded versus OTC

Exchange-traded derivatives

Futures, some options and some other derivatives trade on organized exchanges; the largest by trading volume are the National Stock Exchange of India, B3 (Brazil) and CME Group (United States). The exchange writes **standardized contracts** (size, underlying, dates) and the rules of trading. Exchange members (**market makers** or dealers) mainly earn the **bid-ask spread** rather than speculating.

The **central clearinghouse (CCH)** steps in between the two traders and becomes the buyer to every seller and the seller to every buyer (**novation**). It therefore guarantees that each side's obligations will be performed. To back that guarantee, it collects deposits from both sides when a trade opens and further deposits when a position loses value, which keeps **counterparty credit risk** very low.

Standardization makes exchange contracts more liquid, more transparent and cheaper to trade, and it makes it easy to exit a position by taking an offsetting one. It also simplifies **clearing** (recording the trade and handling payments) and **settlement** (final delivery or payment at maturity).

Over-the-counter (OTC) dealer markets

Forwards, most swaps and some options are customized contracts that dealers create in a market with no central trading venue, the **over-the-counter (OTC) market**. Size, underlying, settlement date and delivery method can be tailored, and the trade can be kept private. Without a central clearinghouse, each side bears the other's counterparty credit risk. Dealers trade with end users and also with each other to lay off their exposure to price changes in the underlying.

Since 2008, a **central clearing mandate** has required many swaps to be cleared through a **central counterparty (CCP)**. Trades reported on a **swap execution facility (SEF)** are replaced by two trades with the CCP. This lowers counterparty risk but concentrates it in the CCP.

KEY CONCEPT

Feature	Exchange-traded	OTC (no central clearing)
Terms	Standardized	Customized
Where traded	Centralized exchange, through members	Dealer network, no central location
Liquidity	Higher	Lower
Transparency	High; trades known to exchange and regulators	Lower
Regulation	Exchange rules; more regulated	Traditionally less regulated; post-2008 rules now cover many OTC derivatives
Counterparty risk	Minimal (CCH guarantee plus deposits)	Each side faces the other
Collateral deposits	Required from both sides (exchange/CCH margin)	No exchange-imposed margin; depends on the contract, the counterparties and regulation
Trading costs	Lower	Higher
Clearing and settlement	Simpler	More difficult

OTC trades that fall under the central clearing mandate have lower counterparty risk and more disclosure, but they remain customizable dealer contracts.

Exam convention: exchange-traded derivatives are standardized, liquid, transparent and margined through the clearinghouse, while OTC derivatives are customized, less transparent, largely unregulated, not subject to collateral deposit requirements and carry more counterparty risk. Current practice: trading outside a CCP does not mean trading outside regulation or without collateral. Bilateral collateral is set by the contract and the counterparties, and for many covered counterparties uncleared-swap margin rules require both initial and variation margin.

Common exam traps

- A derivative's value depends on an underlying. Being a contract, or trading in a particular place, does not make an instrument a derivative.
- A forward has zero value at initiation, but the forward price is not zero.
- Selling forward is not automatically a hedge. The same trade hedges an existing exposure but is speculation for a party that has no such exposure.
- The clearinghouse does not hold an insurance policy or stabilize prices of the underlying. It guarantees performance by being the counterparty to both sides.
- Illiquidity, customized terms and bilateral counterparty risk describe OTC contracts outside central clearing. Exchange-traded futures and options are standardized, liquid and backed by the clearinghouse.

EXAM SHORTCUTS

- A hedger takes now the derivative position that matches the trade it must make later in the cash market: a party that will have to sell the underlying sells forward, and a party that will have to buy it buys forward.

BOTTOM LINE

- A derivative is a contract whose value is derived from an underlying, and every derivative specifies four features: its underlying, contract price, settlement date and contract size.
- The forward price is normally set so that the contract is worth zero to both parties at initiation; at settlement the long receives $S_T - F$ and the short receives $F - S_T$, so one side's gain is the other's loss.
- A deliverable contract exchanges the underlying for the contract price, a cash-settled contract exchanges only the net gain or loss, and ignoring trading costs the two are economically the same.
- Hedging uses a derivative to offset an existing risk, while speculating takes on an exposure the party did not have, so the same trade can be a hedge for one party and speculation for another.
- Compared with a cash market trade, a derivative gives highly leveraged exposure for little cash, can cost much less to trade and may move the underlying's price less.
- For exchange-traded derivatives, the central clearinghouse becomes the buyer to every seller and the seller to every buyer (novation), guaranteeing performance and collecting deposits from both sides, which keeps counterparty credit risk very low.
- Exam convention: exchange-traded derivatives are standardized, liquid, transparent and margined through the clearinghouse, while OTC derivatives are customized, less transparent, largely unregulated, carry more counterparty risk and are not subject to collateral deposit requirements; current practice: an OTC trade outside a CCP still has collateral set by the contract and the counterparties, and for many covered counterparties uncleared-swap margin rules require initial and variation margin.
- Since 2008, a central clearing mandate has required many swaps to be cleared through a central counterparty, which lowers counterparty risk but concentrates it in the CCP.

QUICK CHECK**Question 1**

Harlow Capital agrees with a dealer to buy 200 shares of Pemberton Mills in three months at a forward price of \$48.00 per share. The contract is cash settled. On the settlement date, Pemberton shares trade at \$44.50. The settlement of the contract is:

- A The dealer pays Harlow \$700.
- B Harlow pays the dealer \$700.
- C Harlow pays the dealer \$9,600.

Answer: B

In a cash-settled forward only the gain or loss changes hands. Harlow, the buyer (long), agreed to pay \$48.00 for shares now worth \$44.50, so it has a loss and pays the difference to the dealer (the short).

$$\text{Payoff to long} = (S_T - F) \times \text{shares} = (44.50 - 48.00) \times 200 = -\$700$$

The long (Harlow) has a \$700 loss and pays it to the short (the dealer). Economically this equals delivery: Harlow would pay \$9,600 for shares it could sell for $200 \times 44.50 = \$8,900$.

Why the other options are wrong

- **A.** This reverses the direction. The buyer of a forward loses when the spot price at settlement is below the forward price, so the buyer pays the seller.
- **C.** \$9,600 is the full forward price ($200 \times \$48$) that Harlow would pay in a deliverable contract in exchange for the shares. With cash settlement, only the net difference is paid.

Key takeaway Cash settlement transfers only $|S_T - F| \times \text{size}$, from the losing side to the winning side.

Practice questions**Question 2**

A client asks her adviser how exchange-traded derivatives differ from contracts arranged with dealers. Which of the adviser's statements about exchange-traded derivatives is *least accurate*?

- A They are backed by a central clearinghouse.
- B They are traded by exchange members at a central exchange.
- C They are generally illiquid.

Question 3

A portfolio manager is explaining to a trustee why an interest rate swap held by the fund is classified as a derivative. The defining feature she should point to is that the instrument's value:

- A** is derived from the value of an underlying asset, rate, or index.
- B** is set out in a contract between two counterparties.
- C** is determined in trading that takes place away from an organized exchange.

Answers: 2. C · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/derivative-instrument-and-derivative-market-features/. The full bank has 7 questions on this reading.

credguild.com

DERIVATIVES · READING 71

Forward vs Futures

What you'll learn

LOS 71.a Define swaps, credit default swaps, and call and put options, and describe the rights and obligations of each party.

LOS 71.b Calculate the value at expiration and the profit of long and short call and put positions, including breakeven and maximum gain/loss.

LOS 71.c Contrast forward commitments (forwards, futures, most swaps) with contingent claims (options, credit default swaps).

MODULE 71.1

Forwards and Futures

This reading describes the main derivative instruments, forwards, futures, swaps, credit default swaps and options, and how option values and profits are measured at expiration. A candidate must be able to track a futures margin account through the daily mark-to-market, compute net swap payments, compute option values, profits and breakevens, and classify a derivative as a forward commitment or a contingent claim.

LOS 71.a — Forward and futures contracts

Forward contracts

A **forward contract** is a private agreement in which one party (the buyer, **long**) is obligated to purchase a physical or financial asset from the other (the seller, **short**) at a price fixed today, on a specific future **settlement date**. Both sides are obligated, and neither pays anything at initiation. The long profits if the underlying's price at settlement is above the forward price; the short profits if it is below.

Futures contracts

A **futures contract** is essentially a forward that is standardized and traded on an exchange. The main differences are:

KEY CONCEPT

Feature	Forward	Futures
Terms	Customized	Standardized
Market	OTC dealer market	Organized exchange
Liquidity / secondary market	Limited	Liquid
Regulation and transparency	Lower	Higher
Counterparty credit risk	Each side bears it	Minimal: clearinghouse guarantee
Gains and losses	Usually settled only at maturity	Settled daily (mark-to-market)

Margin and the daily mark-to-market

Futures **margin** is collateral (cash or acceptable securities) that both buyer and seller must post. It protects the clearinghouse. Unlike margin on a stock purchase, it is not a loan, so no interest is charged.

KEY CONCEPT

- **Initial margin** is the amount that must be deposited before a trade. Its size is set at about the largest price move expected in one day on the contract's value.
- **Maintenance margin** is the minimum balance the account must keep. The exchange sets both margin requirements.
- In the **mark-to-market** (marking to market), every account is credited with gains and debited with losses at the end of each day, based on the change in the settlement price.
- The **settlement price** is the average price of trades during a closing period at the end of the session. The exchange sets the length of that period.
- A **margin call** occurs if the balance falls below the maintenance margin. The trader must then deposit enough to bring the account back to the initial margin, or the position is closed out. In an equity margin account, by contrast, the investor only has to restore the maintenance level.

Daily cash settlement of gains and losses, together with the clearinghouse, minimizes counterparty credit risk on futures. Forward contracts typically are not marked to market daily, although they can be when a central clearing party is used.

Example. A trader buys one crude oil futures contract on 1,000 barrels at \$78.00. Initial margin is \$6,000 and maintenance margin is \$4,500.

Day	Settlement price	Daily gain/loss (long)	Balance before deposit	Deposit	Ending balance
0	78.00	—	6,000	—	6,000
1	76.80	$-1.20 \times 1,000 = -1,200$	4,800	0 (above 4,500)	4,800
2	75.90	$-0.90 \times 1,000 = -900$	3,900	$6,000 - 3,900 = 2,100$	6,000

On Day 2 the balance of \$3,900 is below \$4,500, so the deposit must restore the initial margin of \$6,000. The short's account is credited with the same amounts.

Price limits

Many futures have **price limits**: the exchange caps how far the settlement price may move from the previous day's. Trades outside the limits are not allowed, so if the equilibrium price lies beyond a limit, trading stops. Some exchanges use **circuit breakers** that halt trading briefly when a limit is hit.

Common exam traps

- A margin call restores the account to the initial margin. The maintenance margin is only the trigger. On Day 2 of the crude oil example, restoring only the maintenance level would need \$600 instead of the required \$2,100.
- The settlement price is an average over a closing window. It is not the last trade of the day.

BOTTOM LINE

- A forward is a private, customized agreement that obligates both parties, with nothing paid at initiation; the long profits if the underlying's price at settlement is above the forward price, and the short profits if it is below.
- A futures contract is a standardized, exchange-traded forward whose gains and losses are settled daily and whose performance the clearinghouse guarantees, which keeps counterparty credit risk minimal.
- Futures margin is collateral posted by both buyer and seller, not a loan; initial margin must be deposited before a trade, and maintenance margin is the minimum balance the account must keep.
- When a futures account falls below the maintenance margin, the margin call requires a deposit that restores it to the initial margin, or the position is closed out.
- The settlement price used for the daily mark-to-market is the average price of trades during a closing period, and price limits cap how far it may move from the previous day's settlement price.

QUICK CHECK

Question 1

Laszlo Bencze holds a long futures position. The initial margin on his account is \$3,800 and the maintenance margin is \$2,900. After today's mark-to-market, his account balance is \$2,650. If Bencze wants to keep the position open, the deposit he must make is *closest* to:

- A \$250.
- B \$1,150.
- C \$2,900.

Answer: B

Because the balance (\$2,650) has fallen below the maintenance margin (\$2,900), Bencze receives a margin call. In futures trading the deposit must restore the account to the initial margin level, otherwise the position is closed out.

Balance = \$2,650 < \$2,900 maintenance margin $\Rightarrow$ margin call.

$$\text{Deposit} = \text{initial margin} - \text{current balance} = 3,800 - 2,650 = \$1,150$$

Why the other options are wrong

- **A.** \$250 only restores the account to the maintenance margin ($2,900 - 2,650$). That is the rule for equity margin accounts. Futures accounts must be restored to the initial margin.
- **C.** \$2,900 is the maintenance margin itself. There is no rule requiring a deposit equal to the maintenance margin; the deposit is whatever brings the balance back to the initial margin.

Key takeaway After a futures margin call, the account is topped up to the initial margin whenever the balance drops below the maintenance margin.

MODULE 71.2

Swaps and Options

LOS 71.a — Swaps, credit default swaps and options

Interest rate swaps

A **swap** is an agreement to exchange a series of payments on several settlement dates. In a plain **fixed-for-floating interest rate swap**, one party pays a fixed rate (the **swap rate**) and the other pays a floating **market reference rate (MRR)**, both on the same **notional principal**. On each date the two amounts are netted, and only the **net payment** is made, by the party that owes more. The floating rate for a period is set at the start of that period and paid at its end, so the first payment is known when the swap begins.

The swap rate is chosen to give the swap a value of zero when it starts. A swap is equivalent to a **series of forward contracts** on the floating rate. Each forward settles on one swap settlement date, with the fixed rate playing the role of the forward price. Interest rate forwards often settle at the start of the period, paying the present value of the end-of-period swap amount. Swaps trade in dealer markets, so counterparty credit risk exists unless a central counterparty is used; a central counterparty may also require the parties to post margin and make mark-to-market payments.

Example. A 1-year swap has a notional principal of \$40 million, quarterly payments and a fixed rate of 2.50%, so every fixed payment is $40,000,000 \times 0.025/4 = \$250,000$. The MRRs that set the four floating payments turn out as follows:

Quarter	MRR set at start of quarter	Floating payment (end of quarter)	Net payment
1	2.10% (known at inception)	$40,000,000 \times 0.021/4 = \$210,000$	Fixed-rate payer pays \$40,000
2	2.30%	\$230,000	Fixed-rate payer pays \$20,000
3	2.80%	\$280,000	Fixed-rate payer receives \$30,000
4	3.10%	\$310,000	Fixed-rate payer receives \$60,000

The net payment changes direction once MRR rises above the fixed rate.

A firm with a floating-rate loan can pay fixed and receive floating in a swap to convert its liability to a fixed rate. The floating receipts cover the loan's interest, so the swap hedges the uncertainty about future rates. As rate expectations change after inception, the swap takes on positive value for one side and negative value for the other.

Credit default swaps (CDS)

In a **credit default swap**, the **protection buyer** makes a series of fixed periodic payments to the **protection seller**. The seller pays only if the reference security suffers a **credit event** (default, bankruptcy, involuntary restructuring), and the payment covers the resulting drop in the security's value. The periodic payments reflect the credit spread, which compensates for the expected loss from default (probability of default times the loss if default occurs). The buyer is effectively buying insurance against default; a holder of the risky bond can hedge its default risk this way. The seller earns the credit spread and takes on risk similar to owning the reference bond.

Options

- A **call option** gives its buyer (**holder**) the **right, but not the obligation**, to buy the underlying at the **exercise price (strike price)** until or at expiration. The **call writer** (seller) must sell if the holder exercises.
- A **put option** gives its holder the right to sell the underlying at the exercise price, with no obligation to do so. The **put writer** must buy if the holder exercises.
- Options have a cost at initiation: the buyer pays the **option premium** to the writer.
- A **European option** can be exercised only at expiration.

LOS 71.b — Value at expiration and profit for long and short option positions

With S_T the underlying price at expiration and X the exercise price:

KEY CONCEPT

$$c_T = \max(0, S_T - X) \quad p_T = \max(0, X - S_T)$$

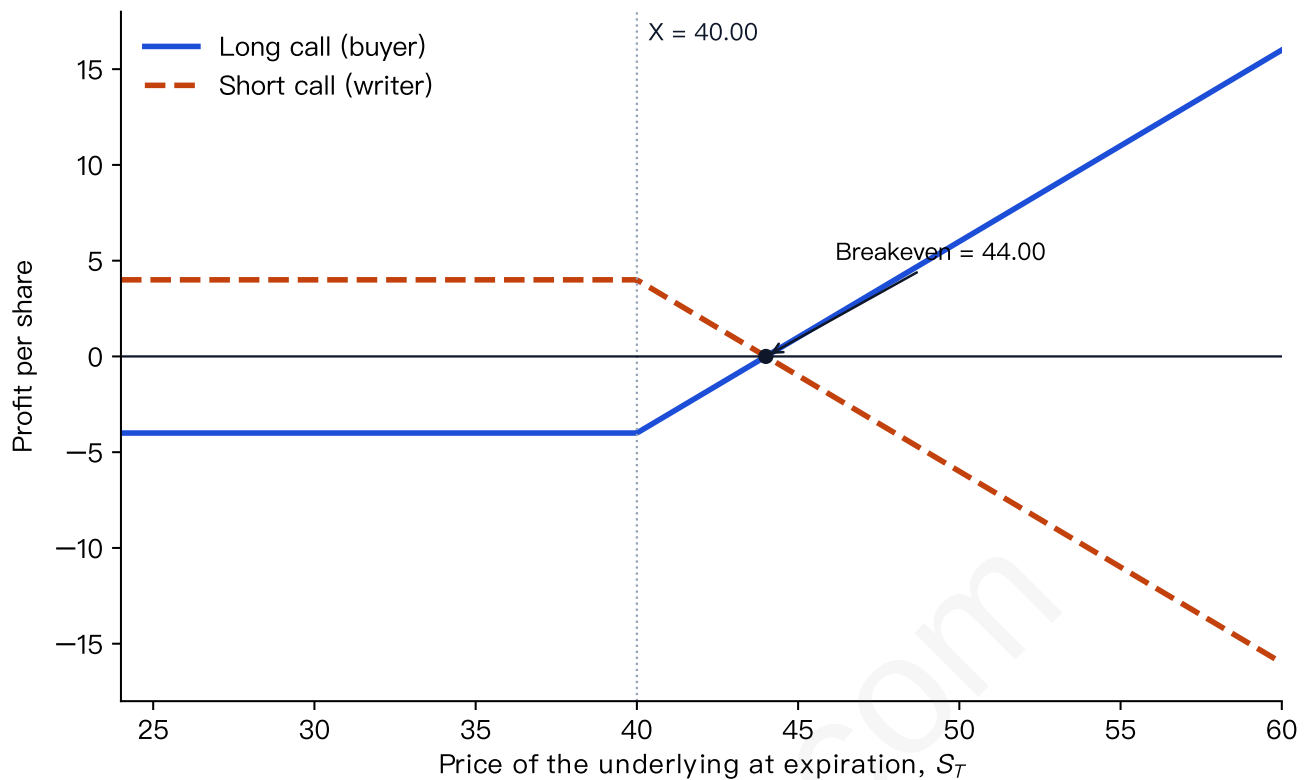
Profit to buyer = value at expiration – premium paid

Profit to writer = premium received – value at expiration

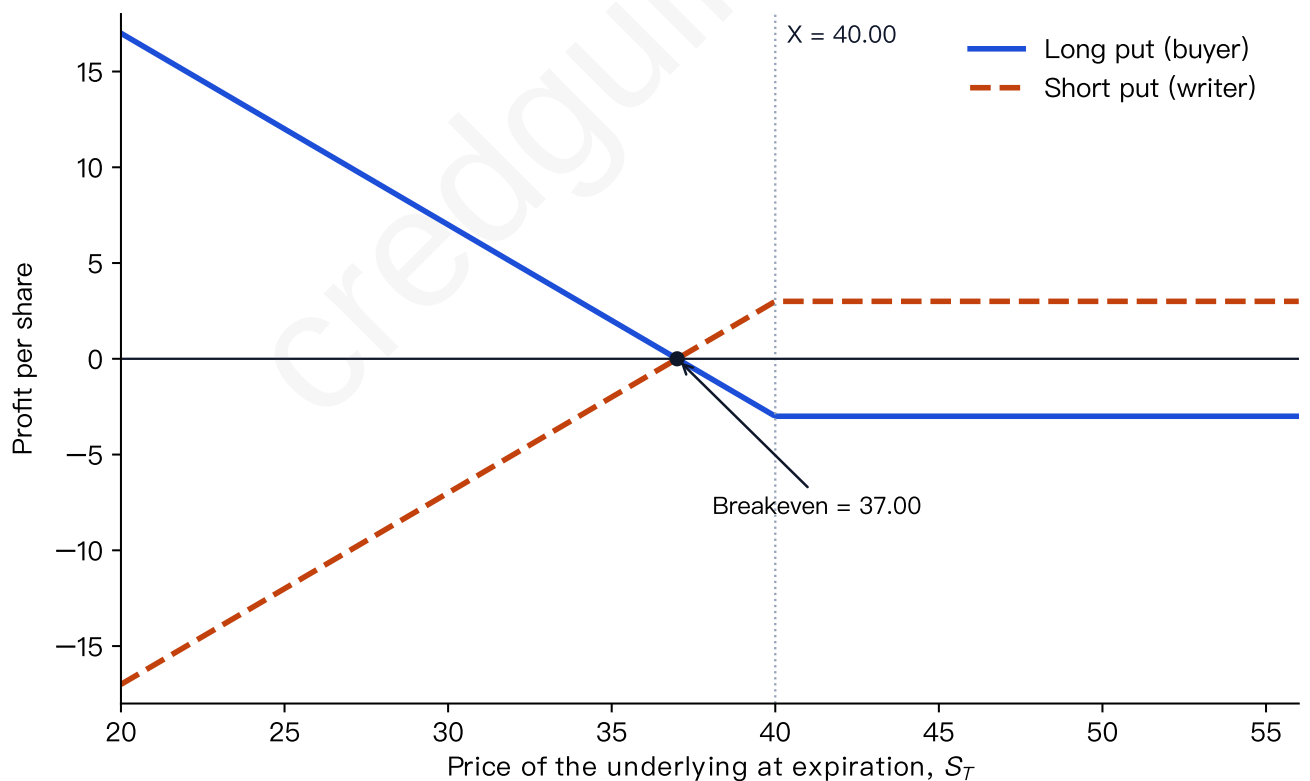
An option's value can never be negative, because the holder simply lets an out-of-the-money option expire. A call is **in the money** when $S > X$ (by $S - X$), and a put is in the money when $S < X$ (by $X - S$). Otherwise the option is **out of the money**, or at the money when $S = X$.

KEY CONCEPT

Position	Max gain	Max loss	Breakeven S_T	Exposure to underlying
Long call	Unlimited	Premium	$X + c_0$	Long
Short call	Premium	Unlimited	$X + c_0$	Short
Long put	$X - p_0$	Premium	$X - p_0$	Short
Short put	Premium	$X - p_0$	$X - p_0$	Long



Profit at expiration of a long and a short call (exercise price 40, premium 4)



Profit at expiration of a long and a short put (exercise price 40, premium 3)

Example. A call and a put both have $X = \$40$. The call premium is \$4 and the put premium is \$3.

S_T	Long call	Short call	Long put	Short put
\$47	$7 - 4 = +3$	-3	$0 - 3 = -3$	+3
\$34	$0 - 4 = -4$	+4	$6 - 3 = +3$	-3

Options are a **zero-sum game**: the writer's profit is always the buyer's loss, and vice versa. The call holder exercises whenever $S_T > X$, even if the premium is not fully recovered, because exercising reduces the loss.

LOS 71.c — Forward commitments versus contingent claims

- A **forward commitment** is a legally binding promise to perform an action in the future (buy, sell, or exchange payments). Forwards, futures and most swaps are forward commitments. The underlying can be an asset, an index, a portfolio or a rate.
- A **contingent claim** is a claim whose payoff is contingent on whether a particular event happens. Options fall into this category (the event being the underlying finishing above or below the strike), and so do credit default swaps (the event being a credit event).

KEY CONCEPT

	Forward commitment	Contingent claim
Obligation	Both parties obligated	Holder has a right; writer is obligated if exercised
Cost at initiation	Usually zero value	Buyer pays a premium
Payoff profile	Linear (symmetric)	One-sided (asymmetric)
Examples	Forwards, futures, interest rate swaps	Calls, puts, credit default swaps

Common exam traps

- A put holder has the right to sell; a put writer has the obligation to buy. A call holder has the right to buy and never an obligation.
- A CDS is called a swap, but it is a contingent claim.
- An option is a contingent claim whatever its underlying. An option on a futures contract is a contingent claim even though the futures contract itself is a forward commitment.
- Profit is value minus premium. When a question asks only for the value at expiration, the premium is not subtracted. At $S_T = \$47$ in the example, the call's value at expiration is \$7; the \$3 figure, after the \$4 premium, is its profit.
- The breakeven for both call parties is $X + c_0$; for both put parties it is $X - p_0$.
- Exposure follows the direction that profits, not the option type. A short call has short exposure and a short put has long exposure.

EXAM SHORTCUTS

- Both sides of a call break even at $X + c_0$ and both sides of a put at $X - p_0$, so the writer's breakeven needs no separate calculation.

BOTTOM LINE

- In a plain fixed-for-floating interest rate swap, the fixed and floating amounts on the same notional principal are netted and only the party that owes more pays; each floating rate is set at the start of its period and paid at the end.
- An interest rate swap works like a set of forwards on the floating rate, one for each settlement date with the fixed rate as the forward price, and the swap rate is chosen to give the swap zero value when it starts.
- In a credit default swap, the protection buyer makes fixed periodic payments and the protection seller pays only if a credit event occurs, covering the drop in the reference security's value.
- $c_T = \max(0, S_T - X)$ and $p_T = \max(0, X - S_T)$; the buyer's profit is the value at expiration minus the premium paid, and the writer's profit is the reverse.
- A long call and a short put have long exposure to the underlying, while a long put and a short call have short exposure.
- A forward commitment (forwards, futures, most swaps) obligates both parties and has a linear payoff, while a contingent claim (options, credit default swaps) gives the holder a right, costs the buyer a premium and has a one-sided payoff.

QUICK CHECK**Question 2**

Bergen Pension Fund owns bonds issued by Holloway Freight. It enters a credit default swap as the buyer of credit protection on those bonds. Under the swap, the fund:

- A** issues a note whose payments come from the cash flows of the Holloway bonds.
- B** exchanges the total return on the Holloway bonds for a fixed or floating rate.
- C** makes a series of periodic payments to the protection seller.

Answer: C

In a credit default swap the protection buyer makes a series of fixed periodic payments to the protection seller. In return, the seller pays an amount that offsets the loss in value if the reference bond suffers a credit event such as default. The fund is effectively buying insurance on its Holloway bonds.

Why the other options are wrong

- A.** A note funded by cash flows from an underlying bond is not a CDS. The CDS protection buyer issues nothing; it pays periodic premiums and receives a payment only after a credit event.
- B.** Exchanging the bond's total return for a fixed or floating rate describes a different credit derivative. In a CDS the buyer pays fixed periodic premiums and receives a payment only after a credit event.

Key takeaway CDS: protection buyer pays periodic premiums; protection seller pays only if a credit event occurs.

Practice questions

Question 3

Tobias Grant buys a call option on Wexford Bio shares with an exercise price of \$36.00 for a premium of \$3.40, when the shares trade at \$38.90. At expiration the shares are priced at \$37.10. Grant's net profit or loss on the call is *closest* to:

- A -\$3.40.
- B -\$2.30.
- C \$2.30.

Question 4

A trainee lists features of forward commitments. Which item on her list is *least accurate*?

- A The underlying can be a stock index or a portfolio.
- B Each party is legally obligated to act at a future date.
- C Either party can walk away at settlement without penalty.

Answers: 3. B · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/forward-vs-futures/. The full bank has 31 questions on this reading.

DERIVATIVES · READING 72

Derivative Benefits, Risks, and Issuer and Investor Uses

What you'll learn

LOS 72.a Describe the benefits (risk transfer, information discovery, operational advantages, market efficiency) and risks (implicit leverage, basis, liquidity, counterparty and systemic risk) of derivatives.

LOS 72.b Compare how issuers (cash flow, fair value and net investment hedges under hedge accounting) and investors use derivatives.

MODULE 72.1

Uses, Benefits, and Risks of Derivatives

This reading weighs the benefits of derivatives against their risks and shows how issuers and investors use them. A candidate must be able to explain the risk-management, information and operational benefits of derivatives, measure the implicit leverage of a margined position, identify basis, liquidity, counterparty and systemic risk, and classify a corporate hedge as a cash flow, fair value or net investment hedge.

LOS 72.a — Benefits and risks of derivatives

Benefits

1. **Changing, transferring and managing risk.** Without buying or selling any cash market security, a user can raise or lower exposure to an index, hedge the currency risk of expected receipts or payments, or turn a floating-rate liability into a fixed-rate one. Derivatives also create exposures unavailable in cash markets. Buying puts on a stock puts a floor under its sale price, and buying calls gives upside exposure with the loss limited to the premium.
2. **Information discovery.** Option prices depend on observable inputs (price of the underlying, exercise price, time to expiration, interest rates) and one unobservable input, expected volatility. Option prices therefore reveal the market's **implied volatility**. Futures and forward prices indicate expected prices of the underlying, and interest rate futures across maturities reveal expected future rates, even the expected number of central bank rate moves.
3. **Operational advantages.**
 - **Ease of short selling:** selling a forward or futures contract is simpler than borrowing an asset to short it, especially where borrowing is hard or short sales are restricted.
 - **Lower transaction costs:** commodity exposure needs no transport, storage or insurance, and entering a swap costs far less than buying back a floating-rate note and replacing it with a new fixed-rate issue.
 - **Greater leverage:** little cash is needed for a large exposure.
 - **Greater liquidity:** the small cash requirement makes very large trades easy.
4. **Improved market efficiency.** Low costs, leverage, liquidity and easy shorting make it cheaper to exploit mispricing, which pushes prices toward fair value.

Risks

Leverage appears on both lists. The small cash requirement behind the greater leverage and liquidity listed as benefits is also the source of the implicit leverage risk in the table below.

KEY CONCEPT

Risk	What it means	Example
Implicit leverage	A small cash deposit controls a large exposure, so small price moves cause large percentage changes in the deposit; opaque structured securities may hide the risk	Margin of 8% of contract value: a 2% adverse price move wipes out $2/8 = 25\%$ of the margin
Basis risk	The derivative's underlying or its settlement date does not match the position being hedged, so the hedge is imperfect	Hedging a 40-stock portfolio with index futures; hedging a harvest due on one date with futures settling weeks later
Liquidity risk	Cash flows on the hedge do not match the cash flows of the hedged position	A producer short futures must meet margin calls when prices rise, before the higher-priced crop is sold
Counterparty credit risk	The other side may fail to pay what it owes	Option buyer is exposed (it may be owed the payoff); option writer is not (it already has the premium); both sides of a forward can be exposed
Systemic risk	Excessive speculation can destabilize markets and institutions	Addressed by regulation such as mandatory central clearing of swaps

With margin equal to a fraction m of contract value, a price change of $\Delta P\%$ changes the margin deposit by approximately

KEY CONCEPT

$$\% \Delta \text{margin} = \frac{\Delta P\%}{m} \quad \text{leverage} = \frac{1}{m}$$

Margins of 3% to 12% imply leverage of roughly 33:1 to 8:1.

Futures carry three protections against counterparty risk: initial margin, daily mark-to-market, and the clearinghouse guarantee. Forwards may have none of these protections unless the contract provides for collateral or central clearing.

LOS 72.b — How issuers and investors use derivatives

Issuers (corporations)

Nonfinancial companies, called issuers in this context, use derivatives to manage the effect of prices, rates and currencies on asset and liability values and on earnings. Where allowed, **hedge accounting** lets a company recognize gains and losses on a qualifying hedge in the same period as the offsetting change in the hedged item. Hedges are classified by purpose:

KEY CONCEPT

Hedge type	Purpose	Typical examples
Cash flow hedge	Reduces variability of future cash flows	Currency forwards on expected foreign-currency receipts; a swap converting a floating-rate liability to fixed rate (interest payments become certain)
Fair value hedge	Offsets changes in the balance-sheet value of an asset or liability	Swap in which the firm pays floating/receives fixed to stabilize the fair value of fixed-rate debt (in effect a floating-rate liability with much lower duration); forwards sold against inventory carried at market value
Net investment hedge	Reduces volatility of the reported (domestic-currency) value of a foreign subsidiary's equity	Currency forwards or futures on the subsidiary's functional currency

The class depends on what is being hedged. Forward contracts appear in all three rows. An interest rate swap can fall in either of the first two, depending on the debt it hedges and which side of the swap the firm takes.

Example. A food company with an outstanding floating-rate bank loan enters a swap paying 4.1% fixed and receiving MRR. Its interest cash flows become fixed, so this is a cash flow hedge. A different company that reports its 6% fixed-rate bonds at fair value enters a swap paying MRR and receiving 6% fixed. When rates change, the gain or loss on the swap offsets the change in the bonds' fair value, so this is a fair value hedge.

Investors

Investors use forward commitments and contingent claims to hedge, modify or increase exposure. For example, they can:

- buy commodity (e.g., silver) forwards to gain price exposure with little or no initial cash;
- lengthen a bond portfolio's duration by entering a swap as fixed-rate receiver and floating-rate payer, which is like issuing floating-rate debt to buy a fixed-rate bond;
- raise equity exposure temporarily by buying index futures, cut it by selling index futures, or protect the downside while keeping the upside by buying index puts.

Common exam traps

- Ease of shorting and implied volatility information are benefits of derivatives.
- The writer of an option bears no counterparty risk once the premium is received; the buyer does.
- Basis risk comes from a mismatch in the underlying or in the dates. Volatility alone does not create it.

BOTTOM LINE

- Derivatives let users change, transfer and manage risk without buying or selling cash market securities, and they create exposures, such as a floor under a sale price, that cash markets do not offer.

- Option prices reveal the market's implied volatility, and futures and forward prices indicate the expected prices of their underlyings, including expected future interest rates.
- Easier short selling, lower transaction costs, greater leverage and greater liquidity make it cheaper to exploit mispricing, which improves market efficiency.
- With margin equal to a fraction m of contract value, a price change of $\Delta P\%$ changes the margin deposit by about $\Delta P\%/m$, so leverage is $1/m$.
- Basis risk arises when the derivative's underlying or settlement date does not match the hedged position, and liquidity risk arises when the hedge's cash flows do not match those of the hedged position.
- An option buyer bears counterparty credit risk while the writer, who already holds the premium, does not, and both sides of a forward can be exposed.
- A cash flow hedge reduces the variability of future cash flows, a fair value hedge offsets changes in the balance-sheet value of an asset or liability, and a net investment hedge reduces the volatility of the reported value of a foreign subsidiary's equity.
- A firm with floating-rate debt that pays fixed in a swap has a cash flow hedge, while a firm with fixed-rate debt carried at fair value that receives fixed and pays floating has a fair value hedge.

QUICK CHECK

Question 1

A manager wants short exposure to a commodity but finds that borrowing physical inventory for a short sale is impractical. Which feature of derivatives is *most useful* to the manager?

- A They eliminate basis risk whenever a derivative is used as a hedge.
- B Short exposure is generally as easy to establish as long exposure.
- C They prevent implicit leverage from magnifying gains and losses on a position.

Answer: B

Selling a forward or futures contract usually creates short exposure without borrowing and selling the physical commodity. This operational advantage is especially important for assets that are difficult to short in the cash market.

Why the other options are wrong

- **A.** A derivatives hedge can still have basis risk when its underlying or settlement date does not match the exposure.
- **C.** Derivatives often provide substantial implicit leverage; they do not remove it.

Key takeaway Derivatives can make short exposure practical when a cash market short sale is difficult or impossible.

Practice questions

Question 2

Which of the following hedges is *most* exposed to basis risk?

- A** A gold refiner sells gold forwards that match the quantity, grade and delivery date of a sale it has already agreed.
- B** A Canadian importer buys a US dollar forward for the amount and settlement date of a dollar invoice it must pay.
- C** A manager of 40 regional bank stocks sells forwards on a broad national equity index equal to the portfolio's value.

Question 3

Marlowe Industries carries its fixed-rate notes payable at fair value and applies hedge accounting. To reduce swings in the reported value of the notes as interest rates change, it enters an interest rate swap in which it pays a floating rate and receives a fixed rate. This hedge is best classified as a:

- A** fair value hedge.
- B** cash flow hedge.
- C** net investment hedge.

Answers: 2. C · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/derivative-benefits-risks-and-issuer-and-investor-uses/. The full bank has 11 questions on this reading.

DERIVATIVES · READING 73

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

What you'll learn

LOS 73.a Explain how arbitrage and replication determine derivative prices, and identify the arbitrage when a forward is mispriced.

LOS 73.b Distinguish the forward price from the expected future spot price, and explain how the cost of carry (financing, holding costs and benefits) sets the no-arbitrage forward price, including with continuous compounding.

MODULE 73.1

Arbitrage, Replication, and Carrying Costs

This reading explains how derivatives are priced by arbitrage and replication, and how the costs and benefits of holding the underlying set a forward price. A candidate must be able to compute a no-arbitrage forward price with discrete or continuous compounding, including carrying costs, benefits and currency interest rates, and describe the trades that exploit a mispriced forward.

LOS 73.a — Arbitrage and replication in derivative pricing

Risky assets are usually valued as the present value of expected cash flows at a risk-adjusted rate. Derivatives are instead priced with a **no-arbitrage condition**.

- **Arbitrage** means simultaneously buying an asset (or portfolio) and selling, at a higher price, another with identical future payoffs in every state. It locks in a risk-free gain with no net investment.
- Arbitrage opportunities may be rare, but when they appear they are exploited rapidly. This forces assets with identical payoffs to the same price (the **law of one price**). Small mispricings can persist when the gain is smaller than the transaction costs of capturing it.
- **Replication** means building a portfolio of cash market positions (the underlying plus risk-free borrowing or lending) that has the same payoff as the derivative for every possible value of the underlying. The derivative must cost the same as its replicating portfolio. A position that combines the derivative with the opposite replicating position has a certain payoff, so the relevant discount rate is the **risk-free rate**, and no assumption about investors' risk preferences is needed.

Replicating forwards (underlying with no costs or benefits of holding)

Position	Replicating portfolio	Payoff at T
Long forward at $F_0(T)$	Borrow S_0 at R_f and buy the underlying	$S_T - S_0(1 + R_f)^T$
Short forward at $F_0(T)$	Short the underlying and lend the proceeds at R_f	$S_0(1 + R_f)^T - S_T$

Matching these payoffs with the forward's payoff $S_T - F_0(T)$ gives the **no-arbitrage forward price**, which is the **future value of the spot price** at the risk-free rate:

KEY CONCEPT

$$F_0(T) = S_0(1 + R_f)^T$$

Owning the asset and selling it forward locks in $F_0(T)$: the combined position has a certain payoff, so it must earn the risk-free rate.

Exploiting a mispriced forward**KEY CONCEPT**

Situation	Forward is...	Arbitrage	Profit at T
$F_0(T) > S_0(1 + R_f)^T$	Too high	Sell the forward , borrow S_0 and buy the underlying	$F_0(T) - S_0(1 + R_f)^T$
$F_0(T) < S_0(1 + R_f)^T$	Too low	Buy the forward , short the underlying and invest the proceeds	$S_0(1 + R_f)^T - F_0(T)$

Arbitrage trading pushes the forward price back toward its no-arbitrage level. The overpriced case can also be read as a return comparison: buying the asset at S_0 and delivering it at $F_0(T)$ earns more than the risk-free rate, so borrowing at R_f to fund the purchase leaves a riskless surplus.

Example. A share paying no dividends trades at \$50, and the 1-year risk-free rate is 6%. The no-arbitrage forward price is $50 \times 1.06 = \$53.00$. If a dealer quotes \$54.00, sell the forward, borrow \$50 and buy the share. In one year, deliver the share for \$54 and repay $50 \times 1.06 = \$53$, a riskless profit of \$1.00 with no initial outlay. If the quote is instead \$52.20, the forward is too cheap: buy the forward, short the share and lend the \$50 proceeds at 6%. In one year, collect \$53.00 from the loan, pay \$52.20 for the share under the forward and return it to close the short, a riskless profit of \$0.80.

LOS 73.b — Spot versus expected future price, and the cost of carry

The no-arbitrage forward price is not a forecast of the expected future spot price. It is the cost of buying the underlying today and carrying it to T : the financing cost at R_f , plus other costs of holding, minus the benefits of holding.

- **Costs of holding** arise mostly for commodities: **storage**, **insurance** and spoilage. Their present value is $PV_0(\text{cost})$. For financial assets these costs are negligible.
- **Benefits of holding** may be monetary (**dividends**, **coupon interest**) or non-monetary. The non-monetary benefit is the **convenience yield**, the advantage of physically holding a commodity, such as being able to sell into a temporary shortage, especially when the asset is hard to short. Their present value is $PV_0(\text{benefit})$.

KEY CONCEPT

$$F_0(T) = [S_0 + PV_0(\text{cost}) - PV_0(\text{benefit})] (1 + R_f)^T$$

Equivalently, the forward price is $S_0(1 + R_f)^T$ plus the future value of the holding costs minus the future value of the benefits. A \$2 dividend paid at settlement, for instance, lowers the forward price by exactly \$2.

- With net costs (costs greater than benefits), the forward price is above $S_0(1 + R_f)^T$; with net benefits, it is below.
- Carrying costs and benefits affect the forward price at initiation and the forward's value during its life. At expiration the value is simply $S_T - F_0(T)$.

A stated annual rate r with continuous compounding corresponds to an effective annual rate of $e^r - 1$. Over T years, $FV = S e^{rT}$ and $PV = S e^{-rT}$. With continuously compounded annual rates (risk-free r , cost c , and benefit b such as a dividend yield):

$$F_0(T) = S_0 e^{rT} \quad F_0(T) = S_0 e^{(r+c)T} \quad F_0(T) = S_0 e^{(r+c-b)T}$$

The first applies with no costs or benefits, the second with costs only, and the third with both.

Compared with the spot price itself, what matters is the total carry, financing included: $F_0(T) = S_0$ when $r + c = b$, $F_0(T) > S_0$ when $r + c > b$, and $F_0(T) < S_0$ only when the benefits exceed financing plus storage costs.

Examples.

- Discrete: spot 200, $PV_0(\text{cost}) = 3$, $PV_0(\text{benefit}) = 1$, $R_f = 5\%$, $T = 0.5$. Then $F_0 = (200 + 3 - 1)(1.05)^{0.5} = 202 \times 1.024695 = 206.99$.
- Continuous: an index at 5,000, dividend yield 2%, risk-free rate 4%, $T = 0.5$. Then $F_0 = 5,000 e^{(0.04-0.02)(0.5)} = 5,000 e^{0.01} = 5,050.25$.

Currency forwards

For an exchange rate quoted as price currency per unit of base currency, the no-arbitrage forward rate reflects the interest rate differential:

$$F = S_0 \times \left(\frac{1 + R_{\text{price}}}{1 + R_{\text{base}}} \right)^T \quad \text{or, continuously compounded,} \quad F = S_0 e^{(r_{\text{price}} - r_{\text{base}})T}$$

Here R_{price} and R_{base} are annual effective risk-free rates and T is in years. For a one-year forward ($T = 1$), the discrete formula reduces to $S_0(1 + R_{\text{price}})/(1 + R_{\text{base}})$. If the rates are instead stated as returns over the forward's own horizon, they are used without the exponent.

The currency with the higher interest rate trades at a forward discount, which offsets its higher interest income. Otherwise, borrowing in one currency and investing in the other would earn an arbitrage profit. If the quoted forward rate (price/base) is above the no-arbitrage rate, borrow the price currency, convert it to the base currency at spot, invest at the base-currency rate and sell the base currency forward. If the quoted rate is below it, reverse every leg. Rates converted to continuous compounding ($r = \ln(1 + R)$) give the same forward rate.

Example. The spot rate is 1.2500 USD/GBP (US dollars per pound), and the 1-year risk-free rates are 4% in US dollars and 5% in pounds (annual effective). The 1-year no-arbitrage forward rate is $1.2500 \times 1.04/1.05 = 1.2381$ USD/GBP. The pound, the base currency, has the higher interest rate, so it buys fewer dollars forward than at spot. A dealer quote of 1.2450 would be too high: borrow dollars, buy pounds at spot, invest them at 5% and sell the pounds forward at 1.2450.

Change (other things equal)	Effect on no-arbitrage forward price
Higher risk-free rate	Increase
Higher storage or insurance costs	Increase
Higher dividends, coupons or convenience yield	Decrease
Longer time to settlement (positive net carry)	Increase

Common exam traps

- Convenience yield is a benefit, so it lowers the forward price even though it is non-monetary. Storage and insurance are costs. In the discrete example, adding the benefit of 1 as if it were a cost gives 209.04 instead of 206.99.
- Derivative pricing assumes that arbitrage is exploited quickly. It does not assume that investors are risk neutral or that arbitrage never occurs.

EXAM SHORTCUTS

- A cash benefit paid at settlement lowers the forward price by exactly its amount, so a \$2 dividend paid at settlement needs no discounting or compounding.
- The currency with the higher interest rate trades at a forward discount, so the direction of the forward rate relative to spot can be read from the two interest rates before any calculation.

BOTTOM LINE

- Arbitrage locks in a risk-free gain with no net investment, so assets with identical payoffs in every state must sell for the same price (the law of one price).
- A derivative must cost the same as the portfolio of the underlying and risk-free borrowing or lending that replicates its payoff, so it is priced with the risk-free rate and no assumption about investors' risk preferences.
- For an underlying that has neither carrying costs nor benefits, the no-arbitrage forward price is the future value of the spot price, $F_0(T) = S_0(1 + R_f)^T$.
- If $F_0(T) > S_0(1 + R_f)^T$, sell the forward, borrow S_0 and buy the underlying; if $F_0(T) < S_0(1 + R_f)^T$, buy the forward, short the underlying and invest the proceeds.
- The forward price is the cost of buying the underlying and carrying it to settlement, $F_0(T) = [S_0 + PV_0(\text{cost}) - PV_0(\text{benefit})](1 + R_f)^T$, and it is not a forecast of the expected future spot price.
- With continuously compounded annual rates for the risk-free rate r , costs c and benefits b , $F_0(T) = S_0 e^{(r+c-b)T}$.
- For an exchange rate quoted as price currency per unit of base currency, $F = S_0[(1 + R_{\text{price}})/(1 + R_{\text{base}})]^T$, and the higher-rate currency trades at a forward discount.
- Other things equal, a higher risk-free rate or higher storage and insurance costs raise the forward price, and higher dividends, coupons or convenience yield lower it.

QUICK CHECK**Question 1**

The no-arbitrage approach used to value derivatives relies on the assumption that:

- A** any arbitrage opportunities that appear are exploited quickly.
- B** investors are risk neutral.
- C** arbitrage opportunities never arise in actual markets.

Answer: A

Derivative valuation assumes that any arbitrage opportunity is quickly traded away when it appears. That trading drives assets with identical future cash flows to the same price. It does not require that arbitrage never happens, and it makes no assumption about investors' attitude to risk.

Why the other options are wrong

- **B.** No-arbitrage pricing discounts at the risk-free rate because a position combining the derivative with its replicating portfolio (one long, the other short) has a certain payoff. The result does not depend on investors' risk preferences, so no assumption that investors are risk neutral is needed.
- **C.** Arbitrage opportunities can and do arise (small ones may even persist when transaction costs exceed the gain); the assumption is that they are quickly eliminated.

Key takeaway No-arbitrage pricing assumes that arbitrage is competed away quickly, so identical payoffs have identical prices.

Practice questions**Question 2**

A share of Halvorsen Robotics, which pays no dividends, trades at \$80.00, and the 1-year risk-free rate is 4%. A dealer quotes a 1-year forward price on the share of \$82.50. Assuming no transaction costs, which strategy earns an arbitrage profit?

- A** Buy the forward, sell the share short, and invest the proceeds at 4%.
- B** Sell the forward and borrow at 4% to buy the share.
- C** None, because the forward price is above the spot price.

Question 3

An analyst compares forwards on two commodities with the same spot price and the same time to settlement. Commodity P has no costs or benefits of holding. For commodity Q, storage and insurance costs exceed its convenience yield. Compared with the forward price on P, the no-arbitrage forward price on Q will be:

- A Lower.
- B Higher.
- C The same.

Answers: 2. A · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/arbitrage-replication-and-the-cost-of-carry-in-pricing-derivatives/. The full bank has 12 questions on this reading.

credguild.com

DERIVATIVES · READING 74

Pricing and Valuation of Forward Contracts and for an Underlying With Varying Maturities

What you'll learn

LOS 74.a Explain how the value and price of a forward contract are determined at initiation, during its life and at expiration, including holding costs and benefits.

LOS 74.b Explain how implied forward interest rates are derived from spot rates and how forward rate agreements (FRAs) are settled and used.

MODULE 74.1

Forward Contract Valuation

This reading separates the price of a forward from its value and applies forward pricing to interest rates through implied forward rates and forward rate agreements. A candidate must be able to value a forward at initiation, during its life and at expiration, compute implied forward rates from spot rates, and compute the settlement payment on an FRA.

LOS 74.a — Value and price of a forward: at initiation, during its life, at expiration

A forward has both a price and a value, and the two are different things:

- The **forward price** $F_0(T)$ is the price written into the contract at which the underlying will change hands at T . It is fixed at initiation and does not change.
- The **value** of the forward is what the position is worth to one party at a point in time. It is normally zero at initiation and then moves with the spot price.

For an underlying with no costs or benefits of holding, priced at its no-arbitrage level $F_0(T) = S_0(1 + R_f)^T$:

KEY CONCEPT

Time	Value of the long forward	Comment
Initiation ($t = 0$)	$V_0 = S_0 - F_0(T)(1 + R_f)^{-T} = 0$	Neither party pays to enter
During the life ($0 < t < T$)	$V_t = S_t - F_0(T)(1 + R_f)^{-(T-t)}$	Spot price minus the present value of the forward price
Expiration ($t = T$)	$V_T = S_T - F_0(T)$	Spot minus forward price

The value to the short is the negative of the value to the long, because one party's gain is the other's loss.

The long can lock in $V_t = S_t - PV(F_0)$ by shorting the asset at S_t and investing $F_0(T)(1 + R_f)^{-(T-t)}$ in a zero-coupon bond. At T , the bond pays $F_0(T)$, which buys the asset under the forward, and the asset is delivered to close the short. The position is fully offset, and the cash left over today is $S_t - F_0(T)(1 + R_f)^{-(T-t)}$.

With costs and benefits of holding (the general case):

KEY CONCEPT

$$V_t(T) = [S_t + PV_t(\text{cost}) - PV_t(\text{benefit})] - F_0(T)(1 + R_f)^{-(T-t)}$$

Example. A 1-year forward on a non-dividend-paying asset was entered at $F_0 = 100$. Three months later, with 9 months remaining, the spot price is 104 and $R_f = 5\%$.

$$V_t = 104 - \frac{100}{1.05^{0.75}} = 104 - 96.41 = 7.59$$

The long has a gain of 7.59 and the short a loss of 7.59. Two common errors give different answers: $104 - 100 = 4.00$ forgets to discount the forward price, and $(104 - 100)/1.05^{0.75}$ wrongly discounts the spot price too.

WORKED EXAMPLE: VALUING A FORWARD ON A DIVIDEND-PAYING SHARE DURING ITS LIFE

Eight months ago an investor went long a 1-year forward on a share at a forward price of 62.40. Today the share trades at 65.10, a dividend of 1.20 will be paid in 2 months, before the forward settles, and the annual risk-free rate is 4.5%. Compute the value of the long forward today and its value to the short.

Step 1. Four months remain, so $T - t = 4/12$ year.

Step 2. Present value of the dividend, a benefit of holding the share: $1.20/1.045^{2/12} = 1.19$.

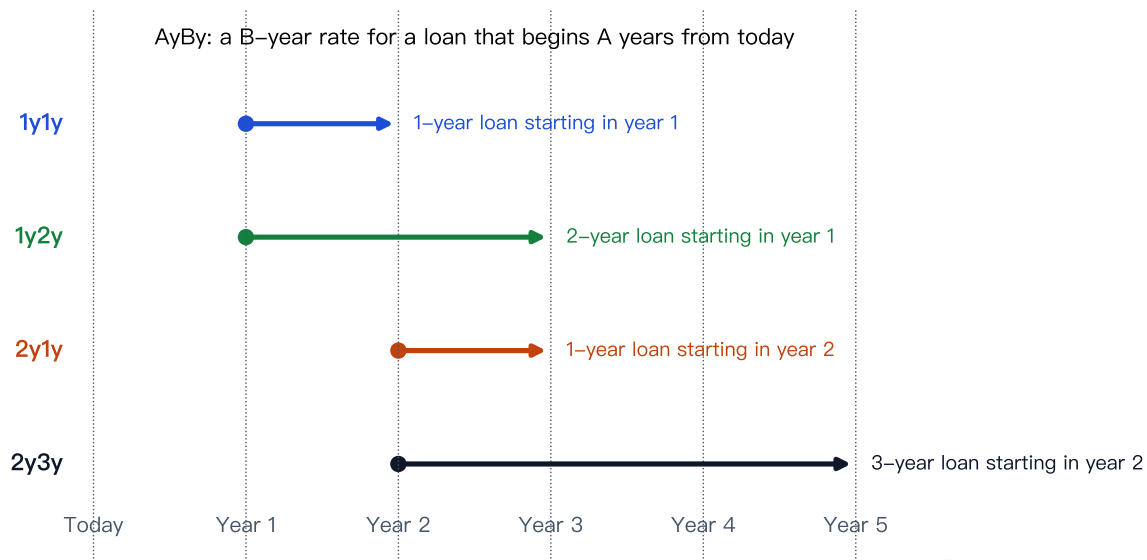
Step 3. Present value of the forward price: $62.40/1.045^{4/12} = 61.49$.

Step 4. $V_t = (65.10 - 1.19) - 61.49 = 2.42$.

Result. The long forward is worth 2.42 today, and the short forward is worth -2.42.

LOS 74.b — Forward interest rates and forward rate agreements (FRAs)**Forward rate notation**

A **forward rate** is an interest rate for a loan that starts in the future. A_yB_y (or $F_{A,B}$) is the rate for a B -year loan beginning A years from today. Thus **1y1y** is a 1-year loan starting in one year, and **2y3y** is a 3-year loan from year 2 to year 5. Money market versions use months: **3m6m** is a 6-month rate three months from now.



Forward rate notation AyBy on a timeline

Implied (no-arbitrage) forward rates

With **spot rates** (zero-coupon yields) Z_n , the **implied forward rate** makes two strategies earn the same: investing to the forward date and rolling over at the forward rate, or investing for the whole period at the longer spot rate.

KEY CONCEPT

$$(1 + Z_{A+B})^{A+B} = (1 + Z_A)^A (1 + F_{A,B})^B \quad \text{so} \quad F_{A,B} = \left[\frac{(1 + Z_{A+B})^{A+B}}{(1 + Z_A)^A} \right]^{1/B} - 1$$

Example. With $Z_1 = 3\%$ and $Z_2 = 4\%$, $F_{1,1} = 1.04^2 / 1.03 - 1 = 5.01\%$. With zero rates only out to year 3, one can derive 1y1y, 1y2y and 2y1y, but no forward rate that ends after year 3.

Strategy 1: lend for 2 years at $Z_2 = 4\%$

$$100 \times 1.04^2 = 108.16$$

Strategy 2 (value 103.00 at year 1)

lend 1 year at $Z_1 = 3\%$

roll over at 1y1y = 5.01%

$$103 \times 1.0501 = 108.16$$



No arbitrage: both strategies end with the same amount, so $(1.04)^2 = (1.03)(1 + 1y1y)$

Implied 1y1y forward rate from spot rates of 3% (one year) and 4% (two years)

Money market rates use simple interest on a 360-day year, and the same logic applies with day-count adjustments (d = days in each period):

$$1 + Z_{\text{long}} \frac{d_{\text{long}}}{360} = \left(1 + Z_{\text{short}} \frac{d_{\text{short}}}{360}\right) \left(1 + F \frac{d_{\text{long}} - d_{\text{short}}}{360}\right)$$

Example. The 3-month MRR is 2.0% and the 9-month MRR is 2.6%. Then $1 + 0.026 \times \frac{270}{360} = 1.0195$ and $1 + 0.020 \times \frac{90}{360} = 1.0050$, so the implied 3m6m rate is $\left(\frac{1.0195}{1.0050} - 1\right) \times \frac{360}{180} = 2.89\%$.

Forward rate agreements (FRAs)

A **forward rate agreement (FRA)** lets a party lock in an interest rate today for borrowing or lending that begins in the future. It is a forward commitment, so both parties are obligated.

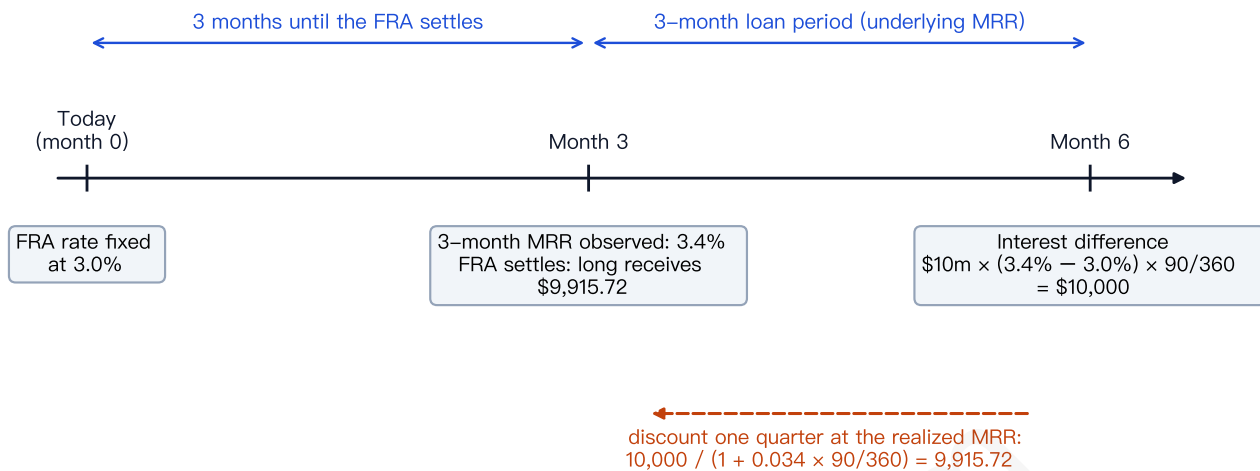
- The long (FRA buyer) pays the fixed contract rate and receives the market reference rate (**MRR**) on a notional principal. It gains if MRR rises. A future borrower goes long to hedge.
- The short (FRA seller) receives fixed and pays MRR. A future lender goes short.
- Only the net amount is paid. Settlement happens at the start of the loan period, so the interest difference, which relates to the end of the period, is discounted at the realized MRR:

$$\text{Payment to long} = \frac{\text{Notional} \times (\text{MRR} - \text{FRA rate}) \times \frac{\text{days}}{360}}{1 + \text{MRR} \times \frac{\text{days}}{360}}$$

Example. A 3-month forward on 3-month MRR (a 3m3m FRA) has a notional principal of \$10 million and an FRA rate of 3.0%. Three months later, 3-month MRR is 3.4%.

1. Interest difference for the loan period, due at its end (month 6): $10,000,000 \times (0.034 - 0.030) \times 0.25 = \$10,000$.
2. Discount it to the settlement date (month 3) at the realized MRR: $10,000 / (1 + 0.034 \times 0.25) = 10,000 / 1.0085 = \$9,915.72$.

3. MRR is above the FRA rate, so the long receives \$9,915.72 from the short.



Timeline of a 3m3m FRA: notional \$10 million, FRA rate 3.0%, realized 3-month MRR 3.4%

FRAs are used mainly by financial institutions to manage the rate sensitivity of their assets and liabilities. An FRA is equivalent to a **single-period swap**. A multi-period **interest rate swap** is a series of FRAs and is used mainly by issuers and investors.

Instrument	What it does
FRA	Locks in one future period's rate (obligation)
Interest rate swap	Exchanges fixed for floating over many periods
Interest rate option	Right, not obligation, to a rate (contingent claim)

Common exam traps

- The forward contract's value is usually zero at initiation; its price is the contract price.
- FRA settlement is discounted at the realized MRR, not at the FRA rate, and it is not paid undiscounted. In the 3m3m example, discounting at the 3.0% FRA rate gives \$9,925.56 and paying undiscounted gives \$10,000, instead of \$9,915.72.
- In *AyBy*, the first number is when the loan starts and the second is how long it lasts.

BOTTOM LINE

- The forward price is written into the contract at initiation and does not change, while the forward's value is normally zero at initiation and then moves with the spot price.
- For an underlying with no costs or benefits of holding, a long forward is worth $V_t = S_t - F_0(T)(1 + R_f)^{-(T-t)}$ during its life and $S_T - F_0(T)$ at expiration, and the short's value is the negative of the long's.
- With costs and benefits of holding, $V_t(T) = [S_t + PV_t(\text{cost}) - PV_t(\text{benefit})] - F_0(T)(1 + R_f)^{-(T-t)}$.

- In AyBy notation, the first number is when the loan starts and the second is how long it lasts, so 2y3y is a 3-year loan from year 2 to year 5.
- $F_{A,B} = [(1 + Z_{A+B})^{A+B} / (1 + Z_A)^A]^{1/B} - 1$, the rate that makes rolling over at the forward rate earn the same as investing at the longer spot rate.
- The long in an FRA pays the fixed FRA rate and receives MRR, so it gains when MRR rises; a future borrower hedges by going long and a future lender by going short.
- The FRA payment to the long is $\text{Notional} \times (\text{MRR} - \text{FRA rate}) \times \frac{\text{days}}{360}$ discounted at $1 + \text{MRR} \times \frac{\text{days}}{360}$, because it is paid at the start of the loan period.
- A multi-period interest rate swap is a series of FRAs, so each FRA amounts to a swap with only one period.

QUICK CHECK

Question 1

Eight months ago, Selma Varga entered a 1-year long forward contract on a share that has no dividends or other costs or benefits of holding, at a forward price of \$55.00. The share now trades at \$57.00 and the annual risk-free rate is 3%. The value of Varga's forward position is *closest* to:

- A \$1.98.
- B \$2.00.
- C \$2.54.

Answer: C

For an underlying that is costless to hold and pays nothing, a long forward's value before expiration is the spot price less the present value of the forward price. Four months remain, so the forward price is discounted for 4/12 of a year.

$$V_t = S_t - \frac{F_0(T)}{(1 + R_f)^{T-t}} = 57.00 - \frac{55.00}{1.03^{4/12}} = 57.00 - \frac{55.00}{1.009902} = 57.00 - 54.46 = \$2.54$$

Why the other options are wrong

- **A.** \$1.98 is the present value of the difference, $(57 - 55)/1.03^{4/12}$. It wrongly discounts the spot price as well as the forward price.
- **B.** \$2.00 is the undiscounted difference between spot and forward prices. That would be the value only at expiration.

Key takeaway Discount only the forward price, for the time remaining $(T - t)$.

Practice questions

Question 2

Annual-pay zero-coupon yields are 2.4% for 1 year, 2.8% for 2 years, and 3.1% for 3 years. The implied 1-year forward 2-year rate (1y2y), stated as an annual rate, is *closest* to:

- A 3.45%.
- B 3.70%.
- C 7.02%.

Question 3

A company expects to invest surplus cash for three months, starting four months from today. It wants to lock in the return on that future investment with an FRA. The company should enter the FRA as the party that:

- A pays fixed and receives MRR.
- B receives fixed and pays MRR.
- C receives both fixed and MRR.

Answers: 2. A · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/pricing-and-valuation-of-forward-contracts-and-for-an-underlying-with-varying-maturities/. The full bank has 9 questions on this reading.

DERIVATIVES · READING 75

Pricing and Valuation of Futures Contracts

What you'll learn

LOS 75.a Compare the price and value of forwards and futures over their lives, and interpret interest rate futures quotes and basis point value.

LOS 75.b Explain why forward and futures prices differ (daily settlement and the correlation of interest rates with futures prices) and the convexity bias of FRAs versus interest rate futures.

MODULE 75.1

Futures Valuation

This reading compares the pricing and valuation of forwards and futures, including interest rate futures, and explains why their prices can differ. A candidate must be able to describe how daily settlement changes a futures contract's price and value, compute an interest rate futures price and its basis point value, and relate the forward–futures price difference to the correlation between interest rates and futures prices and to convexity bias.

LOS 75.a — Value and price of forwards versus futures

Forwards (no mark-to-market). The forward price stays fixed for the whole life of the contract. The forward's value changes as the spot price moves, and the entire gain or loss is paid only at settlement, as the difference between the spot price and the unchanged forward price.

Futures (daily mark-to-market). At the end of each day, the gain or loss since the previous **settlement price** is credited to or debited from each margin account, and the contract is then treated as if it had been re-entered at the new settlement price. As a result:

- the futures price changes every day, because it is reset to the latest settlement price; and
- the value of the futures position goes back to zero after each daily settlement.

Example. A trader is long one futures contract on 100 units bought at 250.

Day	Settlement price	Mark-to-market cash flow (long)	Contract price after MTM	Value after MTM
0	250	—	250	0
1	253	$+3 \times 100 = +300$ credited	253	0
2	247	$-6 \times 100 = -600$ debited	247	0

Over the two days the long has received \$300 and paid \$600, a net $-\$300 = (247 - 250) \times 100$. A forward entered at 250 and settled at 247 would pay the same total, ignoring interest. What differs is the timing: futures gains and losses arrive day by day, and that timing is the source of the forward–futures price differences in LOS 75.b.

A forward that is marked to market daily, for example under a central clearing arrangement, behaves the same way.

Interest rate futures

Interest rate futures are the exchange-traded counterparts of **forward rate agreements (FRAs)**, but they are quoted on a price basis:

KEY CONCEPT

$$\text{futures price} = 100 - (100 \times \text{MRR}_{A, B-A})$$

A quote of 95.80 implies an MRR of 4.20%. The price moves opposite to the rate, so a long futures position gains when rates fall.

The **basis point value (BPV)** is the change in contract value for a 1 bp change in the rate:

KEY CONCEPT

$$\text{BPV} = \text{notional principal} \times \text{period (in years)} \times 0.0001$$

For a 6-month rate on \$2,000,000, $\text{BPV} = 2,000,000 \times 0.5 \times 0.0001 = \100 per basis point.

LOS 75.b — Why forward and futures prices can differ

The key pricing difference is the **daily settlement** of futures gains and losses. Gains above initial margin can be withdrawn and reinvested, and losses that take the account below the maintenance margin must be funded. Forwards usually generate no cash flows until settlement.

KEY CONCEPT

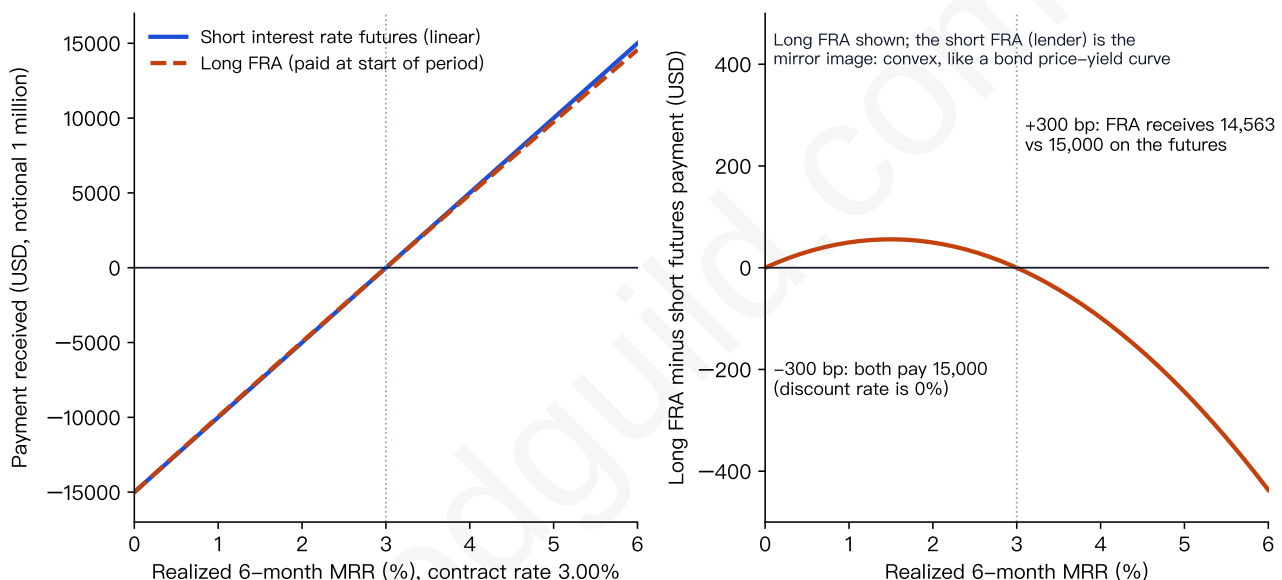
Correlation between interest rates and the futures price	Effect of daily settlement on a long futures	Which is more attractive to a long?	Price relationship
Positive	Cash received when rates are high (reinvested at high rates); cash paid when rates are low (cheap to finance)	Futures	Futures price > forward price
Negative	Cash paid when rates are high (costly); cash received when rates are low (poor reinvestment)	Forward	Futures price < forward price
Zero or constant rates	No systematic effect	Indifferent	Equal

These differences do not create arbitrage opportunities. They reflect the value of the mark-to-market cash flows. In practice the differences are usually too small to observe, because most forwards are short-dated, funds are available near the risk-free rate, and many dealers post margin.

Convexity bias in interest rate forwards versus futures

An FRA settles at the start of the loan period, so its payment is the interest difference discounted at the realized MRR. Interest rate futures pay an undiscounted, linear amount (BPV per basis point). A like-for-like comparison sets the FRA long, which gains when rates rise, against a short futures position, whose value also rises when rates rise because futures prices then fall.

- If rates rise, the FRA long receives the interest saving discounted at a higher realized rate, so it receives slightly less than the futures payment.
- If rates fall by the same amount, the FRA long pays an amount discounted at a lower rate, which is larger than what it would receive for the rise.
- The FRA payoff is therefore a nonlinear function of the rate: a rise in rates changes it by less than an equal fall does. From the side of the FRA short (the lender, whose position resembles a bond), the curve is convex, like a bond's price–yield curve. The long's payoff is the mirror image (concave). This curvature is the FRA's **convexity**. Futures payoffs are linear and have no convexity.



Settlement payment for a rise in MRR: long FRA (discounted, nonlinear) versus the equivalent short interest rate futures position (linear)

This **convexity bias** grows with the length of the period and can make forward and futures prices differ noticeably for longer-term rates.

Common exam traps

- For futures, both the price and the value change each day, and the value is reset to zero. For forwards, the price is fixed and the value fluctuates.
- Positive correlation between rates and futures prices favors the long futures; negative correlation favors the long forward.
- The FRA exhibits convexity; the futures contract is linear.

BOTTOM LINE

- A forward's price stays fixed and its whole gain or loss is paid at settlement, while a futures contract is marked to market daily, so its price resets to each settlement price and its value returns to zero after each daily settlement.
- Ignoring interest, the daily futures cash flows add up to the same total as the payoff of a forward entered at the same price; what differs is the timing.
- An interest rate futures price is quoted as $100 - (100 \times \text{MRR})$, so a long futures position gains when rates fall.
- $\text{BPV} = \text{notional principal} \times \text{period (in years)} \times 0.0001$ gives the change in an interest rate futures contract's value for a 1 bp change in the rate.
- A positive correlation between rates and futures prices makes futures more attractive to a long, so the futures price is above the forward price; a negative correlation makes the forward more attractive and the futures price lower; with zero correlation or constant rates the two are equal.
- Forward-futures price differences reflect the value of the mark-to-market cash flows and do not create arbitrage opportunities, and in practice they are usually too small to observe.
- Because an FRA's settlement is discounted at the realized MRR, its payoff is nonlinear in the rate while a futures payoff is linear; this convexity bias grows with the length of the period and can make prices differ noticeably for longer-term rates.

QUICK CHECK**Question 1**

A trader buys one futures contract on 50 tonnes of cocoa at a price of 4,120 per tonne. At the end of the day, the settlement price is 4,180. After the daily mark-to-market, which statement is *most accurate*?

- A** The contract price and contract value both change.
- B** The contract price is unchanged, but the contract value changes.
- C** The contract price changes, and the contract value returns to zero.

Answer: C

The daily mark-to-market credits the trader's account with the day's gain and resets the futures price to the new settlement price. Because the gain has already been paid, a contract at the new price of 4,180 has a value of zero again.

$$\text{Gain credited} = (4,180 - 4,120) \times 50 = 3,000.$$

$$\text{New contract price} = 4,180; \text{value after mark-to-market} = 0.$$

Why the other options are wrong

- **A.** The price does change, but the change in value is only temporary. Once the gain is settled in cash, the value returns to zero.
- **B.** This describes a forward without mark-to-market, whose price is fixed while its value moves. A futures price is reset every day.

Key takeaway A futures price resets daily and its value goes back to zero; a forward price is fixed while its value fluctuates.

Practice questions

Question 2

Analyst Farah Idris observes that when the price of a commodity rises, short-term interest rates also tend to rise. For an investor seeking long exposure, which statement is *most accurate* about a long futures contract compared with an otherwise equivalent forward that has no mark-to-market?

- A The long futures contract is more attractive than the forward.
- B The long forward contract is more attractive than the futures.
- C Both contracts are equally attractive.

Question 3

A fund manager is long 8 three-month MRR futures contracts, each on a notional principal of \$1,000,000. Over the day, the futures price falls from 96.40 to 96.25. The change in the value of the manager's position is:

- A A loss of \$12,000.
- B A loss of \$3,000.
- C A gain of \$3,000.

Answers: 2. A · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/pricing-and-valuation-of-futures-contracts/. The full bank has 7 questions on this reading.

DERIVATIVES · READING 76

Interest Rate Swap

What you'll learn

LOS 76.a Describe how an interest rate swap is equivalent to a series of FRAs at the swap rate (values summing to zero at inception) and how it can be replicated with cash market positions.

LOS 76.b Contrast the price of a swap (the par swap rate, fixed at initiation) with its value (zero at inception, then changing with expected future rates).

MODULE 76.1

Swap Valuation

This reading treats an interest rate swap as a series of forward rate agreements and separates a swap's price, its fixed rate, from its value. A candidate must be able to explain how the FRAs embedded in a swap differ from market FRAs, replicate a swap with cash market positions, compute the par swap rate from spot rates, and value a swap after initiation.

LOS 76.a — Swaps compared with a series of forward contracts

In a plain **fixed-for-floating interest rate swap**, one party pays a fixed rate and the other pays a floating **market reference rate (MRR)** on the same **notional principal**. On each settlement date only the **net payment** changes hands.

A swap as a series of FRAs

Take a 1-year swap with quarterly settlements. The floating rate for each quarter is set at the start of the quarter, so:

- The first net payment (end of quarter 1) is already known at initiation. It depends on today's MRR and the swap's fixed rate.
- Each later payment (quarters 2, 3 and 4) equals the payoff of a **forward rate agreement** in which the fixed-rate payer is long at a contract rate equal to the **swap fixed rate**. From the fixed-rate payer's perspective, each payment is $(MRR_n - F) \times \frac{1}{4} \times \text{notional}$.
- The timing differs slightly. A standard FRA on the 90-day MRR is cash-settled at the start of the underlying quarter, when the rate is set, and pays the present value of the interest difference: $(MRR_n - F) \times \frac{1}{4} \times \text{notional}$ discounted for one quarter at MRR_n . The FRAs that replicate the swap's payments on days 180, 270 and 360 therefore settle on days 90, 180 and 270. Growing each discounted settlement forward one quarter at MRR_n gives the swap's net payment at the end of the quarter, so the amounts are equivalent and only the payment date differs.

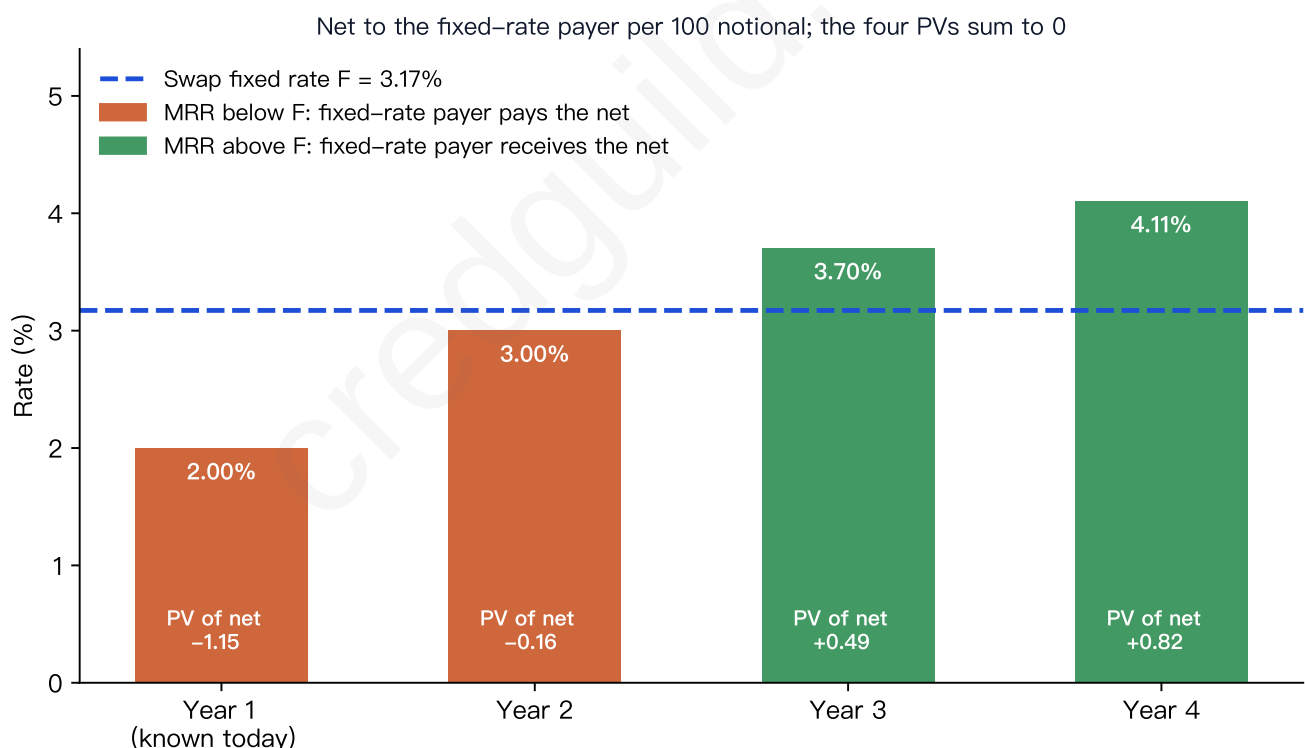
A swap is therefore equivalent to a series of FRAs. Like an FRA, it can produce gains or losses for either side, and each party bears the other's counterparty credit risk unless a central counterparty is used. The one important difference is in the contract rates:

KEY CONCEPT

	Series of market FRAs	FRAs embedded in a swap
Contract rates	Each FRA has its own no-arbitrage rate, so each starts at zero value	All use the same rate: the swap fixed rate
Value of each forward at initiation	Zero	Generally not zero: some positive, some negative
Total value at initiation	Zero	The values sum to zero

With an upward-sloping curve, the early FRAs (swap rate above the forward rate) have negative value to the fixed-rate payer and the later ones have positive value. In total they offset.

Example. For an annual-pay 4-year swap with spot rates of 2.0%, 2.5%, 2.9% and 3.2%, the implied 1-year MRRs are 2.00%, 3.00%, 3.70% and 4.11% (for year 2, $1.025^2/1.02 - 1 = 3.00\%$). The fixed rate that gives the swap zero value, the par swap rate derived in LOS 76.b, is 3.17%. Per 100 of notional, the present values of the net payments to the fixed-rate payer are -1.15, -0.16, +0.49 and +0.82, which sum to zero. For year 1, $(2.00 - 3.17)/1.02 = -1.15$.



An annual-pay 4-year swap as a series of forward contracts that all use one fixed rate

Replicating a swap with cash market positions

Swap position	Cash market replication
Fixed-rate payer (receives floating)	Borrow at a fixed rate (issue a fixed-rate bond) and invest the proceeds at the floating rate (buy a floating-rate note)
Floating-rate payer (receives fixed)	Borrow at a floating rate (issue a floating-rate note) and invest the proceeds in a fixed-rate bond

The floating-rate payer's net cash flows are also equivalent to a series of FRAs (the other side of the fixed-rate payer's FRAs). Those FRAs would all carry the swap rate, so they would not be zero-value FRAs.

LOS 76.b — Price versus value of a swap

- The **price** of a swap is its fixed rate, the **par swap rate**, written into the contract at initiation. It does not change over the swap's life.
- The **value** of the swap is zero at initiation, because the par swap rate is chosen so that the fixed leg and the expected floating leg have equal present values. Neither party pays to enter.
- After initiation, the value changes as expected future MRRs change.

Solving for the par swap rate

Let S_n be the spot rate for period n and MRR_n the floating rate for period n implied by those spot rates (the first equals the current MRR). The par swap rate F (per period) satisfies

$$\sum_{n=1}^N \frac{MRR_n}{(1 + S_n)^n} = \sum_{n=1}^N \frac{F}{(1 + S_n)^n}$$

A floating-rate note paying the implied rates and then returning its principal is worth par. Per unit of notional, the floating payments alone are therefore worth $1 - 1/(1 + S_N)^N$, and the equation simplifies to

KEY CONCEPT

$$F = \frac{1 - \frac{1}{(1 + S_N)^N}}{\sum_{n=1}^N \frac{1}{(1 + S_n)^n}}$$

Example. An annual-pay 2-year swap faces spot rates $S_1 = 1.5\%$ and $S_2 = 2.5\%$. The discount factors are $1/1.015 = 0.985222$ and $1/1.025^2 = 0.951814$, so $F = (1 - 0.951814)/(0.985222 + 0.951814) = 0.048186/1.937036 = 2.49\%$. As a check, the implied 1y1y rate is $1.025^2/1.015 - 1 = 3.51\%$, and $PV(\text{floating}) = 0.015 \times 0.985222 + 0.0351 \times 0.951814 = 0.048186$, which matches $PV(\text{fixed})$.

Value during the life

$$\text{Value to fixed-rate payer} = PV(\text{expected floating payments}) - PV(\text{remaining fixed payments})$$

Example. Take the 2-year swap above with a notional principal of \$50 million and a fixed rate of 2.49%. At the first settlement the fixed-rate payer pays the net $50,000,000 \times (0.0249 - 0.015) = \$495,000$. Just after that payment, 1-year MRR is 3.90%, above the 3.51% implied at initiation, and it sets the final floating payment. One net payment remains, due in a year: $50,000,000 \times (0.0390 - 0.0249) = \$705,000$ to the fixed-rate payer. Its value today is $705,000/1.039 = \$678,537$ to the fixed-rate payer and $-\$678,537$ to the floating-rate payer. Had MRR come in at the expected 3.51%, the remaining payment would be worth $50,000,000 \times (0.0351 - 0.0249)/1.0351 = \$492,706$, about what the fixed-rate payer paid at the first settlement. The rise in MRR above that expectation is the fixed-rate payer's gain and the floating-rate payer's loss.

KEY CONCEPT

Change in expected future MRRs after initiation	Value to fixed-rate payer	Value to floating-rate payer	Swap price (fixed rate)
Increase	Rises (positive)	Falls (negative)	Unchanged
Decrease	Falls (negative)	Rises (positive)	Unchanged

This value is the basis for mark-to-market payments on centrally cleared swaps.

Common exam traps

- A floating-rate payer's position is like issuing floating-rate debt and buying a fixed-rate bond, a long-duration position that gains when rates fall.
- The par swap rate is neither the simple average of the spot rates nor the longest spot rate. For the 4-year swap in the example, the simple average gives 2.65% and the longest spot rate 3.20%, against a par swap rate of 3.17%.

EXAM SHORTCUTS

- The par swap rate needs only the spot-rate discount factors, $F = (1 - \text{last discount factor}) / \text{sum of the discount factors}$, so the implied floating rates do not have to be computed first.

BOTTOM LINE

- An interest rate swap is equivalent to a series of FRAs in which the fixed-rate payer is long at the swap fixed rate, and its first net payment is already known at initiation.
- Market FRAs each carry their own no-arbitrage rate and start at zero value, while the FRAs embedded in a swap all use the swap fixed rate, so their individual values are generally not zero but sum to zero.

- A fixed-rate payer's position replicates borrowing at a fixed rate and investing the money at the floating rate, and a floating-rate payer's position replicates borrowing at a floating rate and putting the money into a fixed-rate bond.
- A swap's price is its fixed par swap rate, which is set at initiation and does not change, while its value is zero at initiation and then changes as expected future MRRs change.
- $F = [1 - 1/(1 + S_N)^N] / \sum_{n=1}^N 1/(1 + S_n)^n$, the fixed rate per period that makes the fixed leg and the expected floating leg equal in present value.
- Value to the fixed-rate payer = PV(expected floating payments) – PV(remaining fixed payments), so a rise in expected MRRs after initiation gives the fixed-rate payer a gain and the floating-rate payer an equal loss.

QUICK CHECK

Question 1

An analyst breaks a fixed-for-floating interest rate swap into a series of forward contracts (FRAs), each with a contract rate equal to the swap's fixed rate. For this series to replicate a swap entered at market terms, the forward contracts must have:

- A** values that are each equal to zero at swap initiation.
- B** values that sum to zero at swap initiation.
- C** values that sum to zero at swap expiration.

Answer: B

Because every FRA in the series uses the same contract rate (the swap fixed rate), the individual FRAs will generally not be zero-value contracts: some have positive and some negative value. A swap entered at market terms has zero value at initiation, so the FRA values must add up to zero at that date.

Why the other options are wrong

- **A.** Zero-value FRAs would each need their own no-arbitrage contract rate. With a single common rate, individual values are generally non-zero; only their sum is zero.
- **C.** The zero-sum condition applies at initiation, when the swap is priced to have zero value. At expiration, the values reflect realized rates and need not sum to zero.

Key takeaway A swap is a series of off-market FRAs whose values sum to zero at inception.

Practice questions

Question 2

Six months after Orsolya Farms entered a 5-year interest rate swap as the fixed-rate payer, expected future MRRs have risen. Which statement is *most accurate*?

- A The swap's value to Orsolya has fallen; the swap price is unchanged.
- B The swap's value to Orsolya has risen; the swap price has also risen.
- C The swap's value to Orsolya has risen; the swap price is unchanged.

Question 3

Rhea Kapoor wants to use cash market transactions to reproduce the net cash flows of the floating-rate payer in a plain vanilla interest rate swap. She could best do so by:

- A taking out a floating-rate loan and buying a fixed-rate bond.
- B taking out a fixed-rate loan plus a strip of zero-value FRAs.
- C taking out a floating-rate loan plus a strip of zero-value FRAs.

Answers: 2. C · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/interest-rate-swap/. The full bank has 6 questions on this reading.

DERIVATIVES · READING 77

Pricing and Valuation of Options

What you'll learn

LOS 77.a Explain an option's exercise (intrinsic) value, moneyness (in, at, out of the money) and time value.

LOS 77.b Contrast no-arbitrage pricing of forward commitments with replication-based pricing of contingent claims, including the upper and lower bounds for European option values.

LOS 77.c Identify the six factors that determine an option's value and the direction in which each affects call and put values.

MODULE 77.1

Option Valuation

This reading covers the building blocks of option pricing: moneyness, exercise value and time value, the no-arbitrage bounds on European option values, and the factors that move option values. A candidate must be able to compute exercise value, time value and the lower and upper bounds of European calls and puts, and state how each of the six factors changes call and put values.

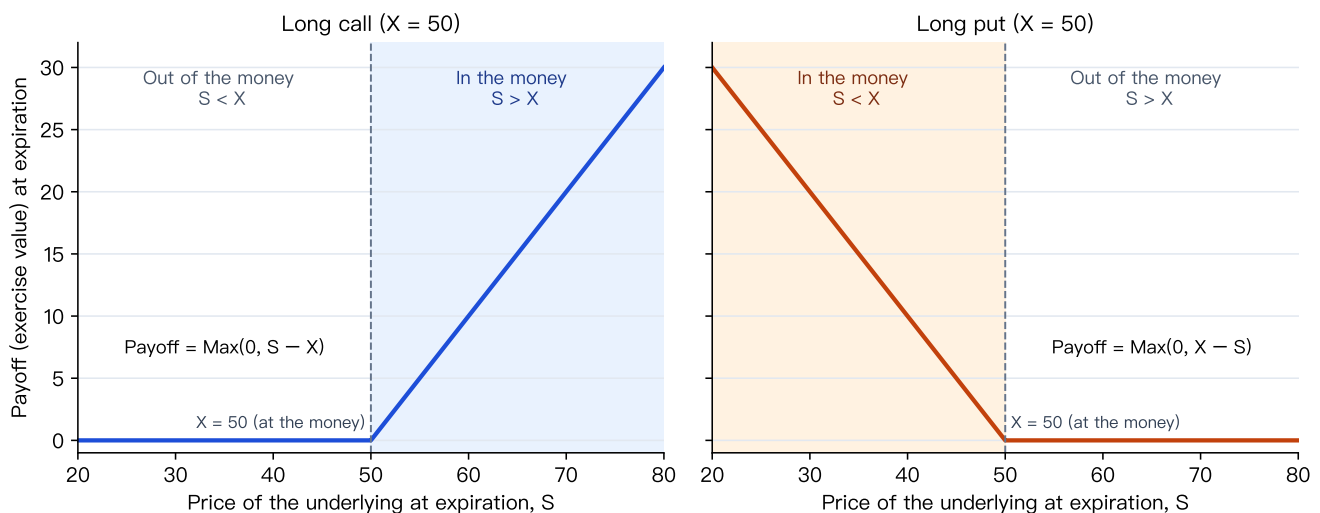
LOS 77.a — Moneyness, exercise value and time value

Moneyness describes what the holder would receive from exercising an option right now. With S = current price of the underlying and X = exercise (strike) price:

KEY CONCEPT

	Call	Put
In the money	$S > X$	$S < X$
At the money	$S = X$	$S = X$
Out of the money	$S < X$	$S > X$

An in-the-money option would produce a positive payoff if exercised immediately. An out-of-the-money option would not, so the holder would simply not exercise it.



Payoff at expiration of a long call and a long put, both with exercise price 50

The **exercise value** (also called **intrinsic value**) is the amount by which the option is in the money, floored at zero:

$$\text{Call exercise value} = \max(0, S - X) \quad \text{Put exercise value} = \max(0, X - S)$$

Before expiration an option normally trades above its exercise value. The difference is the **time value** (sometimes called speculative value):

$$\text{Option premium} = \text{exercise value} + \text{time value}, \quad \text{so} \quad \text{time value} = \text{premium} - \text{exercise value}$$

Three consequences follow:

- For an out-of-the-money or at-the-money option, exercise value is zero, so the entire premium is time value.
- For an in-the-money option, exercise value is positive, so time value is less than the premium.
- Time value comes from the chance that the underlying moves favorably before expiration. It generally shrinks as expiration approaches and is zero at expiration, when the option is worth exactly its exercise value (zero if at or out of the money). Time value is typically positive but can be negative: a deep in-the-money European put can trade below $X - S$, because its holder must wait until expiration to receive X .

Example. A stock trades at \$83. An 80-strike call is \$3 in the money (exercise value \$3), and an 80-strike put is \$3 out of the money (exercise value \$0). If the call's premium is \$5.40, its time value is $5.40 - 3.00 = \$2.40$. If the put's premium is \$0.85, all \$0.85 is time value.

LOS 77.b — Arbitrage and replication for contingent claims

Forward commitments (forwards, futures, swaps) are priced so that their value at initiation is zero to both parties, and both sides face essentially unlimited gains or losses. Options (**contingent claims**) differ in two ways. The buyer pays a premium, so the value at initiation is positive, and the payoffs are one-sided, because the buyer's loss is limited to the premium. Options are therefore priced with a **replication** approach. The option's

payoff is combined with a position in the underlying and a **pure discount bond** (risk-free zero-coupon bond), and no-arbitrage requires that a portfolio whose payoff is never negative must itself have a value of zero or more.

Two such portfolios give the lower bounds:

- *Call*. Buy the call, buy a pure discount bond that pays X at T , and short one share. The payoff at T is $c_T + X - S_T$: zero if $S_T \geq X$ and $X - S_T > 0$ otherwise. Because it is never negative, $c_0 - S_0 + X(1 + R_f)^{-T} \geq 0$, so $c_0 \geq S_0 - X(1 + R_f)^{-T}$.
- *Put*. Buy the put, buy one share, and borrow $X(1 + R_f)^{-T}$. The payoff at T is $p_T + S_T - X$: zero if $S_T \leq X$ and $S_T - X > 0$ otherwise. Hence $p_0 + S_0 - X(1 + R_f)^{-T} \geq 0$, so $p_0 \geq X(1 + R_f)^{-T} - S_0$.

Combined with the fact that no option can have a negative value, this yields boundary values for European options at time t (expiration at T , R_f = risk-free rate):

KEY CONCEPT

Option	Minimum value	Maximum value
European call	$c_t \geq \max[0, S_t - X(1 + R_f)^{-(T-t)}]$	S_t
European put	$p_t \geq \max[0, X(1 + R_f)^{-(T-t)} - S_t]$	$X(1 + R_f)^{-(T-t)}$

Nobody pays more than the share price for the right to buy the share. A European put can never pay more than X , and that payment cannot arrive before expiration, so its ceiling is the present value of X . The lower bounds use the present value of the exercise price because a European option cannot be exercised early.

Example. $S_0 = 83$, $X = 80$, six months to expiration and $R_f = 4\%$. $PV(X) = 80/1.04^{0.5} = 78.45$. The lower bound for the call is $83 - 78.45 = 4.55$; the lower bound for the put is $\max(0, 78.45 - 83) = 0$.

LOS 77.c — Factors that determine option values

Six factors determine option values. Their effects, other things equal, are:

KEY CONCEPT

Increase in ...	Call value	Put value
Price of the underlying	Increase	Decrease
Exercise price	Decrease	Increase
Risk-free rate	Increase	Decrease
Volatility of the underlying	Increase	Increase
Time to expiration	Increase	Usually increase (exceptions for some European puts)
Costs and benefits of holding the asset	Benefits (dividends, interest, convenience yield): decrease. Storage costs: increase	Benefits: increase. Storage costs: decrease

- *Underlying price and exercise price.* These drive exercise value directly: a call gains as S rises or X falls, and a put gains as S falls or X rises. At expiration only these two matter, because time value is zero. Interest rates and volatility affect only time value.
- *Risk-free rate.* A call holder pays X in the future, and a higher rate lowers the present value of that payment, so the call is worth more. A put holder receives X in the future, and a higher rate lowers the present value of that receipt, so the put is worth less. A falling rate therefore lowers calls and raises puts.
- *Volatility.* Volatility makes both calls and puts valuable, because the downside is capped while the potential upside widens. With no volatility, the time value that comes from possible favorable price moves would disappear. Exam convention: option values then equal their exercise values. Current practice: pricing models still discount the exercise price of a European option. With zero volatility the underlying grows at the risk-free rate, so a European call on a non-dividend-paying share is worth $S - PV(X)$ when $S > PV(X)$.
- *Time to expiration.* More time effectively means more volatility, so calls and most puts are worth more. A deep in-the-money European put, however, can be worth less with a longer life, because its upside is almost capped and the payment of X is received later, at a lower present value. The effect is more likely for a put that is deeper in the money, when the risk-free rate is higher, and when the time to expiration is longer.
- *Holding benefits and costs.* Benefits such as a dividend lower the price of the underlying, which hurts calls and helps puts. Storage costs work the other way.

A **long call** profits when the underlying rises, so its profit is positively correlated with the price. A **long put** profits when the underlying falls, so its profit is negatively correlated with the price. Short positions have the opposite sensitivity: a **short put** gains value when the underlying price rises and loses value when volatility increases or, for most puts, when time to expiration increases.

Example. An investor is long a put and long a call on the same share. If the risk-free rate falls, the call loses value and the put gains value, so only one of the two positions benefits. If the investor were instead long the put and short the call, both positions would gain.

Common exam traps

- Moneyness compares the exercise price with the current price. It says nothing about whether the option is now worth more than the holder paid for it.
- The European bounds use $PV(X)$. The put's upper bound is $PV(X)$, not X , and using X in the lower bounds gives the exercise value from immediate exercise instead. In the six-month example, using X gives a call lower bound of 3.00 instead of 4.55 and a put upper bound of 80 instead of 78.45.
- The risk-free rate moves call and put values in opposite directions, while volatility moves them in the same direction.
- A longer life does not always raise a European put's value.

BOTTOM LINE

- A call is in the money when $S > X$ and a put when $S < X$; either is at the money when $S = X$.
- Exercise value is $\max(0, S - X)$ for a call and $\max(0, X - S)$ for a put, and time value is the premium minus the exercise value, so the whole premium of an option with zero exercise value is time value.
- Time value generally shrinks as expiration approaches and is zero at expiration; it is typically positive but can be negative for a deep in-the-money European put.

- European option bounds: $\max[0, S_t - X(1 + R_f)^{-(T-t)}] \leq c_t \leq S_t$ and $\max[0, X(1 + R_f)^{-(T-t)} - S_t] \leq p_t \leq X(1 + R_f)^{-(T-t)}$.
- Other things equal, a higher underlying price or a lower exercise price raises a call's value and lowers a put's, a higher risk-free rate raises calls and lowers puts, and higher volatility raises both.
- A longer time to expiration raises a call's value and usually a put's, but a deep in-the-money European put can be worth less with a longer life, more likely when it is deeper in the money, the risk-free rate is higher and the time to expiration is longer.
- Benefits of holding the underlying, such as dividends, lower call values and raise put values, while storage costs do the opposite.
- At expiration only the underlying price and the exercise price matter, because time value is zero; interest rates and volatility affect only time value.

QUICK CHECK

Question 1

Hana Novak owns two options on the same stock: a call with an exercise price of \$62 and a put with an exercise price of \$70. If the stock currently trades at \$66:

- A neither option is in the money.
- B only the call option is in the money.
- C both options are in the money.

Answer: C

The call is in the money because it lets Novak buy at \$62 a stock worth \$66 ($S - X = 4$). The put is in the money because it lets her sell at \$70 a stock worth \$66 ($X - S = 4$).

Why the other options are wrong

- A. Each option would give a positive payoff if exercised now, so both are in the money.
- B. The call is in the money, but so is the put, because its exercise price is above the market price.

Key takeaway Check each option separately: call ITM if $S > X$, put ITM if $S < X$. With different strikes both can be in the money at once.

Practice questions

Question 2

Which statement about a European put option is *most accurate*?

- A Its exercise value is the stock price less the option's exercise price.
- B Its time value is the sum of its market price and its exercise value.
- C Its exercise value is the exercise price less the stock price, if positive.

Question 3

Compared with forward commitments, why must a different no-arbitrage approach be used to price options?

- A Options have a positive value at initiation and one-sided payoffs.
- B Options cannot be replicated with the underlying and a bond.
- C Options are valued with a risk-adjusted discount rate only.

Question 4

Tomás Reyes has bought a European call option and written a European put option on the same equity index. Holding all other factors constant, an increase in the risk-free interest rate will increase the value of:

- A neither of his positions.
- B only one of his positions.
- C both of his positions.

Answers: 2. C · 3. A · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/pricing-and-valuation-of-options/. The full bank has 21 questions on this reading.

DERIVATIVES · READING 78

Put Call Parity

What you'll learn

LOS 78.a Explain put–call parity for European options (fiduciary call = protective put) and use it to build synthetic stocks, calls, puts and bonds.

LOS 78.b Explain put–call forward parity for European options and the option view of a firm's equity and debt (credit risk as a put).

MODULE 78.1

Put-Call Parity

This reading derives put–call parity and put–call forward parity from no-arbitrage and applies option thinking to a firm's equity and debt. A candidate must be able to use parity to value an option or build a synthetic position, identify and exploit a parity violation, and explain why shareholders hold a call on the firm's assets and debtholders are short a put.

LOS 78.a — Put–call parity

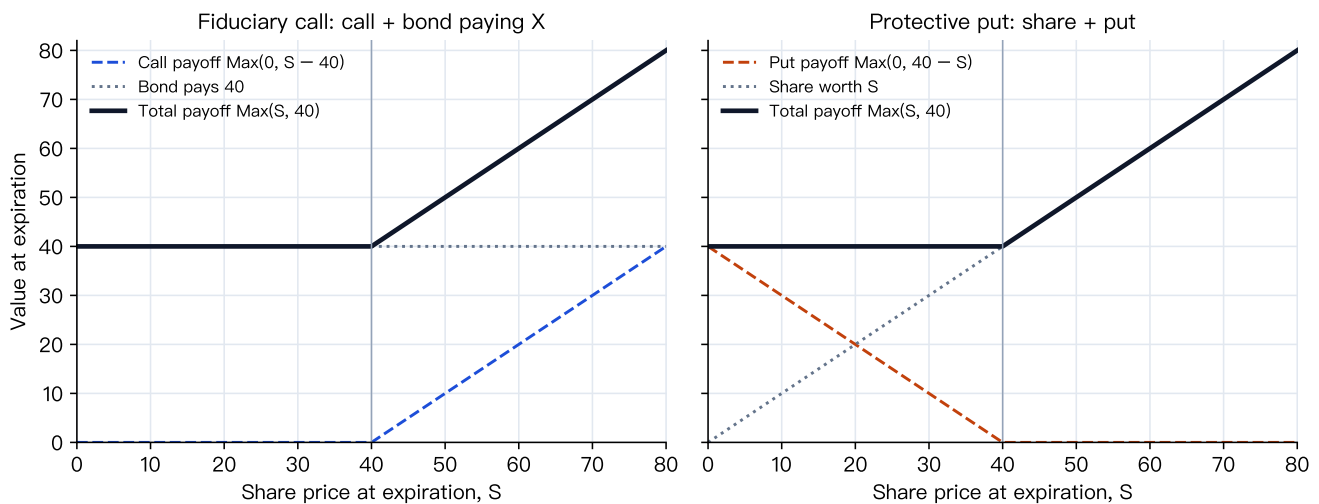
Two portfolios, both built with European options on the same underlying with the same exercise price X and the same expiration T :

- **Fiduciary call** = long call + a risk-free **pure-discount bond** that pays X at expiration.
- **Protective put** = long one share of the underlying + long put.

Payoffs at expiration:

	$S_T \leq X$	$S_T > X$
Fiduciary call	call 0 + bond $X = X$	call $S_T - X$ + bond $X = S_T$
Protective put	share S_T + put $X - S_T = X$	share S_T + put 0 = S_T

Both pay $\max(S_T, X)$ in every state (see the figure below), so no-arbitrage forces them to cost the same today. This is **put–call parity**:



Both portfolios pay 40 if $S \leq 40$ and S if $S > 40$, so $c + X(1 + R_f)^{-T} = S + p$.

Fiduciary call and protective put with exercise price 40: identical payoffs at expiration

KEY CONCEPT

$$c + X(1 + R_f)^{-T} = S + p$$

Synthetic securities

Rearranging parity isolates any one instrument; a + sign means a long position and a – sign a short position (a short bond = borrowing the PV of X ; a long bond = lending it):

KEY CONCEPT

Target (synthetic equivalent)	Replicating portfolio
Synthetic stock: $S = c - p + X(1 + R_f)^{-T}$	long call, short put, long bond
Synthetic put: $p = c - S + X(1 + R_f)^{-T}$	long call, short stock, long bond
Synthetic call: $c = S + p - X(1 + R_f)^{-T}$	long stock, long put, short bond (borrow)
Synthetic bond: $X(1 + R_f)^{-T} = S + p - c$	long stock, long put, short call

Parity in this form holds for **European** options with the same exercise price and expiration on an underlying that pays no income and has no holding costs during the option's life.

Arbitrage. If one side of parity is cheap relative to the other, buy the cheap side and sell the expensive side. Example: if a put trades *below* $c - S + PV(X)$, buy the put and sell the synthetic put: write the call, buy the stock and borrow $PV(X)$. The difference is an arbitrage profit received today.

Example. $S = 36$, $X = 35$, 3 months to expiration, $R_f = 6\%$, put price 1.10. The call implied by parity is

$$c = S + p - \frac{X}{(1 + R_f)^T} = 36 + 1.10 - \frac{35}{1.06^{0.25}} = 37.10 - 34.49 = 2.61$$

If the call is quoted at 3.00 instead, it is expensive relative to the synthetic call worth 2.61. Write the call for 3.00 and build the synthetic call: buy the share for 36, buy the put for 1.10 and borrow 34.49. The synthetic call costs 2.61, so 0.39 is received today. At expiration the share plus the put is worth $\max(S_T, 35)$, the loan repayment is 35 and the written call costs $\max(0, S_T - 35)$. These net to zero in every state, so the 0.39 is an arbitrage profit.

Common exam traps

- Confusing a fiduciary call (call + bond paying X) with a **covered call** (stock + short call) or with call + stock.
- Getting the bond sign wrong: a synthetic call *borrows* (short bond); a synthetic put *lends* (long bond).
- Discounting X at the wrong horizon, or forgetting to discount it at all. In the 3-month example, leaving the 35 undiscounted gives a call value of 2.10, and discounting it for a full year gives 4.08, instead of 2.61.

LOS 78.b — Put-call forward parity and options in corporate finance

A long **forward contract** with price $F_0(T)$ plus a pure-discount bond paying $F_0(T)$ at T replicates the underlying: at T the bond proceeds exactly pay for the asset. The forward costs nothing at initiation, so this **synthetic asset** costs $F_0(T)(1 + R_f)^{-T}$. Substituting it for S in put-call parity gives **put-call forward parity** (at initiation of the forward):

KEY CONCEPT

$$F_0(T)(1 + R_f)^{-T} + p_0 = c_0 + X(1 + R_f)^{-T}$$

Equivalently,

$$p_0 - c_0 = [X - F_0(T)](1 + R_f)^{-T}$$

So a long put plus a short call (same X) has the same value as a risk-free bond that pays $X - F_0(T)$ at expiration. The option pair matches the bond in **value**, not in **payoff**: at expiration it pays $X - S_T$, which is uncertain. The long forward (payoff $S_T - F_0(T)$) removes that uncertainty, so options plus forward pay the fixed amount $X - F_0(T)$; because the forward costs nothing at initiation, adding it leaves the value unchanged. The components are the call, the put, the PV of the exercise price and the **present value of the forward price**. The spot price of the underlying does not appear.

Example. One-year forward price 62, $X = 60$, $R_f = 5\%$: $p_0 - c_0 = (60 - 62)/1.05 = -1.90$, so the call must be worth 1.90 more than the put.

Option view of a firm (equity and debt)

Consider a firm financed by equity and one zero-coupon debt issue with face value D ; firm value V = equity + debt.

- Because of limited liability, **shareholders hold a call option** on the firm's assets with exercise price D : if $V_T > D$ they pay off the debt and keep the rest; if $V_T < D$ they default and walk away.

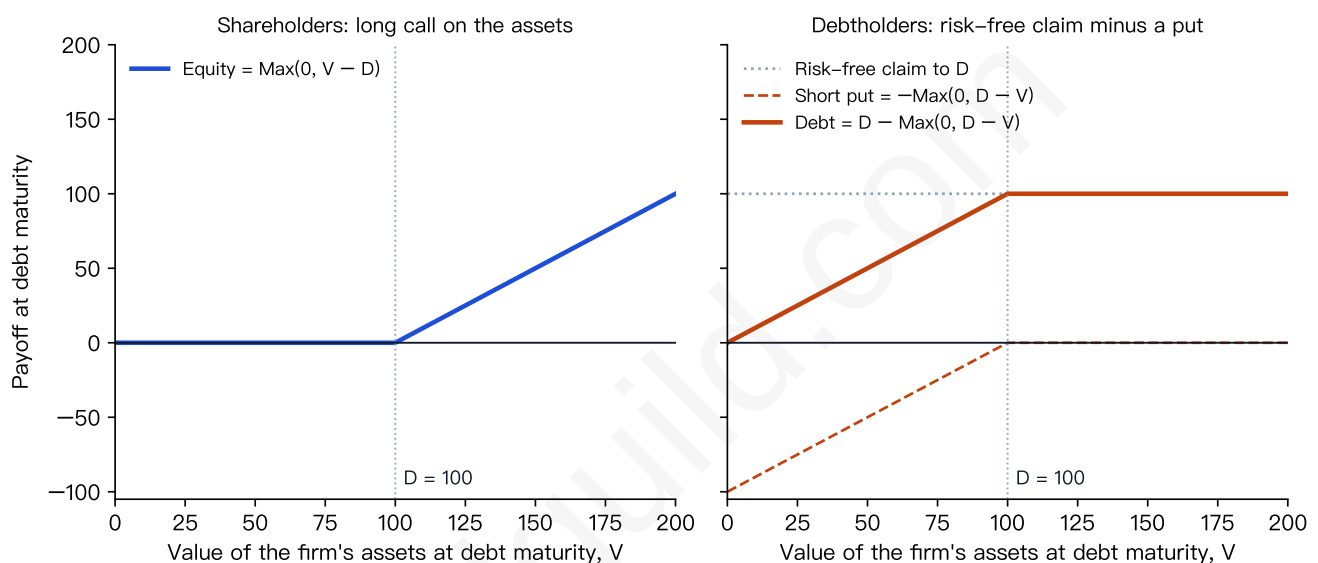
- Equivalently, shareholders own the assets plus a **long put** on the assets with exercise price D . Defaulting amounts to "putting" the assets to the debtholders.
- Debtholders** hold a risk-free claim to D plus a **short put** written to the shareholders. That put is the firm's **credit risk** (default risk): the debt is worth less than $PV(D)$ by the value of the put.

$$V_0 = c_0 + PV(D) - p_0$$

Here c_0 is the equity and $PV(D) - p_0$ is the risky debt.

At debt maturity the two claims pay

$$\text{Equity}_T = \max(0, V_T - D) \quad \text{Debt}_T = \min(V_T, D) = D - \max(0, D - V_T)$$



Payoffs at debt maturity to shareholders and debtholders of a firm with zero-coupon debt of face value 100

Example. With $D = 100$: if $V_T = 130$, shareholders repay 100 and keep 30. If $V_T = 70$, they default, equity is worth 0 and debtholders receive 70, which is the risk-free 100 minus the put payoff of 30.

Common exam traps

- Saying the credit-risk put is written by the shareholders. The debtholders are short the put and the shareholders hold it.
- Including the spot price in put-call forward parity; the forward price (in PV terms) replaces it.
- Using the future value, rather than the present value, of the forward price. In the one-year example, this gives $p_0 - c_0 = -2.00$ instead of -1.90 .
- Claiming that long put + short call alone replicates the bond's payoff; the long forward is also needed.

BOTTOM LINE

- A fiduciary call (long call plus a bond paying X) and a protective put (share plus long put) both pay $\max(S_T, X)$, so $c + X(1 + R_f)^{-T} = S + p$.

- Put–call parity in this form holds for European options with the same exercise price and expiration on an underlying with no income and no holding costs during the option's life.
- Rearranging parity gives synthetic positions: a synthetic call is long the stock, long the put and short the bond (borrowing $PV(X)$), and a synthetic put is long the call, short the stock and long the bond (lending $PV(X)$).
- When one side of parity is cheap relative to the other, buying the cheap side and selling the expensive side earns the price difference today as an arbitrage profit.
- Put–call forward parity replaces the spot price with the present value of the forward price, $F_0(T)(1 + R_f)^{-T} + p_0 = c_0 + X(1 + R_f)^{-T}$, so $p_0 - c_0 = [X - F_0(T)](1 + R_f)^{-T}$.
- For a firm financed by equity and zero-coupon debt with face value D , shareholders hold a call on the firm's assets with exercise price D , or equivalently the assets plus a long put.
- Debtholders hold a risk-free claim to D plus a short put written to the shareholders; that put is the firm's credit risk, and $V_0 = c_0 + PV(D) - p_0$.

QUICK CHECK

Question 1

A portfolio manager wants to reproduce the expiration payoff of a European put without trading any puts. She can trade the stock, a European call with the same strike and expiration, and a risk-free pure-discount bond that pays the exercise price X at expiration. Based on put–call parity, which combination should she hold?

- A** Short the stock, long the call, short a pure-discount bond that pays X .
- B** Short the stock, long the call, long a pure-discount bond that pays X .
- C** Long the stock, short the call, short a pure-discount bond that pays X .

Answer: B

Solving put–call parity for the put gives $p = c - S + X(1 + R_f)^{-T}$: a long call, a short stock position and an investment of $PV(X)$ in the risk-free bond (a long bond position). At expiration the combination pays exactly $\max(0, X - S_T)$.

Why the other options are wrong

- **A.** Borrowing (short bond) has the wrong sign; the put equation adds $PV(X)$, so the bond is held long.
- **C.** This is the negative of the synthetic put (the position of someone who has sold a put synthetically).

Key takeaway Put side of parity: $p = c - S + PV(X)$, so hold a long call, short stock and a long bond.

Practice questions

Question 2

Castellan Group is financed with common equity and a single issue of zero-coupon debt. When options theory is applied to this capital structure, the firm's credit risk can be modeled as a:

- A put option written by the debtholders and held by the shareholders.
- B call option written by the debtholders and held by the shareholders.
- C call option written by the shareholders and held by the debtholders.

Question 3

Shares of Brisa Networks trade at \$73.00. A 6-month European call on the shares with an exercise price of \$70.00 is priced at \$6.20, and the annual risk-free rate is 4%. The shares pay no dividends. Based on put-call parity, the value of a 6-month European put with the same exercise price is *closest* to:

- A \$1.84.
- B \$3.20.
- C \$10.56.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/put-call-parity/. The full bank has 14 questions on this reading.

DERIVATIVES · READING 79

Binomial Option Pricing Model

What you'll learn

LOS 79.a Value a call or put with a one-period binomial model by building a hedged (riskless) portfolio of the option and the underlying using the hedge ratio.

LOS 79.b Describe risk neutrality: value an option as the expected payoff under risk-neutral probabilities, discounted at the risk-free rate.

MODULE 79.1

Binomial Model for Option Values

This reading values options with a one-period binomial model, first by building a riskless hedge portfolio and then by risk-neutral valuation. A candidate must be able to compute a hedge ratio and use it to value a call or a put, compute risk-neutral probabilities, and explain why option values do not depend on investors' risk preferences.

LOS 79.a — One-period binomial model: the hedge (replication) approach

A **binomial model** assumes that over one period the underlying moves to one of just two values. Valuing an option on it requires:

- the value of the underlying today, S_0 ;
- the option's exercise price X (the option expires at the end of the period);
- the size of the moves: the up-move return $R^u = S_1^u/S_0$ and down-move return $R^d = S_1^d/S_0$ (these capture the underlying's volatility);
- the **risk-free rate** over the period.

The valuation needs neither the real-world probabilities of an up-move or down-move nor a risk-adjusted discount rate.

Call: long h shares, short one call

Choose the number of shares h so that the portfolio is worth the same after either move ($V_1^u = V_1^d$):

KEY CONCEPT

$$h S_1^u - c_1^u = h S_1^d - c_1^d \Rightarrow h = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

h is the **hedge ratio**, the number of shares held per option written. Because the end-of-period value is certain, it must earn the risk-free rate: $V_0 = V_1/(1 + R_f)$. With $V_0 = hS_0 - c_0$:

$$c_0 = hS_0 - V_0$$

Put: long h shares, long one put

For a put the hedge portfolio is **long** both the put and the shares (the put gains when the stock falls):

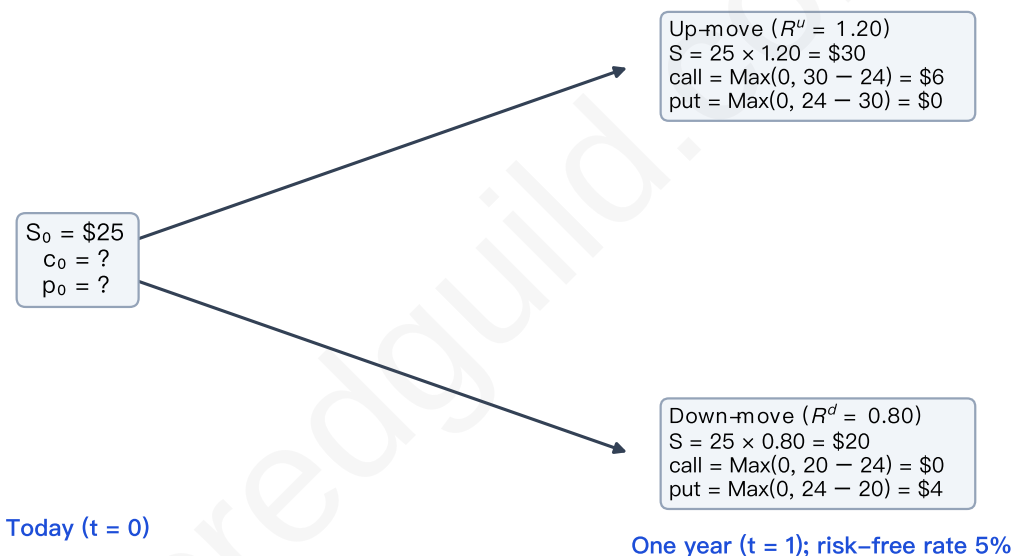
$$h = \frac{p_1^d - p_1^u}{S_1^u - S_1^d}, \quad V_0 = hS_0 + p_0 \Rightarrow p_0 = V_0 - hS_0$$

KEY CONCEPT

The hedge approach in steps:

1. Find the two end-of-period prices of the underlying, S_0R^u and S_0R^d , and the option payoff at each.
2. Compute the hedge ratio h with the call or put formula.
3. Value the hedge portfolio at either node to get V_1 ; both nodes give the same number.
4. Discount V_1 one period at the risk-free rate to get V_0 .
5. Back out the option value from V_0 and hS_0 .

Example.



One-period binomial tree for a stock at \$25 with call and put payoffs ($X = \$24$)

$S_0 = 25$; after one year $S_1^u = 30$ ($R^u = 1.20$) or $S_1^d = 20$ ($R^d = 0.80$); $X = 24$; $R_f = 5\%$.

Call: $c_1^u = 6$ and $c_1^d = 0$, so $h = 6/(30 - 20) = 0.60$. $V_1 = 0.60(30) - 6 = 12 = 0.60(20)$. $V_0 = 12/1.05 = 11.429$. $c_0 = 0.60(25) - 11.429 = 3.57$.

Put: $p_1^u = 0$ and $p_1^d = 4$, so $h = 4/10 = 0.40$. $V_1 = 0.40(30) + 0 = 12 = 0.40(20) + 4$. $V_0 = 11.429$. $p_0 = 11.429 - 0.40(25) = 1.43$.

Check with put-call parity: $c_0 - p_0 = 3.57 - 1.43 = 2.14 = 25 - 24/1.05$.

The hedge portfolio is a pricing device; no cash needs to change hands to form it. Practical models split the option's life into many short periods, which gives many possible prices at expiration; the one-period tree shows the method.

Common exam traps

- Discounting the hedge ratio, or compounding it, at the risk-free rate. h is a pure ratio of payoff range to price range.
- Using a short put in the put hedge (the put hedge is long put + long shares).

LOS 79.b — Risk neutrality

Because the hedged portfolio is riskless, option value does not depend on investors' risk preferences. The option can therefore be valued *as if* investors were risk-neutral: weight the two payoffs by **risk-neutral probabilities**, then discount that expected payoff using the risk-free rate.

KEY CONCEPT

$$\pi_U = \frac{1 + R_f - R^d}{R^u - R^d}, \quad \pi_D = 1 - \pi_U$$

$$c_0 = \frac{\pi_U c_1^u + \pi_D c_1^d}{1 + R_f}, \quad p_0 = \frac{\pi_U p_1^u + \pi_D p_1^d}{1 + R_f}$$

The steps: (1) compute the option payoffs after the up- and down-move; (2) take their probability-weighted average using π_U and π_D ; (3) discount one period at the risk-free rate.

Trees are often built with the down-move factor equal to the reciprocal of the up-move factor ($R^d = 1/R^u$), so an up-move of 1.25 pairs with a down-move of 0.80.

At Level I the risk-neutral probabilities are typically given; what matters is using them to weight the payoffs and discounting at the risk-free rate. π_U and π_D are **pseudo-probabilities**. They are not the actual probabilities of the moves; they come from the no-arbitrage hedge, which is why discounting at the risk-free rate is correct. In the example above, $\pi_U = (1.05 - 0.80)/(1.20 - 0.80) = 0.625$, so $c_0 = 0.625 \times 6/1.05 = 3.57$, the same value as the hedge approach gives.

The name comes from what these probabilities do to the underlying. Weighted by π_U and π_D , the underlying's expected return equals the risk-free rate: $\pi_U R^u + \pi_D R^d = 1 + R_f$. In the example, $0.625 \times 30 + 0.375 \times 20 = 26.25 = 25 \times 1.05$: under these probabilities the share is expected to earn exactly the 5% risk-free rate.

WORKED EXAMPLE: VALUING A PUT BOTH WAYS

A share trades at 40. Over one year it will either rise by a factor of 1.25 or fall by a factor of 0.80, and the 1-year risk-free rate is 3%. Value a 1-year European put with an exercise price of 42 using risk-neutral probabilities, and confirm the value with the hedge approach.

Step 1. End-of-year prices: $40 \times 1.25 = 50$ and $40 \times 0.80 = 32$. Put payoffs: $p_1^u = 0$ and $p_1^d = 42 - 32 = 10$.

Step 2. $\pi_U = (1.03 - 0.80)/(1.25 - 0.80) = 0.23/0.45 = 0.5111$ and $\pi_D = 1 - 0.5111 = 0.4889$.

Step 3. $p_0 = (0.5111 \times 0 + 0.4889 \times 10)/1.03 = 4.889/1.03 = 4.75$.

Step 4. Hedge check: $h = (10 - 0)/(50 - 32) = 0.5556$. The portfolio of 0.5556 shares and one put is worth $0.5556 \times 50 + 0 = 27.78$ after an up-move and $0.5556 \times 32 + 10 = 27.78$ after a down-move.

Step 5. $V_0 = 27.78/1.03 = 26.97$, so $p_0 = 26.97 - 0.5556 \times 40 = 26.97 - 22.22 = 4.75$.

Result. Both approaches value the put at 4.75.

Approach	Uses	Discount rate
Hedge (replication) portfolio	hedge ratio h , certain end value	risk-free
Risk-neutral valuation	π_U, π_D from move sizes and R_f	risk-free

Common exam traps

- Weighting payoffs by actual (real-world) probabilities.
- Forgetting to discount the expected payoff. For the call in the example, this gives 3.75 instead of 3.57.
- Discounting at a risk-adjusted rate instead of the risk-free rate.

EXAM SHORTCUTS

- For European options with the same exercise price and expiration on an underlying with no income or holding costs, once the call is valued the put follows from put–call parity, $p_0 = c_0 - S_0 + X/(1 + R_f)$, with no second hedge calculation.

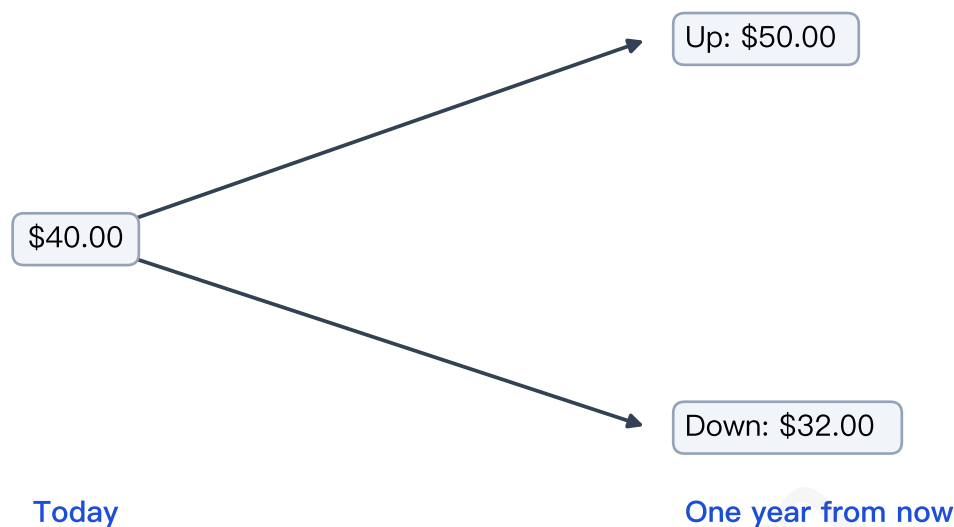
BOTTOM LINE

- A one-period binomial model values an option from the current underlying price, the exercise price, the up-move and down-move factors and the risk-free rate; it needs neither real-world probabilities nor a risk-adjusted discount rate.
- For a call, the hedge portfolio is long h shares and short one call, with $h = (c_1^u - c_1^d)/(S_1^u - S_1^d)$; its certain end value is discounted at the risk-free rate and $c_0 = hS_0 - V_0$.
- For a put, the hedge portfolio is long h shares and long one put, with $h = (p_1^d - p_1^u)/(S_1^u - S_1^d)$ and $p_0 = V_0 - hS_0$.
- $\pi_U = (1 + R_f - R^d)/(R^u - R^d)$ and $\pi_D = 1 - \pi_U$, and an option's value is its payoffs weighted by π_U and π_D and discounted one period at the risk-free rate.
- Risk-neutral probabilities are pseudo-probabilities derived from the no-arbitrage hedge, under which the underlying's expected return equals the risk-free rate; they are not the actual probabilities of the moves.
- The hedge approach and risk-neutral valuation both discount at the risk-free rate and give the same option value.

QUICK CHECK

Question 1

Shares of Solvay Ridge follow the one-period binomial tree shown below. The one-year risk-free rate is 4%.



One-period binomial tree for Solvay Ridge shares

An analyst wants to write one-year European calls on the shares with an exercise price of \$44 and hold shares so that the combined position has the same value at expiration whatever the share price. The number of shares to hold per call written is *closest* to:

- A 0.32.
- B 0.33.
- C 0.35.

Answer: B

The hedge ratio equals the spread of the call's payoffs divided by the spread of the share prices. It is a ratio of payoffs, so no discounting or compounding at the risk-free rate is involved.

Call payoffs: $c_1^u = \max(0, 50 - 44) = 6$; $c_1^d = \max(0, 32 - 44) = 0$.

$$h = \frac{c_1^u - c_1^d}{S_1^u - S_1^d} = \frac{6 - 0}{50 - 32} = \frac{6}{18} = 0.333$$

Check: up-move value = $0.333(50) - 6 = 10.67$; down-move value = $0.333(32) - 0 = 10.67$. The portfolio of 0.333 shares per short call is worth 10.67 in both states.

Why the other options are wrong

- **A.** 0.32 divides the hedge ratio by 1.04 ($0.3333/1.04 = 0.3205$); the risk-free rate is used to value the portfolio and plays no part in computing h .
- **C.** 0.35 multiplies the hedge ratio by 1.04 ($0.3333 \times 1.04 = 0.3467$); again, h is not adjusted for interest.

Key takeaway $h = (c_1^u - c_1^d)/(S_1^u - S_1^d)$. The risk-free rate enters only when the certain portfolio value is discounted.

Practice questions

Question 2

Shares of Keswick Marine trade at \$60. In one year they will be worth either \$78 after an up-move or \$51 after a down-move. The one-year risk-free rate is 3%, and the risk-neutral probability of an up-move is 0.40. An analyst estimates the actual probability of an up-move at 0.55. Using risk-neutral valuation, the value of a one-year European call on the shares with an exercise price of \$65 is *closest* to:

- A \$5.05.
- B \$5.20.
- C \$6.94.

Question 3

In a one-period binomial model, a call is valued by combining one short call with h shares of the underlying so that the portfolio's value at the end of the period is the same after an up-move or a down-move. Rearranging this relationship, a position that replicates a long call consists of:

- A buying h shares and also lending at the risk-free rate.
- B selling h shares short and lending at the risk-free rate.
- C buying h shares, financed partly by borrowing at the risk-free rate.

Answers: 2. A · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/binomial-option-pricing-model/. The full bank has 8 questions on this reading.

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