



CredGuild

STUDY NOTES

CFA® Program Level I

Fixed Income

2027 curriculum · 19 readings · 71 practice questions

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ABOUT THIS EDITION

CredGuild · CFA® Program Level I · Fixed Income
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USING THIS BOOK

These notes are a study aid. Use them alongside the CFA Institute curriculum. The latest version is free at credguild.com.

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Contents

How to use this book	1
Fixed Income	2
51. Bond Indenture	3
52. Zero Coupon Bond	9
53. Fixed-Income Issuance and Trading	18
54. Commercial Paper	26
55. Treasury Bonds	34
56. Yield to Maturity	40
57. Credit Spread	48
58. Yield and Yield Spread Measures for Floating-Rate Instruments	56
59. The Term Structure of Interest Rates: Spot, Par, and Forward Curves	62
60. Interest Rate Risk and Return	71
61. Macaulay Duration	79
62. Bond Convexity	86
63. Curve-Based and Empirical Fixed-Income Risk Measures	93
64. Credit Risk	100
65. Credit Analysis for Government Issuers	108
66. Credit Analysis for Corporate Issuers	113
67. Fixed-Income Securitization	119
68. Asset-Backed Security (ABS) Instrument and Market Features	124
69. Mortgage-Backed Security (MBS) Instrument and Market Features	131
Keep going at credguild.com	139

How to use this book

These are the CredGuild notes for the 2027 CFA® Program Level I curriculum, Fixed Income: 19 readings. They are the same notes as on credguild.com, set out for reading on paper or offline.

What each reading contains

- **What you'll learn:** the learning outcome statements the reading covers.
- **The notes, module by module:** worked examples, key concepts, common exam traps, exam shortcuts and a bottom line.
- **A quick check after each module:** one question with its answer and the full explanation.
- **Practice questions at the end, with the answers.** The full explanation for each one is free on the reading's page at credguild.com; the link is under the answers.

The coloured boxes

KEY CONCEPT

Definitions and formulas to know.

Example. A worked problem, step by step. Try it before you read the working.

COMMON EXAM TRAPS

Mistakes that cost marks.

EXAM SHORTCUTS

Faster ways through a question.

BOTTOM LINE

What to remember from the module.

Using it with the website

Paper is good for reading. Practice works better on the site: it scores each answer as you go, explains every option, keeps a list of the questions you got wrong and spreads the readings across the weeks before your exam. If you are not sure where to start, the free diagnostic test (credguild.com/cfa/level-1/diagnostic/) gives you a score in each topic.

If this book helps you, pass it on to someone else who is studying.

CFA® PROGRAM LEVEL I · TOPIC

Fixed Income

Exam weight 11–14% · 19 readings

Bond features, cash flows and markets; prices, yields and spreads; the term structure of interest rates; duration and convexity as measures of interest rate risk; credit analysis; and securitized debt such as asset-backed and mortgage-backed securities.

Reading 51	Bond Indenture
Reading 52	Zero Coupon Bond
Reading 53	Fixed-Income Issuance and Trading
Reading 54	Commercial Paper
Reading 55	Treasury Bonds
Reading 56	Yield to Maturity
Reading 57	Credit Spread
Reading 58	Yield and Yield Spread Measures for Floating-Rate Instruments
Reading 59	The Term Structure of Interest Rates: Spot, Par, and Forward Curves
Reading 60	Interest Rate Risk and Return
Reading 61	Macaulay Duration
Reading 62	Bond Convexity
Reading 63	Curve-Based and Empirical Fixed-Income Risk Measures
Reading 64	Credit Risk
Reading 65	Credit Analysis for Government Issuers
Reading 66	Credit Analysis for Corporate Issuers
Reading 67	Fixed-Income Securitization
Reading 68	Asset-Backed Security (ABS) Instrument and Market Features
Reading 69	Mortgage-Backed Security (MBS) Instrument and Market Features

FIXED INCOME · READING 51

Bond Indenture

What you'll learn

LOS 51.a Describe the basic features of a fixed-income security: issuer, maturity, par value, coupon rate and frequency, seniority, contingency provisions, and the price-yield relationship.

LOS 51.b Describe what a bond indenture contains, the sources of repayment, and contrast affirmative and negative covenants.

MODULE 51.1

Fixed-Income Instrument Features

This reading introduces the basic features of a fixed-income security and the contents of a bond indenture. A candidate must be able to compute a periodic coupon, classify bonds by issuer, maturity and seniority, and tell affirmative covenants from negative covenants.

LOS 51.a — Features of a fixed-income security

Fixed-income instruments come in two broad forms. A **loan** is a private, nontradable agreement between a borrower and a lender. A **bond** (a **fixed-income security**) is a standardized, tradable security that represents a debt investment. Bond investors lend capital, the **principal**, also called **par value** or **face value**. The issuer promises to repay it together with interest, usually as a periodic **coupon** stated as a percentage of par. Issuers typically use the money raised to fund long-term investments. For a corporate issuer, bonds and long-term loans are classified as **long-term liabilities** on the balance sheet.

Key bond features

Feature	What it specifies
Issuer	Who borrows: sovereign (national) governments, corporations, local governments, supranational entities (e.g., the IMF), quasi-government entities (e.g., a state railway), and special purpose entities (SPEs) that buy financial assets and issue asset-backed securities backed by the cash flows of those assets
Maturity	The date the final cash flow is paid. The time left until then is the tenor . Original maturity of one year or less = money market securities ; more than one year = capital market securities ; no stated maturity = perpetual bonds
Par value	The principal to be repaid — usually in one amount at maturity, but some instruments (e.g., a mortgage) repay it gradually
Coupon rate and frequency	The coupon rate is the <i>annual</i> percentage of par paid as interest; payments may be annual, semiannual, quarterly or monthly
Seniority	Debt ranks ahead of equity in bankruptcy or liquidation; senior debt ranks ahead of junior debt (subordinated debt) , so junior debt carries more credit risk and must offer a higher yield

Feature	What it specifies
Contingency provisions	Some bonds include embedded options, such as a call option , a put option or a conversion right (covered in the next reading)

KEY CONCEPT

Each periodic coupon = $\frac{\text{coupon rate} \times \text{par value}}{\text{payments per year}}$. A bond described as a "6% semiannual coupon bond" pays 3% of par every six months.

Example. A \$5,000 par bond with a 4% coupon paid semiannually pays $0.04 \times 5,000 / 2 = \100 every six months, or \$200 a year. If it paid quarterly, each coupon would be \$50.

Floating-rate notes (FRNs), or **floaters**, pay a coupon equal to a variable **market reference rate (MRR)** plus a fixed margin. The margin is quoted in **basis points** (1 bp = 0.01%).

Zero-coupon bonds (pure discount bonds) pay no interest before maturity. They are sold at a discount to par, and the investor's whole return is the difference between the purchase price and the par value received at maturity. For a given maturity, the lower the price, the higher the return.

Price, yield and the yield curve

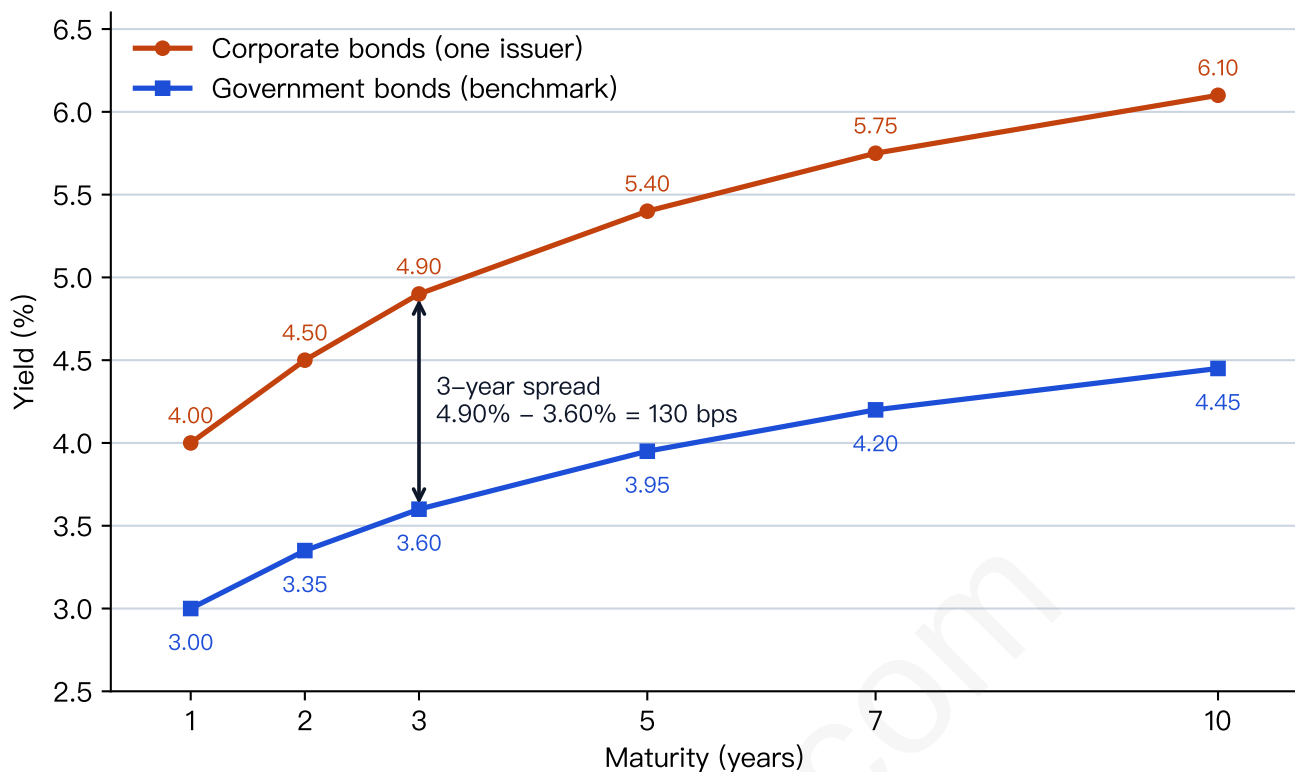
A bond's **yield** is the return an investor expects from buying it at today's price and receiving its promised cash flows. For a fixed-coupon bond, price and yield move in opposite directions. Fixed cash flows can deliver a higher return only if the investor pays less for them.

KEY CONCEPT

- **Premium bond:** price above par (its yield is below its coupon rate).
- **Discount bond:** price below par (its yield is above its coupon rate).

A plot of yields against maturity for one issuer or class of bonds is a **yield curve**. An upward-sloping curve is a **normal yield curve**. It is the most common shape, because investors usually demand a higher yield for longer, more uncertain horizons. A downward-sloping curve is an **inverted yield curve**, which is less common. The US Treasury curve, for instance, was upward sloping in mid-2018 and inverted in mid-2023. Government bonds are usually the lowest-credit-risk bonds in a market, so the government curve is the benchmark, and the extra yield on a riskier bond over a government bond of the same maturity is its **spread**.

Example. A 3-year corporate bond yields 4.9% and the 3-year government bond yields 3.6%, so the spread is $4.9\% - 3.6\% = 1.3\% = 130$ bps.



Government and corporate yield curves; the spread is the vertical gap at one maturity

Common exam traps

- Confusing the per-period coupon with the coupon rate: 2% paid quarterly is an 8% coupon rate. For the \$5,000 par, 4% semiannual bond in the example, applying the full 4% to each payment gives a coupon of \$200 instead of \$100.
- Classifying money market and capital market securities by remaining maturity. The original maturity is what counts.
- Thinking a zero-coupon bond can trade above par when rates fall. As long as its yield is positive, it stays below par until maturity.
- Measuring a spread against a government bond of a different maturity.
- Assuming senior and junior bonds of the same issuer and maturity offer the same yield. The junior bonds are riskier and yield more.

LOS 51.b — The bond indenture and covenants

The **bond indenture** (also called the **trust deed**) is the legal contract between the issuer (borrower) and the bondholders (lenders). It sets out the bondholders' rights and the issuer's obligations and restrictions, and it governs all later dealings between the two sides. It specifies the issuer, maturity, par value, coupon rate and frequency, currency, the **sources of repayment**, any **collateral** pledged, **credit enhancements**, and the **covenants** the issuer must follow.

The indenture does not name the individual bondholders. Bonds are tradable securities, so the holders change over the bond's life.

Sources of repayment

Issuer / bond type	Repaid from
Sovereign bonds	Taxes on economic activity and, in some cases, the power to create currency — hence usually the lowest credit risk in a region
Local government bonds	Local taxes, or revenue from projects such as toll roads
Secured bond	The company's operating cash flow plus a legal claim (lien or pledge) on specific assets (collateral) if the issuer defaults
Unsecured bond	Operating cash flow only
Asset-backed security	Cash flows from the financial assets held by the SPE that issued it

Affirmative versus negative covenants

Bondholders have no votes, but **covenants** written into the indenture protect them.

KEY CONCEPT

	Affirmative covenants	Negative covenants
Nature	Things the issuer must do (mostly administrative)	Things the issuer must not do (restrictions)
Examples	Deliver timely financial reports; use proceeds as stated; pay taxes and comply with laws; insure and maintain assets pledged as collateral; redeem at a premium if acquired; cross-default clause (a default on any other debt is a default on this bond); pari passu clause (this bond ranks equally with the issuer's other senior debt)	Limits on asset sales and sale-and-leaseback deals; no pledging the same collateral twice; no new debt senior to existing debt (negative pledge clause); limits on additional borrowing, share repurchases or dividends, often subject to an incurrence test (e.g., allowed only while debt/EBITDA is below a stated level)

Negative covenants protect bondholders by stopping actions that would raise default risk, but they must not be so tight that the firm cannot respond to opportunities or changing conditions.

Common exam traps

- Keeping collateral insured is affirmative (a required action). Not selling or re-pledging that collateral is negative (a restriction).
- Covenants are provisions inside the indenture. The indenture is the whole contract.
- Wording can mislead. Cross-default and pari passu clauses sound restrictive, yet they are affirmative covenants. A clause such as "dividends may be paid only while debt/EBITDA is below 2.5×" reads like a duty, but it limits what the issuer may do, so it is a negative covenant (an incurrence test).

EXAM SHORTCUTS

- A bond priced above par has a yield below its coupon rate, and a bond priced below par has a yield above it, so comparing the price with par answers a yield-versus-coupon question without any calculation.

BOTTOM LINE

- The coupon rate is the annual percentage of par paid as interest, so each periodic coupon equals the coupon rate times par value divided by the number of payments per year.
- A bond with an original maturity of one year or less is a money market security, one with an original maturity of more than one year is a capital market security, and a bond with no stated maturity is a perpetual bond.
- A zero-coupon (pure discount) bond pays no interest before maturity and is sold at a discount to par, so the investor's whole return is the gap between the price paid and the par value received.
- For a fixed-coupon bond, price and yield move in opposite directions; a premium bond yields less than its coupon rate and a discount bond yields more.
- Senior debt ranks ahead of junior (subordinated) debt in bankruptcy or liquidation, so junior debt carries more credit risk and must offer a higher yield.
- A bond's spread is its extra yield over a government bond of the same maturity, with the government yield curve used as the benchmark.
- The bond indenture (trust deed) is the legal contract between issuer and bondholders; it sets out the sources of repayment, any collateral, credit enhancements and the covenants.
- Affirmative covenants state what the issuer must do, including cross-default and pari passu clauses; negative covenants restrict what the issuer may do, including the negative pledge clause and limits subject to an incurrence test.

QUICK CHECK**Question 1**

A dealer's inventory includes the three fixed-income securities described below. Which one is classified as a money market security?

- A** A note issued three years ago that matures in four months.
- B** A bill issued today that matures in nine months.
- C** A bond issued ten months ago that matures in 14 months.

Answer: B

Money market securities are those with an original maturity of one year or less. The bill issued today with nine months to maturity has an original maturity of nine months, so it is a money market security.

Why the other options are wrong

- **A.** The note had an original maturity of about three years and four months. Its short remaining maturity does not change its classification: it is a capital market security.

- **C.** The bond's original maturity is $10 + 14 = 24$ months, which is longer than one year, so it is a capital market security.

Key takeaway Money market and capital market securities are classified by original maturity at issuance. The time remaining does not change the classification.

Practice questions

Question 2

Which of the following describes a bond that is trading at a premium?

- A** Its coupon rate is below its market yield.
- B** Its market price is above its par value.
- C** Its face value is below its redemption value.

Question 3

Government bond yields are 4.60% for 2 years, 4.10% for 7 years and 3.90% for 10 years. Ardent Rail's 7-year bond yields 5.85%. The spread on Ardent Rail's bond and the shape of the government yield curve are best described as:

- A** 1.25%; inverted yield curve.
- B** 1.75%; normal yield curve.
- C** 1.75%; inverted yield curve.

Answers: 2. B · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/bond-indenture/. The full bank has 17 questions on this reading.

FIXED INCOME · READING 52

Zero Coupon Bond

What you'll learn

LOS 52.a Describe bullet, amortizing and sinking fund cash flow structures and coupon structures, and contrast embedded options that benefit issuers (call) with those that benefit investors (put, conversion).

LOS 52.b Describe how legal, regulatory and tax considerations (domestic, foreign, Eurobond and global bonds; securitization; taxation of interest, capital gains and OID) affect the issuance and trading of fixed-income securities.

MODULE 52.1

Fixed-Income Cash Flows and Types

This reading covers how bonds repay principal and pay coupons, the embedded options that can change those cash flows, and the legal, regulatory and tax factors that shape where and how bonds are issued. A candidate must be able to compute amortizing and balloon payments, floating-rate and inflation-linked coupons and conversion values, and say which party an embedded option benefits.

LOS 52.a — Cash flow structures and contingency provisions

Principal repayment structures

KEY CONCEPT

Structure	Periodic payments	At maturity
Bullet structure (non-amortizing)	Interest only (the coupons)	Final coupon plus the entire principal
Fully amortizing	Equal payments, each part interest and part principal	Nothing left to repay; the last equal payment retires the debt
Partially amortizing	Level payments that repay only part of the principal	Last payment plus a balloon payment of the remaining principal

Example. A \$10,000, 4-year loan at 6% that is fully amortizing needs a level payment of

$$PMT = \frac{P \times i}{1 - (1 + i)^{-n}} = \frac{10,000 \times 0.06}{1 - 1.06^{-4}} = \$2,885.91$$

Calculator (END mode, P/Y = C/Y = 1): N = 4; I/Y = 6; PV = -10,000; FV = 0; CPT PMT = 2,885.91. The amortization schedule is then built one year at a time:

1. Interest for the year = opening balance × 6%.
2. Principal repaid = payment – interest.
3. Closing balance = opening balance – principal repaid. It becomes the next year's opening balance.

Year	Opening balance	Payment	Interest	Principal repaid	Closing balance
1	10,000.00	2,885.91	600.00	2,285.91	7,714.09
2	7,714.09	2,885.91	462.85	2,423.06	5,291.03
3	5,291.03	2,885.91	317.46	2,568.45	2,722.58
4	2,722.58	2,885.93	163.35	2,722.58	0.00

The payment stays level while the balance falls, so the interest part shrinks and a larger part of each payment retires principal. The last payment is 2 cents higher because of rounding.

A partially amortizing version of the same loan that leaves a \$2,000 balloon keeps the same inputs except $FV = 2,000$: $N = 4$; $I/Y = 6$; $PV = -10,000$; $FV = 2,000$; $CPT PMT = 2,428.73$. The final payment is $2,428.73 + 2,000 = \$4,428.73$. The three structures compare as follows:

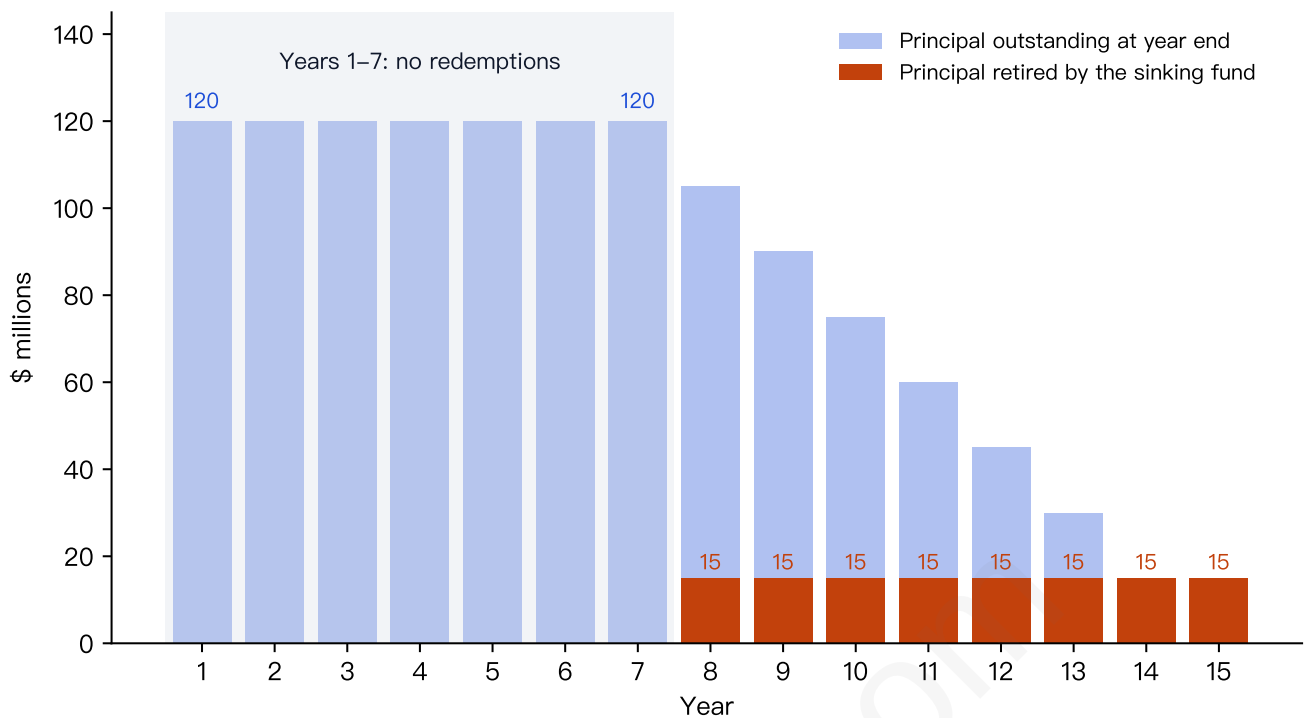
Year	Bullet	Fully amortizing	Partially amortizing (\$2,000 balloon)
1 to 3 (each year)	600.00	2,885.91	2,428.73
4	10,600.00	2,885.93	4,428.73

Compared with a fully amortizing loan, a partially amortizing one has lower regular payments and a higher final payment. It repays less principal along the way, so more principal stays outstanding, and its interest in every year after the first is higher (year-1 interest is the same, because no principal has been repaid yet).

A **sinking fund provision** obliges the issuer to pay off part of the issue according to a set schedule over the bond's life. Typically the bond trustee redeems a fixed amount of principal each year from holders chosen at random. For bondholders, a sinking fund has two effects:

- It lowers **credit risk**, because less principal is left to be repaid at maturity.
- It adds **reinvestment risk**, the risk of receiving cash early and having to reinvest it at lower yields. If rates have fallen, the bonds trade above par, yet holders whose bonds are drawn receive only par. Holders of premium bonds therefore prefer that their bonds are not drawn.

Example. A \$120 million, 15-year issue carries a sinking fund that retires \$15 million of principal each year from year 8 through year 15. Nothing is redeemed in years 1 to 7, and the last \$15 million is retired at maturity.



Sinking fund schedule of an illustrative \$120 million, 15-year issue

Amortizing structures carry the same trade-off, since a sinking fund is one form of amortization. The more principal a structure repays before maturity (bullet, then partially amortizing, then fully amortizing), the lower its credit risk and the higher its reinvestment risk.

A sinking fund is an obligation of the issuer and is not an embedded option. **Waterfall structures**, another form of amortization schedule, are used for ABS and MBS. They pay principal to senior **tranches** first. Junior tranches receive principal only after senior tranches are repaid, although all tranches still receive interest.

Coupon structures

- **Floating-rate notes (FRNs)** pay the **market reference rate (MRR)** plus a fixed margin (the **credit spread**) stated in basis points. Most reset and pay quarterly: the MRR observed at the start of a quarter sets the coupon paid at the end of that quarter. The per-period coupon is $(\text{MRR} + \text{margin}) / \text{payments per year}$.
Example. With an MRR of 3.10%, a margin of 60 bps and quarterly payments, the coupon is $(3.10\% + 0.60\%) / 4 = 0.925\%$ of par.
- **Step-up coupon bonds:** the coupon rises on a predetermined schedule, which gives investors some protection against rising interest rates.
- **Credit-linked coupon bonds** and **credit-linked notes:** the coupon rises if the issuer's credit rating falls and falls if it improves. Leveraged loans may step up if debt/EBITDA rises. Some **green bonds** pay more if environmental targets are missed.
- **Payment-in-kind (PIK) bonds:** the issuer can pay interest by increasing the principal (paying with more bonds). They are used by highly leveraged issuers, so yields are higher.
- **Index-linked bonds** tie coupons or principal to a published index. **Inflation-linked bonds (linkers)** are the most common.
 - **Interest-indexed bonds:** the coupon rate is adjusted for inflation, and the principal is unchanged (not protected).

- **Capital-indexed bonds** (the most common type, e.g., US **Treasury Inflation-Protected Securities (TIPS)**): the coupon rate stays fixed, and the principal is adjusted up for inflation (down for deflation). The fixed coupon rate is effectively a **real rate**. Because the coupons and the principal repaid both rise with the index, the investor's purchasing power is fully protected over the bond's life. Adjusted principal can fall below par during the bond's life, but at maturity TIPS repay the greater of the inflation-adjusted principal and par.
- *Example.* A bond has €5,000 par and a 2% annual coupon paid semiannually. Inflation over the first six months is 1.5%, so the principal becomes $5,000 \times 1.015 = €5,075$ and the coupon is $5,075 \times 0.02/2 = €50.75$.
- **Zero-coupon bonds** make one payment of par at maturity, so they must trade below par to give investors a positive return. They appeal to investors who want to minimize reinvestment risk. A zero-coupon bond is the extreme case of a deferred coupon bond.
- **Deferred coupon bonds** pay no coupons for an initial period after issuance and then pay regular coupons to maturity. In some issues the skipped coupons accrue and are paid as one lump sum when the deferral period ends; otherwise the bond is simply sold at a deeper discount. Like zero-coupon bonds, they often trade below par. The structure suits low-rated issuers or projects that generate cash only later.

Periodic coupons also have to be reinvested, so reinvestment risk rises with the coupon rate, and a call feature adds to it. Holding a coupon bond to maturity does not remove it; only a zero-coupon bond held to maturity has none.

Contingency provisions (embedded options)

A **contingency provision** allows an action to be taken if a specified event occurs. In a bond indenture such provisions are **embedded options**: they are part of the bond contract and are not separate securities. An embedded option benefits whoever holds the right to exercise it. Bonds without them are **straight bonds (option-free bonds)**.

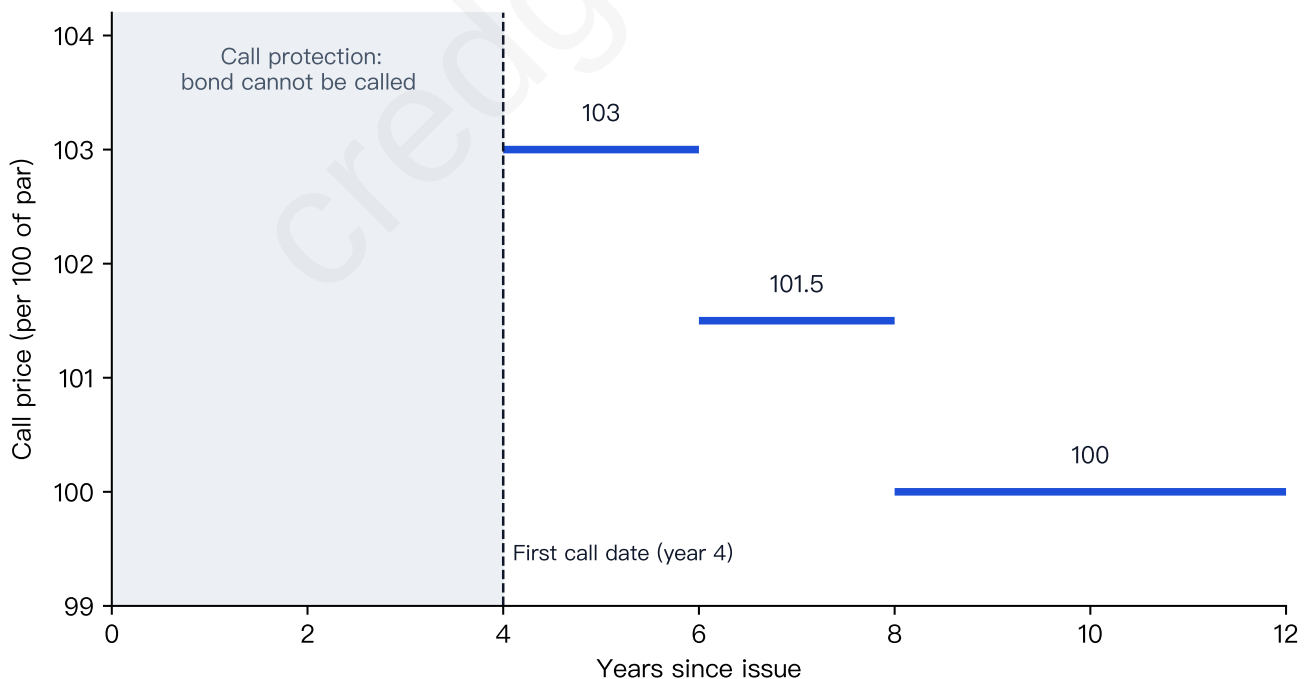
KEY CONCEPT

Bond	Who holds the option	Likely exercised when	Price vs straight bond	Yield vs straight bond
Callable bond (issuer may redeem early at the call price)	Issuer	The bond's market yield has fallen (rates dropped or the issuer's credit improved), so the issuer can refinance more cheaply	Lower: callable = straight – call	Higher
Putable bond (holder may sell back, usually at par)	Bondholder	The bond's price is below the put price (rates rose or the issuer's credit weakened)	Higher: puttable = straight + put	Lower

Bond	Who holds the option	Likely exercised when	Price vs straight bond	Yield vs straight bond
Convertible bond (holder may exchange for common shares)	Bondholder	The share price has risen, lifting the conversion value	Higher	Lower

- **Callable bonds:** the period before the first call date is **call protection**. The call price often steps down over time (e.g., 103, then 101.5, then par). Holders face **call risk**, and during the call period the call price caps the bond's market value. *Example.* A 5.5% bond with \$1,000 par reaches its first call date, callable at 101, just as its market yield falls to 3.5%. The issuer pays \$1,010 per bond and borrows that amount at 3.5%, which cuts annual interest from \$55 to $1,010 \times 0.035 = \$35.35$ per bond. The holder receives \$1,010 early and can reinvest it only at the lower yield.
- **Convertible bonds:** the **conversion price** is the par amount per share received, the **conversion ratio** is par divided by the conversion price, and the **conversion value** is the conversion ratio times the current share price. *Example.* A \$1,000 par bond with a conversion price of \$25 converts into 40 shares. At a share price of \$28, the conversion value is $40 \times 28 = \$1,120$.
- **Warrants** attached to straight bonds let holders buy common shares at a fixed price. They can be detached and traded separately.
- **Contingent convertible bonds (CoCos)** convert automatically into equity if a trigger occurs (e.g., a bank's equity capital falls below a required level).

A typical call schedule steps down toward par after a period of call protection:



Call price schedule of an illustrative 12-year callable bond

Options are worth more when the underlying is more volatile. Higher interest rate volatility therefore raises the value of the call held by the issuer, so the callable bond's value falls. It also raises the value of the put held by the investor, so the puttable bond's value rises.

COMMON EXAM TRAPS

- Level payments that leave nothing owing at maturity mean the bond is fully amortizing. Level payments followed by a balloon mean it is partially amortizing, and "bullet" means all principal is repaid at maturity.
- For a partially amortizing loan, enter the balloon as FV. Leaving $FV = 0$ gives the fully amortizing payment instead. For the \$10,000, 4-year, 6% loan with a \$2,000 balloon, $FV = 0$ gives a payment of \$2,885.91 instead of \$2,428.73.
- A sinking fund is an obligation, and it can hurt holders of premium bonds.
- TIPS have a fixed coupon rate and an adjusted principal. The coupon amount changes because the rate is applied to the adjusted principal. In the €5,000 example, applying the 2% rate to par gives a coupon of €50.00 instead of €50.75.

LOS 52.b — Legal, regulatory and tax considerations

Where bonds are issued and traded

KEY CONCEPT

Type	Definition
Domestic bonds	Issuer domiciled in the same country as the market where the bonds are issued and traded
Foreign bonds	Issued by an issuer from another country and traded in that country's domestic market (e.g., a UK firm selling US dollar bonds that trade in the US)
Eurobonds	Issued outside the jurisdiction of any one country, in any currency; less regulated than domestic bonds in most jurisdictions (they began as a way around US regulation). Named by currency: Eurodollar bonds (USD), Euroyen bonds (yen)
Global bonds	Trade in the Eurobond market and in at least one domestic market

The "Euro" in Eurobond does not mean the bonds are denominated in euros, sold in Europe or issued by a European firm; the name comes from the market's origins in Europe, where most Eurobonds are still traded. Foreign bonds, global bonds and Eurobonds are collectively **international bonds**. **Bearer bonds** (ownership shown simply by possession, with no record kept by the issuer) were once the norm for Eurobonds. Today most bonds are **registered bonds**. Markets also differ in laws, regulation and conventions such as coupon frequency, but the currency of a bond is the factor that best explains yield differences across markets, because a bond's yield is driven by interest rates in the country of its currency. **Sukuk** are Sharia-compliant bonds whose payments are treated as rent on underlying assets.

Taxation of bond income

- Interest income is usually taxed as **ordinary income**. In the US, interest on **municipal bonds** is usually not subject to national income tax, and often not to state tax in the issuing state either.
- Gains or losses on selling a bond before maturity are **capital gains** or losses. They are often taxed at lower rates, and lower still if the gain is long-term.
- **Original issue discount (OID)** bonds (e.g., zero-coupon bonds): the rise in value toward par is really interest, so in many jurisdictions part of the discount is taxed each year as interest income even though no cash is received.
- Some jurisdictions give bonds issued at a **premium** the symmetric treatment: part of the premium is used to reduce the taxable portion of the coupon interest.

COMMON EXAM TRAPS

- A euro-denominated bond sold in the euro area by a local issuer is a domestic bond.
- A bond sold in a country's own currency and market by a foreign issuer is a foreign bond, not a Eurobond.
- Currency alone does not decide the category. A Eurobond is defined by issuance outside the jurisdiction of any one country, and it can be in any currency.
- The accretion on a zero-coupon bond is interest income and is not treated as a capital gain.

EXAM SHORTCUTS

- A partially amortizing payment uses the same calculator entries as the fully amortizing payment, with the balloon entered as FV instead of zero.
- Only a zero-coupon bond held to maturity has no reinvestment risk, so any coupon bond can be eliminated from that answer even if it is held to maturity.
- A callable bond is worth less and a puttable bond more than an otherwise identical straight bond, so the three can be ranked by price, and in reverse by yield, without valuing any of them.

BOTTOM LINE

- A bullet bond pays only interest before maturity and repays all principal at maturity; a fully amortizing bond's level payments retire the debt by the last payment; a partially amortizing bond's level payments leave a balloon payment at maturity.
- A sinking fund provision obliges the issuer to retire part of the issue on a set schedule, which lowers bondholders' credit risk but adds reinvestment risk.
- A floating-rate note pays the market reference rate plus a fixed margin, so its per-period coupon is (MRR + margin) divided by the number of payments per year.
- A capital-indexed bond such as TIPS keeps a fixed coupon rate, effectively a real rate, and adjusts its principal for inflation, so each coupon is that rate applied to the adjusted principal; an interest-indexed bond adjusts the coupon rate and leaves the principal unprotected.
- An embedded option benefits whoever holds the right to exercise it: callable bond = straight bond – call, with a higher yield, and puttable bond = straight bond + put, with a lower yield.
- For a convertible bond, the conversion ratio is par divided by the conversion price, and the conversion value is the conversion ratio times the current share price.

- A foreign bond is issued by a foreign issuer and traded in another country's domestic market, while a Eurobond is issued beyond the jurisdiction of any single country and may be denominated in any currency.
- On an original issue discount bond such as a zero-coupon bond, the rise in value toward par is interest, and many jurisdictions tax part of the discount each year as interest income even though no cash is received.

QUICK CHECK

Question 1

A bond with a par value of \$2,000 carries a 6.8% coupon rate, paid semiannually, and is quoted at a price of 104.35. The dollar amount of each semiannual coupon is:

- A \$136.00.
- B \$70.96.
- C \$68.00.

Answer: C

Coupons are based on par value, and the market price plays no part. Half of the annual coupon is paid every six months.

$$\text{Semiannual coupon} = \frac{\text{coupon rate} \times \text{par}}{2} = \frac{0.068 \times \$2,000}{2} = \$68.00$$

The quoted price of 104.35 (104.35% of par) does not enter the calculation.

Why the other options are wrong

- A. \$136.00 is the full annual coupon; it is not divided by two for semiannual payments.
- B. \$70.96 applies the coupon rate to the market price ($2,000 \times 1.0435$) instead of to the par value.

Key takeaway The price quote is irrelevant to the coupon amount. Each coupon equals the coupon rate times par, divided by the number of payments per year.

Practice questions

Question 2

Which statement about a sinking fund provision is *most accurate*?

- A It obliges the issuer to deposit cash in a reserve that is used to repay the whole issue at maturity.
- B It gives bondholders the right to sell their bonds back to the issuer at par each year.
- C It obliges the issuer to retire part of the issue at scheduled dates before the final maturity.

Question 3

Priya Nair's main concern in her bond portfolio is reinvestment risk. Which of the following is she *least likely* to do?

- A Choose a noncallable bond over an otherwise identical callable bond.
- B Choose a low-coupon bond over a high-coupon bond of the same maturity.
- C Rely on holding a coupon-paying bond to maturity to remove the risk.

Question 4

Which of the following bond issues best fits the definition of a Eurobond?

- A A Mexican firm's Swiss franc bonds sold to investors in Switzerland.
- B An Indian firm's Australian dollar bonds sold to investors in Singapore.
- C A German firm's euro-denominated bonds sold to investors in Germany.

Question 5

Maren Holt buys a newly issued 8-year corporate bond at 106.40 per 100 of par. It pays a 6.5% annual coupon, and she plans to hold it to maturity. Her country taxes coupon interest as ordinary income and gives bonds issued at a premium the treatment that mirrors its original issue discount (OID) provision. Each year, the tax treatment of her coupons is *most likely* that:

- A part of the 6.40 premium reduces the taxable portion of her coupon interest.
- B the full 6.50 coupon is taxed as ordinary income, with no adjustment for the premium.
- C part of the 6.40 premium is added to her taxable interest income.

Answers: 2. C · 3. C · 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/zero-coupon-bond/. The full bank has 42 questions on this reading.

FIXED INCOME · READING 53

Fixed-Income Issuance and Trading

What you'll learn

LOS 53.a Describe how bond markets are segmented (issuer type, credit quality, original maturity and other features) and where issuers and investors sit on the credit/maturity spectrum.

LOS 53.b Describe aggregate and narrower fixed-income indexes and how bond indexes differ from equity indexes (constituents, turnover, weights, benchmark fit).

LOS 53.c Compare primary bond issuance (public offerings, private placements, underwritten, best-efforts, shelf, auctions) and OTC secondary trading with equity markets.

MODULE 53.1

Fixed-Income Issuance and Trading

This reading maps the bond market by issuer type, credit quality and original maturity, shows where different issuers and investors sit on that map, and compares bond indexes and bond trading with their equity counterparts. A candidate must be able to classify a bond by segment, match a funding need or an investor to a part of the spectrum, and explain how primary offerings and dealer markets work.

LOS 53.a — Market segments, issuers and investors

Global bond markets are split mainly along three lines: the **type of issuer** (also called the sector), **credit quality** and **original maturity**. Other ways to classify bonds include currency, the issuer's geography (for example **developed market** versus **emerging market** bonds) and environmental, social and governance (**ESG**) features.

KEY CONCEPT

Dimension	Main categories
Type of issuer	Governments (sovereign and non-sovereign), corporates, and special purpose entities issuing asset-backed securities (ABSs)
Credit quality	Investment-grade bonds : AAA down to BBB- (S&P) or Aaa down to Baa3 (Moody's). High-yield bonds (speculative or "junk"): BB+ or lower (S&P), Ba1 or lower (Moody's)
Original maturity	Short-term, 1 year or less (money market securities); intermediate-term, 1 to 10 years; long-term, over 10 years

Credit ratings come from **credit rating agencies** such as S&P and Moody's.

Exam convention: money market securities have an original maturity of one year or less; the same segment is also described as maturities of less than one year. Current practice: the money market label is used for instruments issued with maturities of up to about one year, such as Treasury bills, commercial paper and repos, so the two descriptions differ only for an original maturity of exactly one year.

The issuer credit/maturity spectrum

Credit segment	Short-term (≤ 1 year)	Intermediate-term (1–10 years)	Long-term (> 10 years)
"Default risk free"	Treasury bills	Treasury notes	Treasury bonds
Investment grade	Repos, commercial paper, asset-backed commercial paper (ABCP)	Unsecured corporate bonds, ABS	Unsecured corporate bonds, MBS
High yield	—	Secured corporate bonds, leveraged loans	—

- Secured new corporate issues sit in the high-yield row. An issuer with less dependable operating cash flows has to pledge collateral to attract lenders.
- ABCP is commercial paper backed by financial assets, so it counts as a short-term asset-backed security. The spectrum places it in the investment-grade row, not in the default-risk-free row.
- **Fallen angels** are bonds of issuers that were investment grade but have since been downgraded to high yield because their credit quality deteriorated. They are part of the high-yield sector.

Issuer choices

What a corporation issues depends on its access to capital markets and on what the money is for. A well-established investment-grade company can match funding to use:

- **Commercial paper** funds short-term (including seasonal) working capital needs.
- **Intermediate-term debt** funds medium-term investments and permanent working capital.
- **Long-term debt** funds capital investment in fixed assets.

Short- and medium-term borrowing is often arranged, for a fee, with a **syndicate** of banks that provides **credit facilities**, so the company can draw funds when it needs them. For a riskier company whose operating cash flows are less stable, access to secured short-term financing and leveraged loans is likely to be limited.

Investor positioning

Investors choose where to sit on the spectrum according to the interest rate and credit risk they want and the timing of the obligations they must meet.

Investor	Typical position	Why
Pension funds, insurance companies	Long-term, investment grade	Match long-dated liabilities (pensions, claims); regulation often bars high-yield holdings
Money market funds	Short-term, at the boundary between default-risk-free and investment-grade instruments	Hold short, high-quality paper for liquidity
Corporations	Short-term investment grade (commercial paper, repos, ABCP)	Earn a return on excess liquidity

Investor	Typical position	Why
Central banks	Intermediate-term Treasury notes	Monetary policy: buying adds to banks' reserves, selling drains them
Bond funds and ETFs	Intermediate investment grade (excluding Treasuries), per mandate	Stated fund objective
Asset managers seeking higher returns, hedge funds, distressed debt funds	Intermediate high yield; distressed debt at the lowest credit quality	Higher expected return
Financial intermediaries (banks)	Treasuries across all maturities	Manage interest rate and liquidity risk

Example. Orchard Valley Foods, rated A, has three funding needs: (1) a build-up of inventory ahead of its peak selling season, repaid within five months; (2) a four-year warehouse-automation project; (3) a processing plant with a 30-year useful life. The natural matches are commercial paper for (1), an intermediate-term note for (2), and a long-term bond for (3).

COMMON EXAM TRAPS

- BBB– (S&P) and Baa3 (Moody's) are the lowest investment-grade ratings. BB+ and Ba1 are the highest high-yield ratings.
- Classifying a bond as a money market security because little time is left to maturity. Only the original maturity (one year or less) counts.
- A fallen angel has been downgraded to high yield. That alone does not make it distressed debt, which requires the issuer to be in, or expected to file for, bankruptcy.
- Placing central banks in the short-term segment. On the investor map they sit in intermediate-term Treasury notes.

LOS 53.b — Fixed-income indexes

Bond indexes differ from equity indexes in three ways:

1. *More constituents.* One corporate issuer may have dozens of bonds outstanding but only two or three share classes. Because a full replica would be impractical, bond index tracker funds use **sampling** to keep transaction complexity reasonable.
2. *Higher turnover.* Bonds mature, get called or retired by sinking funds, and new bonds are issued far more often than new shares appear. Constituents are therefore removed and replaced (**turnover**) more often than in equity indexes, and the bond universe is less stable than the stock universe.
3. *Weights driven by issuance.* Governments are usually the largest borrowers, so broad bond indexes carry large weights in **sovereign bonds**. As issuance trends in maturity and credit quality shift, index weights shift with them.

KEY CONCEPT

Feature	Equity index	Bond index
Number of constituents	Fewer	Many more
Turnover	Lower	Higher
Tracking method	Full replication common	Sampling
Largest weights	Largest listed companies	Usually sovereign issuers

Aggregate indexes hold a broad selection of bonds across many sectors and currencies, but they still apply entry rules. Narrower indexes concentrate on a particular geography, sector, credit quality or maturity range, and some also apply **ESG** screens. Three well-known indexes show the range:

Index	Type	What it holds	Credit rule	Other rules
Bloomberg Barclays Aggregate Index	Aggregate	Bonds from all sectors in 28 currencies	High-yield and unrated issues excluded	Minimum issue size
JP Morgan Emerging Markets Bond Index Plus	Narrower: geography and credit quality	US dollar-denominated emerging market sovereign debt	Baa1/BBB+ or lower	Minimum size and maturity limits
Bloomberg Barclays MSCI Euro Corporate Sustainable SRI Index	Narrower: ESG	Euro corporate bonds	Baa3/BBB– or higher	Minimum ESG rating of BBB; sectors such as alcohol and thermal coal generation screened out

A bond fund's benchmark should have the same sector focus, credit quality and maturity profile as the fund.

COMMON EXAM TRAPS

- Reversing the index comparison. Bond indexes have more constituents and higher turnover than equity indexes.
- An aggregate index is broad but does not hold everything. High-yield, unrated and small issues are left out.
- The benchmark-matching dimensions are sector, credit quality and maturity. Coupon level and return volatility are not on the list.

LOS 53.c — Primary and secondary bond markets**Primary market**

In a primary market transaction the issuer sells newly created bonds and receives the cash. Two routes are available:

- **Public offering:** the issue is registered with securities regulators and sold to the public.

- **Private placement:** the issue is sold only to one investor or a small group of selected investors.

Both routes normally use investment banks as intermediaries. A company selling its first-ever bond is a **debut issuer**. It is typically a growing firm replacing bank loans with bonds. A debut issue needs weeks of investor **roadshows** and is usually slower to complete than later issues. **Repeat issues** are much quicker. When an investment-grade issuer that borrows frequently has a fully underwritten shelf registration, a deal can be agreed, completed and allocated within hours, because investors already know the issuer and the regulatory registrations are already in place. A small high-yield issue with detailed covenants and collateral, sold on a best-efforts basis, takes far longer.

KEY CONCEPT

Method	Price risk	Key feature
Underwritten offering	Intermediaries guarantee the issue price	Bank bears the risk of unsold bonds
Best-efforts offering	No price guarantee	Bank earns a commission for placing the bonds at the best price it can
Shelf registration	Either	Aggregate amount registered once with a master prospectus; bonds issued over time as funds are needed
Public auction	Set by bids	Used for some bonds, especially government bonds

Secondary market

In the secondary market investors trade bonds that are already outstanding. Some trading uses electronic platforms or exchanges, but most bond trading is in **over-the-counter (OTC)** dealer markets. Most equity trading, by contrast, happens on exchanges. Dealers quote a **bid** price (at which they buy) and an **ask** or **offer** price (at which they sell). The ask is always above the bid, and the difference is the **dealer's spread**. It depends on **liquidity**:

- A fraction of a basis point applies to liquid, recently issued (**on-the-run**) developed-market sovereign bonds and to bonds of high-quality frequent corporate issuers.
- 10–20 basis points or more applies to less liquid, smaller or older (**seasoned**) corporate issues.

The dealer's spread therefore signals how liquid an issue is.

Example. A dealer quotes a bond at 97.20 bid and 97.32 ask per 100 of par. The spread is 0.12 per 100 of par, or 12 basis points. That is typical of a seasoned, thinly traded corporate issue. An on-the-run government bond would be quoted far more tightly.

Distressed debt is the debt of issuers that are in bankruptcy or expected to file for it. It has a low credit rating, high risk and a high promised yield. Distressed debt investors often buy from institutions that are not allowed to hold low-rated bonds. They hope to profit from a turnaround, a better-than-expected recovery, or a value-creating restructuring. Entering distress can temporarily lift trading in a normally quiet issue.

COMMON EXAM TRAPS

- In an underwritten offering the price is guaranteed. In a best-efforts offering the bank earns a commission and gives no guarantee. Shelf registration concerns timing (issuing over time) and says nothing about price risk.
- Selling an entire issue to a single investor is a private placement.
- National governments are the typical auction issuers. Corporations and municipalities usually sell through intermediaries.
- A wide bid-ask spread signals illiquidity. On its own it says nothing definite about credit rating or issuer type.

BOTTOM LINE

- Bond markets are segmented mainly by type of issuer (sector), credit quality and original maturity.
- Investment grade means a rating from AAA down to BBB– at S&P, or from Aaa down to Baa3 at Moody's; a rating of BB+ or below (Ba1 or below at Moody's) is high yield.
- Exam convention: money market securities have an original maturity of one year or less, intermediate-term ones between 1 and 10 years, and long-term ones more than 10 years; current practice applies the money market label to instruments issued with maturities of up to about one year, such as Treasury bills, commercial paper and repos.
- Fallen angels are bonds of issuers downgraded from investment grade to high yield because their credit quality deteriorated, while distressed debt is the debt of issuers that are in bankruptcy or expected to file for it.
- A well-established investment-grade company can fund short-term (including seasonal) working capital with commercial paper, medium-term investments and permanent working capital with intermediate-term debt, and fixed-asset investment with long-term debt.
- Compared with equity indexes, bond indexes have many more constituents and higher turnover, tracker funds use sampling, and broad bond indexes usually carry large weights in sovereign bonds.
- In an underwritten offering the intermediaries guarantee the issue price; in a best-efforts offering they earn a commission with no price guarantee; a shelf registration registers the aggregate amount once and lets the bonds be issued over time.
- Most secondary bond trading takes place in over-the-counter dealer markets, and the dealer's spread between ask and bid widens as liquidity falls: a fraction of 1 bp for on-the-run sovereign debt of developed markets, and 10–20 bp or more for seasoned, smaller or less liquid corporate issues.

QUICK CHECK**Question 1**

A bond database lets users group bonds along several dimensions: type of issuer (sector), credit quality, original maturity and currency. Which of the following groups results from sorting bonds by type of issuer?

- A** Bonds denominated in euros.
- B** Short-term money market securities.

C Bonds issued by special purpose entities.

Answer: C

Classification by type of issuer (sector) separates governments (sovereign and non-sovereign), corporates, and special purpose entities issuing asset-backed securities. Bonds issued by special purpose entities are grouped by who the issuer is, so this group results from sorting by issuer. The other two groups come from the currency and original-maturity dimensions.

Why the other options are wrong

- **A.** Bonds denominated in euros are grouped by currency, which is one of the other ways to classify bonds. Any type of issuer can sell bonds in euros.
- **B.** Money market securities are defined by original maturity (one year or less). Any type of issuer can sell them.

Key takeaway Sort the labels: type of issuer (governments, corporates, special purpose entities issuing ABSs), credit quality, original maturity (money market, intermediate, long-term), and other dimensions such as currency, issuer geography and ESG features.

Practice questions

Question 2

An analyst compares bond market indexes with stock market indexes. Which of the following statements is *least accurate*?

- A** A bond index typically contains far more constituents than a stock index.
- B** A single company often has many bonds outstanding but only two or three share classes.
- C** The set of bonds in an index changes less often than the set of stocks in a stock index.

Question 3

Which of the following issuers *most likely* sells its new bonds through a public auction?

- A** A national government.
- B** A corporation.
- C** A municipality.

Question 4

Tamsin Hale wants to buy \$2 million face value of a mid-sized corporate bond that was issued six years ago and now trades only infrequently. She is *most likely* to:

- A** submit a competitive bid in the issuer's next bond auction.
- B** buy from a dealer at its ask price, paying a relatively wide spread.
- C** buy on an exchange order book, paying a spread similar to an on-the-run sovereign bond.

Answers: 2. C · 3. A · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/fixed-income-issuance-and-trading/. The full bank has 26 questions on this reading.

credguild.com

FIXED INCOME · READING 54

Commercial Paper

What you'll learn

LOS 54.a Compare short-term funding sources for nonfinancial corporations (lines of credit, secured loans, factoring, commercial paper) and for financial institutions (deposits, CDs, interbank and central bank funds, CP/ABCP).

LOS 54.b Describe repurchase agreements: mechanics (purchase price, repo rate, initial and variation margin, haircut), uses, repo-rate drivers and risks.

LOS 54.c Contrast long-term funding of investment-grade and high-yield corporate issuers (yields and spreads, covenants, collateral, maturity, standardization, prepayment features).

MODULE 54.1

Fixed-Income Markets for Corporate Issuers

This reading compares the short-term funding sources open to nonfinancial companies and to financial institutions, explains how repurchase agreements work, and contrasts long-term funding for investment-grade and high-yield issuers. A candidate must be able to rank lines of credit by reliability, compute a repo's purchase price, haircut, repurchase price and variation margin, and state how high-yield issues differ from investment-grade issues.

LOS 54.a — Short-term funding alternatives

Nonfinancial corporations

A nonfinancial company funds short-term assets (cash, short-term investments, receivables and inventory) through intermediaries. The intermediaries supply this funding in two forms: loan financing and security-based financing.

Bank lines of credit let a borrower draw funds as needed, usually at a floating **market reference rate (MRR)** plus a fixed credit spread.

KEY CONCEPT

Type	Bank's commitment	Cost	Reliability
Uncommitted line of credit	Bank may refuse to lend if circumstances change	Usually interest only, no fees; flexible, and may be unsecured if the borrower keeps stable cash balances at the bank	Least reliable
Committed (regular) line of credit	Bank commits for a set period	Interest plus a commitment fee (about 50 bp on the full or unused amount)	More reliable; renewal risk at maturity
Revolving (operating) line of credit	Longer commitment, sometimes years	Similar to a committed line; restrictive covenants usually imposed	Most reliable

Banks hold more regulatory reserves against committed lines. They manage that risk by keeping commitments under one year or by lending in a **syndicate**.

- **Secured (asset-backed) loans:** weaker borrowers pledge fixed assets, receivables or inventory as collateral. Receivables can also be assigned to a lender as collateral.
- **Factoring:** the company actually transfers its receivables, together with the credit-granting and collection functions, to a **factor**, which pays less than face value. The discount is effectively the interest rate and depends on the customers' creditworthiness and on collection costs.
- **Commercial paper (CP):** short-term unsecured debt issued by large, highly rated firms, usually maturing in under three months. For these issuers it is cheaper than a bank loan. CP funds working capital and serves as **bridge financing** until permanent (long-term) financing is arranged. CP is normally rolled over at maturity. The danger of being unable to sell new paper to repay maturing paper is **rollover risk**, which issuers manage with **backup lines of credit** (liquidity enhancement). US CP is a pure discount security, like a T-bill. A smaller, less liquid international market is **Eurocommercial paper (ECP)**.

Financial institutions

- Deposits: **demand deposits** (checking accounts, immediate access, usually no interest), **operational deposits** (large customers needing cash management, custody and clearing) and **savings deposits**, which carry a stated term and interest rate. A **certificate of deposit (CD)** has a stated rate and maturity of less than a year. A **nonnegotiable CD** carries a penalty for early withdrawal. A **negotiable CD** can be sold in the open market, and wholesale negotiable CDs, traded in domestic bond markets and in the Eurobond market, are an important bank funding source.
- **Interbank funds:** loans from one bank to another for one day to one year, secured or unsecured, priced off an MRR such as SOFR. The most common secured form is the repo.
- **Central bank funds market:** banks with **excess reserves** at the central bank lend them to banks that are short. The rate is the **central bank funds rate**, which the central bank's open market operations strongly influence. Borrowing directly from the central bank as **lender of last resort** ("discount window lending") costs more and brings extra scrutiny.
- CP and ABCP: financial institutions issue more CP than nonfinancial companies. They also sponsor **asset-backed commercial paper (ABCP)**, which is created in two steps. (1) The bank transfers short-term loans it has made to an off-balance-sheet **special purpose entity (SPE)** in exchange for cash. (2) The SPE sells ABCP to investors, who take on the risk and return of those loans, and the bank provides a backup liquidity line.

COMMON EXAM TRAPS

- Reliability rises from uncommitted to committed to revolving lines of credit, and revolving lines usually add restrictive covenants. Committed and revolving lines carry similar fees, including a commitment fee; an uncommitted line usually costs only the interest on amounts drawn.
- Assigning receivables as collateral is not factoring. Factoring transfers the receivables and their collection to the factor.
- Excess reserves lent bank-to-bank earn the central bank funds rate. Borrowing from the central bank itself is discount window (lender of last resort) lending.

LOS 54.b — Repurchase agreements

In a **repurchase agreement (repo)**, one party sells a security and commits to repurchase it at a later date for a higher price fixed in advance. Economically, the security buyer lends cash to the security seller and holds the security as collateral.

Leg	Security buyer (cash lender)	Security seller (cash borrower)
Opening (today)	Pays the purchase price (loan amount)	Delivers the collateral securities
Closing (the repurchase date)	Returns the collateral	Pays the repurchase price (loan plus interest)

The **repo rate** is the annualized interest rate implied by the gap between the repurchase price and the purchase price. The borrower keeps the economic benefits of the bond, such as coupons, during the term.

Initial margin protects the lender. Collateral worth more than the loan must be posted, so the loan is a discount to the market value:

KEY CONCEPT

$$\text{purchase price} = \frac{\text{market value of collateral}}{\text{initial margin}}$$

$$\text{repurchase price} = \text{purchase price} \times \left(1 + \text{repo rate} \times \frac{\text{days}}{360} \right)$$

$$\text{haircut} = \frac{\text{market value} - \text{purchase price}}{\text{market value}} = 1 - \frac{1}{\text{initial margin}}$$

The loan value grows at the repo rate. If the collateral's value drops below (accrued loan value × initial margin), the lender calls for **variation margin**:

$$\text{variation margin} = \text{initial margin} \times \text{adjusted purchase price} - \text{market value of collateral}$$

A negative result signals overcollateralization, so the borrower may request the return of some collateral.

Example. Collateral is worth \$40 million, the initial margin is 105%, the repo rate is 3% and the term is 30 days (360-day year).

1. Loan amount: the purchase price is $40,000,000 / 1.05 = \$38,095,238$.
2. Haircut: $1 - 1/1.05 = 4.76\%$.
3. Amount due on the repurchase date: $38,095,238 \times (1 + 0.03 \times 30/360) = \$38,190,476$.
4. Margin check during the term. After 10 days the loan has grown to $38,095,238 \times (1 + 0.03 \times 10/360) = \$38,126,984$, so the required collateral is $38,126,984 \times 1.05 = \$40,033,333$. If the collateral is then worth \$39.4 million, the borrower must post variation margin of about \$633,333. If it is worth \$40.5 million instead, variation margin is $40,033,333 - 40,500,000 = -\$466,667$, so the borrower can ask for about \$466,667 of collateral to be released.

Term	Meaning
Overnight repo	Lasts one day
Term repo	Lasts longer than one day
General collateral repo	Any security of a given type is acceptable, such as Treasuries in a maturity range
Master repurchase agreement	The document that holds the contract terms
Tri-party repo	A custodian bank or clearinghouse acts as agent
Bilateral repo	Struck directly between the two parties

Uses

- Dealers and other financial institutions borrow through repos (as security sellers) to finance their securities positions.
- Banks, mutual funds and pension funds lend (as security buyers) to earn the repo rate on excess cash.
- Central banks lend (buy) to add money and borrow (sell) to drain it.
- Short sellers such as hedge funds buy the security and lend cash in a repo, sell the security short, buy it back later and return it at maturity. A trade entered for the purpose of borrowing a security in this way is a **reverse repo** from that participant's side. Asking for a specific security is a **special trade**. For a hard-to-borrow security the lender of cash accepts a lower, sometimes negative, repo rate.

What drives the repo rate?

Factor	Effect on repo rate
Higher rates on alternative money market funding	Higher
Higher credit quality of collateral	Lower
Longer repo term (when longer rates exceed short rates)	Higher
Collateral in high demand or low supply	Lower
Undercollateralized, or collateral specified but not delivered	Higher

Because repos are short-term and collateralized, usually by high-quality sovereign bonds, repo rates are normally below bank loan rates.

A repo is safer than most short-term lending, but it is still debt for the borrower, and heavy reliance on it can lead to financial distress. Repos carry **default risk** (the cash borrower fails to repurchase), **collateral risk**, **margining risk**, **legal risk**, and **netting and settlement risk**. A tri-party repo helps mitigate many of these risks and improves cost efficiency in accessing, valuing and safekeeping collateral, but it does not reduce credit risk.

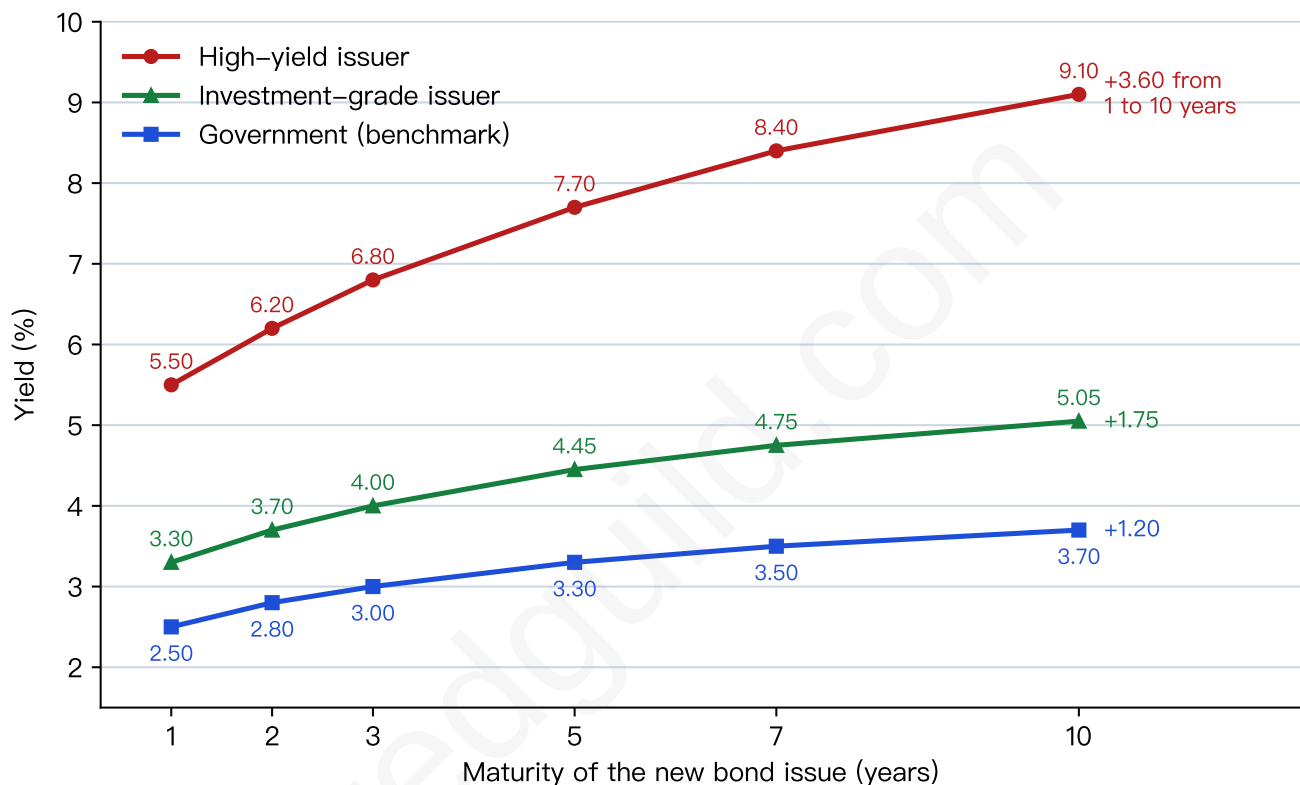
COMMON EXAM TRAPS

- Divide by the initial margin to get the loan amount. The haircut is not the margin in excess of 100%: a 105% initial margin means a haircut of 4.76%, not 5%. Applying a 5% haircut to the \$40 million of collateral in the example gives a loan of \$38,000,000 instead of \$38,095,238.

- The borrower in a repo is the security seller. A participant who buys the security to obtain it is in a reverse repo.
- Scarce ("special") collateral lowers the repo rate the cash lender accepts.

LOS 54.c — Investment-grade vs. high-yield long-term funding

With a normal (upward-sloping) yield curve, both investment-grade and high-yield issuers must pay higher yields at longer maturities. The increase is bigger for high-yield issuers because their credit spreads are larger. Issuing shorter bonds to save on yield adds rollover risk.



Yields on new issues by maturity for government, investment-grade and high-yield borrowers

In the figure, moving from a 1-year to a 10-year issue adds 1.75 percentage points to the investment-grade yield and 3.60 points to the high-yield yield. The government curve rises by only 1.20 points, so most of the extra cost for the high-yield issuer comes from a credit spread that widens with maturity.

KEY CONCEPT

Feature	Investment grade	High yield
Main credit concern of investors	Downgrade risk and a rising probability of future default	Default risk and loss given default
Credit spread as a share of yield	Small; yield tied mostly to benchmark (sovereign) rates	Large

Feature	Investment grade	High yield
Covenants	Few (e.g., limits on liens and sale-and-leaseback of core assets)	Many: debt ratios, limits on additional debt and on distributions such as dividends
Collateral	Usually unsecured	Often secured
Standardization and maturities	Fairly standardized; issued across many maturities, which lowers rollover risk and lets issuers exploit market conditions	Less standardized; usually 10 years or less; less able to refinance when rates fall
Early repayment features	Less common	Leveraged loans with prepayment options or callable bonds, so debt can be refinanced if credit improves
Return profile	Bond-like	More equity-like

COMMON EXAM TRAPS

- Covenants protect lenders, so they set maximums on leverage, debt and dividends.
- An issuer hoping its credit will improve wants the right to repay early (call or prepay). A put option benefits the investor.

EXAM SHORTCUTS

- The haircut depends only on the initial margin, $1 - 1/\text{initial margin}$, so it can be found without the collateral value, the repo rate or the term.

BOTTOM LINE

- Lines of credit become more reliable from uncommitted to committed to revolving; committed and revolving lines carry a commitment fee and revolving lines usually add restrictive covenants, while an uncommitted line usually costs only interest on the amount drawn.
- Factoring transfers receivables, with the credit-granting and collection functions, to a factor at a discount to face value, whereas assigning receivables as collateral keeps them with the borrower as security for a loan.
- Commercial paper is short-term unsecured debt issued by large, highly rated firms, usually maturing in under three months, and issuers manage its rollover risk with backup lines of credit.
- In a repo the security seller borrows cash and the security buyer lends it against the security; $\text{purchase price} = \text{market value of collateral} / \text{initial margin}$ and $\text{repurchase price} = \text{purchase price} \times (1 + \text{repo rate} \times \text{days}/360)$.
- $\text{Variation margin} = \text{initial margin} \times \text{adjusted purchase price} - \text{market value of collateral}$; a negative figure means the loan is overcollateralized and the borrower may ask for collateral back.
- The repo rate is higher when alternative money market rates are higher, when the term is longer and longer rates exceed short rates, and when the repo is undercollateralized or collateral is not delivered; it is lower for higher-quality collateral and for collateral in high demand or low supply.

- With a normal yield curve, both investment-grade and high-yield issuers pay higher yields at longer maturities, and the increase is bigger for high-yield issuers because their credit spreads are larger.
- Compared with investment-grade issues, high-yield issues carry more covenants, are often secured, usually mature in 10 years or less, more often allow early repayment through callable bonds or prepayable leveraged loans, and have more equity-like returns.

QUICK CHECK

Question 1

A corporate treasurer is reviewing the bank credit lines available to her company. Which of the following statements about these lines is *most accurate*?

- A An uncommitted line typically carries a commitment fee on its unused portion.
- B A committed line is the least reliable source of bank funding for a company.
- C A revolving line often runs for years and usually comes with restrictive covenants.

Answer: C

A revolving (operating) line of credit is the most reliable source of bank funding. It is typically arranged for a longer term than a committed line, sometimes several years, and because the bank is committed for that long it usually imposes restrictive covenants that oblige the borrower to take, or to avoid, specified actions, much as a bond indenture does. Its fees and rates are similar to those of a committed line.

Why the other options are wrong

- **A.** An uncommitted line is flexible and usually has no fees beyond interest on the amounts drawn, because the bank can decline to lend if conditions change. Commitment fees (about 50 bp on the full or unused amount) are charged on committed lines, and revolvers carry similar fees.
- **B.** The least reliable form is the uncommitted line, which the bank can decline to honor. A committed line binds the bank for the stated period, although the bank may decline to renew it at maturity (renewal risk).

Key takeaway Reliability rises from uncommitted to committed to revolving lines. Commitment fees apply to committed lines and revolvers, and revolvers usually add restrictive covenants over a multiyear term.

Practice questions

Question 2

A bond dealer finances a position by entering a 60-day repurchase agreement. It delivers government bonds with a market value of \$25,000,000. The lender requires an initial margin of 102% and a repo rate of 3.6%, using a 360-day year. The repurchase price the dealer will pay at the end of the term is *closest* to:

- A \$24,656,863.
- B \$24,647,000.
- C \$24,509,804.

Question 3

An analyst splits the yield of each of three bonds into a benchmark rate and a credit spread. The spread accounts for 4% of Bond P's yield, 12% of Bond Q's yield and 45% of Bond R's yield. Which type of bond is Bond R *most likely* to be?

- A A government bond.
- B An investment-grade corporate bond.
- C A high-yield corporate bond.

Answers: 2. A · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/commercial-paper/. The full bank has 16 questions on this reading.

FIXED INCOME · READING 55

Treasury Bonds

What you'll learn

LOS 55.a Describe how sovereign, non-sovereign (regional/local), quasi-government (agency) and supranational issuers fund themselves, including developed vs. emerging market and domestic vs. external sovereign debt.

LOS 55.b Contrast government and corporate bond issuance and trading: auctions (competitive and noncompetitive bids, cut-off yield, single- vs. multiple-price), primary dealers, on-the-run bonds and noneconomic investors.

MODULE 55.1

Fixed-Income Markets for Government Issuers

This reading describes how sovereign, non-sovereign, agency and supranational issuers fund themselves, and how government bonds are issued at auction and traded. A candidate must be able to classify a government-related issuer, explain the currency risk in emerging market debt and the maturity choices of a government, and work through an auction to find the cut-off yield and each bidder's allocation.

LOS 55.a — Government issuers and their funding choices

Sovereign debt

National governments borrow to pay for public goods and services and for infrastructure. **Sovereign bonds** are backed by the power to tax, so they usually carry the highest credit rating in their home market, and the sovereign is usually the largest issuer in that market.

Public-sector accounting standards differ from private-sector ones: they lean on cash transactions more than on accruals such as depreciation and unfunded liabilities. Analysts should therefore think of an **economic balance sheet** that adds implied assets (expected future tax revenue) and implied liabilities (promised future spending) to the reported financial assets and liabilities.

A government's financial balance sheet and economic balance sheet

	Assets side	Liabilities side
Financial balance sheet (reported)	Financial assets, such as short-term claims, holdings in state-owned firms and foreign exchange reserves	Debt outstanding (short- and long-term, in domestic or foreign currency) and other recorded obligations
Added in the economic balance sheet	Implied asset: expected future tax revenues	Implied liability: promised future expenditures, such as pensions and public services

	Developed market sovereign	Emerging market sovereign
Economy	Stable, diversified	Faster-growing, less stable, more concentrated
Public finances	Consistent, transparent fiscal policy	Less stable tax revenue, sometimes tied to one dominant industry or commodity
Currency of debt	A reserve currency : a major currency held by central banks worldwide and widely used in trade (e.g., US dollar, Chinese renminbi)	Home currency or a foreign reserve currency

Emerging market debt often funds investment in economic growth. It can be:

- **Domestic debt**: issued in the home currency and held by domestic investors. The currency may not be freely convertible, because of illiquidity or capital controls.
- **External debt**: owed to foreign creditors, in the home currency or in a reserve currency. If it is in a reserve currency, foreign investors avoid direct currency risk. They still bear indirect currency risk, because the government must earn enough of the reserve currency to repay.

A government's **debt management policy** sets how much debt it will issue and in what form. To forecast issuance, focus on **fiscal policy**, such as tax cuts and spending increases when the economy is below full employment, and on how cyclical and inflation-sensitive revenues and spending are. Floating-rate or inflation-indexed features and any government guarantees of non-sovereign debt also matter.

Ricardian equivalence says taxpayers expect today's debt to be repaid with future taxes. Under that view a government should be indifferent to the maturity of its debt, but only if taxpayers save more when they expect higher taxes, have rational expectations, can borrow and lend without transaction costs, and pass tax savings on to future generations. These conditions do not hold in practice. Governments therefore issue across the maturity spectrum and keep a fairly stable mix of short- and long-term debt.

- Short-term sovereign debt is seen as safe and highly liquid and can serve as an alternative to bank deposits. That liquidity benefit likely keeps its yields below where they would otherwise be. Relying on it too heavily creates **rollover risk**.
- Longer-term sovereign debt provides **benchmark** yields for pricing the credit risk of other borrowers. It is used for interest rate risk management and as repo collateral, and central banks use it to conduct monetary policy.

Other government-related issuers

KEY CONCEPT

Issuer type	Who issues	Repayment / key feature
Non-sovereign government bonds	States, provinces, counties, cities and entities providing public services (hospitals, airports)	See GO vs. revenue bonds below
General obligation (GO) bonds	Local or regional governments	General public spending, backed by local taxing power

Issuer type	Who issues	Repayment / key feature
Revenue bonds	Local or regional governments	Fund one project; repaid from fees generated by it (e.g., a toll road or bridge)
Agency (quasi-government) bonds	Entities created by a national government for a specific purpose (infrastructure, mortgage finance; e.g., Ginnie Mae)	Repaid from the agency's own revenue (Ginnie Mae: guarantee fees) plus any government backing; if backed by the sovereign, yields and ratings sit close to the government's
Supranational bonds	International institutions set up by several sovereign governments (World Bank, IMF, Asian Development Bank) to promote cooperation, trade or growth	Typically high credit quality; some issues very liquid

COMMON EXAM TRAPS

- Calling the bonds of an institution set up by several governments quasi-government bonds. They are supranational bonds; an agency is created by one national government.
- Mixing classification by geography (developed vs. emerging market) with classification by issuer type (sovereign, non-sovereign, agency, supranational).
- Treating reserve-currency external debt as free of currency risk. Only the direct risk is removed.

LOS 55.b — Issuing and trading government vs. corporate bonds

Corporations raise debt financing as they need it. Sovereign issuers use **regular public auctions**.

How a government bond auction works

KEY CONCEPT

1. **Noncompetitive bids** are filled first. These bidders are guaranteed their allocation at the price the auction sets, but they do not help set it.
2. **Competitive bids** state a yield (price). They are ranked from the highest price (lowest yield) down.
3. Bonds go to competitive bidders from the highest price downward until the offering is filled.
4. The yield on the lowest-priced accepted competitive bid is the **cut-off yield**.

KEY CONCEPT

Auction type	Price paid by successful bidders	Consequences
Single-price auction	Every winner pays the price at the cut-off yield	All bonds issued at one yield, so lower yield volatility. Tends to widen distribution, raise the chance of a successful auction and lower the issuer's funding cost

Auction type	Price paid by successful bidders	Consequences
Multiple-price auction	Each winner pays its own bid	Bids tend to cluster close together and be large in size

Example. A treasury offers 500 million of 7-year notes. It receives 80 million of noncompetitive bids and the competitive bids shown below. Allocation runs down the table, adding up the amounts until the 500 million is used:

Bid	Yield bid	Amount bid	Cumulative amount	Allocated
Noncompetitive	—	80	80	80
Competitive	3.10%	150	230	150
Competitive	3.12%	120	350	120
Competitive	3.15%	200	550	150 (partial)
Competitive	3.18%	100	650	0

The offering runs out within the 3.15% bids, so the cut-off yield is 3.15%. Those bidders receive 150 million of the 200 million they asked for, and the 3.18% bid gets nothing. In a single-price auction every successful bidder pays the price that corresponds to 3.15%. In a multiple-price auction each pays the price of its own bid.

Primary dealers

A sovereign designates certain financial institutions as **primary dealers**. They must bid competitively in auctions, submit bids for third parties, and act as counterparties when the central bank buys (expansionary) or sells (contractionary) government securities to conduct monetary policy. Fiscal policy, meaning spending and taxation, is run by the government.

Secondary trading and yields

Like corporate bonds, government bonds trade mainly in quote-driven **OTC dealer markets**. Trading is heaviest and prices most informative for **on-the-run** bonds, the most recently issued bonds of each maturity. Their yields serve as the default-risk-free **benchmark** yields used to build yield curves.

Some holders of government bonds have **noneconomic objectives**. Central banks hold them for monetary policy, foreign governments hold them as reserves, and regulations require some financial institutions to hold them. This extra demand pushes sovereign yields below those of non-sovereign issuers. Lower credit risk also contributes to the lower yields.

COMMON EXAM TRAPS

- Filling competitive bids before noncompetitive ones, or starting the competitive bids at the highest yield. In the 500 million example, starting from the 3.18% bid gives a cut-off yield of 3.12%, and filling the competitive bids before the noncompetitive ones gives 3.18%, instead of 3.15%.
- Taking the cut-off yield as the lowest accepted yield. It is the highest accepted yield, which matches the lowest accepted price. In the example that error gives 3.10% instead of 3.15%.

- Assuming a competitive bidder is sure of an allocation. Only noncompetitive bidders are, in either auction format.

BOTTOM LINE

- Sovereign bonds are backed by the power to tax, so they usually carry the highest credit rating in their home market, and the sovereign is usually the largest issuer there.
- External debt of an emerging market government in a reserve currency spares foreign investors the direct currency risk but leaves indirect currency risk, because the government must earn enough of that currency to repay.
- Ricardian equivalence would make a government indifferent to the maturity of its debt only if taxpayers save more when they expect higher taxes, have rational expectations, borrow and lend without transaction costs and pass tax savings on to future generations; these conditions do not hold, so governments keep a fairly stable mix of short- and long-term debt.
- Short-term sovereign debt is seen as safe and highly liquid, which likely holds its yields down, but relying on it too heavily creates rollover risk; longer-term sovereign debt supplies the benchmark yields used to price other borrowers' credit risk.
- General obligation bonds are backed by local taxing power, revenue bonds are repaid from the fees of the one project they fund, agency (quasi-government) bonds are issued by entities a national government creates, and supranational bonds are issued by institutions set up by several sovereign governments.
- In a government bond auction, noncompetitive bids are filled first, then competitive bids from the highest price (lowest yield) down until the offering is used up; the cut-off yield is the highest accepted yield, which matches the lowest accepted price.
- In a single-price auction every successful bidder pays the price at the cut-off yield, while in a multiple-price auction each successful competitive bidder pays its own bid.
- On-the-run government bonds, the most recently issued of each maturity, trade most actively and supply the default-risk-free benchmark yields, and demand from holders with noneconomic objectives pushes sovereign yields below non-sovereign yields.

QUICK CHECK

Question 1

A bond analyst wants to group bonds according to the geography of their issuers. Which of the following labels fits that approach?

- A Supranational bonds.
- B Municipal bonds.
- C Emerging market bonds.

Answer: C

Grouping bonds as developed market or emerging market bonds classifies them by the issuer's geography. The label emerging market bonds belongs to this approach.

Why the other options are wrong

- **A.** Supranational bonds are a category of issuer: international institutions owned by several governments. The label is not geographic.
- **B.** Municipal bonds are grouped by type of issuer (local governments), and sometimes by tax status.

Key takeaway Geographic labels are developed market and emerging market. Issuer-type labels are sovereign, municipal, agency, supranational and corporate.

Practice questions

Question 2

The central bank of Valdoria conducts much of its policy through a group of designated primary dealers. Which transaction is it *most likely* to carry out with them?

- A** Expansionary monetary policy by selling government bonds.
- B** Expansionary monetary policy by buying government bonds.
- C** Contractionary fiscal policy by cutting government spending.

Question 3

An analyst is building a government yield curve to use as the default-risk-free benchmark. The yields she uses are *most likely* taken from:

- A** older government bonds that have been outstanding for many years.
- B** the most recently issued government bonds of each maturity.
- C** highly rated supranational bonds.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/treasury-bonds/. The full bank has 11 questions on this reading.

FIXED INCOME · READING 56

Yield to Maturity

What you'll learn

LOS 56.a Calculate a bond's price from its yield to maturity on a coupon date or between coupon dates (accrued interest, flat and full price).

LOS 56.b Identify how price relates to coupon rate, maturity and yield to maturity, including convexity and pull to par.

LOS 56.c Describe matrix pricing for bonds that are not traded or trade infrequently.

MODULE 56.1

Fixed-Income Bond Valuation: Prices and Yields

This reading values a fixed-rate bond as the present value of its promised cash flows, on a coupon date and between coupon dates, and links its price to its coupon rate, maturity and yield. A candidate must be able to compute a bond's price or YTM for any coupon frequency, split a full price into flat price and accrued interest, explain why prices move toward par, and estimate an illiquid bond's yield by matrix pricing.

LOS 56.a — Pricing a bond on and between coupon dates

Price = present value of the promised cash flows

A bond's value is the sum of the present values of every promised coupon and the principal. The single discount rate that makes this sum equal to the market price is the **yield to maturity (YTM)**, also called the **market discount rate**. Given the YTM, the price can be computed; given the price, the YTM can be solved for.

For an **annual-pay bond** with N years left, coupon C and par FV :

$$P = \frac{C}{(1 + YTM)} + \frac{C}{(1 + YTM)^2} + \cdots + \frac{C + FV}{(1 + YTM)^N}$$

For a **semiannual-pay bond**, use the semiannual coupon $C/2$, the periodic rate $YTM/2$ and $2N$ periods:

KEY CONCEPT

$$P = \sum_{t=1}^{2N} \frac{C/2}{(1 + YTM/2)^t} + \frac{FV}{(1 + YTM/2)^{2N}}$$

The same logic works for any **periodicity**: a quarterly-pay bond uses $C/4$, $YTM/4$ and $4N$ periods. A quoted YTM is a **stated annual rate**: the periodic yield times the number of periods per year. It is an effective annual rate only if the bond pays annually. For a semiannual bond the periodic yield is therefore the quoted YTM divided by 2, and no square root is taken.

A **zero-coupon bond** has only one cash flow, so $P = FV/(1 + YTM/m)^{mN}$, where m is the number of periods per year.

Example (annual vs. semiannual). A 6-year, 4.5% bond is valued at a YTM of 5.5% per 100 of par.

- Annual pay: $N = 6$, $I/Y = 5.5$, $PMT = 4.5$, $FV = 100$, $CPT\ PV = -95.00$, so the price is 95.00.
- Semiannual pay: $N = 12$, $I/Y = 2.75$, $PMT = 2.25$, $FV = 100$, $CPT\ PV = -94.95$, so the price is 94.95.

On the **TIBA II Plus**, keep $P/Y = C/Y = 1$ and END mode, enter the periodic values, and clear the worksheet first ([2ND] [CLR TVM]). The PV comes out negative: it is the cash the buyer pays, while PMT and FV are cash received. Enter the rate as a percentage figure (5.5 for 5.5%).

Example (solving for YTM). An 8-year, 3% semiannual bond trades at 94.00: $N = 16$, $PV = -94$, $PMT = 1.5$, $FV = 100$, $CPT\ I/Y = 1.94$. That is the semiannual yield, so the stated $YTM = 1.94\% \times 2 = 3.88\%$. If the PV is entered with the same sign as PMT and FV, the calculator returns an error.

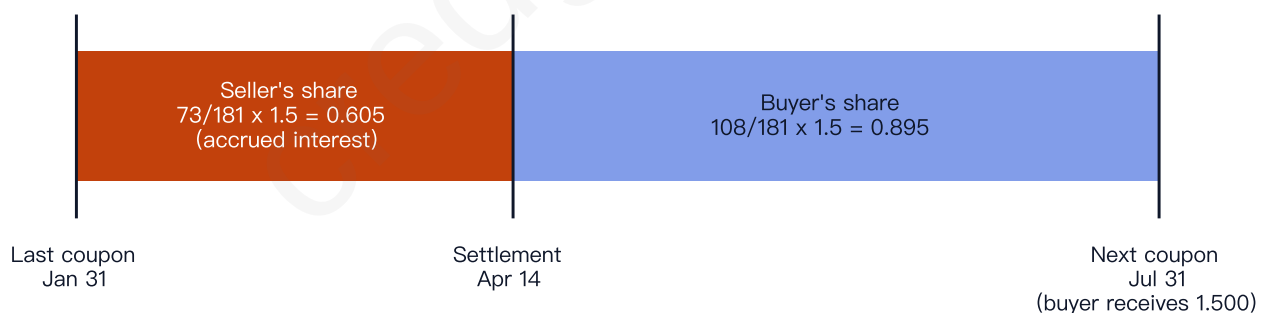
An investor actually earns the YTM only if (1) the bond is held to maturity, (2) the issuer makes every promised payment, and (3) each coupon is reinvested at that same YTM.

Accrued interest, flat price and full price

Trades usually settle between coupon dates. The buyer receives the entire next coupon, but part of it was earned by the seller. That part is the **accrued interest**:

$$AI = \frac{t}{T} \times PMT$$

Here t is the number of days from the last coupon date to the **settlement date**, and T is the number of days in the coupon period. The figure uses the example below.



Splitting a coupon between seller and buyer when a trade settles between coupon dates

Day-count convention	How days are counted	Typical use
Actual/actual	actual calendar days in the accrual period and in the coupon period	government bonds

Day-count convention	How days are counted	Typical use
30/360	every month has 30 days, the year has 360 days	corporate bonds

Example. An annual 3.6% bond paid its coupon on October 12, 2025, and a trade settles on January 3, 2026. Under 30/360 the accrued days are $18 + 30 + 30 + 3 = 81$, so $AI = 3.6 \times 81/360 = 0.810$ per 100. Under actual/actual they are $19 + 30 + 31 + 3 = 83$, so $AI = 3.6 \times 83/365 = 0.819$.

- The **flat price** (also called the **clean price** or **quoted price**) excludes accrued interest. Quoting flat prices stops the price from rising every day and dropping on each payment date.
- The **full price** (also called the **invoice price** or **dirty price**) is what the buyer actually pays: full price = flat price + accrued interest, so flat price = full price – accrued interest.

KEY CONCEPT

To value a bond between coupon dates, compute the full price first and then subtract AI:

- Price the bond on the last coupon date (PV_0), using the remaining coupons.
- Compound that value forward at the periodic YTM r for the fraction of the period elapsed: $PV^{full} = PV_0 \times (1 + r)^{t/T}$.
- Flat price = $PV^{full} - AI$.

Example. A 3% semiannual government bond has coupons on January 31 and July 31 and a YTM of 3.4%. A trade settles on April 14, 2026, with 8 coupons remaining. Actual days are $t = 73$ and $T = 181$. $PV_0: N = 8, I/Y = 1.7, PMT = 1.5, FV = 100, CPT PV = -98.516$. Full price = $98.516 \times 1.017^{73/181} = 99.188$. $AI = 1.5 \times 73/181 = 0.605$. Flat price = $99.188 - 0.605 = 98.583$. The flat price differs from PV_0 .

WORKED EXAMPLE: FULL AND FLAT PRICE OF A CORPORATE BOND

A corporate bond pays a 5.4% coupon semiannually on March 15 and September 15, uses the 30/360 day count and matures on September 15, 2030. A trade settles on June 27, 2026, at a YTM of 4.6%. Compute the accrued interest, the full price and the flat price per 100 of par.

Step 1. The last coupon was paid on March 15, 2026, so 9 semiannual coupons of 2.7 remain. Under 30/360 the accrued days are $30 + 30 + 30 + 12 = 102$ out of 180.

Step 2. Value on the last coupon date, at a periodic yield of $4.6\%/2 = 2.3\%$: $N = 9; I/Y = 2.3; PMT = 2.7; FV = 100; CPT PV = -103.22$.

Step 3. Full price = $103.22 \times 1.023^{102/180} = 104.56$.

Step 4. Accrued interest = $2.7 \times 102/180 = 1.53$.

Step 5. Flat price = $104.56 - 1.53 = 103.03$.

Result. The buyer pays the full price of 104.56 per 100 of par, and the bond is quoted at a flat price of 103.03. Both are above par, as expected for a coupon (5.4%) above the YTM (4.6%).

Common exam traps (56.a)

- Using annual N , I/Y and PMT for a semiannual or quarterly bond: all three must be converted to per-period values. For the 6-year, 4.5% semiannual bond at 5.5% in the example, annual inputs give 95.00 instead of 94.95.
- Counting years from issuance instead of the remaining maturity. When a question asks for a future price, price the bond with the periods left at that future date; the original purchase price is irrelevant.
- Treating the quoted price as the amount paid. The seller receives the full price, which is the quote plus accrued interest.
- Adding accrued interest to the value on the last coupon date, or discounting accrued interest along with the cash flows. Neither is correct. In the April 14 example, adding AI to PV_0 gives a full price of 99.121 instead of 99.188.
- Valuing a **deferred coupon bond** as if its coupons started in Year 1. Discount each coupon from the year in which it is actually paid; the deferral pushes the price well below that of an otherwise identical bond whose coupons start at once.

LOS 56.b — How price, coupon, maturity and yield are related

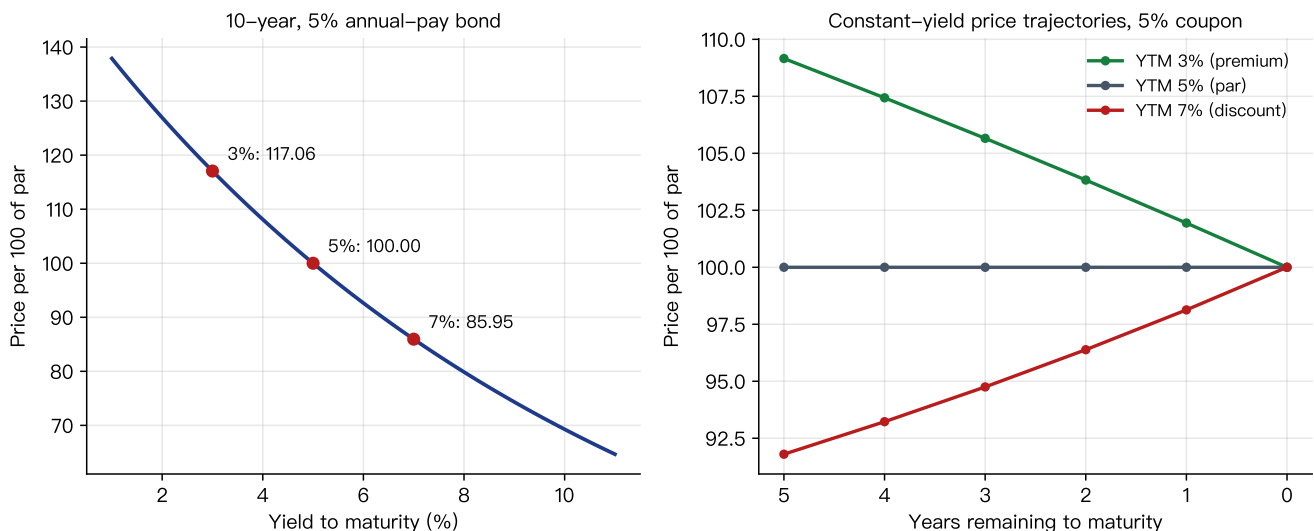
KEY CONCEPT

Coupon rate vs. YTM	Price vs. par	Label
coupon rate > YTM	price > par	premium bond
coupon rate = YTM	price = par	par bond
coupon rate < YTM	price < par	discount bond

A coupon below the YTM is sometimes called a **deficient coupon**, and a coupon above the YTM an **excessive coupon**.

The key relationships for **option-free bonds**, other things equal:

1. *Inverse relationship.* A higher YTM means a lower price, and a lower YTM means a higher price.
2. *Coupon effect.* A lower-coupon bond's price is more sensitive to a given yield change than a higher-coupon bond's price.
3. *Maturity effect.* A longer-maturity bond's price is more sensitive to a given yield change than a shorter-maturity bond's price.
4. **Convexity.** The price-yield curve is convex. For equal yield changes, the price rise when yields fall is larger than the price drop when yields rise. As yield increases, price decreases at a decreasing rate.



Price-yield convexity and constant-yield price trajectories

In the left panel, the 10-year, 5% annual bond is worth 117.06 at a 3% YTM, 100.00 at 5% and 85.95 at 7%. A 2-point fall in yield adds 17.06, while a 2-point rise takes away only 14.05.

Pull to par: whatever its yield, a bond's price converges to par as maturity approaches. If the YTM stays constant, the path the price follows is the **constant-yield price trajectory**:

- A **premium bond**'s price falls toward par over time. In the right panel (YTM 3%) it goes from 109.16 to 107.43 and on down to 100.
- A **discount bond**'s price rises toward par. At a YTM of 7% it goes from 91.80 to 93.23 and on up to 100.
- A **par bond** stays at par while its YTM equals its coupon rate.
- At a positive yield a zero-coupon bond is a discount bond, so with an unchanged yield its price rises steadily toward par.

The same logic runs in reverse. If a discount bond's price is unchanged after some time has passed, its YTM must have risen. The constant-yield price would have moved up, so an unchanged price implies a higher yield. For a premium bond with an unchanged price, the YTM must have fallen.

Common exam traps (56.b)

- Mixing up the sensitivity directions. A low coupon and a long maturity give higher price sensitivity.
- Thinking prices are more sensitive to yield rises than to yield falls. Convexity makes the opposite true.
- Assuming a bond keeps its issue-date premium or discount status. What matters is today's YTM compared with the coupon rate.
- Forgetting that time alone changes the price of a non-par bond even if the yield is unchanged.

LOS 56.c — Matrix pricing

Matrix pricing estimates the YTM (and therefore the price) of a bond that is not traded or trades infrequently, i.e., a bond with low liquidity. It uses the observed YTM's of actively traded bonds with the same or very similar credit quality and similar coupon and maturity. When no comparable bond has exactly the same maturity, the analyst uses **linear interpolation**:

$$YTM_{target} = YTM_{short} + \frac{T_{target} - T_{short}}{T_{long} - T_{short}} (YTM_{long} - YTM_{short})$$

The procedure has four steps:

1. Select the comparable traded bonds and record their YTM.
2. If several comparable bonds share a maturity, average their yields.
3. Interpolate between the two maturities that bracket the target bond's maturity.
4. Price the target bond by discounting its cash flows at the estimated YTM.

Example. A thinly traded 4-year, 5% annual bond is to be priced. Comparable bonds with the same rating yield 3.9% (a 3-year bond) and 5.0% and 5.2% (two 7-year bonds). The 7-year average is 5.1%, the interpolated yield is $3.9\% + \frac{1}{4}(5.1\% - 3.9\%) = 4.2\%$, and the price is $N = 4$, $I/Y = 4.2$, $PMT = 5$, $FV = 100$, CPT $PV = -102.89$.

A spread-based variant is used to set the yield on a new bond:

1. Find an existing bond of similar credit quality from the same issuer or sector.
2. Measure its spread over a benchmark government yield of the same maturity, interpolating the benchmark if needed.
3. Add that spread to the benchmark yield at the new bond's maturity.

Example. Two-year and 4-year government bonds yield 1.60% and 2.20%, and an existing 3-year AA corporate bond yields 3.05%. The interpolated 3-year government yield is 1.90%, so the spread is 1.15%. A new 4-year AA issue should yield about $2.20\% + 1.15\% = 3.35\%$.

Common exam traps (56.c)

- Basing a matrix price on the issuer's cost of capital or on a default-probability model instead of the yields of comparable traded bonds.
- Interpolating from the wrong end, or simply averaging two yields when the target maturity is not the midpoint. In the 4-year example, averaging 3.9% and 5.1% gives 4.5% instead of 4.2%.
- In the spread method, measuring the spread against a benchmark of the wrong maturity. Measuring the 3-year AA bond's spread against the 4-year government yield gives 0.85% and a new-issue yield of 3.05%, instead of 1.15% and 3.35%.

EXAM SHORTCUTS

- Comparing the coupon rate with the YTM shows whether the price is above, at or below par before any calculation, so answer choices on the wrong side of par can be ruled out at once.
- If a discount bond's price is unchanged after time has passed, its YTM must have risen, and if a premium bond's price is unchanged, its YTM must have fallen; no repricing is needed to answer the direction question.

BOTTOM LINE

- A bond's price is the present value of its coupons and principal discounted at the YTM; a semiannual bond uses the coupon $C/2$, the periodic rate $YTM/2$ and $2N$ periods.
- A quoted YTM is a stated annual rate, the periodic yield times the number of periods per year, and it is an effective annual rate only for an annual-pay bond.
- An investor earns the YTM only if the bond is held to maturity, the issuer makes every promised payment and each coupon is reinvested at that same YTM.
- Between coupon dates, the full price is the value on the last coupon date compounded at the periodic YTM, $PV^{full} = PV_0 \times (1 + r)^{t/T}$; accrued interest is $t/T \times PMT$, and subtracting accrued interest from the full price gives the flat (quoted) price.
- Accrued interest is usually counted on an actual/actual basis for government bonds and on a 30/360 basis for corporate bonds.
- A bond whose coupon rate is above its YTM trades at a premium, one whose coupon equals its YTM trades at par, and one whose coupon is below its YTM trades at a discount.
- For option-free bonds, other things equal, a lower coupon and a longer maturity make the price more sensitive to a yield change, and convexity makes the price rise from a yield fall larger than the price drop from an equal yield rise.
- Matrix pricing estimates the YTM of a bond that is not traded or trades infrequently from the yields of traded bonds with very similar credit quality, coupon and maturity, interpolating linearly between the maturities that bracket the target.

QUICK CHECK**Question 1**

Five years ago, Kessler Freight Lines sold 25-year bonds with a 5.5% annual coupon, paid semiannually, to finance new rail terminals. The bonds now yield 5.2%. The price of the bonds per 100 of par is *closest* to:

- A 103.70.
- B 102.32.
- C 96.39.

Answer: A

Only the remaining 20 years (40 semiannual periods) of cash flows matter. Because the 5.5% coupon exceeds the 5.2% yield, the bond trades at a premium.

Remaining maturity = $25 - 5 = 20$ years, so $N = 40$; $PMT = 5.5/2 = 2.75$; periodic yield = $5.2\%/2 = 2.6\%$.

$$P = \sum_{t=1}^{40} \frac{2.75}{1.026^t} + \frac{100}{1.026^{40}} = 103.70$$

BA II Plus (P/Y = C/Y = 1, END mode; clear with [2ND] [CLR TVM]): N = 40; I/Y = 2.6; PMT = 2.75; FV = 100; CPT PV = -103.70.

Why the other options are wrong

- **B.** 102.32 uses $N = 20$, the number of remaining years, as the number of semiannual periods. It should be 40.
- **C.** 96.39 swaps the coupon and the yield (a 5.2% coupon discounted at 2.75% per half-year). This produces a discount price for what is actually a premium bond.

Key takeaway Count the remaining maturity from today and convert it to periods. The issue date does not matter.

Practice questions

Question 2

With respect to a bond traded between coupon dates, accrued interest:

- A** is the interest earned from the last coupon date up to the settlement date.
- B** is the part of the next coupon that the seller has not earned.
- C** is discounted together with the other cash flows to obtain the full price.

Question 3

Holding all other features constant, which of the following statements about option-free bonds is *most accurate*?

- A** A long-maturity bond's price is more sensitive to yield changes than a short-maturity bond's price.
- B** A bond's price is more sensitive to increases in yield than to decreases in yield of the same size.
- C** A high-coupon bond's price is more sensitive to yield changes than a low-coupon bond's price.

Question 4

Marguerite Olsen wants to estimate the yield to maturity of a non-traded 7-year, annual-pay bond rated A. Actively traded A-rated bonds yield 4.10% at a 5-year maturity and 5.60% at a 10-year maturity. Using matrix pricing, the estimated YTM for the non-traded bond is *closest* to:

- A** 5.00%.
- B** 4.85%.
- C** 4.70%.

Answers: 2. A · 3. A · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/yield-to-maturity/. The full bank has 53 questions on this reading.

FIXED INCOME · READING 57

Credit Spread

What you'll learn

LOS 57.a Calculate a bond's annual yield for different compounding periods (periodicity, street vs. true yield, current, simple, YTM, yield to call and yield to worst, option-adjusted yield).

LOS 57.b Compare, calculate and interpret yield spread measures: G-spread, I-spread, Z-spread and option-adjusted spread.

MODULE 57.1

Yield and Yield Spread Measures for Fixed-Rate Bonds

This reading covers the yield measures quoted for fixed-rate bonds and the spreads that compare a bond's yield with a benchmark. A candidate must be able to restate a yield for a different periodicity, compute current yield, simple yield, yield to call and yield to worst, and calculate or interpret G-spreads, I-spreads, Z-spreads and option-adjusted spreads.

LOS 57.a — Annual yields for different compounding periods

Yield to maturity and the semiannual bond basis

The **yield to maturity (YTM)** is the single discount rate that makes the present value of all of a bond's promised cash flows equal to its market price. It is an internal rate of return, so on the calculator it is found by solving for I/Y given the price, the coupon, the number of periods and the redemption value.

For a bond that pays coupons more than once a year, the calculator gives a periodic rate. By convention the quoted YTM is the periodic rate multiplied by the number of periods per year. For a semiannual-pay bond, the quoted YTM is $2 \times$ the semiannual rate. Such a yield is quoted on a **semiannual bond basis**.

Calculator (TI BA II Plus), semiannual bond. Set P/Y = C/Y = 1 and END mode, then N = years $\times 2$, PMT = annual coupon / 2, FV = par, PV = -price (cash paid out is negative), CPT I/Y, and multiply the result by 2.

Periodicity and the effective annual yield

The number of coupon payments per year is the bond's **periodicity**. For a given stated yield, more compounding periods per year produce a higher **effective annual yield (EAY)**:

KEY CONCEPT

$$EAY = \left(1 + \frac{\text{stated annual yield}}{n} \right)^n - 1$$

where n is the periodicity. The same relationship links any **stated annual rate** (also called the nominal rate or APR) with its **effective annual rate (EAR)**: the stated rate divided by n is the **periodic rate**, and compounding the periodic rate n times gives the EAR. The stated rate does not adjust for compounding; the EAR does.

To restate a yield for a different periodicity, keep the EAY fixed:

$$\text{stated yield}_m = m \left[(1 + EAY)^{1/m} - 1 \right]$$

Example. A bond yields 6% on a semiannual bond basis, which is 3% per half-year.

Basis	Calculation	Yield
Annual-pay equivalent (EAY)	$1.03^2 - 1$	6.09%
Semiannual bond basis	given	6.00%
Quarterly basis	$4 (1.03^{0.5} - 1)$	5.96%

All three describe the same return. Holding the EAY constant, higher periodicity means a lower stated yield. Holding the stated yield constant, higher periodicity means a higher EAY. An annual-pay bond's YTM already is an effective annual yield.

Street convention and true yield

A yield computed with the scheduled coupon dates follows the **street convention**. When a scheduled date falls on a weekend or holiday the cash actually arrives on the next business day; a yield computed with these actual payment dates is the **true yield**. Because cash arrives slightly later, the true yield is usually slightly lower than the street convention yield, by a few basis points, and equal to it when no payment date moves.

Current yield and simple yield

Current yield (also **income yield** or **running yield**) considers only coupon income:

$$\text{current yield} = \frac{\text{annual cash coupon}}{\text{flat price}}$$

It ignores capital gains or losses and reinvestment income, and it is the same for annual-pay and semiannual-pay bonds with the same coupon rate and price. *Example.* A 5% bond priced at 96 has a current yield of $5/96 = 5.21\%$.

Simple yield adds (subtracts) the straight-line amortization of a discount (premium) to the coupon:

$$\text{simple yield} = \frac{\text{annual coupon} \pm \text{straight-line amortization}}{\text{flat price}}$$

Example. A 3-year 5% bond at 97 amortizes $3/3 = 1$ per year, so the simple yield is $(5 + 1)/97 = 6.19\%$.

Ranking for option-free bonds:

KEY CONCEPT

Bond priced at	Ranking
Discount (price < par)	coupon rate < current yield < YTM
Par	coupon rate = current yield = YTM

Bond priced at**Ranking**

Premium (price > par) coupon rate > current yield > YTM

Yield to call and yield to worst

For a **callable bond**, the **yield to call** is found like a YTM but with N = periods to the call date and FV = the **call price** instead of par. A yield to call can be computed for each call date. The lowest of the YTM and all the yields to call is the **yield to worst**, the conservative yield to rely on. For a premium bond callable at or near par, a call cuts short the period over which the premium is amortized, so a yield to call is usually the lowest.

Example. A 4-year, 5.5% semiannual bond at 103 is callable at 101 in 2 years. YTM: $N = 8$, $PMT = 2.75$, $FV = 100$, $PV = -103$, $CPT\ I/Y = 2.335$, so the YTM is 4.67%. Yield to call: $N = 4$, $FV = 101$, same PMT and PV , $CPT\ I/Y = 2.200$, so the yield to call is 4.40%. The yield to worst is 4.40%.

Option-adjusted price and option-adjusted yield

A callable bond equals a straight bond minus the call option the investor has sold to the issuer:

$$\text{callable bond value} = \text{straight bond value} - \text{call option value}$$

Adding the call value back gives the **option-adjusted price** (the value of the equivalent option-free bond); the yield computed from it is the **option-adjusted yield**. For a callable bond the option-adjusted yield is lower than the bond's YTM, because investors demand extra yield for bearing the call. "Option-adjusted" means the option has been removed. Because the option's effect is stripped out, option-adjusted yields put bonds with different embedded options, and comparable option-free bonds, on a like-for-like footing.

COMMON EXAM TRAPS

- Forgetting to double the semiannual I/Y , or doubling it for an annual-pay bond. In the callable-bond example, the undoubled figure is 2.335% instead of a YTM of 4.67%.
- Using par instead of the call price as FV in a yield to call. In the callable-bond example, $FV = 100$ gives a yield to call of 3.93% instead of 4.40%.
- Using years instead of periods for N , or the time to maturity instead of the time to the call date. In the callable-bond example, $N = 2$ for the call date gives a yield to call of 3.41% instead of 4.40%.
- Converting in the wrong direction: the annual-pay equivalent of a semiannual-basis yield is higher. For the 6% semiannual-basis yield in the example, converting the wrong way gives 5.91% instead of 6.09%.
- Expecting a negative yield to move toward zero at a higher periodicity. Holding the EAY constant, the stated yield still falls, so a negative yield becomes more negative.
- Treating the current yield as a total return: it ignores the pull to par.

LOS 57.b — Yield spreads for fixed-rate bonds**Benchmark spreads: G-spread and I-spread**

A **yield spread** (or **benchmark spread**) is the bond's yield minus a benchmark yield, usually in basis points. When the benchmark is a government bond, typically the **on-the-run** issue of the same or nearest maturity, the spread is a **G-spread**. If no government bond has exactly the same maturity, the benchmark yield is

interpolated.

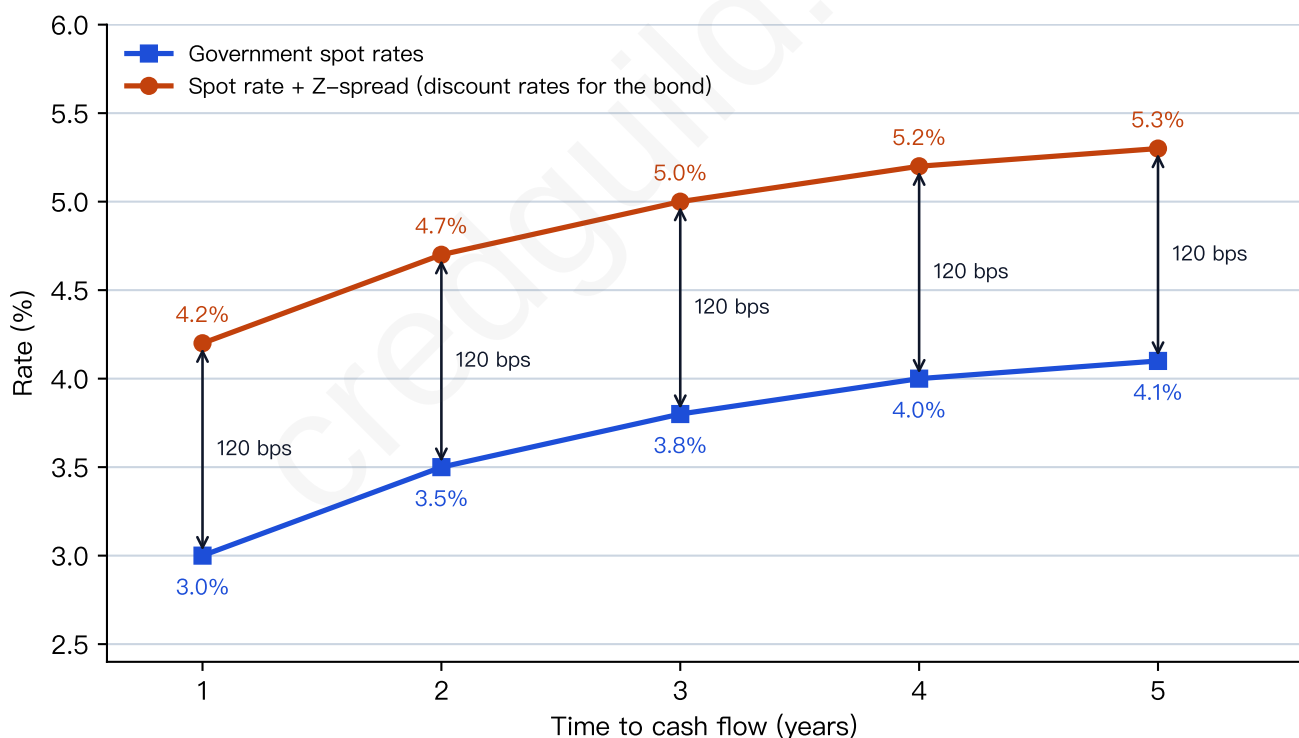
Example. Two-year and 5-year government yields are 3.2% and 3.8%. A 4-year corporate bond yields 5.1%. Interpolated 4-year yield = $3.2\% + (3.8\% - 3.2\%) \times 2/3 = 3.6\%$, so the G-spread is 150 bp.

A spread measured against **swap rates** of the same currency and tenor is an **interpolated spread (I-spread)**. It measures the bond's extra return over the interbank **market reference rates (MRRs)** used in swaps. I-spreads are common for euro-denominated bonds.

If a bond's yield rises and its spread is unchanged, the benchmark yield rose too. **Macroeconomic** factors moved yields in general; the benchmark yield is made up of the real risk-free rate plus expected inflation. If the spread widens, **microeconomic** factors are at work, such as higher issuer **credit risk** or poorer **liquidity**. Differences in taxation can also affect spreads.

Z-spread

G- and I-spreads compare the bond's YTM with a single benchmark yield, so they ignore the shape of the curve. Individual cash flows are discounted at different **spot rates**, and a YTM is only a kind of weighted average of them. The **zero-volatility spread (Z-spread)** is the constant spread that, added to each benchmark spot rate, discounts the bond's cash flows to a present value equal to its market price. The figure shows the idea: one spread is added to every point of the spot curve.



The Z-spread: one constant spread added to every government spot rate

It is found by trial and error:

$$\text{price} = \sum_{t=1}^N \frac{CF_t}{(1 + S_t + Z)^t}$$

The search runs in four steps:

1. Guess a spread Z .
2. Add it to every spot rate and discount each cash flow at its own $S_t + Z$.
3. Compare the total with the market price. A total above the price means the guess is too small; a total below the price means it is too large.
4. Adjust Z and repeat until the present value equals the price.

Example. Spot rates are 3.0% (1 year) and 3.5% (2 years), and a 2-year 5% annual-pay bond trades at 100.58. A first guess of 100 bp gives $5/1.04 + 105/1.045^2 = 100.96$, which is above the price, so the spread must be larger. At 120 bp the value is $5/1.042 + 105/1.047^2 = 100.58$, equal to the price, so the Z-spread is 120 bp.

When the spot curve is flat, every spot rate equals the benchmark YTM, so the G-spread and the Z-spread are the same. YTM-based spreads are theoretically correct only for a flat spot curve.

Option-adjusted spread (OAS)

The **option-adjusted spread (OAS)** is the spread over the benchmark spot curve that remains once the effect of the embedded option is removed. It equals the Z-spread minus the **option value** (also called the option cost), the part of the spread that reflects the embedded option:

$$\text{OAS} = \text{Z-spread} - \text{option value (bp)} \quad \text{option value} = \text{Z-spread} - \text{OAS}$$

For a **callable** bond the option works against the investor, so the investor demands extra spread and the OAS is below the Z-spread. The difference, the option value, is the extra yield that compensates bondholders for the call. *Example.* A Z-spread of 145 bp and an option value of 40 bp give an OAS of 105 bp. The Z-spread compensates for credit, liquidity, taxation and optionality risk, while the OAS compensates for credit, liquidity and taxation risk only. For an option-free bond, $\text{OAS} = \text{Z-spread}$. An option that benefits the bondholder (a conversion or put feature) makes the bond trade at a lower spread than an otherwise identical straight bond.

KEY CONCEPT

Measure	Benchmark	Accounts for curve shape?	Removes option?
G-spread	Government bond YTM (interpolated if needed)	No	No
I-spread	Swap rate of same tenor	No	No
Z-spread	Each government spot rate	Yes	No
OAS	Each government spot rate	Yes	Yes

COMMON EXAM TRAPS

- Measuring the option value as the G-spread minus the OAS. The G-spread stands in for the Z-spread only when the curve is flat.

- Expecting a callable bond's OAS to exceed its Z-spread. That ordering fits a bond whose option helps the investor, such as a puttable bond.
- Comparing a bond with a benchmark of a different maturity. Interpolate instead.
- Subtracting two yields quoted at different periodicities. Restate them on the same periodicity first; for example, a quarterly-basis yield must be converted to a semiannual basis before it is compared with a semiannual-basis yield.
- Blaming the issuer for a higher yield when the spread is unchanged. The benchmark moved, so macro factors caused the rise.

EXAM SHORTCUTS

- For an option-free bond, the price relative to par fixes the order of coupon rate, current yield and YTM, so a ranking question needs no yield calculation.
- A yield to call reuses the YTM inputs; only N, set to the periods to the call date, and FV, set to the call price, change.
- When the spot curve is flat, the Z-spread equals the G-spread, so no trial-and-error search is needed.

BOTTOM LINE

- For a bond that pays more than once a year, the quoted YTM is the periodic yield times the number of periods per year; for a semiannual-pay bond this is the semiannual bond basis.
- $EAY = (1 + \text{stated annual yield}/n)^n - 1$; holding the EAY constant, a higher periodicity means a lower stated yield, and holding the stated yield constant, a higher periodicity means a higher EAY.
- Current yield is the annual cash coupon divided by the flat price and ignores capital gains, losses and reinvestment income; simple yield adds the straight-line amortization of a discount, or subtracts that of a premium, before dividing by the flat price.
- For an option-free bond priced at a discount, coupon rate < current yield < YTM; at a premium the order reverses, and at par all three are equal.
- A yield to call uses the periods to the call date and the call price in place of maturity and par, and the yield to worst is the lowest of the YTM and all the yields to call.
- A G-spread is the bond's yield minus a government benchmark yield of the same maturity, interpolated if needed, while an I-spread is measured against the swap rate of the same currency and tenor.
- The Z-spread is the constant spread that, added to every benchmark spot rate, discounts the bond's cash flows to its market price, and it equals the G-spread when the spot curve is flat.
- $OAS = Z\text{-spread} - \text{option value}$; for a callable bond the OAS is below the Z-spread, and for an option-free bond the two are equal.

QUICK CHECK

Question 1

An analyst discounts every cash flow of a bond at one uniform rate, choosing the rate so that the total present value equals the bond's market price. This rate is the bond's:

- A** Yield to maturity.

- B Current yield.
- C Simple yield.

Answer: A

The yield to maturity is the single discount rate that sets the present value of all of a bond's cash flows equal to its price. It is the bond's internal rate of return.

Why the other options are wrong

- B. Current yield is the annual coupon divided by the price; it does not discount any cash flows.
- C. Simple yield adds straight-line amortization of a discount (or subtracts that of a premium) to the coupon and divides by the price; it is not a discount rate.

Key takeaway YTM = IRR of the bond's cash flows at the market price.

Practice questions

Question 2

Delmont Paper's \$1,000 par value bonds carry a 9% coupon rate and are quoted at a price of \$962. The bonds' current yield is *closest* to:

- A 8.66%.
- B 9.00%.
- C 9.36%.

Question 3

An analyst calculates a bond's yield to maturity using its scheduled coupon dates. Two of those dates fall on a Sunday, so the issuer will pay on the following Monday. The analyst then recalculates the yield using the dates on which the cash will actually be received. The recalculated figure is best described as:

- A the true yield, slightly higher than the first figure.
- B the street convention yield, slightly lower than the first figure.
- C the true yield, slightly lower than the first figure.

Question 4

Which description of a callable bond's yield to worst is *most accurate*? It is the:

- A yield the investor earns if the issuer defaults on its payments.
- B lowest of the bond's yield to maturity and its yields to each call date.
- C yield implied by the lowest price at which the bond has traded.

Question 5

A callable corporate bond has a zero-volatility spread of 160 basis points, and its embedded call option has a positive value. The bond's option-adjusted spread (OAS) is *most likely*:

- A greater than 160 basis points.
- B equal to 160 basis points.
- C less than 160 basis points.

Answers: 2. C · 3. C · 4. B · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-spread/. The full bank has 61 questions on this reading.

credguild.com

FIXED INCOME · READING 58

Yield and Yield Spread Measures for Floating-Rate Instruments

What you'll learn

LOS 58.a Calculate and interpret the quoted margin and discount (required) margin of a floating-rate note and estimate its value on a reset date.

LOS 58.b Calculate and interpret money market yields on discount and add-on bases, 360- and 365-day years, and the bond equivalent yield.

MODULE 58.1

Yield and Yield Spread Measures for Floating-Rate Instruments

This reading covers the spread measures used for floating-rate notes and the yield conventions used for money market instruments. A candidate must be able to value an FRN on a reset date from its quoted and discount margins, solve for the discount margin from a price, and convert money market quotes among discount yields, add-on yields, the bond equivalent yield and a semiannual bond basis.

LOS 58.a — Quoted margin, discount margin and FRN value

How a floating-rate note pays

A **floating-rate note (FRN)** pays a coupon that is reset each period to the current **market reference rate (MRR)** plus a fixed spread. The MRR (typically an interbank or overnight-based rate) is set at the start of the period and paid at the end, so interest is **paid in arrears**. Because the coupon keeps catching up with market rates, an FRN's price is more stable than the price of a fixed-rate bond with the same maturity.

- The fixed spread written into the indenture is the **quoted margin (QM)**. It reflects the issuer's credit risk *at issuance* compared with that of the banks behind the MRR (liquidity and tax treatment can also affect it).
- An issuer riskier than those banks pays MRR plus a margin; an issuer with less credit risk than those banks pays MRR minus a margin, so its coupon rate is below the MRR.
- The spread investors *currently* require over the MRR is the **required margin**, also called the **discount margin (DM)**. It is the margin that would make the FRN price equal to par on a reset date.

QM versus DM: the price rule

KEY CONCEPT

Situation since issuance	Relation	Price on a reset date
Credit quality unchanged	$QM = DM$	At par
Credit quality worsened	$DM > QM$	Below par (discount)
Credit quality improved	$DM < QM$	Above par (premium)

FRNs are normally issued at par, with $QM = DM$. As long as the issuer's credit quality is unchanged, QM stays equal to DM throughout the note's life. The passage of time does not open a gap between them. A change in the MRR itself also does not create a gap, because both the coupon ($MRR + QM$) and the required return ($MRR + DM$) move with it. The analogy with fixed-rate bonds: coupon rate < required yield means a discount price; for an FRN, $MRR + QM < MRR + DM$ means the QM is **deficient** and the note trades below par.

Simplified valuation on a reset date

Project every future coupon using today's MRR plus the QM , and discount at today's MRR plus the DM , with the rates divided by the number of periods per year:

$$V = \sum_{t=1}^N \frac{(MRR + QM) \times FV/m}{\left(1 + \frac{MRR+DM}{m}\right)^t} + \frac{FV}{\left(1 + \frac{MRR+DM}{m}\right)^N}$$

Example. A \$1 million FRN pays annual coupons of $MRR + 60$ bp and has three years left. On a reset date the MRR is 2.80% and investors now require a DM of 90 bp.

Coupon = $(2.80\% + 0.60\%) \times 1,000,000 = \$34,000$; discount rate = $2.80\% + 0.90\% = 3.70\%$.

TI BA II Plus ($P/Y = C/Y = 1$, END mode): $N = 3$, $I/Y = 3.70$, $PMT = 34,000$, $FV = 1,000,000$, CPT $PV = -991,627$. The value is about \$991,627, below par because $DM > QM$. This shortcut ignores expected changes in the MRR; more complex models give better estimates of value.

Solving for the DM. When the price is known, the same formula is run backward:

1. Coupon per period = $(MRR + QM) \times FV/m$.
2. Solve for the periodic discount rate (I/Y) with N , PMT , FV and $PV = -\text{price}$.
3. Multiply the periodic rate by m to annualize it.
4. Subtract the MRR. What remains is the DM .

Example. Suppose the same note were priced at \$988,000. With $m = 1$: $N = 3$, $PMT = 34,000$, $FV = 1,000,000$, $PV = -988,000$, CPT $I/Y = 3.831$, so $DM = 3.831\% - 2.80\% = 1.03\%$, about 103 bp. The DM is above the 60 bp QM , as it must be for a note priced below par on a reset date.

Common exam traps

- Swapping the roles. The coupon uses the QM ; the discount rate uses the DM . In the \$1 million example, swapping them values the note at about \$1,008,421 instead of \$991,627.
- Subtracting the MRR from a quarterly periodic rate. Multiply the periodic rate by 4 first, then subtract the MRR.
- Assuming an FRN always trades at par on a reset date. This holds only when $QM = DM$.
- Expecting the approach of maturity or a move in the MRR to open a gap between the QM and the DM .
- Treating the MRR as a floor on the coupon rate. A lower-risk issuer pays less than the MRR.

LOS 58.b — Money market yield conventions

Money market securities mature in one year or less. Their yields are quoted under two sets of choices:

Choice	Option 1	Option 2
What is annualized	Add-on yield: interest earned ÷ amount invested today	Discount yield: discount from face value ÷ face value
Day-count year	360 days	365 days

KEY CONCEPT

- Annualized add-on rates are the quoting convention for bank CDs, repos and market reference rates.
- Annualized discount yields on a 360-day year are the convention for US Treasury bills and commercial paper.
- The **bond equivalent yield (BEY)** of a money market instrument is an **add-on yield based on a 365-day year**.

Key formulas: money market yields

Holding period yield: $HPY = \frac{\text{amount received at maturity}}{\text{price paid}} - 1$

KEY CONCEPT

$$\text{Add-on yield (365-day)} = HPY \times \frac{365}{\text{days}} \quad \text{Discount yield} = \frac{FV - P}{FV} \times \frac{360}{\text{days}}$$

The 360 in the discount-yield formula becomes 365 when an instrument is quoted as a discount yield on a 365-day year.

Rearranged for pricing, the two conventions run in opposite directions. A discount yield starts from the face value and works back to today's price. An add-on yield starts from the amount invested today and works forward to the amount received at maturity:

$$P = FV \times \left(1 - \text{discount yield} \times \frac{\text{days}}{\text{year}} \right) \quad FV = PV \times \left(1 + \text{add-on yield} \times \frac{\text{days}}{\text{year}} \right)$$

Here "year" is 360 or 365 days, whichever basis the quote uses. Converting an add-on yield from a 360-day to a 365-day basis: multiply by 365/360.

Because a discount yield divides by face value (larger than the price) and uses a 360-day year, the BEY of a discount instrument priced below face value is higher than its quoted discount yield.

Example 1. A 60-day T-bill is quoted at a 3.00% discount yield.

Discount = $3.00\% \times 60/360 = 0.50\%$, so the price is 99.50 per 100 of face value.

$HPY = 100/99.50 - 1 = 0.5025\%$; $BEY = 0.5025\% \times 365/60 = 3.057\%$, above the 3.00% quote.

Example 2. A 45-day interbank deposit quoted at a 2.00% add-on yield on a 360-day basis has a BEY of $2.00\% \times 365/360 = 2.028\%$.

Comparing with a semiannual-pay bond

To compare a money market yield with the YTM of a semiannual-pay bond (a **semiannual bond basis** yield): find the HPY, compound it to an effective annual yield, convert to an effective semiannual rate and double it:

$$EAY = (1 + HPY)^{365/\text{days}} - 1, \quad \text{semiannual bond basis} = 2 \left[(1 + EAY)^{1/2} - 1 \right]$$

Example 3. A 200-day deposit quoted at 3.60% add-on (365-day basis): $HPY = 3.60\% \times 200/365 = 1.9726\%$; $EAY = 1.019726^{365/200} - 1 = 3.6293\%$; semiannual bond basis $= 2(1.036293^{0.5} - 1) = 3.597\%$. With a holding period longer than half a year (periodicity below 2), the semiannual bond basis yield is slightly *below* a simple 365-day add-on quote; with a holding period shorter than half a year it is slightly *above*.

Common exam traps

- Annualizing an HPY by $\text{days}/365$ instead of $365/\text{days}$. For the 60-day T-bill in Example 1, that gives 0.083% instead of a BEY of 3.057%.
- Using 360 instead of 365 when the question asks for a BEY. For the T-bill in Example 1, this gives 3.015% instead of 3.057%.
- Treating a discount yield as a return on the price: a discount yield is computed on face value.
- Compounding when the BEY is required. The BEY is a simple (non-compounded) annualization. For the T-bill in Example 1, compounding gives 3.096% instead of 3.057%.

EXAM SHORTCUTS

- An FRN priced below par on a reset date must have $DM > QM$, and one priced above par must have $DM < QM$, so answer choices on the wrong side of the quoted margin can be eliminated without solving for I/Y .
- An add-on yield quoted on a 360-day basis converts to a 365-day basis, the BEY, by multiplying by $365/360$; the holding period yield is not needed.

BOTTOM LINE

- An FRN's coupon rate is reset each period to the market reference rate set at the start of the period plus the quoted margin, and the coupon is paid at the end of the period, in arrears.
- The quoted margin is fixed in the indenture and reflects the issuer's credit risk at issuance, while the discount (required) margin is the spread investors currently require, the margin that would make the FRN worth par on a reset date.
- On a reset date an FRN trades at par when $QM = DM$, below par when $DM > QM$ because credit quality has worsened, and above par when $DM < QM$ because credit quality has improved.
- While the issuer's credit quality is unchanged, the QM stays equal to the DM; neither the passage of time nor a change in the MRR opens a gap between them.
- The simplified FRN valuation projects each coupon at today's MRR plus the QM and discounts at today's MRR plus the DM, both divided by the periods per year, and it ignores expected changes in the MRR.

- US Treasury bills and commercial paper are quoted as discount yields on a 360-day year, computed on face value, while the convention for bank CDs, repos and market reference rates is an annualized add-on rate.
- A money market instrument's BEY annualizes the add-on return over a 365-day year, $HPY \times 365/\text{days}$, and for a discount instrument priced below face value it is higher than the quoted discount yield.
- To compare a money market yield with a semiannual-pay bond's YTM, compound the HPY to an effective annual yield, $(1 + HPY)^{365/\text{days}} - 1$, and restate it as $2[(1 + EAY)^{1/2} - 1]$.

QUICK CHECK

Question 1

Carrow Logistics has a \$5 million floating-rate note outstanding that pays an annual coupon equal to the market reference rate (MRR) plus a quoted margin of 110 basis points. Today is a reset date with exactly four years remaining to maturity. The MRR is 4.20%, and the discount margin the market currently requires on Carrow's credit is 70 basis points. Using the simplified approach that holds the MRR constant, the estimated value of the note is *closest* to:

- A \$5,071,084.
- B \$5,000,000.
- C \$4,929,572.

Answer: A

Coupons are projected at MRR + QM and discounted at MRR + DM. The QM (110 bp) exceeds the DM (70 bp), so the note is worth more than par.

Coupon rate = 4.20% + 1.10% = 5.30%, so the annual coupon = $0.053 \times \$5,000,000 = \$265,000$.

Discount rate = 4.20% + 0.70% = 4.90%.

$$V = \sum_{t=1}^4 \frac{265,000}{1.049^t} + \frac{5,000,000}{1.049^4} = \$5,071,084$$

TI BA II Plus (P/Y = C/Y = 1, END mode): N = 4, I/Y = 4.90, PMT = 265,000, FV = 5,000,000, CPT PV = -5,071,084.

Why the other options are wrong

- **B.** \$5,000,000 assumes the FRN must trade at par on a reset date. That holds only when the quoted margin equals the discount margin; here QM > DM.
- **C.** \$4,929,572 swaps the two margins: coupons at 4.20% + 0.70% = 4.90% discounted at 4.20% + 1.10% = 5.30%, which produces a discount price.

Key takeaway Coupons come from the QM and the discount rate from the DM. If QM > DM the note trades at a premium; if QM < DM, at a discount.

Practice questions

Question 2

Tenby Savings Bank issues a 91-day negotiable certificate of deposit. An investor who deposits \$1,000 today receives \$1,006.50 at maturity. The certificate's bond equivalent yield is *closest* to:

- A 2.57%.
- B 2.61%.
- C 2.63%.

Question 3

A floating-rate note issued by Penhale Chemicals pays the market reference rate plus 85 basis points. Dealers now price the note at a discount margin of 55 basis points. Which of the following is the *most likely* explanation for the difference?

- A The market reference rate has fallen since the note was issued.
- B The issuer's credit quality has improved since the note was issued.
- C The note is currently priced below its par value.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/yield-and-yield-spread-measures-for-floating-rate-instruments/. The full bank has 15 questions on this reading.

FIXED INCOME · READING 59

The Term Structure of Interest Rates: Spot, Par, and Forward Curves

What you'll learn

LOS 59.a Define spot rates and the spot curve, and price a bond by discounting each cash flow at its own spot rate.

LOS 59.b Define par and forward rates; compute par rates, forward rates from spot rates, spot rates from forward rates, and bond prices using forward rates.

LOS 59.c Compare the spot curve, par curve and forward curve and how they are ordered in normal, inverted and flat environments.

MODULE 59.1

The Term Structure of Interest Rates: Spot, Par, and Forward Curves

This reading prices bonds with a separate spot rate for each cash flow and links spot rates, par rates and forward rates. A candidate must be able to price a bond from spot rates, compute par rates, derive forward rates from spot rates and spot rates from forward rates, price a bond with forward rates, and describe how the spot, par and forward curves relate.

LOS 59.a — Spot rates, the spot curve and pricing a bond with spot rates

A bond's **yield to maturity (YTM)** discounts every cash flow at one rate. In reality the market applies a different rate to each date. The market discount rate for a single payment at a future date is the **spot rate** for that maturity. Because a **zero-coupon bond** makes exactly one payment, its YTM is the spot rate for its maturity. For this reason spot rates are also called **zero-coupon rates** or **zero rates**. At a positive yield, all of a zero-coupon bond's return comes from price appreciation toward par; there are no coupons to reinvest.

The **spot curve** plots spot rates against time to maturity for one type of issuer (e.g., government spot rates). It usually slopes upward, a **normal yield curve**, because investors want higher returns for committing money over longer horizons. When longer-maturity spot rates are lower than shorter ones, the curve slopes downward and is called **inverted**.

Pricing with spot rates

Discount each cash flow at the spot rate that matches its own date and add the present values:

$$P = \frac{PMT}{(1 + S_1)} + \frac{PMT}{(1 + S_2)^2} + \cdots + \frac{PMT + FV}{(1 + S_N)^N}$$

This is the **no-arbitrage price** (the **arbitrage-free valuation** approach). If a bond traded at any other price, a dealer could strip it into zero-coupon pieces (or rebuild it from zeros) and lock in a riskless profit. Absent arbitrage opportunities, the market price therefore equals this value whatever the shape of the curve. The difference between market price and no-arbitrage value is the potential arbitrage profit. A government bond is valued with the **government spot rate** for each cash flow's maturity; the same spot rate applies to a given date whichever bond the cash flow belongs to (the 6-year spot rate discounts the year-6 coupon of a 20-year bond too).

Example. Spot rates: $S_1 = 2.0\%$, $S_2 = 2.8\%$, $S_3 = 3.3\%$. A 3-year, 4% annual-pay bond:

$$P = \frac{4}{1.02} + \frac{4}{1.028^2} + \frac{104}{1.033^3} = 3.922 + 3.785 + 94.348 = 102.055$$

Its YTM ($N = 3$, $PMT = 4$, $FV = 100$, $PV = -102.055$, CPT I/Y) is 3.27%. It is a cash-flow-weighted blend of the spot rates, dominated by S_3 because most of the cash arrives at year 3. The YTM is found as the bond's internal rate of return; a simple or geometric average of spot rates does not give it.

Common exam traps

- Discounting every cash flow at the longest spot rate, or at the first one. For the 3-year, 4% bond in the example, discounting everything at 3.3% gives 101.969 and at 2.0% gives 105.768, instead of 102.055.
- Forgetting the final coupon (discounting only the principal at maturity). For the same bond, discounting only the 100 of principal at year 3 gives 98.426 instead of 102.055.
- Using annual spot rates with semiannual periods: for semiannual cash flows, use $S_t/2$ and count half-years.

LOS 59.b — Par rates, forward rates, and moving between spot and forward rates

Par rates

A **par rate** is the coupon rate that makes a bond of a given maturity price at exactly par, given the spot curve. Equivalently, it is the YTM of a hypothetical **par bond**. For maturity N with coupon c per 100:

$$100 = \frac{c}{1 + S_1} + \frac{c}{(1 + S_2)^2} + \dots + \frac{c + 100}{(1 + S_N)^N} \Rightarrow c = \frac{100 \left[1 - \frac{1}{(1 + S_N)^N} \right]}{\sum_{t=1}^N \frac{1}{(1 + S_t)^t}}$$

With the spot rates above (2.0%, 2.8%, 3.3%), the 3-year par rate is 3.27%, slightly below the 3-year spot rate. On the exam it is often quicker to plug in the middle answer choice: a price below 100 means the par rate is higher; above 100 means it is lower.

Forward rates and notation

A **forward rate** is a rate agreed today for a loan that starts at a future date. The notation **AyBy** means a B-year loan starting A years from now: **1y1y** is a 1-year rate one year from now, **2y1y** a 1-year rate two years from now, **2y2y** a 2-year rate two years from now.

No arbitrage requires that investing for N years at the N -year spot rate grows to the same amount as rolling over at the forward rates:

KEY CONCEPT

$$(1 + S_3)^3 = (1 + S_1)(1 + {}_1y1y)(1 + {}_2y1y) = (1 + S_2)^2(1 + {}_2y1y)$$

The two-year case shows the logic. An investor who buys a 1-year zero at S_1 instead of a 2-year zero at a higher S_2 accepts less in year 1. That choice makes sense only if the rate available for year 2 is high enough to catch up. The forward rate 1y1y is the year-2 rate at which both plans end with the same amount.

Spot from forwards. The spot rate is the **geometric mean** of the one-period rates:

$$S_N = [(1 + S_1)(1 + {}_1y1y) \cdots (1 + {}_{(N-1)y}1y)]^{1/N} - 1$$

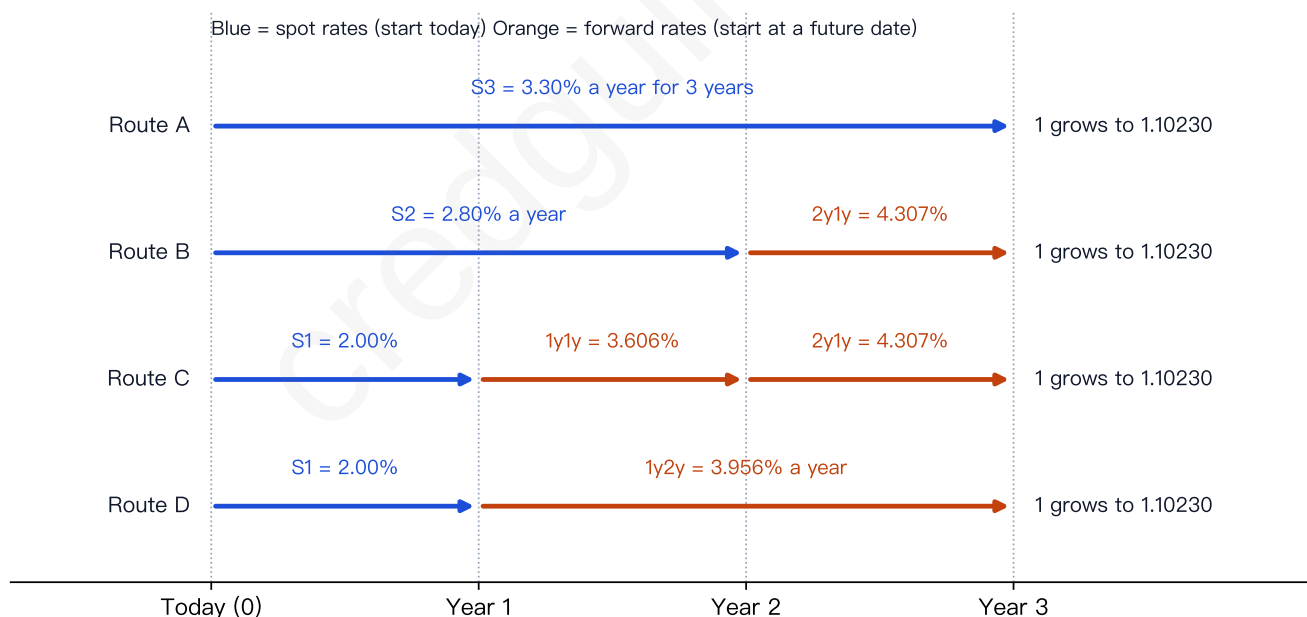
Forward from spots:

KEY CONCEPT

$${}_AyBy = \left[\frac{(1 + S_{A+B})^{A+B}}{(1 + S_A)^A} \right]^{1/B} - 1$$

Example. With $S_1 = 2.0\%$, $S_2 = 2.8\%$, $S_3 = 3.3\%$:

- ${}_1y1y = 1.028^2/1.02 - 1 = 3.606\%$
- ${}_2y1y = 1.033^3/1.028^2 - 1 = 4.307\%$
- ${}_1y2y = (1.033^3/1.02)^{1/2} - 1 = 3.956\%$. The square root annualizes the 2-year forward rate.
- Going back: $S_3 = [(1.02)(1.03606)(1.04307)]^{1/3} - 1 = 3.30\%$.



Four ways to invest from today to year 3 that must grow to the same amount

Each route in the figure turns 1 into 1.10230 by year 3, so any one rate on the timeline can be backed out from the others. A 1-year spot rate, for example, is $1 + S_1 = (1 + S_2)^2 / (1 + {}_1y1y)$.

Approximation check: forward $\approx$ difference of "total" rates, e.g. ${}_2y1y \approx 3 \times 3.3 - 2 \times 2.8 = 4.3\%$. Useful to screen answers; the exact figure uses compounding. The same shortcut works for a multi-year forward:

$${}_2y2y \approx (4S_4 - 2S_2)/2.$$

Semiannual rates: with rates stated on a **semiannual bond basis**, work with half-year rates and periods, then double the result (e.g., the 6-month rate six months forward, *6m6m*).

WORKED EXAMPLE: A 6-MONTH FORWARD RATE AND A SEMIANNUAL BOND PRICE

Spot rates on a semiannual bond basis are 2.3% for 6 months and 3.1% for 1 year. Compute the 6-month rate six months forward (6m6m) on a semiannual bond basis, and price a 1-year bond with a 4.4% coupon paid semiannually, per 100 of par.

Step 1. Half-year spot rates: $2.3\%/2 = 1.15\%$ and $3.1\%/2 = 1.55\%$.

Step 2. Half-year forward rate: $1.0155^2/1.0115 - 1 = 1.9516\%$. On a semiannual bond basis, $6m6m = 2 \times 1.9516\% = 3.903\%$.

Step 3. The bond pays 2.2 after 6 months and 102.2 after 1 year. Price = $2.2/1.0115 + 102.2/1.0155^2 = 2.175 + 99.104 = 101.279$.

Result. The 6m6m forward rate is 3.903%, and the bond is worth 101.279 per 100 of par.

Valuing a bond with forward rates

Each cash flow is discounted by the product of one plus each one-period rate up to its date:

$$P = \frac{PMT}{1 + S_1} + \frac{PMT}{(1 + S_1)(1 + {}_{1y1y})} + \frac{PMT + FV}{(1 + S_1)(1 + {}_{1y1y})(1 + {}_{2y1y})}$$

These products are the spot-rate discount factors, so the price is the same no-arbitrage price. Example: with rates 2%, 3% and 3.5% for years 1–3, a 3-year 4% bond is worth $4/1.02 + 4/(1.02 \times 1.03) + 104/(1.02 \times 1.03 \times 1.035) = 103.37$. A zero-coupon bond is priced the same way with only the final term.

Common exam traps

- Using the arithmetic average of forward rates instead of the geometric mean. The gap widens as the rates spread apart.
- Forgetting to annualize a multi-year forward rate (take the $1/B$ root), or dividing by B instead of taking the root. For 1y2y in the example, leaving out the root gives 8.069%, and dividing by 2 gives 4.034%, instead of 3.956%.
- Off-by-one exponents: the year-4 to year-5 forward uses $(1 + S_5)^5/(1 + S_4)^4$.
- Discounting a later cash flow at only one forward rate instead of the compounded chain. With the 2%, 3% and 3.5% rates in the example, discounting each cash flow at its own year's rate alone gives 108.29 instead of 103.37.

LOS 59.c — Comparing the spot, par and forward curves

Curve	What it plots	Observed or derived?
Spot curve (zero curve, strip curve)	Spot rates (YTM of zero-coupon bonds) vs maturity	From zero-coupon or stripped Treasury prices
Yield curve for coupon bonds	YTM of actively traded coupon bonds of the same or similar issuer vs maturity	Observed; gaps filled by linear interpolation

Curve	What it plots	Observed or derived?
Par curve	Coupon rates at which bonds of each maturity would price at par	Derived (hypothetical) from the spot curve
Forward curve	Forward rates (e.g., 1-year rates) for successive future periods	Derived from spot rates

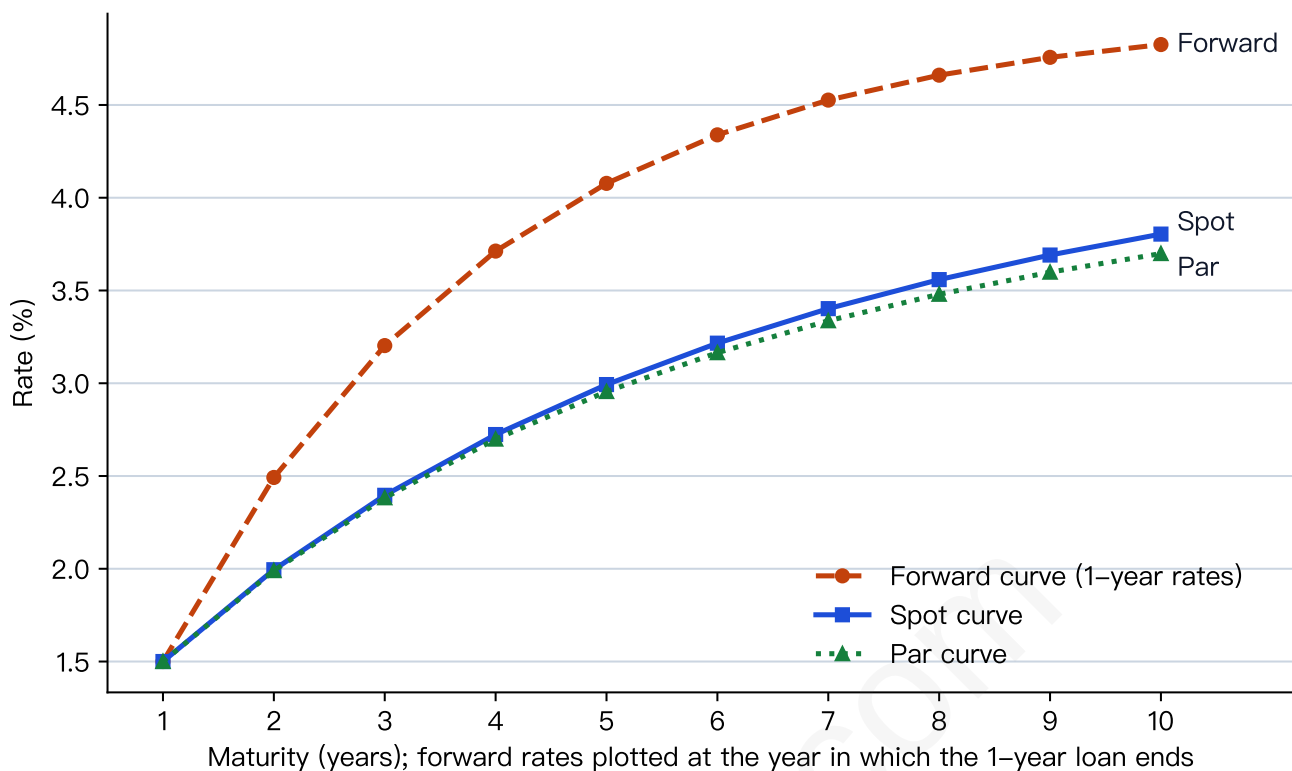
A coupon-bond yield curve built from market prices suffers from illiquidity and from tax distortions for bonds trading away from par; using **on-the-run** issues helps with liquidity, but there may be too few of them. A **par curve** derived from the spot curve avoids these problems. Rates on these curves are usually quoted on a **semiannual bond basis**.

How the curves relate

Forward rates drive spot rates (a spot rate is a geometric mean of forwards), and spot rates drive par rates (a par rate is a weighted average of spot rates). The par rate is weighted most heavily toward the final cash flow.

KEY CONCEPT

Environment	Ordering
Normal (upward-sloping)	forward > spot > par beyond the first year; spot rises more slowly than forward; par just below spot
Inverted (downward-sloping)	forward < spot < par beyond the first year; par just above spot
Flat	forward = spot = par at all maturities



Forward, spot and par curves when forward rates rise with maturity

An inverted curve is consistent with the market expecting short-term rates to fall.

Common exam traps

- Saying the par curve is observed in the market. It is theoretical, derived from spot rates.
- Reversing the ordering in an inverted environment: there, par yields sit above spot rates and forwards are lowest.
- Confusing "the yield curve" of coupon bonds (same issuer, different maturities) with a plot across issuers.

EXAM SHORTCUTS

- A forward rate is roughly the difference of the total rates, for example $2y1y \approx 3S_3 - 2S_2$; the exact figure needs compounding, but the approximation screens out wrong answer choices quickly.
- To find a par rate from answer choices, plug in the middle choice: a price below 100 means the par rate is higher, and a price above 100 means it is lower.

BOTTOM LINE

- A spot rate is the market discount rate for a single payment at a future date, which is why it equals the YTM of a zero-coupon bond of that maturity.
- The no-arbitrage price of a bond discounts each cash flow at the spot rate for its own date, and the bond's YTM is the internal rate of return at that price, not a simple or geometric average of the spot rates.

- A par rate is the coupon rate at which a bond of a given maturity would be priced at par, given the spot curve.
- $A_{yBy} = [(1 + S_{A+B})^{A+B} / (1 + S_A)^A]^{1/B} - 1$, where the $1/B$ root annualizes a multi-year forward rate.
- A spot rate is the geometric mean of the one-period rates up to its maturity: $S_N = [(1 + S_1)(1 + {}_1y_{1y}) \cdots (1 + {}_{(N-1)}y_{1y})]^{1/N} - 1$.
- Discounting each cash flow by the compounded chain of one-period forward rates up to its date gives the same no-arbitrage price as discounting at spot rates.
- Beyond the first year, forward > spot > par when the curve is normal (upward-sloping), forward < spot < par when it is inverted, and all three are equal when it is flat.
- The par curve and the forward curve are derived from the spot curve, while the yield curve for coupon bonds is observed from the YTM's of actively traded bonds, with gaps filled by linear interpolation.

QUICK CHECK

Question 1

Which of the following statements about zero-coupon bonds and spot rates is *most accurate*?

- A** If a 4-year zero-coupon bond yields 5.2%, the 4-year spot rate is 1.3%.
- B** Spot rates are the same for every maturity on the term structure.
- C** At a positive yield, a zero-coupon bond's whole return is price appreciation.

Answer: C

A zero-coupon bond pays no coupons: at a positive yield it is bought below par and pays par at maturity, so its entire return is price appreciation and there is no uncertainty about reinvesting coupons. Its yield to maturity is the spot rate for its maturity.

Why the other options are wrong

- **A.** The YTM of a zero-coupon bond *is* the spot rate for that maturity, so the 4-year spot rate is 5.2%; it is not divided by the number of years.
- **B.** Spot rates generally differ across maturities, and that variation is what the spot curve shows. Only in a flat environment are they all equal.

Key takeaway Zero-coupon bond: YTM = spot rate; at a positive yield the whole return is the price rising toward par; no reinvestment risk.

Practice questions

Question 2

Which of the following *best* describes how the arbitrage-free approach values an option-free bond?

- A It discounts the cash flows at a set of spot rates that reflects the current term structure.
- B It discounts the cash flows at the geometric average of the current spot rates.
- C It discounts all the cash flows at a single discount rate.

Question 3

Government spot rates are 2.1% for one year, 2.9% for two years and 3.6% for three years (annual compounding). The coupon rate at which a 3-year annual-pay government bond would be priced at par (the 3-year par rate) is *closest* to:

- A 2.87%.
- B 3.57%.
- C 3.60%.

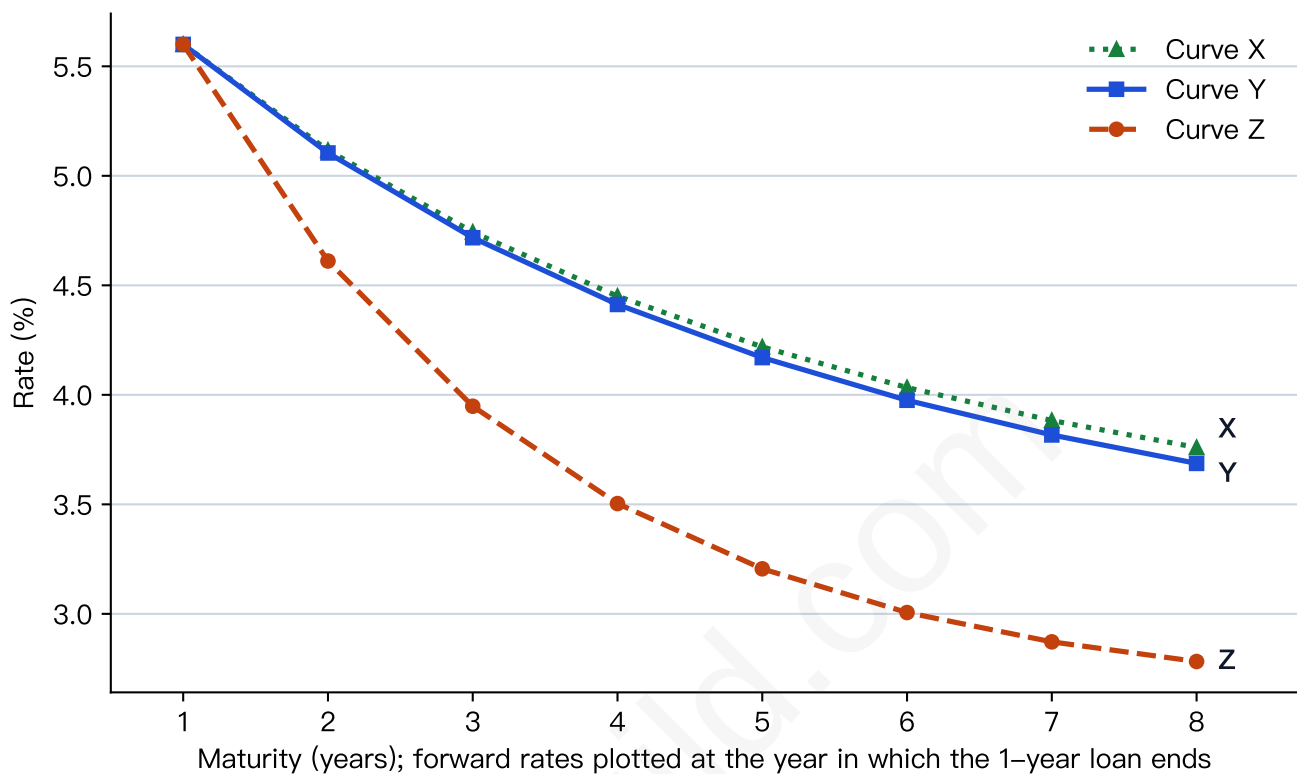
Question 4

Current market rates (annual compounding) are as follows: the 1-year spot rate is 2.8%, the 1-year forward rate one year from now is 4.6%, and the 1-year forward rate two years from now is 6.2%. The price per 100 of par of a 3-year zero-coupon bond is *closest* to:

- A 92.05.
- B 87.57.
- C 83.49.

Question 5

The chart below shows the spot curve, the par curve and the forward curve of 1-year rates, all derived from the same set of government bond prices (annual compounding). The curves are labeled only X, Y and Z.



Three curves derived from the same government bond prices

Curve X is *most likely* the:

- A par curve.
- B spot curve.
- C forward curve.

Answers: 2. A · 3. B · 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/the-term-structure-of-interest-rates-spot-par-and-forward-curves/. The full bank has 41 questions on this reading.

FIXED INCOME · READING 60

Interest Rate Risk and Return

What you'll learn

LOS 60.a Calculate and interpret the sources of return from a fixed-rate bond: coupons and principal, reinvestment income, and capital gains or losses relative to carrying value.

LOS 60.b Describe how a bond's holding period return depends on its Macaulay duration relative to the investment horizon (duration gap).

LOS 60.c Define, calculate, and interpret Macaulay duration as the present-value-weighted average time to receipt of a bond's cash flows.

MODULE 60.1

Interest Rate Risk and Return

This reading traces the sources of return on a fixed-rate bond and shows how market price risk and reinvestment risk trade off over the investment horizon. A candidate must be able to compute a horizon yield and a capital gain or loss measured against carrying value, explain the duration gap, and calculate and interpret Macaulay duration.

LOS 60.a — Where a fixed-rate bond's return comes from

An investor in a fixed-rate bond can earn return from three sources:

1. **Coupon and principal payments** promised by the issuer.
2. **Reinvestment income**: the interest earned by reinvesting coupons over the investor's holding period.
3. A **capital gain or loss** if the bond is sold before it matures.

Throughout this reading we assume the issuer makes every payment on time (no credit risk) and that coupons are reinvested at the bond's prevailing **yield to maturity (YTM)**.

Horizon yield

The **investment horizon** is how long the investor plans to hold the bond; it can be shorter than the bond's maturity. The **horizon yield** (annualized horizon return) is the compound annual rate that turns the purchase price into the total value at the horizon:

$$\text{Horizon yield} = \left(\frac{\text{sale price (or principal)} + \text{coupons} + \text{reinvestment income}}{\text{purchase price}} \right)^{1/H} - 1$$

where H is the horizon in years. Coupons plus reinvestment income equal the future value of the coupon annuity at the reinvestment rate. Working backward, the reinvestment income needed to reach a target yield is the purchase price compounded at that yield over the horizon, minus the coupons and the principal (or sale price) received.

Example. A 4-year, 3% annual-coupon bond is bought at a YTM of 4%, so its price is 96.370 ($N = 4$, $I/Y = 4$, $PMT = 3$, $FV = 100$, CPT PV). Held to maturity with coupons reinvested at 4%, the coupons grow to $3(1.04)^3 + 3(1.04)^2 + 3(1.04) + 3 = 12.739$, of which $12.739 - 12 = 0.739$ is reinvestment income.

Ending value = 112.739, and $(112.739/96.370)^{1/4} - 1 = 4.00\%$, the YTM at purchase.

Five key results

KEY CONCEPT

Situation (any change in YTM occurs before the first coupon date)	Realized return vs. YTM at purchase
YTM unchanged, bond held to maturity	Equal to the YTM
YTM unchanged, bond sold before maturity	Equal to the YTM (sale at the carrying value)
YTM rises (falls), bond held to maturity	Higher (lower); only reinvestment income changes
YTM rises, short horizon (below the bond's Macaulay duration, LOS 60.b)	Lower; the lower sale price dominates
YTM falls, long horizon (beyond the bond's Macaulay duration, LOS 60.b)	Lower; lost reinvestment income dominates

Continuing the example: if the YTM moves to 4.75% immediately after purchase and the bond is held to maturity, coupons grow to 12.882 and the realized return is 4.03% ($> 4\%$). If instead the bond is sold after one year, its price is 95.212 and the one-year return is $(95.212 + 3)/96.370 - 1 = 1.91\%$ ($< 4\%$). With a move to 3.25%, the hold-to-maturity return is 3.97% but the one-year return is $(99.296 + 3)/96.370 - 1 = 6.15\%$. In every hold-to-maturity case the realized return lies between the original YTM and the new reinvestment rate.

Two sources of interest rate risk

- **Market price risk** (price risk): uncertainty about the price at which the bond can be sold at the horizon. Higher yields mean lower prices.
- **Reinvestment risk**: uncertainty about the income from reinvesting coupons. Higher yields mean more reinvestment income.

A rise in yields therefore *hurts* the sale price but *helps* reinvestment income, and a fall does the opposite. The investment horizon decides which effect is larger. An investor who holds to maturity has no price risk (the bond pays par), only reinvestment risk. An investor who sells on the first coupon date has not yet earned any interest on reinvested coupons, so that investor faces price risk only.

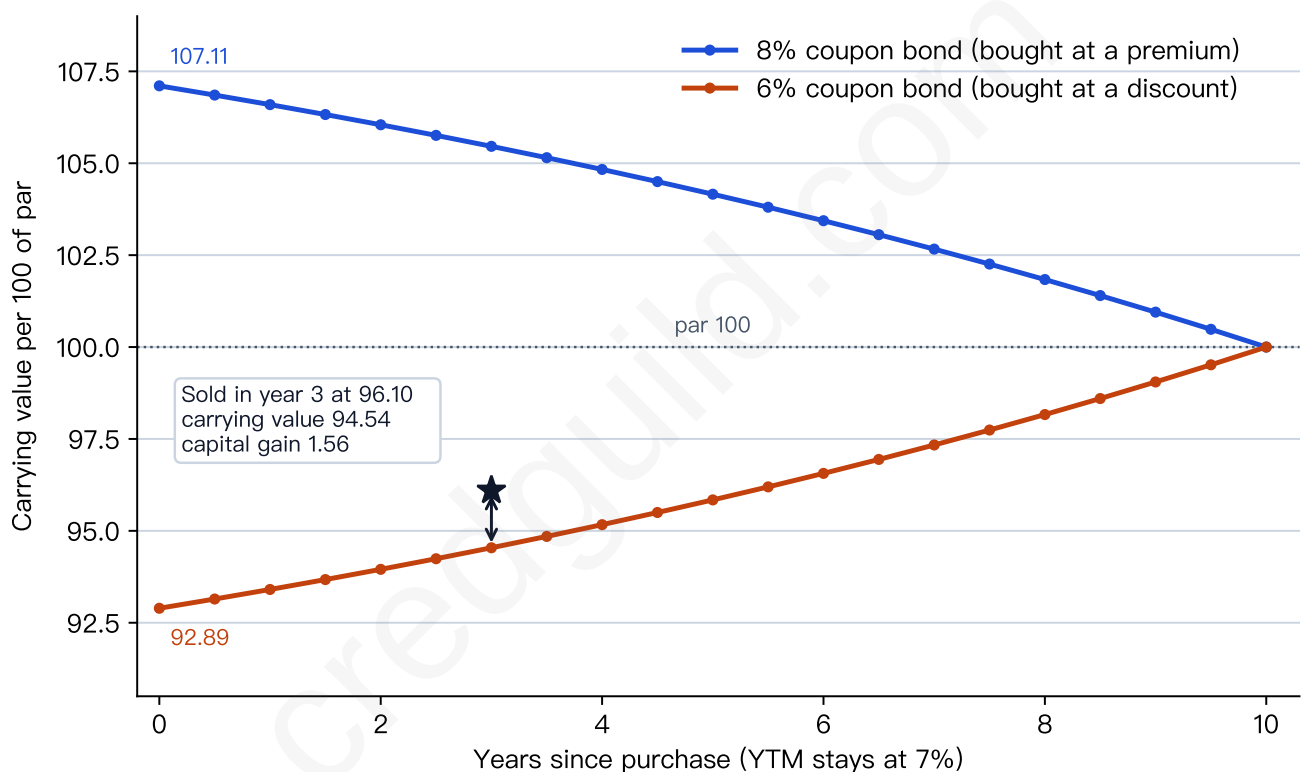
Horizon	Dominant risk
Short	Market price risk > reinvestment risk
Long (to maturity)	Reinvestment risk > market price risk

Other things equal, reinvestment risk is greater for bonds with higher coupons (more cash to reinvest) and longer maturities (a longer reinvestment period). A zero-coupon bond held to maturity has no reinvestment risk.

Capital gains and losses: carrying value

If the YTM does not change, a bond's price moves along its **constant-yield price trajectory** toward par: a discount bond's price rises toward par and a premium bond's price falls toward par. The price on that path at any date is the bond's **carrying value**: the purchase price plus the amortized discount (or minus the amortized premium). A capital gain or loss is measured as sale price minus carrying value, not sale price minus purchase price. Movement along the trajectory is part of interest income (it adds to a discount bond's interest and reduces a premium bond's), so it never counts as a capital gain or loss.

Example. A 10-year, 6% semiannual-coupon bond is bought at a YTM of 7%. Three years later (14 semiannual periods left) it is sold for 96.10. Carrying value: $N = 14$, $I/Y = 3.5$, $PMT = 3$, $FV = 100$, $CPT PV = -94.54$. Capital gain = $96.10 - 94.54 = 1.56$ per 100 of par. A bond held to maturity, or sold at the purchase YTM, has no capital gain or loss.



Constant-yield price trajectories: carrying value of a premium and a discount bond as time passes

The figure shows both paths for 10-year bonds bought at a 7% YTM, together with the sale from the example.

A zero-coupon bond follows the same rule: the accretion toward face value is interest income, and a sale before maturity produces a capital gain or loss only if a change in YTM has moved the price away from the carrying value.

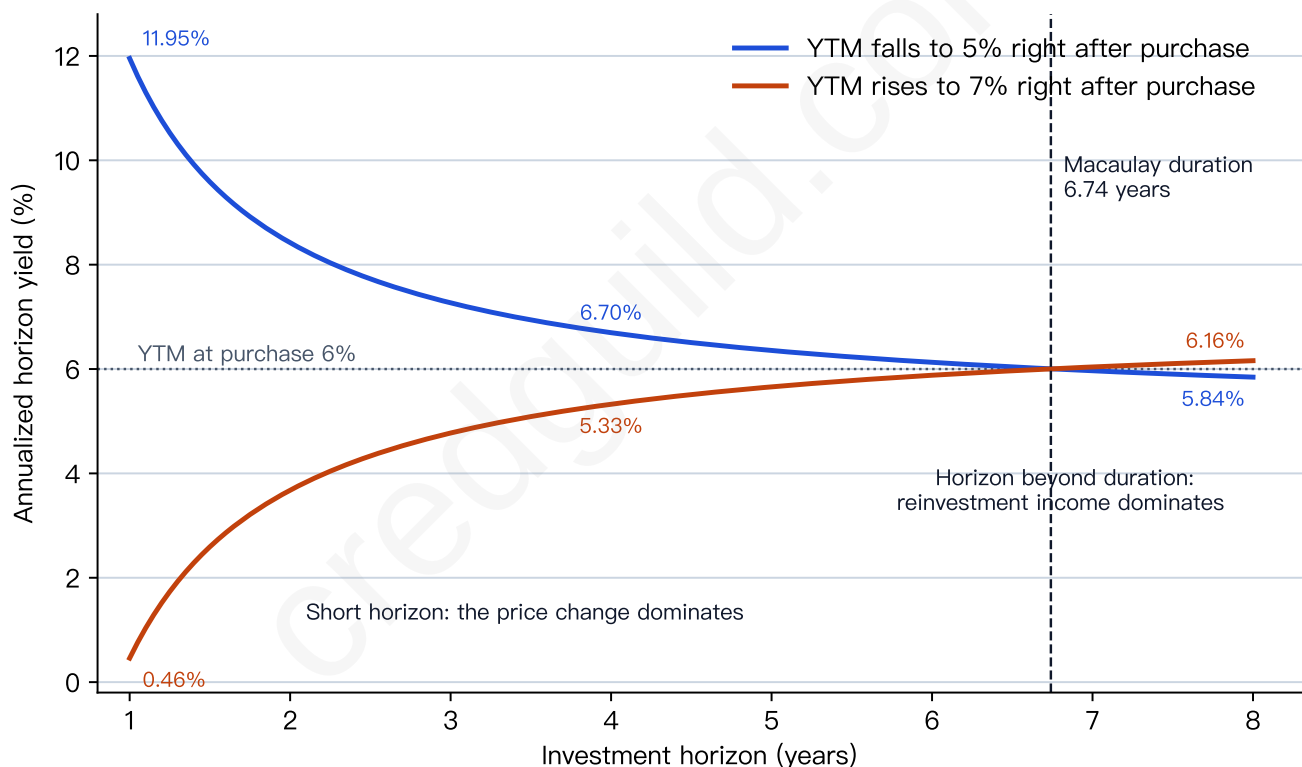
Common exam traps

- Assuming the YTM is earned whatever the reinvestment rate. Held to maturity with coupons reinvested at the coupon rate, a premium bond earns more than its YTM and a discount bond less.

- Measuring a capital gain against the purchase price or par. The 8% bond in the figure, bought at 107.11 and sold after three years at 106.00, shows a capital gain of $106.00 - 105.46 = 0.54$ even though it was sold below its purchase price. For the 10-year, 6% bond in the carrying-value example, measuring against the purchase price of 92.89 gives a gain of 3.21, and measuring against par gives a loss of 3.90, instead of a gain of 1.56.
- Expecting a rise in YTM to lower the return on a bond held to maturity. Only reinvestment income changes, so the return rises.

LOS 60.b — Holding period return, Macaulay duration and the investment horizon

Because price risk and reinvestment risk move in opposite directions, there is an investment horizon at which they roughly cancel. That horizon is the bond's **Macaulay duration**, the weighted average time until its cash flows are received (calculated under LOS 60.c). If the horizon equals the Macaulay duration, a one-time change in YTM right after purchase leaves the horizon yield approximately equal to the YTM at purchase: the gain (loss) on reinvestment income offsets the loss (gain) on the sale price.



Horizon yield against investment horizon after a one-time change in YTM right after purchase

The figure follows an 8-year, 5% annual-coupon bond bought at a 6% YTM (price 93.79, Macaulay duration 6.74 years) whose YTM moves to 5% or to 7% right after purchase. Over a one-year horizon the change in price dominates: the horizon yield is 11.95% if the YTM falls and 0.46% if it rises. As the horizon lengthens, the price effect shrinks because the bond is sold closer to maturity, when its price is nearer par. The reinvestment effect grows because more coupons are reinvested for longer. The two effects cancel at a horizon equal to the Macaulay duration, where both lines pass through 6%. Beyond that point the ranking reverses, and held to maturity the bond earns 6.16% if the YTM rose and 5.84% if it fell.

The **duration gap** is:

$$\text{Duration gap} = \text{Macaulay duration} - \text{investment horizon}$$

KEY CONCEPT

Duration gap	Horizon vs. MacDur	Main exposure	Hurt by
Positive duration gap	Horizon < MacDur	Market price risk	Rising rates
Zero	Horizon = MacDur	Risks roughly offset	—
Negative duration gap	Horizon > MacDur	Reinvestment risk	Falling rates

Example. An investor with a 7-year horizon holds bonds with a Macaulay duration of 5.5 years. The gap is $5.5 - 7 = -1.5$ years (negative). A fall in yields lowers the realized return, because reinvestment income falls by more than the sale price rises.

Common exam traps

- A horizon shorter than *maturity* does not by itself mean price risk dominates; compare the horizon with Macaulay duration.
- A negative duration gap means falling rates are the threat; it does not minimize reinvestment risk.
- Any nonzero duration gap leaves an exposure. Only a horizon equal to the Macaulay duration makes the two risks roughly offset.

LOS 60.c — Calculating and interpreting Macaulay duration

The (annual) Macaulay duration is a weighted average of the number of years until each promised cash flow is paid. Each weight is the present value of a cash flow (discounted at the YTM) divided by the bond's full price:

KEY CONCEPT

$$\text{MacDur} = \sum_{t=1}^N t \times w_t, \quad w_t = \frac{CF_t / (1+r)^t}{\text{full price}}, \quad \sum_t w_t = 1$$

Each cash flow contributes its weight times the number of years until it is paid. For a typical coupon bond, the final payment (last coupon plus principal) has the largest weight and contributes most to the duration.

Steps. (1) Price the bond at its YTM. (2) Discount each cash flow at the YTM. (3) Divide each present value by the price to get its weight; the weights sum to 1. (4) Multiply each weight by the time until its cash flow and add the products.

Example (the 4-year, 3% bond at 4%, price 96.370):

Year	Cash flow	PV at 4%	Weight
1	3	2.885	0.0299

Year	Cash flow	PV at 4%	Weight
2	3	2.774	0.0288
3	3	2.667	0.0277
4	103	88.045	0.9136

$$\text{MacDur} = 0.0299(1) + 0.0288(2) + 0.0277(3) + 0.9136(4) = 3.82 \text{ years.}$$

For a semiannual-pay bond, the same calculation gives a duration in semiannual periods; divide by 2 to state it in years. The Macaulay duration of a zero-coupon bond equals its maturity; a coupon bond's is shorter than its maturity.

Interpretations

Measure	Meaning
Macaulay duration	Weighted average time to receipt of the cash flows; the horizon at which price risk and reinvestment risk offset
Modified duration (Reading 61)	Approximate percentage price change for a 1% change in YTM

Why the offset happens at this horizon: a zero-coupon bond held to its maturity has neither risk, because nothing is reinvested and nothing is sold early. Macaulay duration condenses a coupon bond's cash flows into one equivalent payment date, so an investor whose horizon matches that date is roughly in the position of a zero-coupon holder.

Common exam traps

- Macaulay duration is a time measure (years) and is the duration matched to the investment horizon. The "percentage price change per 1% yield change" reading belongs to modified duration.
- Macaulay duration is not the time needed to recover the principal. The principal comes back at maturity.
- Use present-value weights rather than undiscounted cash flows, and do not divide the result by $1 + \text{YTM}$ (that gives modified duration). For the 4-year, 3% bond in the example, undiscounted weights give 3.84 years and dividing by 1.04 gives 3.68, instead of 3.82 years.

EXAM SHORTCUTS

- For a bond held to maturity after a one-time change in YTM right after purchase, the realized return lies between the original YTM and the new reinvestment rate, so any answer outside that range can be ruled out.
- A zero-coupon bond's Macaulay duration equals its maturity, so it needs no weighting calculation.

BOTTOM LINE

- A fixed-rate bond's return comes from the promised coupons and principal, the income from reinvesting coupons, and any capital gain or loss on a sale before maturity.
- With every payment made on time and coupons reinvested at the YTM, the realized return equals the YTM at purchase whether the bond is kept until it matures or sold earlier at an unchanged YTM.

- Higher yields lower the sale price but raise reinvestment income; over a short horizon market price risk dominates, and over a long horizon reinvestment risk dominates.
- A capital gain or loss is the sale price minus the carrying value on the constant-yield price trajectory, so movement along that trajectory counts as interest income rather than as a gain or loss.
- When the investment horizon equals the bond's Macaulay duration, a one-time change in YTM right after purchase leaves the horizon yield approximately equal to the YTM at purchase.
- Duration gap = Macaulay duration – investment horizon; a positive gap leaves the investor exposed to rising rates through price risk, and a negative gap leaves the investor exposed to falling rates through reinvestment risk.
- Macaulay duration is the weighted average time until a bond's cash flows are received, with each weight equal to the cash flow's present value at the YTM divided by the full price; a coupon bond's Macaulay duration is shorter than its maturity.
- Other things equal, reinvestment risk is greater for bonds with higher coupons and longer maturities, and a zero-coupon bond held to maturity has none.

QUICK CHECK

Question 1

A wealth manager tells a client: "When you invest in a bond for a given horizon, your return comes from the coupons you collect plus the difference between what you paid and what you receive when you sell the bond or it matures." The wealth manager's description is:

- A** incomplete, because it leaves out interest earned on reinvested coupons.
- B** complete, because it captures all sources of bond return.
- C** incomplete, because a bond held to maturity produces no capital gain or loss.

Answer: A

A fixed-rate bond has three sources of return: coupon and principal payments, reinvestment income on the coupons, and any capital gain or loss if it is sold before maturity. The manager's description covers the coupons and the price/principal difference but omits the interest earned by reinvesting coupons over the horizon.

Why the other options are wrong

- **B.** The description omits reinvestment income, which can be a large part of the return on a long-horizon, high-coupon bond.
- **C.** It is true that a bond held to maturity has no capital gain or loss, but that is not what is wrong with the description. It already allows for the bond to mature at par.

Key takeaway Three sources of bond return: coupons and principal, reinvestment income, and capital gain or loss on a sale before maturity.

Practice questions

Question 2

Dmitri Volkov bought a 15-year, 4% semiannual-coupon bond at a yield to maturity of 5.2%. Four years later, he sells it for 92.10 per 100 of par. Per 100 of par, Volkov realizes:

- A a capital gain of 2.06.
- B a capital gain of 4.49.
- C a capital loss of 7.90.

Question 3

A pension fund must make a single payment in eight years. The trustees want any change in the income from reinvesting coupons to be roughly offset by an opposite change in the price at which the bonds are sold at the horizon. Which measure of the bonds' duration should they set equal to eight years?

- A Modified duration.
- B Effective duration.
- C Macaulay duration.

Question 4

Rafael Quintero calculates a Macaulay duration of 3.46 for a four-year, option-free corporate bond. The best interpretation of 3.46 is that it measures:

- A the bond's price sensitivity to a change in interest rates.
- B the average time until the bond's cash flows are received.
- C the time the investor needs to recover his principal.

Answers: 2. A · 3. C · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/interest-rate-risk-and-return/. The full bank has 23 questions on this reading.

FIXED INCOME · READING 61

Macaulay Duration

What you'll learn

LOS 61.a Define, calculate, and interpret modified duration (exact and approximate), money duration, and the price value of a basis point (PVBP).

LOS 61.b Explain how maturity, coupon rate, yield level (and embedded options) affect a bond's duration and interest rate risk.

MODULE 61.1

Yield-Based Bond Duration Measures and Properties

This reading turns duration into measures of price sensitivity and explains what makes a bond's price more or less sensitive to yield changes. A candidate must be able to compute modified duration, approximate modified duration, money duration and the PVBP, estimate a price change from them, and explain how maturity, coupon, yield level and embedded options affect interest rate risk.

LOS 61.a — Modified duration, money duration and the PVBP

Modified duration

Modified duration (ModDur) converts Macaulay duration (Reading 60) into a measure of price sensitivity. Divide Macaulay duration by one plus the bond's periodic yield:

$$\text{ModDur} = \frac{\text{MacDur}}{1 + r} \quad (\text{annual-pay: } r = \text{YTM})$$

$$\text{ModDur} = \frac{\text{MacDur}}{1 + \text{YTM}/2} \quad (\text{semiannual-pay, YTM on a semiannual bond basis, durations in years})$$

Whenever the yield is positive, modified duration is *smaller* than Macaulay duration.

Modified duration is the approximate percentage change in price for a 1% (100 bp) change in YTM:

KEY CONCEPT

$$\% \Delta P \approx -\text{ModDur} \times \Delta \text{YTM}$$

Example. A 6-year, 4% annual-pay bond priced at 94.924 to yield 5% has a Macaulay duration of 5.435. $\text{ModDur} = 5.435/1.05 = 5.18$. If the YTM rises by 30 bp, the price should fall by about $5.18 \times 0.30\% = 1.55\%$. For a semiannual bond with MacDur 7.2 years and a 6% YTM, $\text{ModDur} = 7.2/1.03 = 6.99$.

The conversion also runs backward: $\text{MacDur} = \text{ModDur} \times (1 + r)$. This is the step needed when a question gives modified duration but asks about the duration gap (Reading 60), which uses Macaulay duration. *Example.* A semiannual-pay bond with a 5% YTM has a modified duration of 9.20, so its Macaulay duration is $9.20 \times$

$1.025 = 9.43$ years. An investor with a 7-year horizon has a duration gap of $9.43 - 7 = +2.43$ years and is exposed mainly to rising rates.

Approximate modified duration

Modified duration can be estimated by repricing the bond at a slightly lower and a slightly higher yield:

KEY CONCEPT

$$\text{ApproxModDur} = \frac{V_- - V_+}{2 \times V_0 \times \Delta\text{YTM}}$$

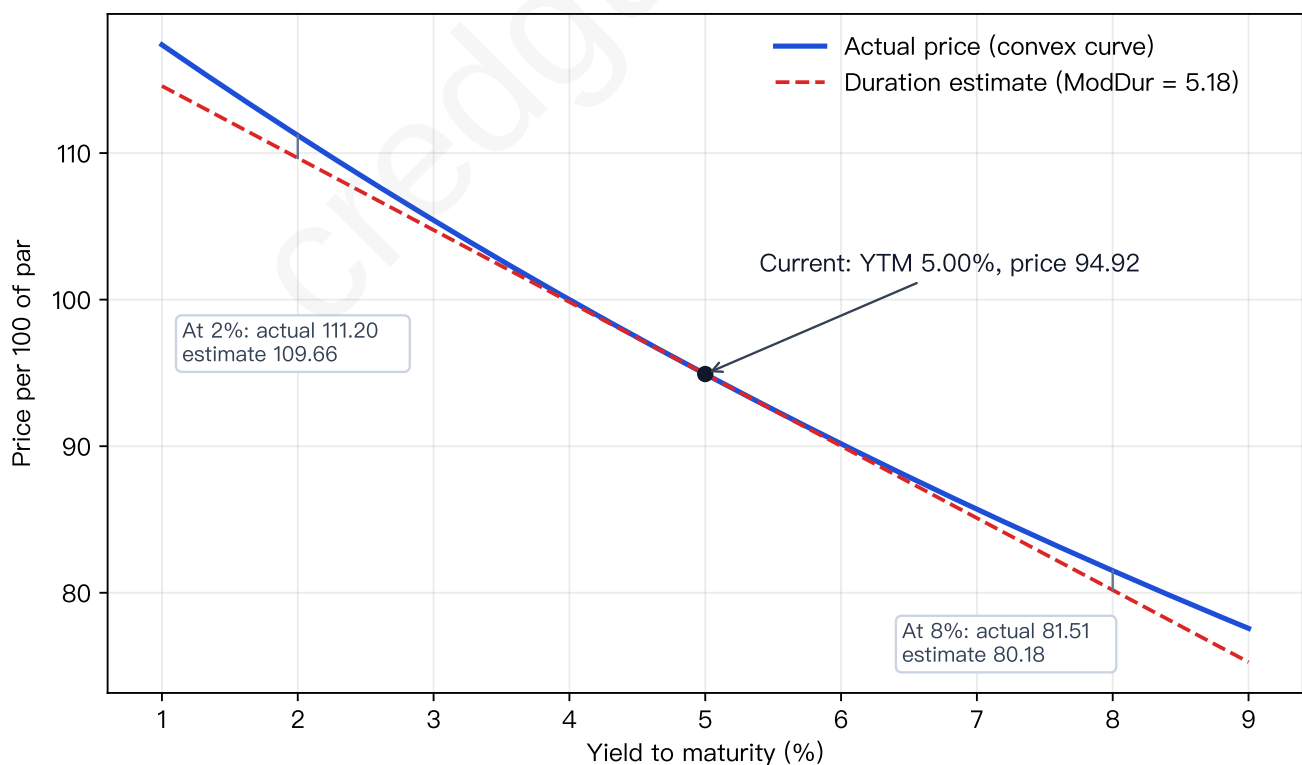
- V_- = price if the YTM falls by ΔYTM ; V_+ = price if it rises by ΔYTM ; V_0 = current price.
- The 2 averages the price rise and the price fall; V_0 turns the change into a percentage.

Geometrically, the straight line joining the two repriced points has almost the same slope as the tangent to the price-yield curve at the current price. Dividing the size of that slope by V_0 therefore approximates modified duration, and the approximation improves as the yield shift gets smaller.

For the 6-year bond: at 4.9% the price is 95.4172, at 5.1% it is 94.4345, so $\text{ApproxModDur} = (95.4172 - 94.4345) / (2 \times 94.9243 \times 0.001) = 0.9827 / 0.18985 = 5.18$.

Duration is a straight-line estimate

The true price-yield relationship of an option-free bond is **convex**. Duration draws a straight tangent line at the current yield, so it works well for small yield changes and less well for large ones. For large changes, duration alone underestimates the price increase when yields fall and overestimates the price decrease when yields rise.



Actual (convex) price-yield curve vs. the straight-line modified-duration estimate

Money duration and PVBP

Money duration (also called **dollar duration**) expresses sensitivity in currency units:

$$\text{Money duration} = \text{ModDur} \times \text{full price of the position}$$

$$\text{Money duration per 100 of par} = \text{ModDur} \times \text{full price per 100}$$

$$\Delta V \approx -\text{Money duration} \times \Delta \text{YTM}$$

The **price value of a basis point (PVBP)** is the currency change in a bond's full price for a 1 bp change in YTM. Reprice at ± 1 bp and average:

$$\text{PVBP} = \frac{V_- - V_+}{2}, \quad \text{PVBP} \approx \text{Money duration} \times 0.0001$$

When V_- and V_+ are prices per 100 of par, the PVBP of a position is $[(V_- - V_+)/2] \times \text{par value} \times 0.01$.

Example. For the 6-year bond, with the unrounded modified duration of 5.176, money duration per 100 of par = $5.176 \times 94.924 = 491.3$. A \$3 million par position is worth \$2,847,720 and has a money duration of about \$14.74 million, so a 1 bp rise cuts its value by about \$1,474 (the PVBP).

Common exam traps

- Express ΔYTM as a decimal: 50 bp = 0.005. Entering it as 0.05 makes the duration 10 times too small, and entering it as 0.5 makes it 100 times too small. For the 6-year bond, entering the 10 bp shift as 0.01 gives 0.518 instead of 5.18.
- Do not forget the 2 in the approximate ModDur denominator, and use V_0 (not V_- or V_+) as the base. For the 6-year bond, leaving out the 2 gives 10.35, and using V_- as the base gives 5.15, instead of 5.18.
- For semiannual bonds, reprice with $N = \text{years} \times 2$ and $I/Y = \text{YTM}/2$, but state the yield shift in annual terms. For the PVBP, a 1 bp shift in the annual yield moves the periodic rate by only 0.5 bp, and ΔYTM in the approximate ModDur formula is the annual change.
- Modified duration uses the YTM in the denominator. The coupon rate plays no part in the divisor. For the 6-year, 4% bond, dividing by 1.04 gives 5.23 instead of 5.18.
- Using par value instead of the full price misstates money duration whenever the bond does not trade at par. For the \$3 million par position in the example, using par gives about \$15.53 million instead of \$14.74 million.

WORKED EXAMPLE: APPROXIMATE MODIFIED DURATION AND PVBP OF A SEMIANNUAL BOND

A 5-year bond pays a 4.4% coupon semiannually and yields 5.6% on a semiannual bond basis. It is valued on a coupon date, so its full price equals its quoted price. An investor holds \$2,000,000 of par. Estimate the bond's modified duration by repricing it at yields 10 bp below and 10 bp above the current yield, then compute the position's money duration and PVBP.

Step 1. Current price: $N = 10$; $I/Y = 2.8$; $\text{PMT} = 2.2$; $\text{FV} = 100$; $\text{CPT PV} = -94.8292$.

Step 2. At a YTM of 5.5% the periodic rate is 2.75%: $N = 10$; $I/Y = 2.75$; $PMT = 2.2$; $FV = 100$; CPT PV = -95.2480. At 5.7% it is 2.85%: $N = 10$; $I/Y = 2.85$; $PMT = 2.2$; $FV = 100$; CPT PV = -94.4127.

Step 3. $\text{ApproxModDur} = (95.2480 - 94.4127) / (2 \times 94.8292 \times 0.001) = 0.8353 / 0.18966 = 4.404$. The shift enters as 0.001, the annual change, although each periodic rate moved by only 5 bp.

Step 4. Position value = $2,000,000 \times 0.948292 = \$1,896,584$. Money duration = $4.404 \times 1,896,584 = \$8,352,556$.

Step 5. $\text{PVB} \approx 8,352,556 \times 0.0001 = \835 .

Result. The approximate modified duration is 4.40, the position's money duration is about \$8.35 million, and a 1 bp change in yield changes its value by about \$835.

LOS 61.b — What drives interest rate risk

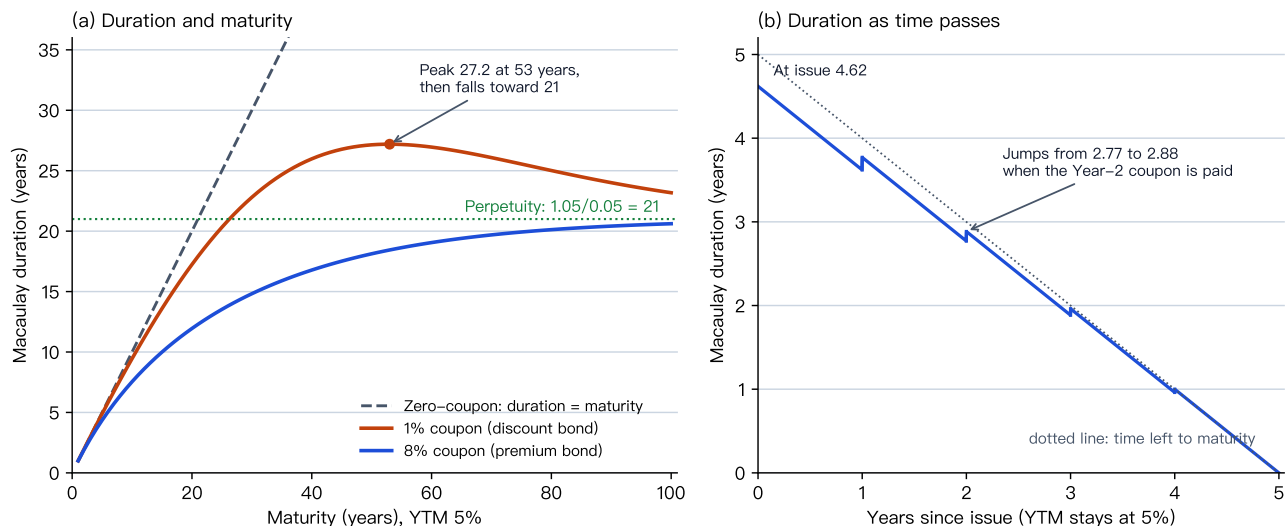
Other things equal:

KEY CONCEPT

Feature	Effect on duration / interest rate risk	Why
Longer maturity	Usually higher	Distant cash flows are more sensitive to discount-rate changes
Higher coupon rate	Lower	More value comes from early cash flows
Higher yield to maturity	Lower	The price-yield curve is flatter at higher yields
Passage of time	Duration falls smoothly between coupon dates, then jumps up slightly on each coupon date	Time to the next payment resets to a full period

The slope of the price-yield curve is a picture of interest rate risk. When bonds on one chart have the same price at the current yield (for example, all at par), the one whose curve is steepest there has the highest modified duration.

- A **zero-coupon bond**'s Macaulay duration equals its maturity, so for the same maturity and yield it has more interest rate risk than any coupon bond.
- The maturity effect holds only usually: for **discount bonds** at long maturities, duration can fall as maturity rises, approaching the duration of a **perpetuity**, $(1 + \text{YTM}) / \text{YTM}$. For example, at an 8% yield a perpetuity's Macaulay duration is $1.08 / 0.08 = 13.5$.
- A **floating-rate note (FRN)** resets its coupon to a **market reference rate (MRR)**, so it has little price risk. Its Macaulay duration is roughly the time until the next reset date. *Example.* An FRN resets every three months and one month of the current period has passed. Its Macaulay duration is about 2 months, or $2/12 = 0.17$ year, whatever its final maturity.



Macaulay duration against maturity (a) and as time passes (b)

Panel (a) shows the maturity effect for annual-pay bonds at a 5% YTM, with the zero-coupon bond as the reference line. A premium bond's duration rises more slowly and approaches the perpetuity value of 21. A deep-discount bond's duration overshoots that value, peaking at about 27.2 for a 53-year maturity, and then declines toward 21. Panel (b) shows the saw-tooth path of a 5-year, 4% bond's Macaulay duration as time passes with its YTM unchanged.

Bonds with embedded options (preview of Reading 63)

A **callable bond** has less interest rate risk (lower duration) than an otherwise identical option-free bond. When yields fall, the call price acts as a ceiling on the bond's price. This limits price gains and exposes the investor to reinvestment risk if the bond is called. A **puttable bond** has less price volatility at high yields, because the put becomes valuable and acts as a floor on the price. At low yields the put is worth little and the bond behaves like an option-free bond.

Common exam traps

- When bonds differ in more than one feature, weigh the drivers together. A large maturity difference usually outweighs a small coupon difference.
- Final maturity alone can mislead. An FRN with a long final maturity has very little duration, and a zero-coupon bond can have a higher duration than a coupon bond that matures somewhat later.

EXAM SHORTCUTS

- A floating-rate note's Macaulay duration is roughly the time to its next reset date, whatever its final maturity, so no cash flow weighting is needed.
- When bonds on one price-yield chart have the same price at the current yield, the bond whose curve is steepest at that yield has the highest modified duration.

BOTTOM LINE

- $\text{ModDur} = \text{MacDur} / (1 + r)$, where r is the periodic yield ($\text{YTM}/2$ for a semiannual-pay bond with durations in years), so with a positive yield modified duration is smaller than Macaulay duration.

- $\% \Delta P \approx -\text{ModDur} \times \Delta \text{YTM}$; this straight-line estimate works well for small yield changes, and for large changes it understates the price rise when yields fall and overstates the price fall when yields rise.
- $\text{ApproxModDur} = (V_- - V_+) / (2 \times V_0 \times \Delta \text{YTM})$, with ΔYTM entered as the annual yield change in decimal form.
- Money duration is modified duration times the full price of the position, $\Delta V \approx -\text{Money duration} \times \Delta \text{YTM}$, and the PVBP, the change in full price for a 1 bp yield change, is approximately money duration $\times 0.0001$.
- Other things equal, a higher coupon rate or a higher YTM lowers duration, and a longer maturity usually raises it; for discount bonds at long maturities, duration can fall as maturity rises toward the perpetuity duration $(1 + \text{YTM})/\text{YTM}$.
- A zero-coupon bond's Macaulay duration equals its maturity, so for the same maturity and yield it has more interest rate risk than any coupon bond.
- A callable bond has a lower duration than an otherwise identical option-free bond, and a puttable bond has less price volatility at high yields, where the put acts as a floor on its price.

QUICK CHECK

Question 1

A 7-year, option-free, annual-pay bond with a 5% coupon is priced at \$944.18 for each \$1,000 of face value. Its Macaulay duration is 6.0444. Its modified duration is *closest* to:

- A 5.868.
- B 5.757.
- C 5.702.

Answer: C

Modified duration is Macaulay duration divided by one plus the bond's periodic yield. The YTM must first be found from the price; for an annual-pay bond the periodic yield is the YTM itself.

YTM (P/Y = C/Y = 1, END): N = 7, PV = -944.18, PMT = 50, FV = 1,000, CPT I/Y = 6.00.

$$\text{ModDur} = \frac{6.0444}{1.06} = 5.702$$

Why the other options are wrong

- A. 5.868 divides by $1 + 0.06/2 = 1.03$, a semiannual adjustment; this bond pays annually.
- B. 5.757 divides by one plus the coupon rate (1.05) instead of one plus the YTM (1.06).

Key takeaway ModDur divides by one plus the periodic yield (the coupon rate plays no part), and the periodic yield matches the payment frequency.

Practice questions

Question 2

An analyst uses modified duration alone to estimate how an option-free bond's price will respond to a 150 basis point decline in its yield. Compared with the actual price change, the estimate will *most likely*:

- A understate the price increase.
- B overstate the price increase.
- C match the price increase closely.

Question 3

A portfolio holds \$6 million par value of an option-free bond with a full price of 98.75 per 100 of par and an annual modified duration of 5.4. If the bond's yield to maturity rises by 15 basis points, the estimated change in the value of the position, based on its money duration, is *closest* to:

- A -\$479,925.
- B -\$48,600.
- C -\$47,993.

Question 4

An option-free 20-year bond trades at a premium to par. If market interest rates rise, the bond's modified duration:

- A increases.
- B does not change.
- C decreases.

Question 5

Relative to an otherwise identical option-free bond, which statement about a bond with an embedded call feature is *least accurate*? The call feature:

- A caps the price gains the bondholder can earn when yields decline.
- B exposes the bondholder to greater reinvestment risk.
- C lengthens the bond's duration and so raises its price risk.

Answers: 2. A · 3. C · 4. C · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/macaulay-duration/. The full bank has 40 questions on this reading.

FIXED INCOME · READING 62

Bond Convexity

What you'll learn

LOS 62.a Calculate and interpret convexity (cash-flow and approximate methods) and describe the convexity adjustment.

LOS 62.b Estimate a bond's percentage (or money) price change for a given yield change using duration and convexity.

LOS 62.c Calculate portfolio duration and convexity as weighted averages and explain the limitations of these measures.

MODULE 62.1

Yield-Based Bond Convexity and Portfolio Properties

This reading adds convexity to duration. It shows how convexity is measured, how the convexity adjustment improves a duration-based estimate of a bond's price change, and how duration and convexity are combined for a portfolio.

LOS 62.a — Convexity and the convexity adjustment

Modified duration provides a straight-line (first-order) estimate of how a bond's full price responds to a change in its yield to maturity. The true **price-yield relationship** of an option-free bond is curved. It is **convex** toward the origin, so the straight line lies below the actual curve. For small yield changes the gap is negligible; for large changes a duration-only estimate becomes noticeably wrong. **Convexity** measures the curvature of the price-yield relationship, and the **convexity adjustment** (a second-order term) corrects the duration estimate.

Convexity from the cash flows

Convexity can be built cash flow by cash flow, exactly like Macaulay duration. For the cash flow paid at period t , with periodic yield r (the annual YTM divided by the periods per year):

$$\text{convexity}_t = \frac{t(t+1)}{(1+r)^2}$$

The bond's convexity is the **weighted average** of these figures, where each weight is the cash flow's present value divided by the bond's price (the same weights used for Macaulay duration). If coupons are paid more than once a year, the result is in periods squared, so annualize by dividing by the periodicity squared (by $2^2 = 4$ for a semiannual bond, by $4^2 = 16$ for a quarterly bond).

Worked example. A 3-year, 5% annual-pay bond yields 7%, so its price is 94.751.

t	Cash flow	PV at 7%	Weight	$t(t+1)/1.07^2$
1	5	4.673	0.0493	1.747
2	5	4.367	0.0461	5.241
3	105	85.711	0.9046	10.481

Convexity = $0.0493 \times 1.747 + 0.0461 \times 5.241 + 0.9046 \times 10.481 \approx 9.81$. No annualizing is needed because the bond pays annually.

Approximate convexity

A pricing model can reprice the bond after a yield decrease (V_-) and an equal yield increase (V_+):

KEY CONCEPT

$$\text{approximate convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta YTM)^2 V_0}$$

The numerator, $V_- + V_+ - 2V_0 = (V_- - V_0) - (V_0 - V_+)$, is how much larger the price rise is than the price fall. That gap exists only because the slope of the curve changes, which is what curvature means. The companion measure uses the same three prices: **approximate modified duration** = $(V_- - V_+) / (2V_0 \Delta YTM)$. When the shifts are applied to a benchmark yield curve rather than to the bond's own YTM, the same formula gives **effective convexity** (Reading 63).

Worked example. $V_0 = 104.20$; after a 30 bp fall in yield the price is 105.48, after a 30 bp rise it is 102.95. Approximate convexity = $(105.48 + 102.95 - 208.40) / (0.003^2 \times 104.20) = 0.03 / 0.00093780 \approx 31.99$; approximate modified duration = $2.53 / (2 \times 104.20 \times 0.003) \approx 4.05$.

What drives convexity

The same features that raise duration raise convexity: a longer maturity, a lower coupon rate and a lower YTM all increase convexity. Between two bonds with the same duration, the one whose cash flows are more dispersed over time has more convexity (a barbell of cash flows is more convex than a bullet).

Positive and negative convexity

	Option-free bond (positive convexity)	Callable bond at low yields (negative convexity)
Shape	Curve bends away from the tangent line	Price is capped near the call price
Equal yield fall vs. rise	Price gain exceeds price loss	Price gain is smaller than price loss
As yields fall	Price rises at an increasing rate	Price rises at a decreasing rate

Callable bonds (and mortgage-backed securities, because of prepayments) exhibit **negative convexity** at low yields: the issuer is likely to call, so the price cannot rise much above the call price. At higher yields the call is unlikely and the callable bond behaves like an option-free bond with positive convexity. Option-free bonds, including government and zero-coupon bonds, always have positive convexity.

Common exam traps

- Forgetting to divide by periodicity squared when annualizing a semiannual convexity.
- Using $V_- - V_+$ (the duration numerator) in the convexity formula. With the prices in the example this gives about 2,698 instead of 31.99.

- A negative approximate/effective convexity is a legitimate answer for a callable bond.
- The larger the convexity, the larger the error of a duration-only estimate; short-term bonds have little convexity.

LOS 62.b — Price change using duration and convexity

KEY CONCEPT

$$\% \Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta YTM + \frac{1}{2} \times \text{AnnConvexity} \times (\Delta YTM)^2$$

The first term is the **duration effect** (sign opposite to the yield change). The second is the **convexity adjustment**; because $(\Delta YTM)^2$ is always positive, the adjustment is positive for both increases and decreases in yield when convexity is positive. Duration alone understates the price gain when yields fall and overstates the price loss when yields rise, and the positive convexity term improves both estimates. Enter ΔYTM as a decimal (40 bp = 0.0040).

Worked example. A bond has ModDur 5.8 and convexity 42, and its yield falls 80 bp: duration effect = $-5.8 \times (-0.008) = +4.64\%$; convexity effect = $0.5 \times 42 \times 0.008^2 = +0.1344\%$; estimate = $+4.7744\%$.

How much the convexity term adds. A 10-year, 5% annual-pay bond priced at par has a modified duration of 7.72 and a convexity of 75.0. Its estimated and actual percentage price changes are:

Yield change	Actual	Duration only	Duration + convexity
+100 bp	-7.36%	-7.72%	-7.35%
-100 bp	+8.11%	+7.72%	+8.10%
+200 bp	-14.05%	-15.44%	-13.94%
-200 bp	+17.06%	+15.44%	+16.94%

A duration-only estimate is symmetric, but the actual price rises more than it falls. The convexity term is +0.375% for a 100 bp move and +1.50% for a 200 bp move in either direction. Because it grows with the square of the yield change, doubling the move roughly quadruples the error of a duration-only estimate.

Money convexity

Money convexity is the currency counterpart of money duration:

$$\text{MoneyCon} = \text{AnnConvexity} \times PV^{Full} \text{ of the position}$$

$$\Delta PV^{Full} \approx -(\text{MoneyDur} \times \Delta YTM) + \frac{1}{2} \times \text{MoneyCon} \times (\Delta YTM)^2$$

Maturity, par value and coupon do not enter the money convexity formula directly. Only the annual convexity and the full value of the position do. The money-based estimate and the percentage-based estimate are the same calculation expressed in different units, so they give equivalent estimated prices (apart from rounding).

Worked example. A position with full value \$3,900,000, ModDur 5.8 and convexity 42: MoneyDur = \$22,620,000; MoneyCon = \$163,800,000. For an 80 bp fall in yield: $22,620,000 \times 0.008 = \$180,960$ plus $0.5 \times 163,800,000 \times 0.008^2 = \$5,241.60$, a gain of about \$186,202 (4.7744% of \$3,900,000).

WORKED EXAMPLE: ESTIMATING A PRICE CHANGE FROM THREE PRICES

An 8-year, 4% annual-pay bond yields 4.40% and is priced at 97.351 on a coupon date, so its full price equals its flat price. A pricing model gives 98.996 if the yield falls 25 bp and 95.739 if it rises 25 bp. Estimate the percentage price change for a 75 bp rise in yield and the dollar change for a position of \$5,000,000 par.

Step 1. Approximate modified duration = $(98.996 - 95.739) / (2 \times 97.351 \times 0.0025) = 6.691$.

Step 2. Approximate convexity = $(98.996 + 95.739 - 2 \times 97.351) / (0.0025^2 \times 97.351) = 54.24$.

Step 3. For a 75 bp rise in yield: duration effect = $-6.691 \times 0.0075 = -5.0183\%$; convexity adjustment = $0.5 \times 54.24 \times 0.0075^2 = +0.1526\%$; estimate = -4.8657% .

Step 4. Repricing the bond at 5.15% gives 92.612, a change of -4.8680% . The duration-only estimate misses by about 0.15 percentage points; with the convexity adjustment the miss is about 0.002 percentage points.

Step 5. A position of \$5,000,000 par is worth $5,000,000 \times 0.97351 = \$4,867,550$, so its estimated loss is $0.048657 \times 4,867,550 \approx \$236,840$.

Common exam traps

- Leaving out the $\frac{1}{2}$, or subtracting the convexity term. For the 80 bp fall in the example, leaving it out gives +4.9088% and subtracting the term gives +4.5056%, against +4.7744%.
- Getting the sign of the duration effect wrong: yields down means price up.

LOS 62.c — Portfolio duration and convexity

Two approaches exist:

Approach	How	Comment
Aggregate cash flows	One duration/convexity from all portfolio cash flows, discounted at the portfolio's cash flow yield	Theoretically more correct, harder to use; cannot handle bonds whose cash flows are uncertain
Weighted average	portfolio duration = $\sum w_i D_i$, w_i = full market value of bond i / portfolio value	Used in practice: easier, and can use effective durations of bonds with embedded options

Portfolio convexity is weighted the same way, with weights based on full market values.

Worked example. Consider a portfolio holding \$3.5 million par of Bond P (full price 100.00, duration 3.2, convexity 12) and \$7.0 million par of Bond Q (full price 92.86, duration 8.6, convexity 88).

1. Market values: $3.5 \times 1.0000 = \$3.50$ million and $7.0 \times 0.9286 = \$6.50$ million, a total of \$10.00 million.

2. Weights: 35% in P and 65% in Q (par weights would give 33.3% and 66.7%).
3. Portfolio duration = $0.35(3.2) + 0.65(8.6) = 6.71$; portfolio convexity = $0.35(12) + 0.65(88) = 61.4$.

Limitations. The weighted-average measure is a valid estimate of the percentage value change per 1% change in yield only if every bond's yield changes by the same amount, which is a **parallel shift** of the yield curve. Real curves steepen, flatten and twist. Portfolio duration also inherits the straight-line limitation from LOS 62.a: without the convexity term it is accurate only for small yield changes.

To estimate a dollar change in portfolio value: $\Delta V \approx -D_{port} \times \Delta y \times V$.

Price value of a basis point. The **price value of a basis point (PVBP)** is the change in a bond's price when its yield changes by one basis point (0.01%): $PVBP \approx \text{modified duration} \times V \times 0.0001$. Convexity is ignored because a 1 bp change is too small for curvature to matter. The product of value and modified duration (without the 0.0001) is money duration.

Common exam traps

- Averaging durations equally, or with par-value weights, instead of with market-value weights. For bonds P and Q, equal weights give 5.90 and par weights give 6.80, against 6.71.
- Calling the weighted-average approach the theoretically correct one. It is the practical choice; the aggregate cash-flow approach is the theoretically correct one.

EXAM SHORTCUTS

- For an option-free bond the convexity adjustment is positive whether yields rise or fall.
- The convexity term grows with the square of the yield change, so doubling the change roughly quadruples it.

BOTTOM LINE

- Convexity measures the curvature of the price-yield relationship; for an option-free bond, duration alone understates the price gain when yields fall and overstates the price loss when yields rise.
- $\% \Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta YTM + \frac{1}{2} \times \text{AnnConvexity} \times (\Delta YTM)^2$.
- Approximate convexity is $(V_- + V_+ - 2V_0) / [(\Delta YTM)^2 V_0]$.
- A longer maturity, a lower coupon and a lower yield raise convexity; with the same duration, more dispersed cash flows give more convexity.
- Callable bonds and mortgage-backed securities can have negative convexity at low yields; option-free bonds always have positive convexity.
- Money convexity is annual convexity $\times$ the full value of the position.
- Portfolio duration and convexity come either from the aggregate portfolio cash flows, the theoretically correct approach, or in practice as averages weighted by full market value, which give the portfolio's percentage change only for a parallel shift of the yield curve.

QUICK CHECK**Question 1**

A 1.5-year option-free bond pays an 8% coupon semiannually and is priced to yield 6.00%. Using the present values of its cash flows as weights, its annualized convexity is *closest* to:

- A 2.69.
- B 5.37.
- C 10.75.

Answer: A

Convexity is the present-value-weighted average of each cash flow's convexity, $t(t+1)/(1+r)^2$, using the periodic yield. For a semiannual bond the result is in half-years squared and must be divided by $2^2 = 4$ to annualize.

Periodic yield $r = 3\%$; cash flows 4, 4, 104.

PVs: $4/1.03 = 3.8835$; $4/1.03^2 = 3.7704$; $104/1.03^3 = 95.1747$; price = 102.8286.

Weights: 0.0378, 0.0367, 0.9256. Cash-flow convexities: $1 \times 2/1.03^2 = 1.8852$; $2 \times 3/1.03^2 = 5.6556$; $3 \times 4/1.03^2 = 11.3112$.

Periodic convexity = $0.0378(1.8852) + 0.0367(5.6556) + 0.9256(11.3112) \approx 10.748$; annualized = $10.748/4 \approx 2.69$.

Why the other options are wrong

- B. 5.37 divides by the periodicity (2) instead of the periodicity squared (4).
- C. 10.75 is the convexity in semiannual periods, before annualizing.

Key takeaway Annualize convexity by dividing by periodicity squared (4 for semiannual).

Practice questions**Question 2**

Three option-free bonds are priced at the same yield to maturity. Which one is *most likely* to have the highest convexity?

- A A 10-year bond with a 7% coupon.
- B A 25-year bond with a 2% coupon.
- C A 10-year bond with a 2% coupon.

Question 3

Which statement about an option-free bond with positive convexity is *most accurate*?

- A A given fall in yield raises its price by more than the same rise in yield lowers it.
- B An increase in its yield leads to an increase in its price.
- C Its price rises and falls at a faster rate than its yield changes.

Question 4

A portfolio holds the three option-free bonds shown below (prices per 100 of par).

Portfolio holdings

Bond	Par value (\$)	Price (per 100 of par)	Duration
Bond 1	1,000,000	112	3.1
Bond 2	1,200,000	86	10.8
Bond 3	800,000	101	6.0

Using the weighted-average method, the portfolio's duration is *closest* to:

- A 6.58.
- B 6.63.
- C 6.95.

Answers: 2. B · 3. A · 4. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/bond-convexity/. The full bank has 26 questions on this reading.

FIXED INCOME · READING 63

Curve-Based and Empirical Fixed-Income Risk Measures

What you'll learn

LOS 63.a Explain why effective duration and effective convexity (measured against benchmark curve shifts) are the right risk measures for bonds with embedded options.

LOS 63.b Estimate a bond's percentage price change for a benchmark yield change using effective (or modified) duration and convexity.

LOS 63.c Define key rate duration and use it to measure sensitivity to nonparallel changes in the benchmark yield curve.

LOS 63.d Describe the difference between empirical duration and analytical duration.

MODULE 63.1

Curve-Based and Empirical Fixed-Income Risk Measures

This reading measures interest rate risk against the benchmark yield curve instead of a bond's own yield, which is needed when cash flows depend on future rates. A candidate must be able to compute and use effective duration and effective convexity, explain negative convexity in callable bonds, use key rate durations to measure shaping risk, and contrast empirical with analytical duration.

LOS 63.a — Why effective duration and effective convexity for bonds with embedded options

Yield-based measures (Macaulay duration, **modified duration**, **approximate modified duration**, convexity) assume the bond's cash flows are fixed and known. A bond with an **embedded option** does not satisfy that assumption:

- From the investor's perspective, a **callable bond** = option-free bond – a call option: the investor is short the embedded call and the issuer holds (is long) the call right; the issuer redeems early when rates fall.
- A **puttable bond** = option-free bond + a long put held by the investor; the investor sells back when rates rise.
- A **mortgage-backed security (MBS)** behaves like a callable bond because borrowers can prepay.

Because the cash flows (and their timing) depend on the level and path of future interest rates, such a bond has no single well-defined yield (for example, yield to maturity versus yield to each call date). Interest rate risk is therefore measured against shifts in a **benchmark yield curve** (e.g., government par rates), with a pricing model producing the values V_- and V_+ :

KEY CONCEPT

$$\text{effective duration (EffDur)} = \frac{V_- - V_+}{2V_0 \Delta\text{Curve}} \quad \text{effective convexity (EffCon)} = \frac{V_- + V_+ - 2V_0}{(\Delta\text{Curve})^2 V_0}$$

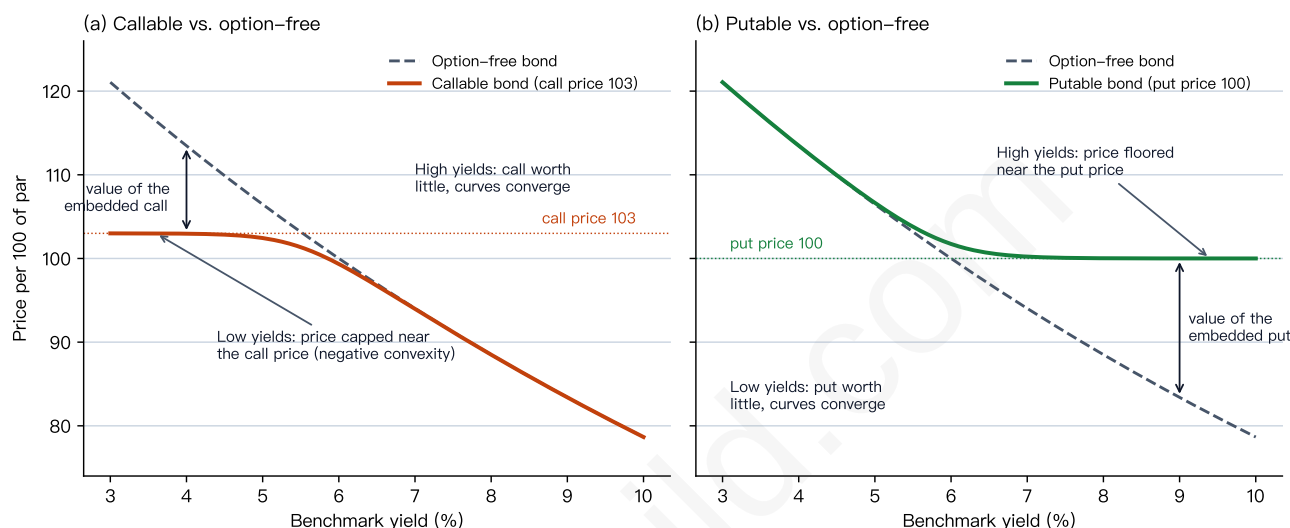
Effective duration separates benchmark-rate risk from spread risk: it measures sensitivity to the benchmark curve only and holds the credit and liquidity spread constant. Modified duration makes no such distinction. Because effective duration is a curve-based measure, it can be used for option-free bonds too; for an option-

free bond, modified and effective duration are close but not identical unless the curve is flat (a parallel shift in the par curve moves the spot rates by different amounts).

Worked example. A callable bond is worth 99.10. A 50 bp parallel fall in the benchmark curve lifts its model value to 101.60; a 50 bp rise lowers it to 96.40.

$\text{EffDur} = (101.60 - 96.40) / (2 \times 99.10 \times 0.005) \approx 5.25$. $\text{EffCon} = (101.60 + 96.40 - 198.20) / (0.005^2 \times 99.10) \approx -80.7$, which is negative convexity.

Callable and putable bonds



Stylized price-yield curves of a callable bond (a) and a putable bond (b) against an otherwise identical option-free bond

In panel (a), the vertical gap between the curves is the value of the embedded call, which the investor has sold; it widens as yields fall. In panel (b) the gap is the value of the embedded put, which the investor owns; it widens as yields rise. At the other end of the yield range the option is worth little and each bond trades close to the option-free bond.

KEY CONCEPT

Feature	Callable bond	Putable bond
Option held by	Issuer	Investor
When the option matters	Low yields (call likely)	High yields (put likely)
Convexity	Negative convexity at low yields; positive at high yields	Always positive convexity
Effective duration vs. option-free bond	Lower at low yields (price capped near the call price)	Lower at high yields (price floored near the put price)

With negative convexity, the price gain from a fall in yield is smaller than the price loss from an equal rise in yield.

Common exam traps

- A negative effective convexity is the expected result for a callable bond or an MBS at low yields, not a calculation error.

LOS 63.b — Percentage price change for a change in the benchmark yield

The estimate has the same form as with yield-based measures:

$$\% \Delta PV^{Full} \approx -\text{EffDur} \times \Delta \text{Curve} + \frac{1}{2} \times \text{EffCon} \times (\Delta \text{Curve})^2$$

and for an option-free bond with yield-based measures, $\% \Delta PV \approx -\text{ModDur} \times \Delta YTM + \frac{1}{2} \times \text{Convexity} \times (\Delta YTM)^2$. The first term is the **duration effect**; the second is the **convexity effect** (positive for positive convexity, whatever the direction of the rate change). Convert basis points to decimals (35 bp = 0.0035).

Worked example. A bond has EffDur 7.1 and EffCon 62, and the benchmark curve falls 90 bp: duration effect = +6.39%; convexity effect = $0.5 \times 62 \times 0.009^2 = +0.2511\%$; estimate = +6.6411%. With duration alone the estimate would be +6.39%, too low.

Worked example (negative convexity). The callable bond from LOS 63.a has EffDur 5.25 and EffCon -80.7. For a 100 bp parallel shift the convexity effect is $0.5 \times (-80.7) \times 0.01^2 = -0.40\%$ in both directions. A rise in the benchmark curve gives $-5.25\% - 0.40\% = -5.65\%$; a fall gives $+5.25\% - 0.40\% = +4.85\%$, the asymmetry described in LOS 63.a.

To get a price, multiply the current price by $(1 + \% \Delta)$; to get a currency change, multiply the current value by $\% \Delta$.

Unlike yield-based measures, effective duration and convexity do not necessarily give better estimates for smaller rate changes, because factors other than the level of benchmark rates (credit spreads on a corporate bond, the principal outstanding on a mortgage) affect whether the option is exercised.

Common exam traps

- Forgetting the $\frac{1}{2}$, or giving the convexity effect the sign of the rate change instead of the sign of convexity. For the 90 bp fall in the example (EffDur 7.1, EffCon 62), leaving out the $\frac{1}{2}$ gives +6.8922%, and giving the convexity effect a negative sign gives +6.1389%, instead of +6.6411%.
- Sign errors in the duration effect: a fall in rates raises prices. In the same example, reversing the duration effect gives -6.1389% instead of +6.6411%.
- Using modified duration for a callable bond when an effective duration is given.

LOS 63.c — Key rate duration and shaping risk

Effective duration captures only **parallel shifts** of the benchmark curve. A **key rate duration** (also called a **partial duration**) is the sensitivity of a bond's or portfolio's value to a change in the benchmark yield at one specific maturity, holding all other yields constant. The key rate durations sum to the effective duration.

Key rate durations measure **shaping risk**: the effect of a nonparallel shift (steepening, flattening, twist) on a portfolio. The effect of each key rate's change is estimated separately and the effects are added:

$$\% \Delta V \approx - \sum_k \text{KeyRateDur}_k \times \Delta r_k$$

For a portfolio of cash flows, the key rate duration at a maturity is the modified duration of the cash flow at that maturity multiplied by its weight in the portfolio.

Worked example. Half the portfolio is in a 2-year zero yielding 3%, half in a 7-year zero yielding 4.5% (annual compounding). $\text{KRD}(2) = 0.5 \times 2/1.03 = 0.971$; $\text{KRD}(7) = 0.5 \times 7/1.045 = 3.349$. If the 2-year yield rises 40 bp and the 7-year yield falls 10 bp: $-0.971(0.004) - 3.349(-0.001) = -0.388\% + 0.335\% \approx -0.05\%$.

The two key rate durations add up to $0.971 + 3.349 = 4.320$, the portfolio's effective duration. A parallel 40 bp rise would cost about $4.320 \times 0.40\% = 1.73\%$. The twist costs far less because the two key rates moved in opposite directions.

Example: same effective duration, different shaping risk. The three portfolios below all have an effective duration of 5.00, so a parallel shift affects them equally.

Key rate durations of three portfolios with the same effective duration (illustrative)

Key rate maturity	Portfolio X	Portfolio Y	Portfolio Z
2-year	2.10	0.50	1.25
5-year	0.30	0.60	1.25
10-year	0.20	3.40	1.25
20-year	2.40	0.50	1.25
Sum = effective duration	5.00	5.00	5.00

Suppose only the 5-year and 10-year key rates rise by 20 bp. Each portfolio loses about $(\text{KRD}_5 + \text{KRD}_{10}) \times 0.20\%$: X loses $0.50 \times 0.20\% = 0.10\%$, Y loses $4.00 \times 0.20\% = 0.80\%$ and Z loses $2.50 \times 0.20\% = 0.50\%$. A manager expecting this change would prefer X, whose exposure sits at the maturities that do not move.

KEY CONCEPT

Measure	Yield change it captures
Macaulay duration	None directly: a weighted-average time in years, the horizon at which price and reinvestment risk offset
Modified duration	Change in the bond's own YTM
Money duration and PVBP	Change in the bond's own YTM, measured in currency units

Measure	Yield change it captures
Effective duration	Parallel shift in the benchmark curve
Key rate duration	Change in one maturity's benchmark rate
Empirical duration (LOS 63.d)	Benchmark changes as observed in historical prices, including any spread moves that came with them

LOS 63.d — Empirical versus analytical duration

Macaulay, modified and effective duration are **analytical durations**: they come from mathematical analysis of the bond's cash flows and assume, for credit-risky bonds, that the spread does not change when benchmark yields move (spread and benchmark changes are uncorrelated). **Empirical duration** is estimated statistically from observed data, that is, from how bond prices have actually responded to past changes in benchmark yields.

When the uncorrelated-spread assumption fails, empirical duration is more appropriate. In a **flight to quality**, government yields fall while credit spreads widen, so corporate bond prices rise less than analytical duration predicts (or even fall). The empirical duration of a corporate bond portfolio is then lower than its analytical duration. For a portfolio of government bonds, spreads are not a factor, so empirical and analytical durations should be similar and analytical duration remains appropriate.

Common exam traps

- Treating effective duration as empirical because it comes from a pricing model. It is analytical; only a duration estimated from historical price data (for example, by regression) is empirical.

EXAM SHORTCUTS

- For a portfolio of zero-coupon bonds with annual compounding, each key rate duration is the zero's portfolio weight times its maturity divided by one plus its yield, as in the 2-year and 7-year example, so no repricing is needed.

BOTTOM LINE

- Yield-based duration and convexity assume fixed, known cash flows, so callable bonds, putable bonds and mortgage-backed securities are measured with effective duration and effective convexity against shifts in a benchmark yield curve.
- $\text{EffDur} = (V_- - V_+) / (2V_0 \Delta\text{Curve})$ and $\text{EffCon} = (V_- + V_+ - 2V_0) / [(\Delta\text{Curve})^2 V_0]$, with V_- and V_+ produced by a pricing model.
- Effective duration measures sensitivity to the benchmark curve only and holds the credit and liquidity spread constant, while modified duration does not separate benchmark changes from spread changes.
- A callable bond has negative convexity and a lower effective duration than the option-free bond at low yields, where the call price caps its price; a putable bond always has positive convexity and has a lower effective duration at high yields, where the put price floors its price.
- $\% \Delta PV^{\text{Full}} \approx -\text{EffDur} \times \Delta\text{Curve} + \frac{1}{2} \times \text{EffCon} \times (\Delta\text{Curve})^2$, and the convexity effect takes the sign of the convexity whatever the direction of the rate change.

- A key rate duration is the sensitivity of value to the benchmark yield at one maturity with all other yields held constant; key rate durations sum to effective duration and measure shaping risk from nonparallel shifts.
- Analytical durations assume that credit spreads do not change when benchmark yields move, while empirical duration is estimated from historical prices; in a flight to quality a corporate bond portfolio's empirical duration is lower than its analytical duration, and for government bonds the two should be similar.

QUICK CHECK

Question 1

Effective duration is a better measure than modified duration of a bond's price sensitivity to interest rates when:

- A the yield curve shifts in a nonparallel way.
- B the bond has an embedded option.
- C the bond has a long maturity and a low coupon rate.

Answer: B

Effective duration uses model values that reflect how an embedded option changes expected cash flows as rates move; modified duration ignores that effect. For option-free bonds the two measures are similar. Both, however, assume a parallel shift in the curve.

Why the other options are wrong

- **A.** Neither effective nor modified duration handles nonparallel shifts; key rate durations do.
- **C.** A long maturity and low coupon raise duration and convexity, but for an option-free bond modified duration is still appropriate.

Key takeaway Effective duration matters when cash flows depend on rates (embedded options).

Practice questions

Question 2

Interest rates fall by 65 basis points. Given a modified duration of 6.72 and a convexity of 55.30, the estimated percentage change in the bond's price is *closest* to:

- A 4.48%.
- B 4.37%.
- C 4.25%.

Question 3

A key rate duration measures the sensitivity of a bond's value to a:

- A parallel shift in the benchmark yield curve.
- B change in the bond's expected cash flows.
- C change in the benchmark yield at one specific maturity.

Question 4

To estimate the interest rate sensitivity of a corporate bond fund, an analyst regresses the fund's past returns on past changes in benchmark government yields. The resulting measure is *best* described as:

- A modified duration.
- B analytical duration.
- C empirical duration.

Answers: 2. A · 3. C · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/curve-based-and-empirical-fixed-income-risk-measures/. The full bank has 34 questions on this reading.

FIXED INCOME · READING 64

Credit Risk

What you'll learn

LOS 64.a Describe credit risk, the Cs of credit analysis, and how probability of default and loss given default combine into expected loss.

LOS 64.b Describe how credit ratings are used and the limitations of relying on them.

LOS 64.c Describe macroeconomic, market (liquidity) and issuer-specific factors that affect the level and volatility of yield spreads.

MODULE 64.1

Credit Risk

This reading defines credit risk and its components, explains what credit ratings do and do not tell an investor, and identifies what drives the level and volatility of yield spreads. A candidate must be able to compute expected loss and a fair credit spread from the probability of default and the recovery rate, split a yield spread into liquidity and credit parts, and estimate the price effect of a spread change.

LOS 64.a — Credit risk, probability of default and loss given default

Credit risk is the risk that a borrower does not make its promised interest or principal payments, which leaves fixed-income investors with a loss. A borrower that fails to service its debt is in **default**.

The Cs of credit analysis

Analysts sort the drivers of credit risk into two groups.

KEY CONCEPT

Bottom-up credit analysis (borrower-specific)

Capacity — the borrower's ability to make its debt payments on time (profitability, coverage, leverage)

Capital — other resources that reduce the borrower's reliance on debt

Collateral — the value of assets pledged as security for the lender if the borrower defaults

Covenants — the legal terms and conditions agreed between borrower and lenders in the bond issue

Character — the borrower's integrity (for a company, its management) and commitment to pay

Top-down credit analysis (environment)

Conditions — the general economic environment that affects every borrower's ability to pay

Country — the geopolitical, legal and political setting that applies to the debt

Currency — exchange-rate moves that affect the ability to service foreign-currency debt

Sources of repayment. At bottom, credit risk is the chance that the borrower's sources of repayment do not generate enough cash to service the debt.

Borrower	Primary sources of repayment	Secondary sources	Credit risk rises with
Corporate issuer	Operating cash flows and investments. Secured debt also has the cash flows from the specific collateral pledged; unsecured debt has no such claim	Asset sales, divestitures of subsidiaries, new debt or equity	Weak economic and market conditions, stronger competition, low profitability, excessive debt
Sovereign	Taxes, tariffs and other fees	New borrowing, privatizations (sales of public assets)	A weak economy, political uncertainty, fiscal deficits, high debt relative to the size of the economy

An issuer that is **illiquid** (unable to raise cash to pay now) is not necessarily **insolvent** (assets worth less than debt), yet an illiquid issuer can still default.

A **cross-default clause** makes a default on one bond a default on all issues; a **pari passu** clause makes bonds of the same type rank equally in default. With both clauses in place, a default on any unsecured issue gives all unsecured holders a claim on the issuer's general assets. Secured holders get a claim on the general assets and on their pledged collateral, so they suffer a credit loss only if the collateral is worth less than the pari passu secured debt it backs.

Measuring credit risk

$$\text{expected loss} = \text{probability of default} \times \text{loss given default}$$

- **Probability of default (PD)**: the (usually annualized) probability that the borrower misses a payment of interest or principal.
- **Loss given default (LGD)**: the loss suffered if default occurs, as a money amount or as a rate (**LGD%**).
- **Recovery rate**: the proportion of the claim investors recover after default.
- **Loss severity**: the proportion not recovered: $1 - \text{recovery rate}$.
- **Expected exposure (exposure at default)**: the money amount the investor is owed (principal plus accrued interest) net of available collateral value.

$$\text{LGD (in money)} = \text{expected exposure} \times (1 - \text{recovery rate})$$

$$\text{LGD\% (rate)} = \text{loss severity} = 1 - \text{recovery rate}$$

The **expected loss** in money terms is $\text{PD} \times \text{exposure} \times (1 - \text{RR})$, and the **expected loss rate** (per unit of exposure) is $\text{PD} \times (1 - \text{RR})$. Because expected loss is the compensation investors need for bearing default risk, the expected loss rate is used as an estimate of the fair **credit spread** (a rate, so no money amount enters):

KEY CONCEPT

$$\text{credit spread} \approx \text{PD} \times \text{LGD\%} = \text{PD} \times (1 - \text{recovery rate})$$

If the bond's actual spread over a risk-free benchmark is above this estimate, investors are more than fairly compensated; if it is below, they are under-compensated and should avoid the bond.

Exam convention: loss given default is often given as a rate (LGD%) and multiplied directly by the probability of default, which gives expected loss as a percentage. Current practice: LGD is stated either as a money amount or as a percentage of exposure at default, and the percentage form is the loss severity, $1 - \text{recovery rate}$.

Worked example. With a PD of 2% and a recovery rate of 60%, LGD% is 40% and expected loss = $0.02 \times 0.40 = 0.80\%$. A bond trading at a spread of 1.10% more than covers this estimate.

Worked example (money terms). A bondholder is owed \$5,000,000 of principal plus \$100,000 of accrued interest, and collateral worth \$1,500,000 is available to repay the claim.

1. Expected exposure = $5,100,000 - 1,500,000 = \$3,600,000$.
2. With a 30% recovery rate, loss severity is 70%, so $\text{LGD} = 3,600,000 \times 0.70 = \$2,520,000$.
3. With an annual PD of 1.5%, expected loss = $0.015 \times 2,520,000 = \$37,800$ a year.

What drives the inputs. PD depends on capacity metrics: high EBIT margin, high EBIT/interest coverage, low debt/EBITDA and high cash flow relative to debt all indicate a low PD. LGD depends on security and seniority: senior secured debt loses less in default than junior unsecured debt. Investment grade issuers have a lower PD than high-yield issuers (rating categories defined under LOS 64.b), but high-yield issuers often issue secured debt, so their LGD can be lower than that of an investment grade issuer's unsecured bonds. For unsecured investment grade debt the main worry is a rise in PD as the issuer's finances deteriorate.

Common exam traps

- Expected loss uses the loss severity, $1 - \text{recovery rate}$, not the recovery rate itself. A higher recovery rate lowers expected loss and spreads. In the first worked example, using the 60% recovery rate gives 1.20% instead of 0.80%.
- Yield volatility is an input to interest rate (price) risk. It plays no part in expected credit loss.

LOS 64.b — Uses and limitations of credit ratings

Credit rating agencies assign forward-looking ratings to issuers and to individual issues. Ratings are used to compare credit risk across issuers, industries and bond types; to track changing credit conditions and **credit migration risk** (the risk that a downgrade lowers a bond's value and triggers contractual clauses); and to satisfy regulatory, statutory or contractual rules. A credit-linked coupon, which rises when the issuer is downgraded (Reading 52), is one example of a contractual clause that a downgrade can trigger.

KEY CONCEPT

Scale (S&P and Fitch / Moody's)	Category
AAA/Aaa down to BBB-/Baa3	Investment grade
BB+/Ba1 and below	Non-investment grade (high-yield, "junk")
D (S&P, Fitch); Moody's lowest category C	In default

The full scale, notch by notch:

Credit rating scales of the three major agencies

Moody's	S&P	Fitch	Category
Aaa	AAA	AAA	Investment grade (highest rating)
Aa1, Aa2, Aa3	AA+, AA, AA-	AA+, AA, AA-	Investment grade
A1, A2, A3	A+, A, A-	A+, A, A-	Investment grade
Baa1, Baa2, Baa3	BBB+, BBB, BBB-	BBB+, BBB, BBB-	Investment grade
Ba1, Ba2, Ba3	BB+, BB, BB-	BB+, BB, BB-	Non-investment grade (high-yield)
B1, B2, B3	B+, B, B-	B+, B, B-	Non-investment grade (high-yield)
Caa1, Caa2, Caa3	CCC+, CCC, CCC-	CCC	Non-investment grade (high-yield)
Ca	CC	CC	Non-investment grade (high-yield)
C (also covers bonds in default)	C	C	Non-investment grade (high-yield)
(included in C)	D	D	In default

Limitations of ratings

- Ratings lag market pricing. Spreads and prices move faster than ratings, and two bonds with an identical rating may still trade at different yields, since ratings focus on expected loss while distressed-debt prices focus on default timing and recoveries.
- Some risks are hard to assess: litigation, natural disasters, environmental events, acquisitions and debt-financed buybacks. Agencies may disagree, producing **split ratings**.
- Agencies make mistakes, e.g., subprime mortgage securities before 2008–2009, and sudden defaults after corporate fraud.

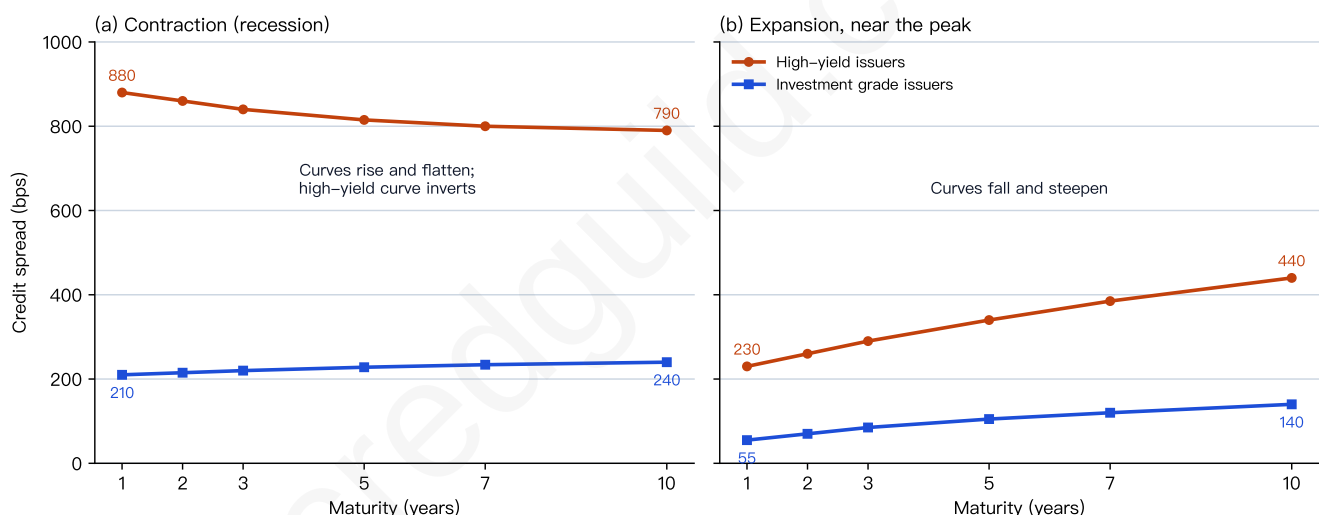
Investors should also carry out their own due diligence; those who correctly anticipate rating changes earn better results than those who trade only after a change is announced.

LOS 64.c — Factors that drive the level and volatility of yield spreads

Credit spread risk means the risk that yield spreads widen and prices of credit-risky bonds fall. For investment grade investors it is the main practical concern, because a sudden default is unlikely.

Macroeconomic factors

- Strong growth and high profits lower PD, so spreads narrow; a recession raises PD, so spreads widen.
- Investment grade spreads are lower than high-yield spreads (lower expected loss), and spreads usually rise with maturity because the probability of default grows over longer horizons. A **credit spread curve** plots credit spread against maturity, so it normally slopes upward.
- In a contraction, credit curves rise and flatten, and the high-yield curve may even invert. In an expansion, credit curves fall and steepen; they are lowest and steepest at the peak of the cycle.
- High-yield spreads are more dispersed across issuers and fluctuate more than investment grade spreads. In a crisis, investors sell risky assets and buy safe ones (**flight to quality**), so high-yield spreads can widen dramatically. Their bid-offer spreads widen more too, and the rise in risk aversion can spill over into wider bid-offer spreads on investment grade bonds.
- Why hold high-yield anyway: the larger spread, diversification (low or negative correlation with investment grade bonds), capital appreciation in recoveries, and, on some evidence, equity-like returns with lower volatility than equities.
- Other systematic factors that push spreads up: tighter regulation of dealers (higher cost of holding bond inventory), funding stresses that raise risk aversion, and heavy new issuance that is not matched by demand. Conversely, strong demand and light supply narrow spreads.



Stylized credit spread curves in a contraction (a) and near the peak of an expansion (b)

The figure sketches the credit spread curves of both rating groups in a contraction and near the peak of an expansion, with illustrative numbers.

Issuer-specific factors

An issuer with problems servicing its debt trades at a wider spread than the average for its rating; comparing an issuer's spread with its rating-category average highlights issuer-specific concerns.

Market factors: liquidity

Market liquidity risk is the risk that trading a bond is costly because the investor sells below (or buys above) the bond's fair midprice. It shows up in the dealers' **bid-offer spread**. It is higher for less actively traded bonds, lower-rated issuers and issuers with little debt outstanding. The two "spreads" differ: the **yield spread** is extra yield over the benchmark; the bid-offer spread is a price difference.

Splitting a yield spread. Compute the yield at the bid price and at the offer price; the difference is the **liquidity spread**, and the remainder of the yield spread is attributed to credit risk.

Worked example. A 5-year, 4% annual-coupon bond is quoted 99.00 bid / 99.80 offer; the benchmark yields 2.80%.

- At the mid price of 99.40 the yield is 4.135%, so the yield spread is about 1.335% (133.5 bp).
- Bid yield ($P/Y = C/Y = 1$, END mode): $N = 5$; $PV = -99.00$; $PMT = 4$; $FV = 100$; CPT $I/Y = 4.226$.
Offer yield, changing only the price: $PV = -99.80$; CPT $I/Y = 4.045$.
- Liquidity spread $\approx 4.226\% - 4.045\% = 0.181\%$, or 18.1 bp; credit component $\approx 133.5 - 18.1 = 115.4$ bp.

Building blocks of a corporate bond's yield (worked example above)

Building block	Size	How it is found
Benchmark yield	2.800%	yield on the government benchmark
• Liquidity spread	0.181%	yield at the bid price minus yield at the offer price
• Credit spread	1.154%	yield spread minus liquidity spread
= Yield spread	1.335%	bond yield at the mid price minus benchmark yield
Bond yield	4.135%	benchmark yield plus yield spread

Price impact of a spread change. Duration and convexity apply to spread changes just as to yield changes:

$$\% \Delta P_{\text{full}} \approx -\text{ModDur} \times \Delta \text{Spread} + \frac{1}{2} \text{Convexity} \times (\Delta \text{Spread})^2$$

E.g., ModDur 4.5, convexity 30, spread +40 bp: $-4.5(0.004) + 0.5(30)(0.004)^2 = -1.776\%$.

The formula can also be worked backward to find the spread change behind an observed price change. Start from the duration-only estimate, $-\% \Delta P / \text{ModDur}$, then test candidate values in the full formula. In the example, a 1.776% price fall gives a duration-only estimate of $1.776\% / 4.5 = 0.395\%$, or 39.5 bp of widening. The convexity term cushions the fall, so the spread in fact widened by slightly more, 40 bp. For a price gain the error runs the other way: the duration-only estimate overstates how far the spread narrowed.

Because modified duration multiplies the spread change, a longer-duration bond loses more value than a shorter one for the same widening of its spread.

Common exam traps

- The liquidity spread is bid yield minus offer yield (the bid price is lower, so its yield is higher). Express it in the same units as the total spread before computing a share.
- An expected expansion, which narrows spreads, favors corporates over government bonds; an expected contraction favors government bonds.
- The convexity term is always added (positive for option-free bonds), whether spreads rise or fall. In the +40 bp example, subtracting it gives -1.824% instead of -1.776% .

EXAM SHORTCUTS

- Bid and offer yields come from the same TVM entries with only PV changed, so the second yield needs one new entry and one compute.
- For an option-free bond, with the benchmark yield and all other factors unchanged, back out a spread change from a price change by starting from $-\% \Delta P / \text{ModDur}$; for a price fall the true widening is slightly larger than this, and for a price gain the true narrowing is slightly smaller.

BOTTOM LINE

- Bottom-up credit analysis looks at the borrower's capacity, capital, collateral, covenants and character, and top-down analysis looks at conditions, country and currency.
- Expected loss = $\text{PD} \times \text{LGD}$, where in money terms $\text{LGD} = \text{expected exposure} \times (1 - \text{recovery rate})$ and the loss severity is $1 - \text{recovery rate}$.
- Exam convention: loss given default is often given as a rate (LGD%) and multiplied directly by the probability of default to give expected loss as a percentage; current practice states LGD as a money amount or as a percentage of exposure at default, the percentage form being the loss severity.
- The expected loss rate, $\text{PD} \times (1 - \text{recovery rate})$, estimates the fair credit spread; an actual spread above it means investors are more than fairly compensated, and one below it means they are under-compensated.
- Ratings from AAA/Aaa down to BBB-/Baa3 are investment grade and ratings of BB+/Ba1 and below are non-investment grade (high-yield).
- Credit ratings lag market prices, struggle with risks such as litigation, natural disasters and debt-financed buybacks, and can be wrong, so ratings are no substitute for an investor's own due diligence.
- Spreads narrow when growth and profits are strong and widen in a recession; in a contraction credit curves rise and flatten, and in an expansion they fall and steepen.
- $\% \Delta P_{\text{full}} \approx -\text{ModDur} \times \Delta \text{Spread} + \frac{1}{2} \text{Convexity} \times (\Delta \text{Spread})^2$, and the liquidity spread inside a yield spread equals the yield at the bid price less the yield at the offer price.

QUICK CHECK**Question 1**

To judge how likely Morrow Chemicals is to default, an analyst calculates its EBIT margin, its EBIT-to-interest expense ratio, and its debt-to-EBITDA ratio. This analysis is *most likely* an evaluation of the company's:

- A capacity.
- B collateral.
- C covenants.

Answer: A

Capacity is the borrower's ability to make its debt payments on time. It is assessed with quantitative measures of profitability, coverage and leverage. A high EBIT margin, high interest coverage and low debt/EBITDA indicate strong capacity and a low probability of default.

Why the other options are wrong

- **B.** Collateral analysis looks at the value of the specific assets pledged as security rather than at the issuer's income and leverage ratios.
- **C.** Covenant analysis reviews the legal terms and conditions of the bond agreement.

Key takeaway Ratio analysis of profits, coverage and leverage assesses capacity; pledged assets are collateral; legal terms are covenants.

Practice questions

Question 2

The trustees of the Linwood Teachers' Pension Plan propose to select corporate bonds using agency credit ratings alone. Which of the following is a limitation of relying solely on credit ratings?

- A** Agencies rate bond issuers but not individual bond issues.
- B** Bond prices and credit spreads can move well before a rating changes.
- C** Ratings let investors compare credit risk across industries and bond types.

Question 3

A portfolio manager at Corvel Asset Management expects the economy to move out of recession into a strong recovery over the coming year. Which change to her bond portfolio is *most appropriate*?

- A** Buy corporate bonds and sell government bonds.
- B** Sell corporate bonds and buy government bonds.
- C** Replace lower-rated corporate bonds with higher-rated ones.

Question 4

A corporate bond has a modified duration of 6.2 and a convexity of 52. If its credit spread widens by 60 basis points while benchmark yields are unchanged, the estimated percentage change in the bond's full price is *closest* to:

- A** -3.63%.
- B** -3.72%.
- C** -3.81%.

Answers: 2. B · 3. A · 4. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-risk/. The full bank has 20 questions on this reading.

FIXED INCOME · READING 65

Credit Analysis for Government Issuers

What you'll learn

LOS 65.a Explain the qualitative and quantitative factors in sovereign credit analysis and the special features of non-sovereign government issuers (agencies, supranationals, regional GO and revenue bonds).

MODULE 65.1

Credit Analysis for Government Issuers

This reading explains what to look for when assessing the credit of sovereign governments and of non-sovereign government issuers. A candidate must be able to name and apply the five qualitative and three quantitative factors of sovereign credit analysis, compute debt burden and debt affordability ratios, and compare general obligation bonds with revenue bonds.

LOS 65.a — Credit analysis of sovereign and non-sovereign government issuers

Sovereign (national government) debt

A national government services its **sovereign debt** mainly through its power to tax economic activity within its borders. Sovereign credit analysis therefore asks whether the country has the conditions for stable growth with low inflation. The evidence falls into five qualitative and three quantitative factors.

Five qualitative factors

KEY CONCEPT

Factor	What the analyst looks for
Institutions and policy	Rule of law, property rights, a culture of repaying debt, transparent and consistent data, business-friendly policy, political stability and peaceful relations with neighbors. The government's willingness to pay also counts, which matters because of sovereign immunity (bondholders usually cannot sue a government that refuses to pay)
Fiscal flexibility	Ability to raise taxes or cut spending to keep making debt payments
Monetary effectiveness	A credible central bank able to vary money supply and interest rates to support stable growth; an independent central bank is less likely to print money to pay government debt, lowering the risk of high inflation and a weak currency
Economic flexibility	Growth trends, income per capita, and diversity of the sources of growth
External status	Standing of the currency internationally. A reserve currency (one that central banks around the world widely hold in their foreign exchange reserves) gives a country more ability to issue debt to foreign investors in its own currency and to maintain larger budget deficits and debt; also geopolitical risk

Three quantitative factors

KEY CONCEPT

Factor	Stronger credit shown by
Fiscal strength	Low debt burden (debt/GDP, debt/revenue) and good debt affordability (low interest/GDP, low interest/revenue)
Economic growth and stability	High real GDP growth, large economy, high GDP per capita, low volatility of growth
External stability	High FX reserves relative to GDP and to external debt; low long-term external debt/GDP; low external debt due within 12 months relative to GDP

FX reserves typically build up in countries with a **current account surplus**. If exports are concentrated in one commodity, the sovereign's credit risk becomes tied to that commodity's price.

Worked example. A country has GDP of 2,000, government debt of 1,100, government revenue of 640 and annual interest cost of 38 (all in billions). Debt burden: $1,100/2,000 = 55\%$ of GDP; debt affordability: $38/640 = 5.9\%$ of revenue. A peer with debt/GDP of 50% but interest/revenue of 8% has the lighter debt burden yet the weaker debt affordability, so the two measures can point in different directions.

Non-sovereign government debt

Issuer type	Credit profile
Agencies (quasi-government entities, e.g., for infrastructure)	Backed by law with implicit government support; ratings usually close to the sovereign
Government sector banks / financing institutions (e.g., issuing green bonds)	Implied government support; ratings similar to the sovereign
Supranational issuers (e.g., the World Bank), set up by groups of sovereign governments	Ratings depend on the implicit support of the sponsoring governments and institutions
Regional governments (provinces, states, local governments — municipal bonds in the United States)	Analyzed on their own tax base or project cash flows

KEY CONCEPT

Regional government bonds are mostly of two kinds:

- **General obligation (GO) bonds:** unsecured bonds backed by the **full faith and credit** of the issuing government, i.e., by its taxing power.
- **Revenue bonds:** issued to finance a specific project (airport, toll bridge, hospital, power plant) and serviced from that project's revenues.

Regional governments, unlike sovereigns, have no monetary policy tools (they cannot print money) and usually must balance their operating budgets. Their ability to service GO debt depends ultimately on the local economy (the tax base). Revenue bonds often carry more credit risk than GO bonds, since repayment comes only from

the single project.

Revenue bond analysis resembles corporate credit analysis: it focuses on the project's cash flows and the **debt-service coverage ratio**, which is revenue after operating costs divided by required interest and principal payments. The higher the ratio, the safer the bond.

Common exam traps

- Taxing power does not separate a sovereign from a state or city, since both levy taxes. The difference is that a sovereign that issues its own currency can print money.
- A revenue bond is not safer because a named project backs it. One project is a narrower source of repayment than a whole tax base.
- The direction of each ratio matters: a higher debt/GDP or interest/revenue ratio means weaker fiscal strength, while higher FX reserves relative to external debt mean stronger external stability.
- Three pairs of factor names look alike but belong to different groups: fiscal flexibility (qualitative) and fiscal strength (quantitative); economic flexibility (qualitative) and economic growth and stability (quantitative); external status (qualitative) and external stability (quantitative).

BOTTOM LINE

- A national government services its debt mainly through its power to tax, so sovereign credit analysis asks whether the country has the conditions for stable growth with low inflation.
- The five qualitative factors of sovereign credit are institutions and policy (including the government's willingness to pay, given sovereign immunity), fiscal flexibility, monetary effectiveness, economic flexibility and external status.
- The three quantitative factors are fiscal strength (a low debt burden and good debt affordability), economic growth and stability, and external stability (high FX reserves relative to GDP and to external debt, and low external debt relative to GDP).
- A higher debt/GDP or interest/revenue ratio means weaker fiscal strength, and debt burden and debt affordability can point in different directions for the same country.
- A country with a reserve currency can more easily issue debt to foreign investors in its own currency and can maintain larger budget deficits and debt.
- Agencies and government sector banks carry implied government support and ratings close to the sovereign's, while supranational ratings depend on the implicit support of the sponsoring governments and institutions.
- General obligation bonds are unsecured and backed by the issuing government's taxing power, while revenue bonds are serviced only from one project's revenues and often carry more credit risk.
- Regional governments cannot print money and usually must balance their operating budgets, and revenue bonds are analyzed like corporate bonds, with a higher debt-service coverage ratio (project revenue net of operating costs, divided by the interest and principal due) meaning a safer bond.

QUICK CHECK**Question 1**

Salvarra's central bank operates independently of the government and has a credible record of adjusting interest rates and the money supply to keep inflation low. In a sovereign credit analysis of Salvarra, this strength is best classified under which qualitative factor?

- A** Fiscal flexibility.
- B** External status.
- C** Monetary effectiveness.

Answer: C

Monetary effectiveness concerns the central bank's ability to vary the money supply and interest rates credibly to support stable growth. An independent central bank is less likely to print money to service government debt, which reduces the risk of high inflation and currency weakness.

Why the other options are wrong

- **A.** Fiscal flexibility is the government's ability to raise taxes or cut spending to keep up debt payments. That is a budget issue rather than a central bank issue.
- **B.** External status concerns the international standing of the currency (e.g., reserve currency status) and geopolitical risk.

Key takeaway Central bank credibility and independence belong to monetary effectiveness, room to tax or cut spending to fiscal flexibility, and reserve currency status to external status.

Practice questions

Question 2

An analyst compares the fiscal strength of two sovereign issuers using the following data.

Selected fiscal data for two sovereign issuers (USD billions)

Item	Kestria	Varnholm
Nominal GDP	1,200	500
Government debt	780	290
Government revenue	420	145
Annual interest expense on government debt	29.4	13.05

Based on the debt burden (government debt to GDP) and debt affordability (interest expense to government revenue), which statement is *most accurate*?

- A Kestria is stronger on both debt burden and debt affordability.
- B Varnholm has the lighter debt burden, but Kestria has better debt affordability.
- C Varnholm is stronger on both debt burden and debt affordability.

Question 3

The regional government of Varenna sets up a transit authority to build a light-rail line. The authority issues bonds whose interest and principal are to be paid from the line's fare revenue. The national government also guarantees the bonds as a backstop. These bonds are *best* described as:

- A general obligation bonds.
- B revenue bonds.
- C sovereign bonds.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-analysis-for-government-issuers/. The full bank has 8 questions on this reading.

FIXED INCOME · READING 66

Credit Analysis for Corporate Issuers

What you'll learn

LOS 66.a Describe the qualitative (business model, competition, business risk, governance) and quantitative factors used to assess a corporate borrower's creditworthiness.

LOS 66.b Calculate and interpret EBITDA, FFO, RCF and the profitability, coverage and leverage ratios used in credit analysis.

LOS 66.c Describe seniority rankings, secured versus unsecured debt, the priority of claims in bankruptcy, and how they affect issue ratings (notching).

MODULE 66.1

Credit Analysis for Corporate Issuers

This reading applies credit analysis to corporate issuers: the qualitative and quantitative factors, the financial ratios analysts use, and how seniority and collateral affect recovery and ratings. A candidate must be able to compute and interpret profitability, coverage and leverage ratios, rank debt by priority of claims, work out recoveries in a liquidation, and explain notching and structural subordination.

LOS 66.a — Qualitative and quantitative factors in corporate credit analysis

Qualitative factors

Factor	Signs of high credit quality
Business model	Stable, predictable cash flows; for long-dated debt, consider whether the model must change to stay competitive (change adds business risk)
Industry competition	Less intense competition, now and over the life of the debt
Business risk	Low risk of unexpected deviations from expected revenues and margins (from issuer, industry or external sources)
Corporate governance	Fair and legal treatment of debtholders, judged through covenants and accounting policies

Covenants. Unsecured investment-grade issuers typically have mainly **affirmative covenants** (comply with laws, maintain assets, pay taxes), so analysts must judge the risk that management adds debt that dilutes existing lenders. High-yield issuers usually also have **negative covenants** restricting dividends and new debt. For them, analysts look for past evidence of favoring shareholders over lenders, such as a debt-financed buyback that led to a downgrade.

Accounting policies. Fraud is the obvious concern. Other warning signs about management's character include aggressive revenue recognition, heavy off-balance-sheet financing, a strong preference for capitalizing rather than expensing, and frequent changes of auditor or CFO.

Quantitative factors

Analysts forecast financial statements and cash flows to see what drives the **probability of default** and **loss given default** over the economic cycle. Investors in unsecured investment-grade debt worry mostly about the probability of default rising. Investors in secured high-yield debt also focus on loss given default, because default is more likely for high-yield issuers. Inputs can be **top-down** (macro cycle, industry size, market share, event risk), **bottom-up** (issuer revenue, costs, assets, liabilities, cash flows), or a **hybrid** of both.

All else equal, credit quality is higher with strong operating profits and recurring revenues, low leverage, high coverage of debt service by income, and high liquidity for short-term obligations.

In a bottom-up analysis these features come from the three financial statements: revenue and operating profit from the income statement, leverage and liquidity from the balance sheet, and the cash available to service debt from the cash flow statement. The profitability, coverage and leverage ratios in LOS 66.b measure the first three features.

LOS 66.b — Financial ratios used in credit analysis

Ratios let analysts judge a company's creditworthiness, follow trends over time and compare the company with industry averages and peers. The main groups are profitability, coverage and leverage.

Building blocks

KEY CONCEPT

Measure	Definition
EBITDA	Operating income + depreciation and amortization. Drawback: ignores capital expenditures and working capital needs
Cash flow from operating activities (CFO)	Net income + noncash charges – increase in working capital (from the cash flow statement)
Funds from operations (FFO)	Net income from continuing operations + depreciation + amortization + deferred taxes + other noncash items (like CFO but excludes working capital changes)
Free cash flow (FCF)	CFO – fixed asset expenditures + net interest expense; the discretionary cash that could go to providers of financing once all the company's obligations are met
Retained cash flow (RCF)	Operating cash flow (CFO, FFO or another chosen measure) – dividends

Key ratios

KEY CONCEPT

Type	Ratio	Higher credit quality shown by
Profitability	EBIT margin = EBIT / revenue	Higher
Coverage	EBIT to interest expense	Higher
Leverage	Debt to EBITDA	Lower
Leverage	RCF to net debt = RCF / (debt – cash and marketable securities)	Higher

Example. A company reports (USD millions) revenue 4,000; EBIT 600; D&A 200; interest expense 75; total debt 2,400; cash 400; CFO 700; dividends 150.

- $\text{EBIT margin} = 600/4,000 = 15.0\%$
- $\text{EBIT/interest} = 600/75 = 8.0\times$
- $\text{Debt/EBITDA} = 2,400/(600 + 200) = 3.0\times$
- $\text{RCF/net debt} = (700 - 150)/(2,400 - 400) = 27.5\%$

Reading the results. Suppose a peer reports an EBIT margin of 11.0%, EBIT/interest of 9.0×, debt/EBITDA of 2.2× and RCF/net debt of 36%. The company is more profitable than the peer but more heavily indebted: its coverage is slightly weaker, its debt/EBITDA is higher and its RCF/net debt is lower. Mixed results like these are common, so the analyst weighs profitability against leverage instead of relying on any single ratio.

A multi-year average ratio can be taken as the simple average of the yearly ratios. Dividing total debt by total EBITDA over the same years gives a slightly different figure.

LOS 66.c — Seniority, secured vs unsecured debt, priority of claims and ratings

A bond's place in the **priority of claims** is its **seniority ranking**. **Secured debt** is backed by **collateral** and ranks ahead of **unsecured debt**, which is a general claim on the issuer's assets and cash flows.

KEY CONCEPT

Rank (highest first)	Category
1	First lien / first mortgage
2	Senior secured (second lien)
3	Junior secured
4	Senior unsecured
5	Senior subordinated
6	Subordinated

Rank (highest first)**Category**

7

Junior subordinated

Debt in the same category ranks **pari passu** (equal priority). Any part of a secured claim not covered by its collateral ranks *pari passu* with senior unsecured claims.

Example. A company is liquidated. Its senior secured lenders are owed \$200 million and their collateral sells for \$150 million. Senior unsecured bondholders are owed \$300 million and subordinated noteholders \$100 million. After the collateral is sold, \$270 million of other assets remain.

1. The secured lenders take the \$150 million from their collateral. Their \$50 million shortfall joins the senior unsecured claims.
2. The senior unsecured pool is $50 + 300 = \$350$ million, and it shares the \$270 million pro rata: $270/350 = 77.1\%$ of each claim.
3. Nothing is left for the subordinated notes or the shareholders (under strict priority).

The secured lenders recover $150 + 0.771 \times 50 = \188.6 million (94.3%), the senior unsecured bondholders 77.1%, and the subordinated noteholders nothing. Recovery rates fall, and credit risk rises, with each step down in seniority. In practice, strict priority is often not applied. Junior creditors and even shareholders may receive something before seniors are paid in full, because a negotiated settlement shortens a costly bankruptcy during which asset values tend to erode.

Issuer vs issue ratings. The **issuer credit rating**, called the **corporate family rating (CFR)**, reflects the borrower's overall creditworthiness and is typically based on its senior unsecured debt. **Issue credit ratings** apply to specific bonds (individual debt obligations). Issue credit ratings are also called **corporate credit ratings (CCRs)**. Setting an issue rating above or below the issuer rating is **notching**. Seniority and covenants (including collateral) drive it.

Issue feature**Likely notching**

Secured, strong protective covenants (e.g., limits on additional debt)

Upward (e.g., one notch above the CFR)

Subordinated, weak protection

Downward

Structural subordination. In a holding company group, a subsidiary's covenants may block cash transfers to the parent until the subsidiary's own debt is serviced. The subsidiary's bonds then have the first claim on the subsidiary's cash flows. The parent's bonds are therefore **effectively subordinated** to them with respect to those cash flows, even though they are not formally junior. Rating agencies consider structural subordination when notching.

Notching is more common for lower-rated issuers. When the probability of default is high, differences in expected recovery between issues matter more. For highly rated issuers they matter little, so the issues may not be notched at all.

Common exam traps

- EBITDA adds D&A to operating income (EBIT). Dividing debt by EBIT overstates leverage. In the example, debt/EBIT gives 4.0× instead of a debt/EBITDA of 3.0×.

- FFO starts from net income from continuing operations. It does not start from CFO and does not subtract capex or dividends.
- Debt/EBITDA is the one key ratio for which a lower value means stronger credit. Higher earnings, all else equal, lower it and raise the other three.
- RCF/net debt uses net debt. Dividing RCF by total debt, without subtracting cash and marketable securities, understates the ratio. In the example, dividing by total debt gives 22.9% instead of 27.5%.
- Second lien is still secured debt and ranks above all unsecured debt; subordinated debt ranks below senior unsecured.
- Bonds of the same seniority rank *pari passu* whatever their maturity or coupon, so a 3-year and a 20-year senior unsecured bond of one issuer have the same priority of claims in bankruptcy.
- Notching can be upward or downward.
- The rating typically based on senior unsecured debt is the issuer rating (CFR), not an issue rating.

BOTTOM LINE

- The qualitative factors in corporate credit analysis are the business model, industry competition, business risk and corporate governance, the last judged through covenants and accounting policies.
- All else equal, credit quality is higher with strong operating profits and recurring revenues, low leverage, high coverage of debt service by income and high liquidity for short-term obligations.
- EBITDA is operating income plus depreciation and amortization, FFO starts from net income from continuing operations and adds back noncash items, and RCF is the chosen operating cash flow measure minus dividends.
- Higher EBIT margin, EBIT/interest expense and RCF/net debt indicate higher credit quality, while for debt/EBITDA a lower value indicates higher credit quality; net debt is debt minus cash and marketable securities.
- The priority of claims runs first lien, senior secured (second lien), junior secured, senior unsecured, senior subordinated, subordinated, junior subordinated; debt in the same category ranks *pari passu*, and any secured claim not covered by collateral ranks with senior unsecured claims.
- Recovery rates fall and credit risk rises with each step down in seniority, although in practice strict priority is often not applied because a negotiated settlement shortens a costly bankruptcy.
- The issuer credit rating (corporate family rating) is typically based on senior unsecured debt, and issue ratings are notched above or below it according to seniority and covenants, with notching more common for lower-rated issuers.
- Under structural subordination, a subsidiary's own bonds have the first claim on its cash flows, so the parent's bonds are effectively subordinated to them for those cash flows even though they are not formally junior.

QUICK CHECK

Question 1

All else equal, which of the following would indicate *higher* credit quality for a corporate issuer?

- A** Higher leverage
- B** Higher liquidity

C Lower coverage of debt service**Answer: B**

Other things equal, companies have higher credit quality when they have strong operating profits and recurring revenues, low leverage, high coverage of debt service by income, and high liquidity to meet short-term debt payments. Higher liquidity means more cash and liquid assets to meet obligations that fall due soon.

Why the other options are wrong

- **A.** Higher leverage means greater reliance on debt in the capital structure, which points to lower credit quality.
- **C.** Lower coverage means income covers interest and principal payments less comfortably, which points to lower credit quality.

Key takeaway Signs of stronger corporate credit: strong, recurring profits; low leverage; high coverage; high liquidity.

Practice questions

Question 2

Tamworth Mills is being liquidated. It has three classes of debt outstanding: senior subordinated notes, first mortgage bonds and senior unsecured notes. Which list ranks them from highest to lowest priority of claims?

- A** First mortgage bonds, senior unsecured notes, senior subordinated notes.
- B** Senior unsecured notes, first mortgage bonds, senior subordinated notes.
- C** First mortgage bonds, senior subordinated notes, senior unsecured notes.

Question 3

How does an issuer credit rating *most likely* differ from an issue credit rating?

- A** An issuer credit rating reflects the borrower's overall creditworthiness.
- B** An issue credit rating is always set below the issuer credit rating.
- C** An issue credit rating is the one based on senior unsecured debt.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-analysis-for-corporate-issuers/. The full bank has 13 questions on this reading.

FIXED INCOME · READING 67

Fixed-Income Securitization

What you'll learn

LOS 67.a Explain the benefits of securitization for originators, investors, economies and financial markets, and the risks to ABS investors.

LOS 67.b Describe the securitization process and the roles of the seller, SPE (issuer/trust), servicer and trustee.

MODULE 67.1

Fixed-Income Securitization

This reading explains how securitization turns pools of loans into asset-backed securities and who takes part in the process. A candidate must be able to describe the benefits of securitization to originators, investors and the wider economy, and explain the roles of the seller, the SPE, the servicer and the trustee and why the SPE is bankruptcy remote.

LOS 67.a — Benefits of securitization

KEY CONCEPT

Securitization turns a pool of loans or receivables into tradable securities:

1. An **originator** (a bank making loans, or a company extending credit to customers) builds a pool of debt-based assets.
2. It sells the pool, called the **collateral**, to a separate legal entity, the **special purpose entity (SPE)**.
3. The SPE issues **asset-backed securities (ABS)** whose payments come from the borrowers' loan repayments, and sells them to investors such as pension and fixed-income funds.

Securitization links investors directly with the borrowers in the pool and takes the originator out of the intermediation chain.

The word "issuer" is used in two ways. In lists of the benefits of securitization, "issuers" are the originators. The legal issuer of the ABS is the SPE.

KEY CONCEPT

Who benefits	Benefit
Originators (issuers)	Increased business activity: sale proceeds are re-lent, so a bank can lend more than its own balance sheet would allow
	Improved profitability: fees for originating the loans and for selling them to the SPE
	Lower capital reserves: selling the loans removes their credit risk from the bank's balance sheet, so regulators require less capital
	Improved liquidity: illiquid loan portfolios can be sold

Who benefits	Benefit
Investors	Tailored risk and return to match their needs (e.g., long-horizon investors in long mortgage pools)
	Access to loan pools without the expertise to originate and service loans
	Liquidity: ABS are easier to sell than the underlying loans
Economies and financial markets	Decreased liquidity risk and improved market efficiency (tradable ABS allow equilibrium prices)
	Lower financing costs for originators than issuing their own debt or equity
	Lower leverage for originators, which can grow without adding debt to their balance sheets

Risks to ABS investors. Cash flows are uncertain in timing and size; for example, borrowers may prepay at an unexpected rate. The credit risk of the collateral also passes through to ABS investors, who bear defaults in the pool. A system-wide buildup of this risk contributed to the 2007–2009 financial crisis.

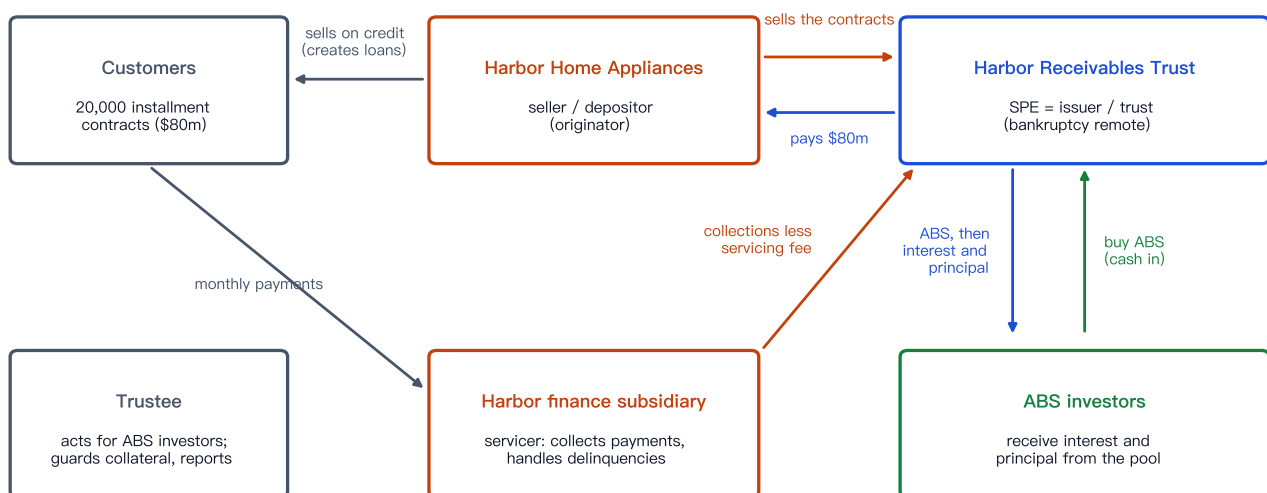
LOS 67.b — The securitization process and its parties

Example. Harbor Home Appliances sells washing machines on installment credit. It bundles 20,000 installment contracts worth \$80 million and sells them for \$80 million to Harbor Receivables Trust, an SPE created solely to buy the contracts and issue ABS. Harbor keeps collecting the monthly payments through a finance subsidiary.

KEY CONCEPT

Party	Role in the example
Seller / depositor (originator)	Harbor Home Appliances creates the loans and sells the pool to the SPE; it is the firm raising funds through the securitization
Issuer / trust (SPE, also special purpose vehicle (SPV) or special purpose company)	Harbor Receivables Trust buys the loans and issues ABS to investors
Servicer	Harbor's finance subsidiary collects payments, sends delinquency notices, and repossesses and sells collateral when borrowers default
Trustee	An independent (disinterested) party that safeguards the collateral and cash flows due to ABS holders and reports to them

The seller and servicer are often, but not always, the same company. The figure traces the flows in the example.



Structure of the Harbor Home Appliances securitization

Flow of cash. Collections of principal and interest on the pool first pay the **servicing fee** and other administrative fees. The rest goes to ABS investors as interest and principal, so investors receive less than the total payments collected from the pool.

Bankruptcy remote. The SPE is a **separate legal entity** that owns the collateral outright, so the assets are legally isolated from the seller. A decline in the seller's finances, even its bankruptcy, does not affect the ABS holders' claim on the collateral, and the seller's creditors cannot reach the pool. The SPE is therefore **bankruptcy remote**. In return, ABS holders have no claim on the seller's other assets. This separation can allow senior ABS to carry a higher credit rating than the seller's own corporate bonds, whose holders are exposed to all of the seller's risks.

Key documents. Besides the indenture and covenants, a securitization relies on the **purchase agreement** (the terms on which the SPE buys the collateral) and the **prospectus** (fees paid to servicers and administrators, and how collateral cash flows are distributed to ABS investors).

Common exam traps

- Removing liabilities from the balance sheet is not a benefit of securitization. Only assets (loans, receivables) are sold; deposits and bonds stay with the originator.
- Securitization does not make loans more collectible or protect investors from defaults in the pool. Credit enhancement may reduce that risk but does not eliminate it.
- The SPE's independence from the seller is legal: it is a separate legal entity that has bought the collateral outright. An SPE sponsored or even owned by the seller can still be bankruptcy remote if the structure isolates the assets; being a subsidiary or a joint venture is not what the requirement asks for.
- ABS do not rank "senior" to the seller's bonds in the seller's bankruptcy. They sit outside it, and their higher rating comes from the SPE structure and any credit enhancement.
- The servicer collects from borrowers; the trustee protects investors' interests.

BOTTOM LINE

- In a securitization, an originator sells a pool of loans or receivables, the collateral, to a special purpose entity, which issues asset-backed securities paid from the borrowers' loan repayments.
- Securitization lets originators increase business activity, earn fees, hold lower capital reserves and turn illiquid loan portfolios into cash.
- Investors gain risk and return tailored to their needs, access to loan pools without the expertise to originate and service loans, and securities that are easier to sell than the underlying loans.
- For economies and financial markets, securitization decreases liquidity risk, improves market efficiency and gives originators lower financing costs and lower leverage.
- ABS investors bear uncertainty in the timing and size of cash flows, for example from unexpected prepayments, and the credit risk of the collateral passes through to them.
- The seller (depositor) originates and sells the loans, the SPE (issuer or trust) buys them and issues the ABS, the servicer collects from borrowers, and the trustee safeguards the collateral and cash flows for ABS holders.
- The SPE owns the collateral outright as a legally separate entity, which makes it bankruptcy remote: the seller's creditors cannot reach the pool, and ABS holders have no claim on the seller's other assets.
- Collections from the pool pay the servicing fee and other administrative fees first, so ABS investors receive less than the total payments collected.

QUICK CHECK**Question 1**

Kestrel Savings Bank funds its mortgage lending mainly with customer deposits. It sells \$400 million of its mortgages to a special purpose entity, which pays for them with the proceeds of an ABS issue. As a direct result of the sale, which item on Kestrel's balance sheet is *least likely* to change?

- A** Mortgage loans.
- B** Customer deposits.
- C** Cash.

Answer: B

Securitization sells financial assets, not liabilities. Kestrel gives up \$400 million of mortgage loans and receives \$400 million of cash, so both of these asset lines change. The deposits that funded the loans stay on the balance sheet, because the SPE buys the loans and takes on none of the bank's obligations to its depositors. The bank benefits from the cash it can re-lend and from the lower capital it must hold once the loans are sold; removing deposits or other liabilities is not a benefit of securitization.

Effect of the sale on Kestrel's balance sheet:

- Mortgage loans: —\$400 million (sold to the SPE).
- Cash: +\$400 million (sale proceeds).
- Customer deposits and other liabilities: no change.

Why the other options are wrong

- **A.** The mortgages are the assets sold to the SPE, so this line falls by \$400 million.

- **C.** The SPE pays \$400 million for the mortgages, so cash rises by that amount until the bank re-lends it.

Key takeaway In a securitization the originator swaps loans for cash on the asset side; its deposits and other liabilities are not transferred.

Practice questions

Question 2

Brenlow Bank sells a pool of its auto loans to a newly created trust, which issues asset-backed securities (ABS). The senior ABS receive a higher credit rating than Brenlow's own senior bonds. The *most likely* reason is that:

- A** ABS are always investment grade, whereas bank bonds need not be.
- B** the trust is a special purpose entity that is bankruptcy remote from Brenlow.
- C** the ABS rank senior to Brenlow's bonds if Brenlow goes bankrupt.

Question 3

An investor buys ABS backed by a pool of consumer loans originated by Farrow Bank. Which of the following risks does the investor *most likely* bear?

- A** The credit risk of Farrow Bank's other lending businesses.
- B** The credit risk of the borrowers in the collateral pool.
- C** The risk that Farrow Bank's creditors seize the collateral.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/fixed-income-securitization/. The full bank has 9 questions on this reading.

FIXED INCOME · READING 68

Asset-Backed Security (ABS) Instrument and Market Features

What you'll learn

LOS 68.a Describe the characteristics and risks of covered bonds (including hard-bullet, soft-bullet and conditional pass-through) and how they differ from other ABS.

LOS 68.b Describe internal credit enhancement structures: overcollateralization, excess spread and subordination (credit tranching).

LOS 68.c Describe the cash flows and risks of non-mortgage ABS, such as credit card ABS and solar ABS.

LOS 68.d Describe collateralized debt obligations (CBOs, CLOs), their collateral manager, cash flows and risks.

MODULE 68.1

Asset-Backed Security (ABS) Instrument and Market Features

This reading compares covered bonds with other asset-backed securities, explains the internal credit enhancements that protect ABS investors, and describes non-mortgage ABS and collateralized debt obligations. A candidate must be able to contrast covered bonds with typical ABS, allocate collateral losses through a tranche waterfall, and describe credit card ABS, solar ABS and CLOs.

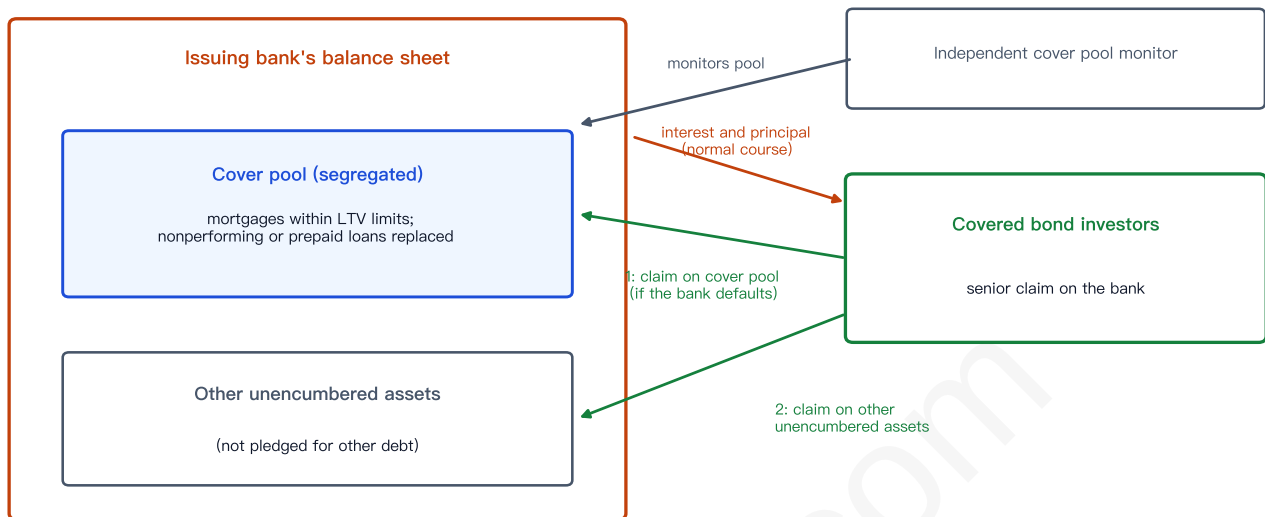
LOS 68.a — Covered bonds versus other ABS

Covered bonds are senior debt obligations of financial institutions (mainly European, Asian and Australian banks) backed by a segregated pool of assets, the **cover pool**, which usually consists of mortgages. No SPE is created, so the cover pool stays on the issuer's balance sheet. Because the bond is the bank's own debt, in the normal course the bank pays interest and principal from its general resources, and the cover pool is security that bondholders can turn to if the bank defaults.

KEY CONCEPT

Feature	Covered bond	Typical ABS
Where the assets sit	On the issuer's balance sheet (segregated)	Sold to a bankruptcy-remote SPE
Investor recourse	Dual recourse: the cover pool and the issuer's other unencumbered assets	The collateral pool only
Asset pool	Dynamic: the issuer must replace or add to nonperforming or prepaid assets so the pool keeps supporting the promised payments	Fixed at issuance
Credit tranching	Typically none	Common
Issuer's capital relief	None (assets stay on balance sheet)	Yes

Feature	Covered bond	Typical ABS
Yield	Generally lower than comparable ABS (higher credit quality)	Higher



Dual recourse, no SPE, dynamic pool: why covered bonds usually yield less than comparable ABS (illustrative).

How a covered bond is structured

Covered bond investors have further protection from loan-to-value limits on the mortgages in the pool, from overcollateralization (pool value above the bonds outstanding), and from monitoring of the pool by a third party.

If the issuer misses a payment, the outcome depends on the bond's terms:

KEY CONCEPT

Type	Treatment
Hard-bullet covered bond	In default as soon as a scheduled payment is missed; payments to bondholders are accelerated
Soft-bullet covered bond	The original maturity can be postponed by as much as a year, delaying default and acceleration
Conditional pass-through covered bond	Converts to a pass-through bond at maturity if payments remain due; later recoveries from the cover pool are passed through to investors

LOS 68.b — Internal credit enhancement

Internal credit enhancements are features built into the ABS structure to absorb defaults in the collateral pool.

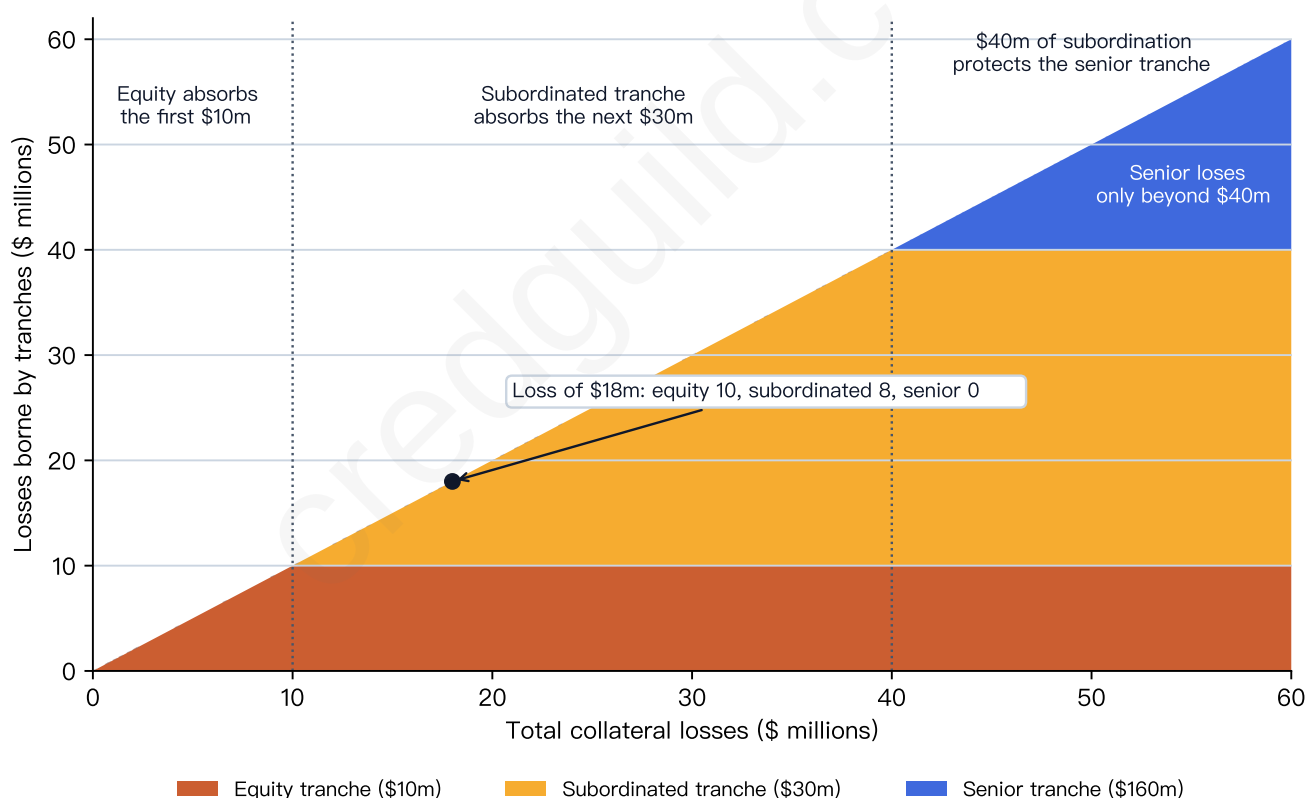
KEY CONCEPT

1. **Overcollateralization:** the collateral value exceeds the ABS face value. For example, if collateral of \$250 million backs ABS of \$225 million, the \$25 million excess can absorb losses of $25/250 = 10\%$ of the collateral value before any ABS investor loses money.

2. **Excess spread:** the collateral earns more interest than the ABS coupons require, and the surplus builds a reserve that absorbs losses.
3. **Subordination (credit tranching):** the ABS is split into **tranches** with different priority. Junior tranches absorb losses first, up to their principal, which protects the senior tranches. The larger the subordinated share, the greater the protection for the senior tranche.

This loss-absorption order is a **waterfall structure**: each junior tranche receives only what is left after the more senior tranches are paid. The lowest-ranking tranche has a residual claim and is often called the **equity tranche**. Credit tranching and the bankruptcy-remote SPE together can allow senior tranches to be rated above the originator.

Example. A deal backed by \$200 million of collateral has a senior tranche of \$160 million, a subordinated tranche of \$30 million and an equity tranche of \$10 million. Collateral losses are \$18 million. The equity tranche absorbs the first \$10 million and is wiped out. The subordinated tranche absorbs the other \$8 million, so its face value falls from \$30 million to \$22 million. A subordinated tranche investor's coupon is paid on the reduced face value, so it falls to $22/30 = 73.3\%$ of its previous amount. The senior tranche loses nothing.



Allocation of collateral losses across the senior, subordinated and equity tranches

The figure extends the example to any level of collateral losses: the senior tranche is hit only once losses exceed the \$40 million of subordination, which is 20% of the \$200 million collateral.

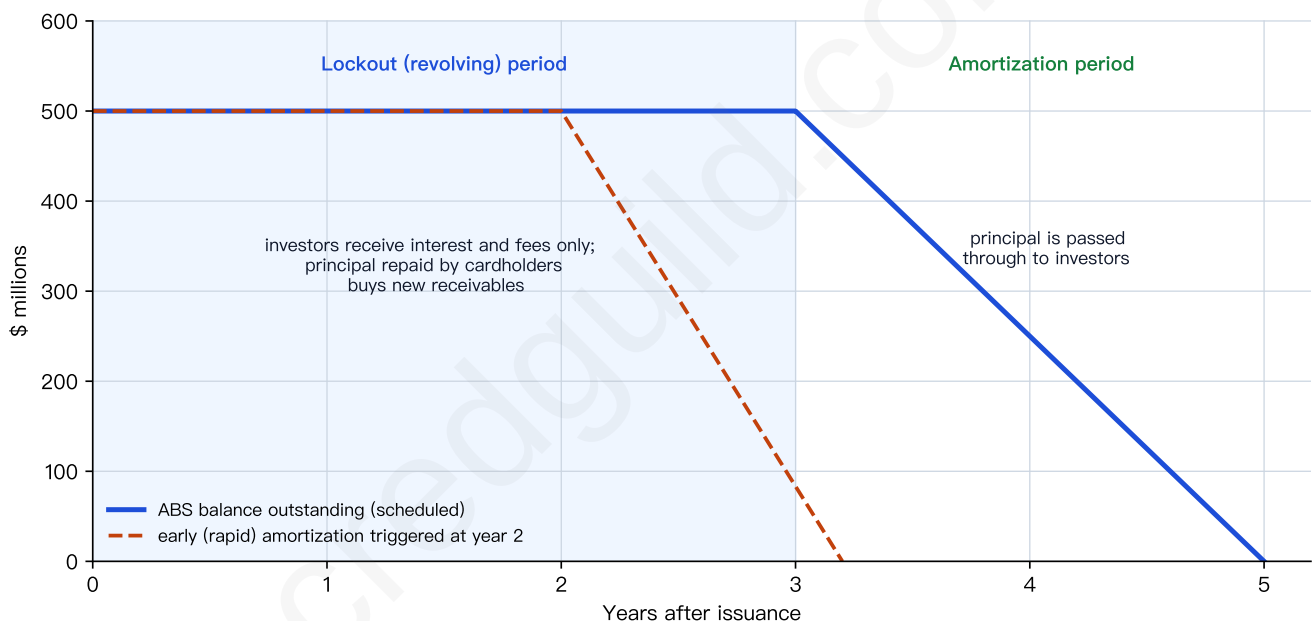
Credit tranching redistributes default risk. **Time tranching**, covered with MBS, redistributes prepayment risk.

LOS 68.c — Non-mortgage ABS

Any asset that generates future cash flows can be securitized: business loans, accounts receivable and auto loans, and even music royalties or franchise license payments. A key distinction is whether the loans are **amortizing** (scheduled principal paydown) or **nonamortizing** (no schedule).

Credit card ABS

- Backed by credit card receivables of banks, retailers and others. The collateral is nonamortizing, though cardholders may repay principal at will.
- Cash flows consist of finance charges (interest), fees (membership, late payment) and principal. Rates may be fixed or floating and are typically subject to a cap.
- During the **lockout period (revolving period)**, investors receive only interest and fees. Principal repaid by cardholders buys new receivables, which keeps the pool size stable, so investors face no prepayment risk during this period.
- After the lockout, the **amortization period** passes principal through to investors. An **early (rapid) amortization provision** accelerates principal repayment if needed to protect credit quality.



Credit card ABS: balance outstanding in the lockout and amortization periods (illustrative)

Solar ABS

- Backed by loans to homeowners to install solar energy systems, made by specialist finance companies or the finance arms of solar energy companies. The asset class draws investors who weigh environmental, social and governance (**ESG**) factors.
- The loans are secured on the system itself or on the home as a junior mortgage. Borrowers usually have good credit and save on energy bills, so credit risk is thought to be low, although the asset class has not yet been tested through a full credit cycle.
- Internal credit enhancement is common. Many deals have a **pre-funding period** during which the trust invests newly raised funds in additional solar loans, building a larger, more diversified pool.

LOS 68.d — Collateralized debt obligations

A **collateralized debt obligation (CDO)** is a structured security that an SPE issues against a pool of debt obligations.

Type	Collateral
Collateralized bond obligation (CBO)	Corporate and emerging market debt
Collateralized loan obligation (CLO)	Leveraged loans: senior secured bank loans to companies with high existing debt or a poor credit history

A CDO differs from ordinary ABS in having a **collateral manager**, who actively buys and sells securities in the pool to generate the cash needed to pay investors. Ordinary ABS rely on a static pool. CDOs are tranching like ABS, and the structure must still offer an attractive return to the equity tranche after paying the senior tranches.

Example. A CLO buys \$400 million of leveraged loans yielding 8% and funds them with \$360 million of debt tranches at an average cost of 5.5% plus a \$40 million equity tranche. Collateral income is $0.08 \times 400 = \$32.0$ million and interest on the debt tranches is $0.055 \times 360 = \$19.8$ million, which leaves \$12.2 million, a 30.5% return on the equity before fees. If defaults cost \$6 million in a year, the residual falls to \$6.2 million and the equity return roughly halves to 15.5%, while the debt tranches are still paid in full. The equity tranche earns the gap between the collateral yield and the cost of the debt tranches, magnified by leverage, and it absorbs the first losses.

Since the 2007–2009 crisis, CDOs have become simpler, requirements for collateral quality have become more stringent, and most are now backed by leveraged loans, so the CLO is the most common form.

CLO type	How investors are paid
Cash flow CLO	From the cash flows of the underlying collateral
Market value CLO	From trading the collateral at market value
Synthetic CLO	Exposure is created with credit derivative contracts; the trust does not own the collateral

CLO collateral is subject to protective tests: coverage of payments to investors by collateral cash flows; overcollateralization limits for each tranche (a breach diverts cash to buy more collateral or to repay the most senior tranche); diversification; and limits on CCC rated holdings.

Common exam traps

- A covered bond differs from a typical ABS on three points that are easy to reverse: it gives recourse to the issuer, uses no SPE, and has a dynamic rather than fixed pool. It usually yields less.
- A hard bullet means immediate default; a soft bullet allows a maturity extension of up to a year.
- The lockout period belongs to credit card ABS; the pre-funding period belongs to solar ABS.
- The collateral manager distinguishes a CDO. Tranching and the use of an SPE are common to ABS in general.

EXAM SHORTCUTS

- Under subordination, the senior tranche loses nothing until collateral losses exceed the total of the tranches ranked below it, so smaller losses need only be allocated among the junior tranches.

BOTTOM LINE

- Covered bonds are senior debt of financial institutions backed by a segregated cover pool, usually mortgages, that stays on the issuer's balance sheet because no SPE is created.
- Covered bond investors have dual recourse to the cover pool and to the issuer's unencumbered assets, the cover pool is dynamic, and covered bonds generally yield less than comparable ABS.
- When an issuer misses a payment, a hard-bullet covered bond defaults at once and payments are accelerated, a soft-bullet one lets the maturity slip by up to a year, and a conditional pass-through one becomes a pass-through security at maturity if payments are still owed.
- Internal credit enhancements are overcollateralization, excess spread and subordination, under which junior tranches absorb losses first, up to their principal, to protect the senior tranches.
- Credit tranching redistributes default risk, while time tranching redistributes prepayment risk.
- Credit card ABS are backed by nonamortizing receivables; during the lockout (revolving) period investors receive only interest and fees, and principal repaid by cardholders buys new receivables.
- Solar ABS are backed by loans to homeowners to install solar energy systems, and many deals have a pre-funding period in which the trust adds new solar loans to the pool.
- A CDO differs from ordinary ABS by having a collateral manager who actively buys and sells securities in the pool, and the CLO, backed by leveraged loans, is now its most common form.

QUICK CHECK**Question 1**

Compared with otherwise similar asset-backed securities, covered bonds issued by Eskdale Bank *most likely* offer investors:

- A** recourse to the issuing bank.
- B** higher yields.
- C** lower credit quality.

Answer: A

Covered bondholders have dual recourse: a claim on the cover pool and a claim on the issuing bank's other unencumbered assets. Ordinary ABS investors can look only to the collateral held by the SPE. The extra recourse, together with overcollateralization and loan-to-value limits, gives covered bonds higher credit quality and lower yields than comparable ABS.

Why the other options are wrong

- **B.** Because covered bonds are safer, investors accept lower yields than on comparable ABS.
- **C.** Dual recourse and a dynamic, overcollateralized cover pool give covered bonds higher credit quality than comparable ABS.

Key takeaway A covered bond has dual recourse, no SPE and a dynamic cover pool, which give it higher credit quality and a lower yield than comparable ABS.

Practice questions

Question 2

Halden Card Trust issued asset-backed securities backed by credit card receivables, and the securities are still in their lockout period. Which event would *most likely* cause the ABS holders to start receiving principal payments ahead of schedule?

- A Cardholders repay a larger share of their balances each month.
- B A trigger in the early (rapid) amortization provision is breached.
- C The interest rate charged on the card balances reaches its cap.

Question 3

Which feature *most clearly* distinguishes a collateralized debt obligation (CDO) from most other asset-backed securities?

- A It is issued through a special purpose vehicle.
- B A collateral manager actively trades the securities in its collateral pool.
- C It is structured with senior and subordinated tranches.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/asset-backed-security-abs-instrument-and-market-features/. The full bank has 17 questions on this reading.

FIXED INCOME · READING 69

Mortgage-Backed Security (MBS) Instrument and Market Features

What you'll learn

LOS 69.a Define prepayment risk (contraction and extension risk) and describe how time tranching reallocates it.

LOS 69.b Describe the features of residential mortgage loans that are securitized: prepayment penalties, recourse, LTV and DTI, agency vs non-agency RMBS.

LOS 69.c Describe mortgage pass-through securities and CMOs (sequential pay, PAC and support, Z, IO/PO, floating-rate tranches) and their cash flows and risks.

LOS 69.d Describe the characteristics and risks of commercial MBS, including DSCR, LTV, call protection and balloon risk.

MODULE 69.1

Mortgage-Backed Security (MBS) Instrument and Market Features

This reading covers mortgage-backed securities: prepayment risk, the features of residential mortgage loans, pass-throughs and CMOs, and commercial MBS. A candidate must be able to explain contraction and extension risk, compute LTV, DTI, WAC, WAM and DSCR, and describe how sequential pay, PAC, support, IO and PO tranches reallocate prepayment risk.

LOS 69.a — Prepayment risk and time tranching

Prepayments are principal repayments above the scheduled amortization of a mortgage, usually because borrowers sell their homes or refinance. MBS are priced on an assumed prepayment rate, and **prepayment risk** is the risk that actual prepayment speeds differ from that assumption. The **average life** of an MBS is the average time until its principal is received, weighted by the amount of principal paid at each date. Faster prepayments shorten it and slower prepayments lengthen it.

Example. A borrower's scheduled monthly payment is \$1,500, of which \$900 is interest and \$600 is scheduled principal. If the borrower pays \$2,300, the extra \$800 is a prepayment, and the loan balance falls by \$1,400 instead of \$600.

KEY CONCEPT

Risk	What happens	Typical trigger	Why it hurts investors
Contraction risk	Prepayments faster than expected; cash arrives sooner, average life shortens	Interest rates fall (refinancing)	Principal must be reinvested at lower rates; MBS prices rise less than option-free bonds when rates fall (negative convexity , like a callable bond)

Risk	What happens	Typical trigger	Why it hurts investors
Extension risk	Prepayments slower than expected; average life lengthens	Interest rates rise	Cash flows are received later and discounted over more periods at a higher rate

Time tranching creates bond classes with different maturities. It reallocates prepayment risk among them without removing it. Tranches that mature first offer more protection against extension risk; longer tranches offer more protection against contraction risk. **Sequential pay tranching** (LOS 69.c) is the standard example.

LOS 69.b — Features of residential mortgage loans

A **residential mortgage loan** is secured by residential real estate. The lender's legal claim on the property is a **first lien**: if the borrower defaults, the lender can take possession of the property and sell it, a process called **foreclosure**.

- A **prepayment penalty** is an extra payment by a borrower who prepays when rates have fallen. It reduces the lender's prepayment risk. Prepayment penalties are common in Europe and rare in the United States.
- With a **recourse loan**, the lender can also claim the borrower's other assets for any shortfall after foreclosure. With a **nonrecourse loan**, the lender can look only to the property. When a home is worth less than the loan (an **underwater mortgage**, or **negative equity**), a nonrecourse borrower has a stronger incentive to **strategically default**, walking away even though they could pay. Most mortgages in Europe are recourse loans. US rules differ from state to state, but most US loans are nonrecourse.
- The **loan-to-value ratio (LTV)** is the loan divided by the property value. A lower LTV means more borrower equity, which lowers default risk and raises recovery.
- The **debt-to-income ratio (DTI)** is monthly debt payments divided by monthly gross pretax income. A lower DTI means lower default risk.

Example. A \$360,000 loan on a \$450,000 home has an LTV of 80%. At 5.5% for 30 years with monthly payments, the payment is \$2,044.04. On a financial calculator in END mode with $P/Y = C/Y = 1$; $N = 360$, $I/Y = 5.5/12$, $PV = 360,000$, $FV = 0$, $CPT\ PMT = -2,044.04$ (the loan received is positive, the payment negative). With gross income of \$96,000 a year (\$8,000 a month), $DTI = 2,044.04/8,000 = 25.6\%$.

Prime loans are made to borrowers whose credit record is strong and whose LTV and DTI are both low.

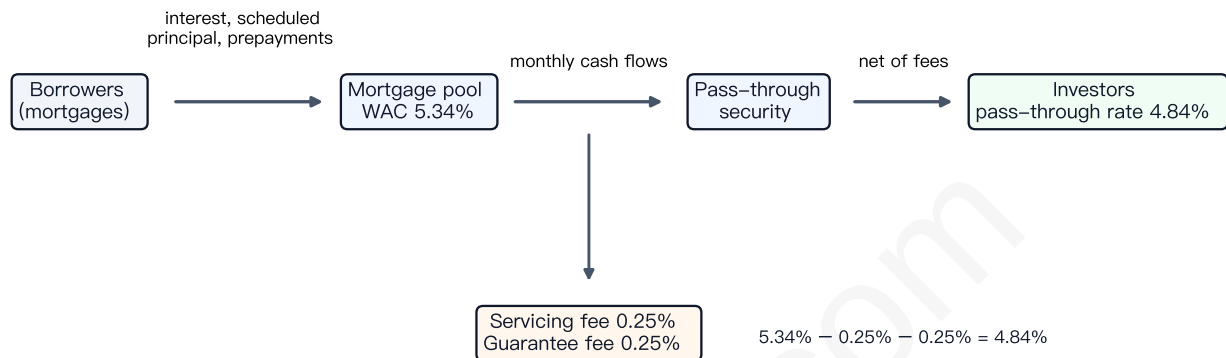
Subprime loans involve weaker credit, a higher DTI or LTV, or a lower-priority lien.

Agency RMBS (U.S.) are guaranteed by the government or by a **government-sponsored enterprise (GSE)**. A GSE is legally separate from the government, so securities it guarantees carry only the GSE's own guarantee, not the **full faith and credit** of the government. The collateral must meet minimum underwriting standards. **Non-agency RMBS** are issued by private entities such as banks without these guarantees and rely on credit enhancement (insurance, letters of credit, tranching, private guarantees). They are widely blamed for helping to trigger the 2007–2009 financial crisis: senior tranches rated investment grade, backed by pools of subprime loans, took large losses when many of those loans defaulted. New issuance almost stopped in the years after the crisis as regulation tightened.

LOS 69.c — Mortgage pass-throughs and CMOs

A **mortgage pass-through security** is a claim on the cash flows of a mortgage pool, net of fees.

- The **weighted average maturity (WAM)** and **weighted average coupon (WAC)** weight each mortgage's maturity or rate by its share of the pool's outstanding balance.
- Investors receive the pool's monthly cash flows minus servicing and guarantee or insurance fees, so the **pass-through rate** (net coupon) is below the WAC.



Cash flows of a mortgage pass-through (pool WAC 5.34%; fees illustrative)

With w_i the share of mortgage i in the pool's outstanding balance:

$$WAC = \sum_i w_i \times \text{mortgage rate}_i \quad WAM = \sum_i w_i \times \text{remaining term}_i$$

$$\text{pass-through rate} = WAC - \text{fees}$$

Example. A pool holds balances of 150, 250 and 100 (total 500) at rates of 4.8%, 5.4% and 6.0%, with remaining terms of 18, 26 and 12 years.

$$WAC = \frac{150(4.8) + 250(5.4) + 100(6.0)}{500} = 5.34\%,$$

$$WAM = \frac{150(18) + 250(26) + 100(12)}{500} = 20.8 \text{ years}$$

Collateralized mortgage obligations (CMOs) are backed by pass-throughs and mortgage pools. They split the cash flows into tranches with different exposure to prepayment risk. The total prepayment risk of the collateral is unchanged; it is reallocated to suit investors with different concerns (some worry mainly about extension risk, others about contraction risk). This widens the market for securitized mortgages and may lower funding costs.

KEY CONCEPT

CMO structure

How it works / who it suits

Sequential pay

All principal (scheduled and prepaid) goes to the first ("short") tranche until it is retired, then to the next. This is the time tranching of LOS 69.a, so the short tranche suits investors most concerned about extension risk

Planned amortization class (PAC)

Makes scheduled payments as long as prepayments stay within a range, reducing **both** contraction and extension risk

Support tranche

Absorbs the variability: receives excess principal when prepayments are fast and has its principal payments curtailed when they are slow, so PAC schedules are met. Bears high **prepayment risk**. If speeds move outside the range, the protection breaks down

Z-tranche (accrual / accretion bond)

Receives no cash interest during an accrual period; interest is added to principal (e.g., \$200 at 4% becomes \$208). Typically the lowest-ranking tranche

Principal-only (PO)

Pays only principal (a zero-coupon-like security bought at a discount). Its return is the gap between the price paid and the principal received, so the sooner principal arrives, the higher the return: it **benefits from falling rates and faster prepayments**

Interest-only (IO)

Pays only interest on the principal still outstanding. Faster prepayments shrink that balance and so the total interest received: it is **hurt by falling rates and faster prepayments**

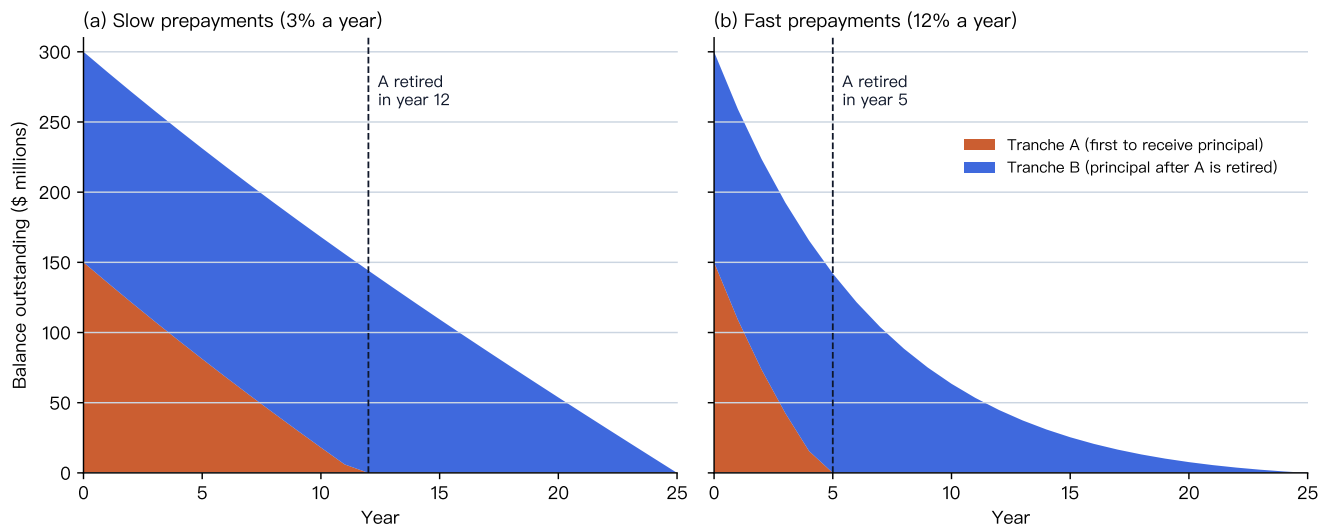
Floating-rate / inverse floater

Coupon tied to a market reference rate (often capped/floored); an inverse floater's coupon rises as rates fall

Residual

Most junior, like an ABS equity tranche

Example: sequential pay. A \$300 million pool of 6%, 25-year mortgages backs Tranche A (\$150 million) and Tranche B (\$150 million). All principal, scheduled and prepaid, goes to A until A is retired. If 3% of the balance is prepaid each year, A is paid off in year 12. If 12% is prepaid each year, A is paid off in year 5, and B starts receiving principal seven years earlier. Fast prepayments shorten A's life sharply (contraction risk), and slow prepayments leave B waiting much longer for its principal (extension risk). Tranche A has the shorter expected average life, so it suits investors who prefer shorter-term securities.



Tranche balances in a two-tranche sequential-pay CMO at slow and fast prepayment speeds

Example: PAC and support. A PAC tranche is scheduled to receive \$20 million of principal this year, and the support tranche has \$40 million outstanding. The PAC schedule therefore holds as long as the pool's principal for the year is between \$20 million and \$60 million.

Pool principal this year	PAC tranche receives	Support tranche receives
\$24 million (slow)	\$20 million	\$4 million
\$40 million (as expected)	\$20 million	\$20 million
\$58 million (fast)	\$20 million	\$38 million
\$15 million (below the range)	\$15 million, \$5 million short of schedule	\$0

Inside the range the support tranche takes all the variation. In the last row it has nothing left to give up, so the PAC tranche falls short. Above \$60 million the support tranche would be paid off, and the excess principal would go to the PAC tranche ahead of schedule.

LOS 69.d — Commercial mortgage-backed securities

CMBS are backed by mortgages on income-producing property (multifamily, industrial, retail, office, health care, hotels). Pools contain fewer loans than RMBS pools, sometimes just one, so diversification is lower. A pool may hold loans from one lender or several loans to one borrower. In an RMBS pool of thousands of small, similar home loans, one default barely affects investors; in a CMBS pool, the default of a single large loan can. U.S. CMBS coupons are typically fixed; European CMBS usually float. The loans are repaid by property investors who depend on tenants' rents, and the pool's regular income, the **weighted average proceeds from the mortgages (WAMP)**, plays the role the WAC plays for RMBS. Commercial mortgages are **nonrecourse loans**, so the lender's recovery depends on the property and its income. Analysis therefore focuses on the credit risk of the property rather than that of the borrower. CMBS are tranced so that credit losses are absorbed by the lowest-priority tranches first, in sequence.

KEY CONCEPT

$$\text{DSCR} = \frac{\text{net operating income}}{\text{debt service}} \quad \text{LTV} = \frac{\text{loan amount}}{\text{appraised value}}$$

Net operating income (NOI) is rental income minus cash operating expenses and a replacement reserve. A higher **debt service coverage ratio (DSCR)** and a lower LTV indicate better credit quality. For example, NOI of \$1.3 million and debt service of \$1.0 million give a DSCR of 1.30×; a \$12 million loan on a \$16 million property has an LTV of 75%.

Call protection (protection against prepayment) is a key difference from RMBS:

- At the loan level: a **prepayment lockout** (typically 2–5 years), **prepayment penalty points** (1 point = 1% of the principal prepaid), and **defeasance** (the borrower buys government securities that replicate the remaining loan payments, freeing the property from the lien).
- At the structure level: sequential pay tranching of the pool.

Balloon payments. Commercial loans are usually not fully amortizing, which leaves a **balloon payment** at maturity. If the borrower cannot refinance it, the lender may extend the loan in a workout, so **balloon risk** is a form of extension risk.

Common exam traps

- Contraction and extension describe the average life, not the stated maturity: contraction comes with falling rates and faster prepayments, extension with rising rates and slower prepayments.
- Residential mortgages in agency RMBS pools can usually be prepaid freely; CMBS loans typically carry call protection.
- A support tranche's high risk is prepayment risk in both directions, not credit risk; with agency collateral its credit risk is low.
- Prepayment speed changes the timing of cash flows, not the net coupon; the gap between the WAC and the pass-through rate comes from fees.

BOTTOM LINE

- Contraction risk is the risk that prepayments run faster than expected, typically when rates fall, which shortens the average life and forces reinvestment at lower rates; extension risk is the risk that they run slower, typically when rates rise, which lengthens it.
- Time tranching reallocates prepayment risk among bond classes without removing it, and the tranches that mature first give more protection against extension risk.
- A lower loan-to-value ratio and a lower debt-to-income ratio mean lower default risk, and a borrower with an underwater nonrecourse loan has a stronger incentive to default strategically.
- A GSE is legally separate from the government, so agency RMBS it guarantees carry only the GSE's own guarantee, while non-agency RMBS rely on credit enhancement.
- A pass-through's net coupon is below the pool's weighted average coupon because servicing and guarantee or insurance fees are deducted, and the WAC and WAM weight each mortgage by its share of the outstanding balance.

- A PAC tranche makes its scheduled payments as long as prepayments stay within a range, which reduces both contraction and extension risk, while the support tranche absorbs the variation and bears high prepayment risk.
- A principal-only security benefits from falling rates and faster prepayments, while an interest-only security is hurt by them.
- Commercial mortgages are nonrecourse, so CMBS analysis focuses on the property: a higher $DSCR = NOI/\text{debt service}$ and a lower LTV indicate better credit quality, call protection is common, and balloon risk is a form of extension risk.

QUICK CHECK

Question 1

A sequential-pay CMO has two tranches. Class J receives all principal payments until it is fully repaid, and principal then goes to Class K. Relative to Class K, Class J has:

- A less contraction risk and more extension risk.
- B more contraction risk and more extension risk.
- C more contraction risk and less extension risk.

Answer: C

In a sequential-pay structure the first tranche receives all scheduled principal and prepayments first. If prepayments speed up, it is repaid sooner (more contraction risk); if they slow, it is still first in line, so it is well protected against extension, which falls mainly on the later tranche.

Why the other options are wrong

- A. This is the profile of the later tranche (Class K), which waits longest for principal.
- B. Time tranching reallocates prepayment risk: the short tranche gains extension protection in exchange for bearing more contraction risk, so it cannot have more of both.

Key takeaway First-pay tranche: contraction risk. Last-pay tranche: extension risk.

Practice questions

Question 2

The Linwell pass-through security is backed by the three mortgages shown below. Servicing and guarantee fees total 0.50% a year.

Mortgages in the Linwell pass-through pool

Mortgage	Current balance (\$ thousands)	Mortgage rate	Remaining term (years)
1	120	5.00%	22
2	80	6.50%	15
3	200	5.75%	27

The pass-through rate on the security is *closest* to:

- A 5.18%.
- B 5.25%.
- C 5.68%.

Question 3

When comparing the loans backing different CMBS pools, which combination of credit ratios *most likely* indicates better credit quality?

- A Higher DSC ratios and lower LTV ratios.
- B Lower DSC ratios and higher LTV ratios.
- C Higher DSC ratios and higher LTV ratios.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/mortgage-backed-security-mbs-instrument-and-market-features/. The full bank has 16 questions on this reading.

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