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STUDY NOTES

CFA® Program Level I

Complete Study Notes

2027 curriculum · 102 readings · 10 topics · 398 practice questions

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ABOUT THIS EDITION

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2027 curriculum · Version v2027.1 · Generated October 4, 2026

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How to use this book

These are the CredGuild notes for the 2027 CFA® Program Level I curriculum: all 102 readings in 10 topics. They are the same notes as on credguild.com, set out for reading on paper or offline.

What each reading contains

- **What you'll learn:** the learning outcome statements the reading covers.
- **The notes, module by module:** worked examples, key concepts, common exam traps, exam shortcuts and a bottom line.
- **A quick check after each module:** one question with its answer and the full explanation.
- **Practice questions at the end, with the answers.** The full explanation for each one is free on the reading's page at credguild.com; the link is under the answers.

The coloured boxes

KEY CONCEPT

Definitions and formulas to know.

Example. A worked problem, step by step. Try it before you read the working.

COMMON EXAM TRAPS

Mistakes that cost marks.

EXAM SHORTCUTS

Faster ways through a question.

BOTTOM LINE

What to remember from the module.

Using it with the website

Paper is good for reading. Practice works better on the site: it scores each answer as you go, explains every option, keeps a list of the questions you got wrong and spreads the readings across the weeks before your exam. If you are not sure where to start, the free diagnostic test (credguild.com/cfa/level-1/diagnostic/) gives you a score in each topic.

If this book helps you, pass it on to someone else who is studying.

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CFA® PROGRAM LEVEL I · TOPIC

Quantitative Methods

Exam weight 11–14% · 11 readings

How to measure and compare returns, discount cash flows, describe return data with statistics and probability distributions, test hypotheses and fit a simple regression. It ends with simulation and an introduction to financial data science.

Reading 1	Rate of Return
Reading 2	Types of Financial Returns
Reading 3	Benchmarking Returns
Reading 4	Time Value of Money
Reading 5	Skewness and Kurtosis
Reading 6	Lognormal Distribution
Reading 7	Null Hypothesis
Reading 8	Portfolio Standard Deviation
Reading 9	Simulation of Financial Asset Prices and Returns
Reading 10	Least Squares Regression
Reading 11	Machine Learning in Finance

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QUANTITATIVE METHODS · READING 1

Rate of Return

What you'll learn

LOS 1.a Describe, compare and interpret returns: price, capital distribution and total return; ex ante vs. ex post and realized vs. unrealized returns; arithmetic and geometric means; annualized and continuously compounded returns.

LOS 1.b Describe, compare and interpret required returns, real and nominal risk-free rates, risk premia and inflation, and the real, excess, gross/net, after-tax and leveraged measures of return.

MODULE 1.1

Rates of Return

This reading measures returns over a single period and over several periods: the holding period return, the arithmetic and geometric means, annualized returns and continuously compounded returns. It then breaks a required return into the real risk-free rate, expected inflation and risk premia, and computes real, excess, gross, net, after-tax and leveraged returns.

LOS 1.a — Measuring and comparing returns

1. Assets, instruments and indicators

Returns can be measured for three overlapping categories of items:

Category	What it is	Typical examples	Generates cash flows itself?
Financial assets	Things of value that an investor can own	Cash, equity stakes, loans/debt (an asset for the lender), hybrid securities	Usually (dividends, interest, principal)
Financial instruments	Standardized, tradable vehicles through which financial assets change hands in markets	Equity and debt securities (shares, bonds), exchange-traded futures and other standardized contracts	Securities may pay dividends or coupons; standardized contracts need not pay anything
Financial indicators	Observable measures of value that pay nothing to anyone	Interest rates, exchange rates, market index levels	No; they are used to value other items

A machine, a building or a truck is a *real* (physical) asset, so it is neither a financial instrument nor a financial indicator.

2. Return over a single period

An asset's return has two sources: the change in its price (the **price return**, also called the **capital appreciation return**) and any cash it pays out. The **total return** for a period adds the two.

$$\text{Price return} = \frac{P_1 - P_0}{P_0} \qquad \text{Capital distribution return} = \frac{D}{P_0}$$

KEY CONCEPT

$$\text{Total (holding period) return} = \frac{P_1 - P_0 + D}{P_0} = \text{price return} + \text{capital distribution return}$$

The **capital distribution return** measures a period's cash income relative to the beginning price. For a share it is the **dividend yield**; for a bond it is the **current yield**. Every component is divided by the *beginning* price P_0. D is the total cash actually received between purchase and sale (add up every coupon or dividend paid in the window, and nothing paid outside it).

Example. A bond is bought at 97.00, pays coupons of 1.50 and 1.50 over the year and is sold at 98.20. Price return $= 1.20/97 = 1.24\%$; distribution return $= 3.00/97 = 3.09\%$; total return $= 4.20/97 = 4.33\%$.

A **holding period return** describes the whole holding period, however long it is. The length of the period matters only when the result has to be annualized.

Other labels for returns

Pair	Meaning
Actual return (**ex post return**) vs. **expected return** (**ex ante return**)	An ex post return uses prices and income already observed; an ex ante return is an estimate for a future period. Decisions rest on expected returns, and the gap between expected and actual returns represents risk.
Realized returns vs. **unrealized return**	Realized = cash already in hand (income received, sale proceeds). Unrealized = paper gain or loss on a position still held, valued at today's market price. Taxes are usually levied on income when received and on price gains only when realized by a sale.
Real vs. realized	*Real* means adjusted for inflation (LOS 1.b). Do not confuse it with *realized*.

3. Averaging returns over several periods

KEY CONCEPT

$$\bar{R}_{\text{arith}} = \frac{R_1 + R_2 + \cdots + R_n}{n} \qquad R_{\text{geo}} = \left[(1 + R_1)(1 + R_2)\cdots(1 + R_n)\right]^{1/n} - 1$$

- The **geometric mean return** (**compound average return**) is the constant per-period rate that turns the starting value into the ending value: $(1 + R_{\text{geo}})^n$ equals 1 plus the holding period return for all n periods together. It is the measure to use for an annualized return or a **compound annual growth rate**.
- The **arithmetic mean return** adds the periodic returns and divides by their number. It ignores compounding: it implicitly assumes the amount invested stays at its starting level, although gains, losses and cash flows change it. For this reason it can overstate what the investor actually earned.

- Under the **arithmetic-geometric mean inequality principle**, $R_{\text{geo}} \leq \bar{R}_{\text{arith}}$. The gap widens as the returns become more dispersed, and the two are equal only when every period's return is identical.
- Each period enters the product as a **wealth relative**, $1 + R$. A loss keeps its sign: a −6% year enters as 0.94.

> *Example.* Returns of +25% and −20%: arithmetic mean $= 2.5\%$, but $\sqrt{1.25 \times 0.80} - 1 = 0\%$. €1,000 grows to €1,250 and falls back to €1,000. Wealth did not change, and only the geometric mean shows this.

The root used in a geometric mean equals the count of periods. If the periods are quarters, the result is an average *quarterly* compound return. To get an annual rate from several sub-annual returns, take the root equal to the number of *years* covered. For eight quarterly returns, the annual rate is $\left[\prod(1 + R_q)\right]^{1/2} - 1$. Calculator (TI BA II Plus) for a cube root: `1.1 [y^x] 3 [1/x] [=]`.

4. Annualizing a holding period return

Interest rates and market returns are normally quoted as **annualized returns**, whatever the actual length of the period. Annualizing makes investments with different horizons comparable. For a period shorter than a year, it assumes that the same return is earned again in every remaining period of the year, which actual returns rarely do.

$$\text{Annualized return} = (1 + HPR)^{365/\text{days}} - 1 \qquad \text{or} \qquad (1 + R_{\text{period}})^{c} - 1$$

where c counts how many such periods make up a year (12 for monthly, 4 for quarterly, 2 for semiannual). The direction of the adjustment depends on the sign of the HPR:

KEY CONCEPT

Holding period	Positive HPR	Negative HPR (between −100% and 0)
Longer than a year (exponent < 1)	Annualized return is smaller than the HPR	Annualized return is less negative (closer to zero) than the HPR
Shorter than a year (exponent > 1)	Annualized return is larger than the HPR	Annualized return is more negative than the HPR

A zero HPR annualizes to zero. Rule of thumb: annualizing a long period pulls the return toward zero; annualizing a short period pushes it away from zero.

> *Example.* 2% earned over 146 days: $1.02^{365/146} - 1 = 1.02^{2.5} - 1 = 5.08\%$. A 0.6% monthly return: $1.006^{12} - 1 = 7.44\%$ (not $12 \times 0.6\% = 7.2\%$). A loss of 15.36% over exactly two years: $0.8464^{1/2} - 1 = -8.00\%$ per year, which is *greater* (less negative) than −15.36%.

5. Compounding frequency and continuous compounding

For a given positive stated (quoted) annual rate, more frequent compounding means a higher **effective annual rate**, a higher future value and a lower present value. The limiting case is **continuous compounding**, whose effective annual rate, $e^{r} - 1$, is higher than that of any discrete frequency.

With m compounding periods a year, the rate for each period is the stated annual rate divided by m. Compounding that rate m times gives

$$\text{Effective annual rate} = \left(1 + \frac{r_s}{m}\right)^m - 1 \qquad PV = \frac{FV}{(1 + r_s/m)^m} \text{ (one year ahead)}$$

The first formula is the annualizing formula of section 4 with $c = m$.

Example. The present value of 2,000 due in one year at a stated 4% is 1,923.08 with annual compounding, 1,921.96 with quarterly ($2{,}000/1.01^4$, an effective annual rate of $1.01^4 - 1 = 4.06\%$), 1,921.71 with monthly, 1,921.58 with daily, and $2{,}000e^{-0.04} = 1{,}921.58$ with continuous compounding (effective annual rate $e^{0.04} - 1 = 4.08\%$).

To compare a continuously compounded rate with a discrete one, convert both to effective annual rates. Going the other way, the continuously compounded stated rate that gives a chosen effective annual rate is $r_{cc} = \ln(1 + \text{EAR})$. *Example.* A deposit quoting 5% compounded quarterly has an effective annual rate of $1.0125^4 - 1 = 5.09\%$. A continuously compounded deposit matches it at a stated rate of $\ln(1.0509) = 4.97\%$.

Moving from a holding period return to a **continuously compounded return** (also called a **logarithmic return**, or log return) uses the natural logarithm:

$$r_{cc} = \ln(1 + HPR) \qquad \text{and back:} \qquad HPR = e^{r_{cc}} - 1$$

Because the HPR is a total return, the log return built from it includes income. With a single distribution D received at the end of the period, $r_{cc} = \ln\left(\frac{P_1 + D}{P_0}\right)$. Only when no distributions are paid does this simplify to $r_{cc} = \ln(P_1/P_0)$, which on its own is a *price* log return. *Example.* Bought at 100, sold at 100, with a 5 dividend: the price log return is $\ln(100/100) = 0$, but the total log return is $\ln(105/100) = 4.88\%$.

- For a gain, $r_{cc} < HPR$; for a loss, r_{cc} is *more* negative than the HPR (e.g., a fall from 100 to 90: HPR $= -10\%$, $\ln 0.9 = -10.54\%$).
- Log returns are additive across periods. Wealth relatives multiply, $P_2/P_0 = (P_1/P_0)(P_2/P_1)$, and the logarithm of a product is the sum of the logarithms. So with no distributions, $\ln(P_2/P_0) = \ln(P_1/P_0) + \ln(P_2/P_1)$ (in general, the sub-period values $\ln(1 + HPR_t)$ add up). Keep the sign of each sub-period (a fall contributes a negative term).

Example. A price moves from 50 to 60 and then to 54: $\ln(1.2) = 18.23\%$, $\ln(0.9) = -10.54\%$, sum $= 7.70\% = \ln(54/50)$.

COMMON EXAM TRAPS

- Dividing by the *ending* price instead of the beginning price. In the bond example, dividing the 4.20 total gain by the selling price of 98.20 gives 4.28% instead of 4.33%.
- Leaving out (or double counting) a dividend or coupon; check which payments fall inside the holding period.
- Annualizing when the question asks for the holding period return (or vice versa).
- Using simple multiplication ($c \times R$) instead of compounding $(1 + R)^c - 1$; using 360 days when the question says 365; inverting the exponent (days/365). For the 2% earned over 146 days, inverting the exponent gives $1.02^{146/365} - 1 = 0.80\%$ instead of 5.08%.
- Reporting the arithmetic mean when the compound (annualized) rate is asked for, forgetting to take the root, or dropping the minus sign on a loss.

- Using $e^{HPR} - 1$ instead of $\ln(1 + HPR)$ for a continuously compounded return, or dropping a distribution by using $\ln(P_1/P_0)$ when income was paid. For the fall from 100 to 90, $e^{-0.10} - 1 = -9.52\%$ instead of $\ln 0.9 = -10.54\%$.
- Assuming annualization always shrinks a multi-year return. It does so only for a gain.

EXAM SHORTCUTS

- The direction of an annualized return is known before calculating: a positive HPR over more than a year annualizes to a smaller figure and over less than a year to a larger one, while a negative HPR (between −100% and 0) becomes less negative over more than a year and more negative over less than a year.
- The geometric mean return cannot exceed the arithmetic mean return of the same returns, so an answer with a higher geometric mean can be ruled out.
- A log return is below the HPR for a gain and more negative than the HPR for a loss, which rules out answers on the wrong side of the HPR.

BOTTOM LINE

- Total (holding period) return is $(P_1 - P_0 + D)/P_0$, the price return plus the capital distribution return, with each component divided by the beginning price P_0.
- An ex post (actual) return uses prices and income already observed, while an ex ante (expected) return is an estimate for a future period.
- The geometric mean return $[(1 + R_1)(1 + R_2) \cdots (1 + R_n)]^{1/n} - 1$ is the measure for an annualized return or a compound annual growth rate; it cannot exceed the arithmetic mean, and the two are equal only when every period's return is identical.
- A holding period return is annualized with $(1 + HPR)^{365/\text{days}} - 1$, or $(1 + R_{\text{period}})^{c} - 1$ with c periods in a year; for a period shorter than a year this assumes the same return is earned in every remaining period.
- For a given positive stated annual rate, more frequent compounding gives a higher effective annual rate, $(1 + r_s/m)^m - 1$, and continuous compounding gives the highest, $e^r - 1$.
- The continuously compounded return is $r_{cc} = \ln(1 + HPR)$, which reduces to $\ln(P_1/P_0)$ only when no distributions are paid, and log returns are additive across periods.

QUICK CHECK

Question 1

A macro strategist's dashboard displays the central bank policy interest rate, the USD/JPY exchange rate and the level of the Euro Stoxx 50 index. These three items are best classified as:

A financial assets.

B financial instruments.

C financial indicators.

Answer: C

Interest rates, exchange rates and market index levels are financial indicators: observable measures of value that do not produce any cash flows on their own. They matter because they are used to value other things, such as the securities and contracts that are financial instruments.

Why the other options are wrong

- **A.** Financial assets are items an investor can own that carry value, such as cash, shares or loans. Nobody owns an interest rate or an index level, and these items pay nothing to anyone.
- **B.** Financial instruments are standardized vehicles for trading financial assets in markets, such as listed shares, bonds or standardized derivative contracts. An index *level* or an exchange *rate* is a measurement and cannot itself be traded.

Key takeaway An indicator is an observed measure (rates, FX, index levels); an instrument is a standardized, tradable vehicle; an asset is something an investor owns that has value.

MODULE 1.2

Components and Measures of Return

LOS 1.b — Required returns, risk-free rates, risk premia, inflation and other return measures

1. Three ways to read an interest rate

An interest rate expresses the time value of money; differences in risk across securities explain why their equilibrium rates differ. An interest rate can be seen as:

- a **required rate of return**: the minimum return that savers must expect before they will willingly lend or invest their money (the market's equilibrium rate for that investment);
- a **discount rate**: the rate used to translate future cash flows into today's value (someone who can borrow at 7% should discount future payments at 7%);
- an **opportunity cost of current consumption**: the return given up by spending today instead of saving.

The terms interest rate and discount rate are often used interchangeably. A **risk premium** may be part of an interest rate, but "a measure of risk" is not one of these interpretations. When a question describes the *minimum rate needed to persuade investors to commit funds*, the best label is the required rate of return.

2. Building blocks of a required return

KEY CONCEPT

$$\text{Required return} = \underbrace{\text{real risk-free rate} + \text{expected inflation}}_{\text{nominal risk-free rate}} + \text{risk premium(s)}$$

Rate	Contains expected inflation?	Contains default/liquidity/other risk premia?
Real risk-free rate	No	No; there is no uncertainty about the cash flows

Rate	Contains expected inflation?	Contains default/liquidity/other risk premia?
Nominal risk-free rate (e.g., a short-term government bill)	Yes	No
Required return on a risky asset	Yes	Yes

- The **real risk-free rate** reflects only **time preference**, that is, how strongly people prefer consuming now over consuming the same amount later.
- A short-term government bill is (practically) free of default risk, but its yield carries an **inflation premium** because expected inflation is not zero, so its yield is a nominal risk-free rate rather than a real one.
- Exact link: $(1 + r_{\text{nominal}}) = (1 + r_{\text{real}})(1 + \pi^e)$. Common approximation: $r_{\text{real}} \approx r_{\text{nominal}} - \pi^e$.
- The real rate therefore moves with the nominal risk-free rate and against expected inflation. If the bill yield falls while expected inflation rises, the real rate falls.

Typical risk premia

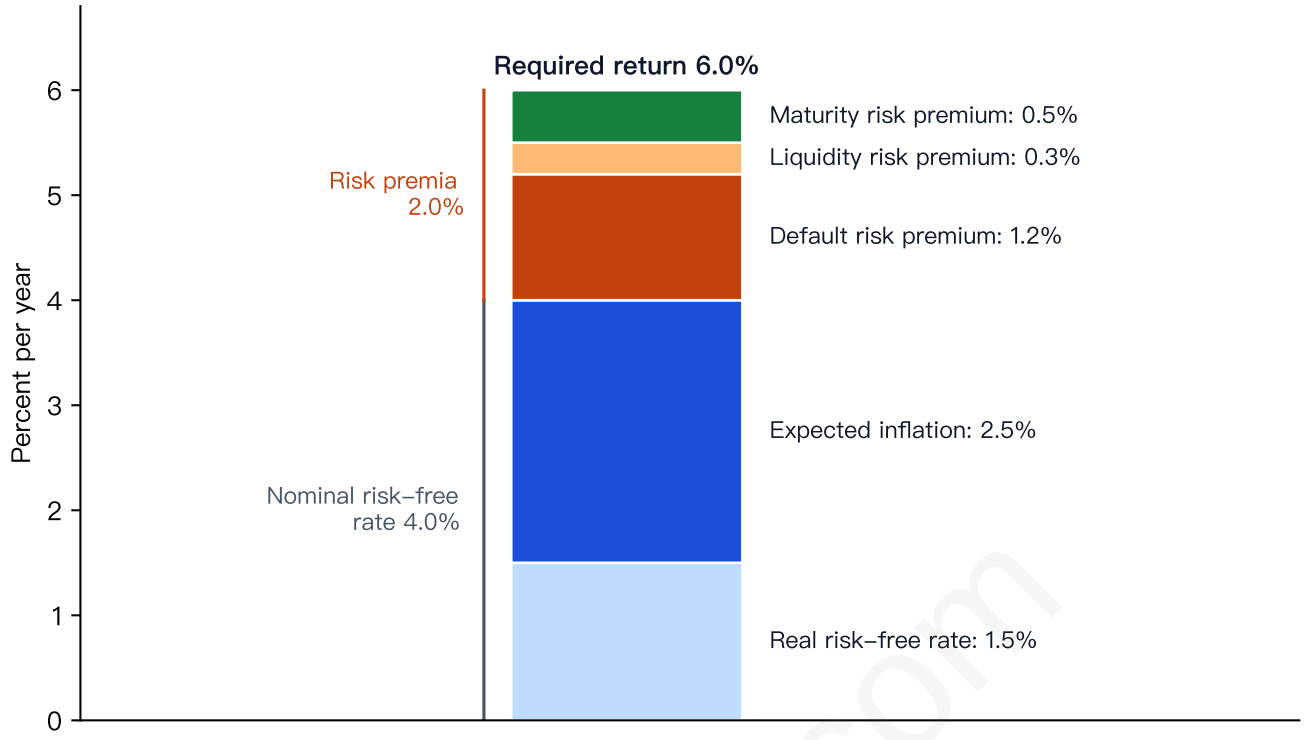
Each additional type of risk a security carries adds its own premium and raises the required return. The lists below are examples, not complete lists.

Bonds	Equities
Default risk premium: the borrower may not pay on time or in full	**Size risk premium**: return gap between small-cap and large-cap stocks
Liquidity risk premium: a quick sale for cash may fetch less than fair value	**Value risk premium**: return gap between high book-to-market (value) and low book-to-market (growth) stocks
Maturity risk premium: longer bonds' prices react more to rate changes	

The **market risk premium**, used in later readings, is the return on the market portfolio (every risky asset) minus the return on a risk-free asset.

> *Example.* Real risk-free 1.5%, expected inflation 2.5%, default risk premium 1.2%, liquidity risk premium 0.3%, maturity risk premium 0.5%: required yield ≈ 1.5 + 2.5 + 1.2 + 0.3 + 0.5 = 6.0%. Equivalently, nominal risk-free 4.0% + 2.0% of premia = 6.0%.

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Building a required return: nominal risk-free rate plus risk premia (example numbers)

3. Other return measures

Real return

The **real return** measures the growth in purchasing power:

$$1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + \pi} \qquad \text{so} \qquad \pi = \frac{1 + r_{\text{nominal}}}{1 + r_{\text{real}}} - 1$$

With positive inflation the real return is below the **nominal return**; with deflation ($\pi < 0$) it is above it.

> *Example.* A portfolio earns a nominal 6.0% while inflation is 2.5%. The approximate real return is $6.0\% - 2.5\% = 3.5\%$; the exact real return is $1.060/1.025 - 1 = 3.41\%$. In purchasing-power terms, €1,000 grows to €1,060 while a basket of goods rises in price from €1.00 to €1.025, so the investor can buy $1{,}060/1.025 = 1{,}034.1$ baskets instead of 1,000, that is, 3.41% more.

Excess return

The **excess return** is the return over a benchmark or reference rate: approximately $r - r_b$, exactly $(1 + r)/(1 + r_b) - 1$. For a fund return of 11% against a benchmark return of 7%, the excess return is about 4% by subtraction and $1.11/1.07 - 1 = 3.74\%$ exactly.

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Gross vs. net return

KEY CONCEPT

Measure	Deduct trading commissions and other costs of generating the return?	Deduct management and administration fees?
Total return	No	No
Gross return	Yes	No
Net return	Yes	Yes

Pretax vs. after-tax nominal return

The **pretax nominal return** is earned before taxes; the **after-tax nominal return** is what remains after the tax liability. The rates that apply can depend on the tax laws of the country where the investment is held and on those of the investor's home country. Dividends, interest, short-term gains and long-term gains may each be taxed at a different rate, and many countries tax long-term gains more lightly than short-term gains. To get the after-tax nominal return with one tax rate on capital gains (t_{cg}) and another on distributions such as dividends and interest (t_d):

$$r_{\text{after-tax}} = \text{price return} \times (1 - t_{cg}) + \text{distribution return} \times (1 - t_d)$$

Example. Buy at 50, receive 1.50 of dividends, sell at 54. Price return 8%, distribution return 3%, pretax 11%. With $t_{cg} = 15\%$ and $t_d = 30\%$: $8\% \times 0.85 + 3\% \times 0.70 = 6.80\% + 2.10\% = 8.90\%$. If the whole return is price gain, only the capital gains rate applies. In practice, capital gains are usually taxed only when realized, and capital losses can often be used to offset gains.

Leveraged return

Leverage means using borrowed money or derivatives to control more of an asset than the investor's own cash would buy. The resulting **leveraged return** is the gain or loss as a percentage of the investor's cash investment. Suppose an investor puts up equity V_E, borrows V_D at rate r_D and earns the **unleveraged return** r_P on the whole amount:

KEY CONCEPT

$$r_L = \frac{r_P(V_E + V_D) - r_D V_D}{V_E} = r_P + \frac{V_D}{V_E}(r_P - r_D)$$

Example. $V_E = 600$, $V_D = 200$, $r_D = 5\%$. If $r_P = 9\%$: $(72 - 10)/600 = 10.33\%$. If $r_P = -3\%$: $(-24 - 10)/600 = -5.67\%$. Both results lie further from zero than the unleveraged 9% and −3%. The second form of the formula shows why: leverage adds $(V_D/V_E)(r_P - r_D)$ to the unleveraged return. A loss is always magnified, because the interest cost adds to it; a gain is magnified only if r_P exceeds r_D.

Derivatives such as futures and options also give leveraged returns: the cash deposited (margin or premium) is only a small fraction of the value of the underlying exposure.

COMMON EXAM TRAPS

- Adding expected inflation to a *nominal* risk-free rate (double counting), or leaving it out when starting from a *real* rate. In the required-yield example, adding the 2.5% expected inflation to the 4.0% nominal risk-free rate gives 8.5% instead of 6.0%.
- Calling a government bill yield a *real* rate. It is a nominal rate.
- Adding a maturity risk premium for a short-term instrument. Commercial paper or another short-term instrument maturing with the government bill carries no extra maturity risk, so only its default (and any liquidity) premium is added.
- Mixing up the size risk premium (small vs. large cap) and the value risk premium (high vs. low book-to-market), or building an equity premium into a bond's required yield.
- Deducting transaction costs again when moving from gross to net return.
- Swapping the two tax rates, or applying one blended rate to the total return. In the tax example, swapping them gives $8\% \times 0.70 + 3\% \times 0.85 = 8.15\%$ instead of 8.90%.
- In the leverage formula: charging interest on the whole portfolio instead of only on V_D, adding the interest instead of subtracting it, or dividing by V_D or $V_E + V_D$ instead of V_E. In the example with $r_P = 9\%$, charging 5% interest on all 800 gives 5.33% and dividing 62 by 800 gives 7.75%, instead of 10.33%.

EXAM SHORTCUTS

- The sign of $r_P - r_D$ shows whether borrowing raises or lowers a gain before any calculation: a gain is magnified only if r_P exceeds r_D, and a loss is always magnified.
- The real rate moves with the nominal risk-free rate and against expected inflation, so a falling bill yield with rising expected inflation means a lower real rate without computing it.

BOTTOM LINE

- An interest rate can be read as a required rate of return, a discount rate or an opportunity cost of current consumption; a measure of risk is not one of these interpretations.
- Required return = real risk-free rate + expected inflation + risk premia, where the real risk-free rate plus expected inflation is the nominal risk-free rate, such as a short-term government bill yield.
- The real risk-free rate reflects only time preference and is linked to the nominal rate exactly by $(1 + r_{\text{nominal}}) = (1 + r_{\text{real}})(1 + \pi^e)$, or approximately by $r_{\text{real}} \approx r_{\text{nominal}} - \pi^e$.
- Gross return deducts trading commissions and other costs of generating the return but not management and administration fees; net return deducts both.
- With one tax rate on capital gains and another on distributions, the after-tax nominal return is price return $\times(1 - t_{cg})$ + distribution return $\times(1 - t_d)$.
- The leveraged return is $r_L = r_P + (V_D/V_E)(r_P - r_D)$, a gain or loss measured against the investor's own cash V_E.

QUICK CHECK

Question 2

Which statement about the real risk-free interest rate is *most accurate*? It:

A includes a premium for expected inflation but no premium for default risk.

B excludes both expected inflation and any uncertainty about the amount or timing of cash flows.

C includes a liquidity risk premium because instruments paying this rate may be hard to sell quickly.

Answer: B

The label "risk-free" rules out any doubt over the amount or timing of the cash flows (so no default risk), while "real" rules out any allowance for expected inflation. The real risk-free rate therefore contains neither, reflecting only time preference.

Why the other options are wrong

- **A.** A rate that includes an inflation premium but has zero probability of default is a *nominal* risk-free rate, not a real one.
- **C.** A liquidity risk premium is compensation for a type of risk. A risk-free rate contains no risk premiums of any kind, including a liquidity risk premium.

Key takeaway Real risk-free rate: no inflation, no risk premia. Adding expected inflation gives the nominal risk-free rate; adding risk premia to that gives the required return on a risky asset.

Practice questions

Question 3

Olivia Brandt put €50,000 into a sector fund that rose 20% in year one and then dropped 20% in year two. To express the true average yearly change in her wealth, she should use the:

A arithmetic mean return.

B geometric mean return.

C capital distribution return.

Question 4

A company must pay a supplier €250,000 in exactly one year. Its treasurer discounts this payment at a stated annual rate of 6%. Under which compounding convention will the present value of the payment be *lowest*?

A Annual compounding.

B Monthly compounding.

C Semiannual compounding.

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Question 5

The managers of the Arcadia Opportunities Fund are considering borrowing to raise the fund's investment exposure from €100 million of investor capital to €150 million. Compared with remaining unleveraged, this borrowing will *most likely*:

A magnify gains but limit losses.

B diminish gains and magnify losses.

C magnify both gains and losses.

Answers: 3. B · 4. B · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/rate-of-return/. The full bank has 74 questions on this reading.

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QUANTITATIVE METHODS · READING 2

Types of Financial Returns

What you'll learn

LOS 2.a Calculate, compare and interpret the types of return (price, capital distribution, total, domestic-currency) earned on financial assets, instruments and indicators, and relate them to risk and asset allocation.

MODULE 2.1

Financial Assets, Instruments, and Indicators

This reading splits investment returns into price return and capital distribution return and shows which source dominates for equities, bonds, derivatives, cash and hybrid assets. It then computes the return on a T-bill, the domestic return on a foreign asset and index returns, and closes with the long-run risk-return ranking of asset classes, systematic and unsystematic risk, asset allocation and the weighted-average expected return of a portfolio.

LOS 2.a — Return components of financial assets, instruments and indicators

1. Two sources of return

Every investment return comes from one or both of two sources:

- **Price return** (also called **capital appreciation return**) is the gain or loss from a change in the market value of the investment.
- **Capital distribution return** is the income the investment pays out during the period, such as **dividends** or **coupons**. A distribution can be *fixed* (the rate is set at the outset, as with most bond coupons) or *variable* (the amount is uncertain, e.g. discretionary dividends or a coupon linked to a market reference rate).

KEY CONCEPT

$$\text{Total return} = \text{capital appreciation return} + \text{capital distribution return}$$

Capital distributions are **realized** when received. Price gains or losses are realized only if the asset is sold. If the asset is still held at the end of the period, they are **unrealized**, but they still count in total return.

2. Main types of financial instruments

KEY CONCEPT

Instrument	What it is	Dominant source of return
Equities (stocks, shares)	Ownership stake in the issuer	**Price return**, driven by factors that affect the company's earnings (company-specific news and broad economic factors). Dividends are variable and discretionary.

Instrument	What it is	Dominant source of return
Bonds (fixed income, debt securities)	Tradable loan; holder receives **coupons** and the **principal** (par value, face value) at **maturity**	**Fixed capital distribution** (the coupon); price return is usually the smaller part.
Derivatives	Contracts whose value derives from the performance of a specified reference (underlying) asset	Performance of the **underlying** asset (e.g., a contract fixing a future price for a commodity gains for the buyer if the commodity price rises).
Cash (notes, coins, non-interest-bearing deposits)	Financial asset held for liquidity	No return; its purchasing power falls as prices rise.
Hybrid assets (e.g., **convertible bonds**, **preferred stock**)	Blend of debt and equity features	A mix of debt-like and equity-like returns, e.g. a fixed coupon plus the right to convert into common shares.

3. Financial indicators

Financial indicators, such as **interest rates**, **foreign exchange rates** and **market indexes**, influence returns without producing any cash flows of their own.

Interest rates and the risk-free rate

Interest rates are paid to savers who lend funds. They are seen most directly in the rates banks quote on interest-bearing deposits and, in capital markets, in the returns offered by securities that borrowers issue in the fixed-income market.

The yield on short-term government securities of a developed economy is treated as a **risk-free rate**, because the issuer is assumed to make the promised payments (no **credit risk**). A **Treasury bill** (T-bill) is issued at a discount and pays its face value at maturity, so its holding period return and annualized rate (365-day year, with compounding) are:

$$HPR = \frac{\text{Face value} - \text{Price}}{\text{Price}}, \qquad r_{\text{annual}} = (1 + HPR)^{365/t} - 1$$

Example. A 91-day bill with face value \$2,000 sells for \$1,983.00.
$HPR = 17.00/1{,}983.00 = 0.8573\%$; annualized $= 1.008573^{365/91} - 1 = 3.48\%$.
If the annualized rate then rises to 4.5%, the price falls to $2{,}000/1.045^{91/365} = \$1{,}978.17$, a price loss of about \$4.83. When market interest rates rise, fixed-income prices fall (a negative price return), and the effect is larger for longer-dated bonds. A rise in the issuer's perceived default risk also lowers bond values; government debt is usually regarded as less exposed to default than corporate debt.

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Foreign exchange rates

A **foreign exchange rate** is the price of one country's currency stated in units of another currency (for example, 1 EUR = 1.08 USD). Exchange-rate moves change the returns of investors who hold assets denominated in a foreign currency.

The return on a foreign asset measured in its own currency is the **local currency return**; the return in the investor's home currency is the **domestic currency return**:

KEY CONCEPT

$$1 + r_{\text{domestic}} = (1 + r_{\text{local}})(1 + r_{FX})$$

where r_{FX} is the percentage change in the value of the foreign currency (negative if it depreciates). A strengthening foreign currency raises the domestic return; a weakening one lowers it. *Example.* A UK investor earns a 9% local return on US shares while the dollar appreciates 2% against the pound: $1.09 \times 1.02 - 1 = 11.18\%$. Simply adding (9% + 2% = 11%) ignores the cross-product and is only an approximation. Exchange-rate changes affect both parts of a foreign asset's return: the price change and the income are each converted at the new rate.

Market indexes

Market indexes track the average performance of a set of investments (a market, sector, region or asset class). Because an index is more diversified than a single security, its returns tend to be less volatile. Volatility lowers compound (geometric) returns, so over many periods an index tends to have a higher geometric mean relative to its arithmetic mean than a volatile single stock does.

Example. Stock returns +30%, −20%, +10%; index returns +8%, +2%, +5%.

	Arithmetic mean	Geometric mean
Single stock	6.67%	$(1.30 \times 0.80 \times 1.10)^{1/3} - 1 = 4.59\%$
Index	5.00%	$(1.08 \times 1.02 \times 1.05)^{1/3} - 1 = 4.97\%$

The stock "wins" on the arithmetic mean but the index wins on the compound (geometric) return. The **arithmetic mean** ignores compounding, so it does not capture this **volatility drag** and overstates multiperiod performance, most of all for the more volatile asset (the individual stock).

To compare two compound returns, measure the relative outperformance on the same compound basis: $(1 + R_A)/(1 + R_B) - 1$. *Example.* Compound returns of 9.40% for one asset and 7.10% for another give $1.0940/1.0710 - 1 = 2.15\%$, slightly less than the 2.30-point simple difference.

COMMON EXAM TRAPS

- Reporting the price change alone as the total return, which leaves out the distributions.
- Treating dividends as the main source of equity return, or price change as the main source of a traditional bond's return.

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- T-bill annualization: dividing by face value instead of the purchase price, or multiplying by $365/t$ instead of compounding. In the 91-day example, dividing the $17.00 by the $2,000 face value gives an annualized 3.45%, and multiplying 0.8573% by 365/91 gives 3.44%, instead of 3.48%.
- Domestic return: treating a depreciating foreign currency as an appreciation, or adding the two returns instead of compounding them. A depreciation enters as $(1 - \text{depreciation})$. In the UK investor example, treating the 2% dollar appreciation as a depreciation gives $1.09 \times 0.98 - 1 = 6.82\%$ instead of 11.18%.
- Saying that arithmetic means understate the multiperiod returns of volatile assets. Higher volatility lowers the geometric mean, so the arithmetic mean overstates them.

EXAM SHORTCUTS

- A strengthening foreign currency puts the domestic return above the local return and a weakening one puts it below, so answers on the wrong side of the local return can be ruled out before compounding.

BOTTOM LINE

- Total return is the capital appreciation (price) return plus the capital distribution return; distributions are realized when received, price gains only when the asset is sold, and unrealized gains still count in total return.
- Equity returns come mainly from price return, with variable and discretionary dividends, while a traditional bond's return comes mainly from its fixed coupon and price return is usually the smaller part.
- Interest rates, market indexes and foreign exchange rates are financial indicators: they influence returns but produce no cash flows of their own.
- A T-bill's holding period return is $(\text{Face value} - \text{Price})/\text{Price}$, annualized as $(1 + HPR)^{365/t} - 1$; when market interest rates rise, fixed-income prices fall, and longer-dated bonds fall more.
- The domestic currency return is $(1 + r_{\text{local}})(1 + r_{FX}) - 1$, where r_{FX} is the change in the value of the foreign currency and is negative if it depreciates.
- Volatility lowers the geometric (compound) return, so the arithmetic mean overstates multiperiod performance, most of all for the more volatile asset.

QUICK CHECK

Question 1

Tomasz Wierzbicki holds a portfolio of traditional fixed-coupon bonds. Which statement *best* describes how his total return is typically made up?

A Most of the return comes from the fixed coupon payments, with price return making up a smaller share.

B Most of the return comes from changes in the bonds' market prices, with the coupons contributing a smaller share.

C The fixed coupons and the price return typically contribute roughly equal shares.

Answer: A

A bond's coupon is a fixed capital distribution, and for traditional bonds it is usually the largest part of total return. The bond's market price can move while it is held, which creates some price return, but that is normally the smaller part.

Why the other options are wrong

- **B.** Price return as the dominant source describes equities, whose value is driven by changes in earnings prospects. It does not fit traditional fixed-coupon bonds.
- **C.** There is no general tendency toward an equal split. For traditional bonds the fixed coupon usually dominates.

Key takeaway Bonds: return mainly from fixed capital distributions (coupons). Equities: return mainly from price return.

MODULE 2.2

Long-Term Returns on Financial Assets

LOS 2.a — Long-term returns, risk and asset allocation

1. The risk-return tradeoff

The **risk-return tradeoff** says rational investors should demand higher long-run returns for bearing more risk. Applied to the main asset classes in a developed market, expected long-term returns rank from lowest to highest:

KEY CONCEPT

$$\text{short-term government bills} < \text{government bonds} < \text{corporate bonds} < \text{large-cap equities} < \text{small-cap equities}$$

(Government and corporate bonds are compared at similar maturities. Bills rank lowest because they are both short-term and free of credit risk.)

- *Bonds vs. equities.* Bonds are less risky (steadier, contractual distributions), so they are expected to earn less than equities.
- *Government vs. corporate bonds.* Corporate bonds carry more default risk; the extra return they offer over government bonds is the **credit spread**.
- *Short vs. long maturity.* Longer-term fixed-income securities face more inflation risk and interest rate risk, so they are expected to earn more than shorter-term ones.
- *Large-cap vs. small-cap equities.* Small-cap stocks are more volatile, so investors should demand more from them.

Relative performance depends on the period studied. The long-run record (1979–2023) falls into three phases: after 1995 equity returns and volatility rose while bond total return indexes grew steadily; in the 2008 global financial crisis all risky asset classes fell, recovering in the early 2010s; after 2015 equity returns and volatility again rose relative to bonds. Over the whole record several predictions hold. Equities outperformed bonds over the long run, longer-term bonds beat shorter-term bonds, and very short T-bills earned almost zero **real return**.

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Equity **dividend yields** have fallen over time as companies return more cash through **share buybacks**. Buybacks support share prices and create capital gains, which are often taxed less than dividends. Bond distribution returns have also fallen with the multidecade fall in interest rates that lasted until the early 2020s. Two results in the same 1979–2023 data run against expectations: large-cap stocks outperformed small-cap stocks, and government bonds outperformed corporate bonds. These results reflect the period studied. Investors should still expect higher long-run returns from small caps than from large caps, and from corporate bonds than from government bonds, because those assets carry more risk.

2. Risk: systematic vs. unsystematic

Risk is the chance of earning a return different from the one expected.

KEY CONCEPT

	Systematic risk (macro risk)	Unsystematic risk (micro risk)
Affects	All investments to some degree	A particular security, issuer or industry
Reduced by diversification?	No	Yes
Examples	**Inflation risk**: rising prices reduce real returns. **Interest rate risk**: rising rates reduce the value of investments (especially long-dated bonds, and equities via higher financing costs and a lower present value of future cash flows). **Market risk**: a market downturn causes losses.	**Issuer risk**, including **business risk** (earnings volatility from business conditions, e.g. supply-chain problems), **financial risk** (use of financial leverage) and **credit risk** (a debt issuer's ability to meet debt payments when due). **Sector risk** or **industry risk**: a downturn specific to one industry.

Measuring risk

Risk is measured by how much returns vary around their expected value. The simplest measure is the **range**, the highest return minus the lowest return. A wider range means more variability and more risk. Standard deviation, a more common measure, is introduced in later readings. *Example.* Returns of 12%, −6%, 9% and 3% have a range of $12\% - (-6\%) = 18$ percentage points.

The extra return investors demand for bearing extra risk is a **risk premium**. Riskier assets such as equities have earned higher risk premiums over the long run than fixed income.

3. Asset allocation

A portfolio manager builds an optimal portfolio given the client's **objectives and constraints**:

Client profile	Tilt the allocation toward
Needs high returns / capital growth, higher risk tolerance	Riskier assets that deliver **price return** (equities, including small caps)
Needs steady periodic income, low risk tolerance (e.g., a retiree)	Assets with **fixed capital distributions**, i.e. **fixed-income securities** (bonds)
Wants both growth and income	A balanced mix of equities and fixed income

Non-interest-bearing cash cannot meet an income need.

4. Portfolio expected return

Each component of a portfolio's expected return is the **weighted average** of the asset-class components, using the portfolio weights:

KEY CONCEPT

$$E(R_p) = \sum_i w_i E(R_i),$$

$$\text{total return} = \text{capital appreciation return} + \text{capital distribution return}$$

Example. 30% equities (capital appreciation 7.0%, distribution 2.0%) and 70% bonds (capital appreciation 0.4%, distribution 4.0%):

- Capital appreciation: $0.30 \times 7.0\% + 0.70 \times 0.4\% = 2.10\% + 0.28\% = 2.38\%$
- Capital distribution: $0.30 \times 2.0\% + 0.70 \times 4.0\% = 0.60\% + 2.80\% = 3.40\%$
- Total: $2.38\% + 3.40\% = 5.78\%$

COMMON EXAM TRAPS

- Ignoring the weights. A simple average of the asset-class returns is correct only when the weights are equal, and a plain sum of the returns is not an average at all. In the 30/70 example, a simple average of the equity and bond total returns (9.0% and 4.4%) gives 6.70% instead of 5.78%.
- Answering with the wrong component: capital distribution return, capital appreciation return and total return are three different figures.
- Calling inflation or interest rate risk unsystematic because its effect differs from issuer to issuer. Both affect all investments and cannot be diversified away.
- Confusing credit risk (debt payments not made when due) with business risk (earnings volatility) or interest rate risk (rate changes moving values).

BOTTOM LINE

- Under the risk-return tradeoff, expected long-term returns in a developed market rank short-term government bills < government bonds < corporate bonds < large-cap equities < small-cap equities, with bonds compared at similar maturities.
- The credit spread is the extra return corporate bonds offer over government bonds for their greater default risk, and longer-term fixed-income securities are expected to earn more than shorter-term ones because they face more inflation and interest rate risk.
- Systematic (macro) risk, such as inflation, interest rate and market risk, affects all investments to some degree and is not reduced by diversification; unsystematic (micro) risk, such as issuer and sector risk, is.
- The range, the highest return minus the lowest return, is the simplest measure of risk, and a wider range means more variability and more risk.

- A client who needs capital growth and tolerates risk tilts toward equities, a client who needs steady income with low risk tolerance tilts toward fixed-income securities, and a client who wants both holds a balanced mix.
- Each component of a portfolio's expected return is the weighted average of the asset-class components, $E(R_p) = \sum w_i E(R_i)$, and total return is the capital appreciation return plus the capital distribution return.

QUICK CHECK

Question 2

Rafael Oduya's retirement account holds 65% in equities and 35% in government bonds. He expects a total return of 9.0% on the equities and 3.8% on the bonds. The account's expected total return is *closest* to:

A 7.18%.

B 6.40%.

C 5.62%.

Answer: A

A portfolio's expected return is the weighted average of the expected returns of its components, using the allocation weights.

$$E(R_p) = 0.65 \times 9.0\% + 0.35 \times 3.8\% = 5.85\% + 1.33\% = 7.18\%$$

Adding the two returns without weights, $9.0\% + 3.8\% = 12.80\%$, counts each asset class as if it had a 100% allocation; the weights must sum to 100%, which gives the weighted 7.18%.

Why the other options are wrong

- **B.** 6.40% is the simple average $(9.0\% + 3.8\%)/2$. It would only be right with a 50/50 allocation.
- **C.** 5.62% swaps the weights, $0.35 \times 9.0\% + 0.65 \times 3.8\%$, putting 65% on the bonds. Each weight must go with its own asset class.

Key takeaway Portfolio expected return $= \sum_i w_i E(R_i)$; with non-negative weights it must lie between the lowest and highest component returns.

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Practice questions

Question 3

An analyst compares the multiyear return history of Corvane Mining shares with that of a broad, diversified market index. Which conclusion is *most accurate*?

A Because the index is less volatile, the arithmetic mean will overstate the index's multiperiod return by more than it overstates the stock's.

B Because the stock is more volatile than the index, its geometric mean return tends to be pulled down relative to the index's.

C The arithmetic mean return will tend to understate the stock's multiperiod performance relative to the index.

Answers: 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/types-of-financial-returns/. The full bank has 19 questions on this reading.

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QUANTITATIVE METHODS · READING 3

Benchmarking Returns

What you'll learn

LOS 3.a Calculate and compare money-weighted and time-weighted rates of return and judge which one fits a given performance-evaluation situation.

LOS 3.b Describe index construction choices and weighting methods (price, equal, market-capitalization, float-adjusted, fundamental), and calculate and interpret index values, divisors, weights, and price and total returns, including rebalancing and reconstitution.

MODULE 3.1

Time-Weighted and Money-Weighted Returns

This reading compares the money-weighted return, the IRR of a portfolio's cash flows, with the time-weighted return, which links subperiod returns and is unaffected by deposits and withdrawals. It then covers how a security market index is built and maintained, how price, equal, market-capitalization and fundamental weighting differ, and how to calculate an index's value, price return and total return.

LOS 3.a — Money-weighted vs. time-weighted returns

When money moves into or out of a portfolio during the evaluation period, "the portfolio's return" can mean two different things: the growth of the *investor's money*, given when it was put in, or the growth that the *manager's decisions* produced on each dollar invested. The **money-weighted rate of return** measures the first and the **time-weighted rate of return** the second.

1. Money-weighted rate of return (MWR)

The **money-weighted return** (also called the **dollar-weighted return**) is the **internal rate of return (IRR)** of the portfolio's cash flows. It is the single discount rate that gives all the flows a **net present value (NPV)** of zero:

KEY CONCEPT

$$\sum_{t=0}^{T} \frac{CF_t}{(1+MWR)^t} = 0$$

Steps:

1. Choose a period length, usually one year, such that every deposit and withdrawal falls at the start or end of a period.
2. Write down the net cash flow on every date from $t = 0$ to the end, with the signs in the table below. Enter 0 for a date on which no money moves in or out. The calculator treats the entries as equally spaced, so leaving out an empty date would move every later cash flow one period earlier.
3. Solve for the IRR, for example with the calculator's cash flow worksheet (keystrokes below).

Sign convention (from the investor's point of view; the account's point of view simply reverses every sign):

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KEY CONCEPT

Item	Sign
Beginning value / initial purchase	outflow (−)
Additional deposits or share purchases	outflow (−)
Dividends or income withdrawn (not reinvested), withdrawals	inflow (+)
Ending value / sale proceeds	inflow (+)

A dividend received on the same date as a new purchase is netted with it into one cash flow. For example, a dividend of 2 and a purchase of 50 give $CF = -48$. The IRR is unchanged if every sign is flipped, as long as money going in and money coming out carry opposite signs.

Calculator (TI BA II Plus): `[CF] [2ND] [CLR WORK]`, enter CF0, C01, C02 … (each frequency = 1), then `[IRR] [CPT]`. The resulting IRR is a rate per period. If the periods are years, it is already an annual rate.

2. Time-weighted rate of return (TWR)

The **time-weighted return** measures **compound growth**: the rate at which one unit of money invested at the start would have grown. Deposits and withdrawals do not affect it. Steps:

1. Value the portfolio immediately before each significant deposit or withdrawal, and split the evaluation period into **subperiods** at those dates.
2. Calculate each subperiod's **holding period return (HPR)**. When there is a cash flow, the beginning value of the next subperiod is the value *after* that deposit or withdrawal:

$$HPR_i = \frac{V_{end,i} - V_{begin,i} + D_i}{V_{begin,i}}$$

3. **Geometrically link** the subperiod returns. If the total period is longer than one year, take the geometric mean to annualize, using the number of years (not the number of subperiods):

$$TWR_{annual} = [(1 + HPR_1)(1 + HPR_2)\cdots(1 + HPR_n)]^{1/\text{years}} - 1$$

For a single share bought and later added to, the per-share HPRs can be used directly: $HPR_1 = (P_1 - P_0 + D_1)/P_0$, and so on.

3. Example

An investor opens an account with $1,000. One year later the account is worth $1,150 (+15%), and she deposits another $500, bringing it to $1,650. At the end of year 2 the account is worth $1,782 (+8% in year 2).

- **TWR**: $[(1.15)(1.08)]^{1/2} - 1 = 11.45\%$ per year.
- **MWR**: solve $-1{,}000 - \frac{500}{1+r} + \frac{1{,}782}{(1+r)^2} = 0$, which gives $r = 10.81\%$. (CF0 = −1,000; C01 = −500; C02 = +1,782.)

The MWR is lower than the TWR because the extra $500 was invested just before the weaker year. The MWR gives more weight to periods in which more money is invested.

4. Comparing the two measures

KEY CONCEPT

Feature	Money-weighted return	Time-weighted return
Method	IRR of all cash flows	Geometric linking of subperiod HPRs
Affected by the timing and size of external cash flows?	Yes	No
Deposit made just before strong performance	MWR higher than TWR	unaffected
Deposit made just before poor performance	MWR lower than TWR	unaffected
No external cash flows during the period (no deposits, withdrawals or paid-out income)	equal to TWR	equal to MWR
Best use	Manager (or investor) controls the cash flows	Manager does not control the cash flows

- The investment management industry prefers the time-weighted return because managers usually do not decide when clients add or withdraw money. The TWR isolates the manager's skill in selecting investments.
- The **Global Investment Performance Standards (GIPS)** are voluntary. A firm that claims compliance must present performance using **time-weighted returns**, except in specific situations where **money-weighted returns** are more appropriate. These require that the manager controls external cash flows and, in addition, that the portfolio is closed-end, has a fixed life, calls committed capital from investors, or holds mainly illiquid investments (typical of private equity).
- Withdrawals work the opposite way to deposits. Taking money out just before a strong period lowers the MWR relative to the TWR.

COMMON EXAM TRAPS

- Leaving out dividends that are paid out and not reinvested. For the MWR they are inflows on their payment date. For the TWR they belong in the numerator of that subperiod's HPR.
- Computing a TWR subperiod return without first adding the deposit just made to (or removing the withdrawal from) the beginning value, which mixes new money with investment results. In the example, measuring year 2 from $1,150 instead of the $1,650 after the deposit gives 54.96% instead of 8%.
- Annualizing over the number of subperiods instead of the number of years. Three quarterly subperiods inside one year need no root.
- Averaging subperiod returns arithmetically. The TWR links them geometrically. In the example, $(15\% + 8\%)/2 = 11.50\%$ instead of the TWR of 11.45%.
- The "required return" of an investor is irrelevant to both calculations. It is a hurdle rate for judging the result.
- If the subperiods are of equal length and each earns the same rate, both measures equal that rate whatever the cash flows. Differences come from cash flows combined with uneven subperiod returns.

EXAM SHORTCUTS

- With no deposits, withdrawals or paid-out income during the period, the MWR equals the TWR, so no IRR has to be computed.
- A deposit made just before the stronger subperiod puts the MWR above the TWR, and one made just before the weaker subperiod puts it below; withdrawals work the opposite way, so comparison questions need no IRR.
- If the subperiods are of equal length and each earns the same rate, the MWR and the TWR both equal that rate whatever the cash flows.

BOTTOM LINE

- The money-weighted return is the IRR of the portfolio's cash flows, the rate that gives them an NPV of zero, with money going in and money coming out entered with opposite signs.
- The time-weighted return geometrically links the holding period returns of subperiods formed at each significant deposit or withdrawal, and is annualized with a root equal to the number of years, not the number of subperiods.
- The MWR depends on the timing and size of external cash flows: a deposit just before strong performance makes it higher than the TWR and a deposit just before poor performance makes it lower, while the TWR is unaffected.
- The MWR and the TWR are equal when there are no external cash flows during the period.
- The investment management industry prefers the TWR because managers usually do not control when clients add or withdraw money; the MWR suits a manager or investor who controls the cash flows.
- Firms claiming compliance with the voluntary GIPS must present time-weighted returns, except where the manager controls external cash flows and the portfolio is also closed-end, has a fixed life, calls committed capital or holds mainly illiquid investments.

QUICK CHECK

Question 1

Hannah Kowalczyk makes the following trades in Silverleaf Foods shares, with no taxes or transaction costs:

- Today: buys one share at $30.00.
- One year from today: buys a second share at $36.00.
- Two years from today: sells both shares at $41.00 each.

Silverleaf pays a dividend of $1.50 per share at the end of year 1 and $2.00 per share at the end of year 2. Dividends are not reinvested. Kowalczyk's required return is 10%. Her money-weighted rate of return is *closest* to:

A 15.9%.

B 21.3%.

C 22.2%.

Answer: B

The money-weighted return is the internal rate of return of the cash flows the investor actually pays and receives. At the end of year 1 the $1.50 dividend on the first share is netted against the $36.00 purchase. At the end of year 2 she receives the sale proceeds and the dividends on two shares. The 10% required return is not an input.

t	Cash flow
0	−30.00
1	+1.50 − 36.00 = −34.50
2	2 × 41.00 + 2 × 2.00 = +86.00

$$-30.00 - \frac{34.50}{1+r} + \frac{86.00}{(1+r)^2} = 0 \;\Rightarrow\; r = 0.2131 \approx 21.3\%$$

TI BA II Plus: `[CF] [2ND] [CLR WORK]`; CF0 = −30; C01 = −34.5, F01 = 1; C02 = 86, F02 = 1; `[IRR] [CPT]` = 21.31.

Why the other options are wrong

- **A.** 15.9% is the IRR after ignoring both dividends (CF1 = −36, CF2 = +82). Dividends that are paid out are cash received by the investor.
- **C.** 22.2% is the time-weighted return: $HPR_1 = (36 - 30 + 1.5)/30 = 25.0\%$ and $HPR_2 = (41 - 36 + 2)/36 = 19.4\%$, so $\sqrt{1.250 \times 1.194} - 1 = 22.2\%$. This measure ignores the fact that more money was invested in year 2.

Key takeaway For an MWR share problem, net the dividend received against any purchase on the same date, and include dividends on all shares held at the end.

MODULE 3.2

Security Market Indexes

LOS 3.b — Security market indexes: construction, weighting and returns

1. What an index is and how its return is measured

A **security market index** represents the performance of an asset class, a security market or a segment of a market. It is built as a portfolio of **constituent securities**. Its value at any time is calculated from the market prices of those securities. The percentage change in the index value over a period is the **index return**.

KEY CONCEPT

Version	What it uses	Return it produces
Price index (price return index)	Constituent prices only	**Price return**

Version	What it uses	Return it produces
Return index (total return index)	Prices plus income (dividends, interest, other distributions), assumed reinvested	**Total return**

When constituents pay cash distributions, the total return is greater than the price return, and a total return index cannot grow more slowly (or fall faster) than the matching price index. With no distributions, the two are identical.

Returns for successive periods are **geometrically linked**:

KEY CONCEPT

$$R_{\text{period}} = (1 + R_1)(1 + R_2)\cdots(1 + R_n) - 1 \qquad V_n = V_0(1 + R_1)(1 + R_2)\cdots(1 + R_n)$$

Example. Monthly index returns of +1.8%, −0.6% and +2.1% link to $(1.018)(0.994)(1.021) - 1 = 3.31\%$. Their sum, 3.3%, is only an approximation. Starting from 500, the index ends at $500 \times 1.0331 = 516.6$.

2. Building an index: five decisions, in order

1. **Target market / objective**: which market or segment the index should measure. This is always the first decision, and it drives everything else.
2. **Constituent securities**: the selection criteria and the securities that meet them.
3. **Weighting method**: price, equal, market-capitalization (possibly float-adjusted) or fundamental weighting.
4. **Calculation**: the starting index value and how the index is computed on each later date (**divisor** and **base value**).
5. **Maintenance**: rebalancing, reconstitution and periodic review of the method.

Common index types by target market (preview of Reading 41, LOS 41.d):

Type	What it covers
Broad-based equity index	A whole market: its largest listed companies, usually float-adjusted and market-cap-weighted. A **composite market index** measures the overall market and usually holds more than 90% of its total value.
Multi-market index	Several national markets (e.g. a regional, emerging-market or global index)
Sector index	One industry or economic sector (e.g. financials, health care), in one country or globally
Investment theme index	Stocks tied to an investor trend beyond industry or company size (e.g. an ESG-screened index)

3. Weighting methods

Price-weighted index

A **price-weighted index** is the sum of the constituent prices divided by a divisor:

$$\text{Index} = \frac{\sum P_i}{D} \qquad w_i = \frac{P_i}{\sum P_j}$$

- At launch, $D = \sum P_i / \text{base value}$.
- The divisor must be changed whenever a **stock split**, **reverse stock split** or **stock dividend** occurs, or when the constituents change, so that the event alone does not move the index. A stock dividend works like a split stated as a percentage: after a 10% stock dividend, one share becomes 1.1 shares. Unlike a cash dividend, it pays out nothing and leaves each holder's ownership unchanged. For a split, recompute the sum with the post-split price and solve $D_{new} = \sum P_{new} / \text{index value}$.
- Higher-priced stocks have more weight. A 10% move in the highest-priced stock moves the index most, whatever the company's size.
- A portfolio holding an equal number of shares of each constituent replicates the index.
- It is simple to compute, but a share price says nothing about a company's value, because share counts differ widely. It is therefore a suitable benchmark mainly for a portfolio whose results depend disproportionately on the price changes of particular securities. Examples: the DJIA (30 US stocks) and the Nikkei 225.

> *Example.* Prices of \$14, \$38 and \$73 (sum \$125) with a base value of 100 give $D = 1.25$. If the \$73 stock splits 2-for-1, the sum becomes $14 + 38 + 36.50 = 88.50$, and the new divisor is $88.50/100 = 0.885$.

Equal-weighted index

In an **equal-weighted index**, every constituent gets the same weight $1/N$ at each rebalancing.

- At inception, each stock is given the same **reference value**. The number of shares held is reference value ÷ price. The **aggregate reference value** is $N \times$ reference value, and $D = \text{aggregate reference value} / \text{base value}$.
- For a period that starts at equal weights, the index return is the **arithmetic mean** of the constituent returns. The geometric mean does not apply, because the stocks are held side by side.
- Weights drift as prices move. Winners grow above $1/N$ and losers fall below it. To restore equal weights, the index must be rebalanced frequently, which means selling winners and buying losers. This is costly for a tracking portfolio.
- Small companies are overweighted relative to their size. The index is a suitable benchmark for a strategy that limits exposure to the largest companies, or for a fund that invests equal amounts in each holding. Examples: Value Line Composite Average, FT Ordinary Share Index.

Market-capitalization-weighted (value-weighted) index

In a **market-capitalization-weighted index**, each constituent's weight is its share of the total market value of the constituents:

$$w_i = \frac{P_i N_i}{\sum P_j N_j} \qquad \text{Index} = \frac{\sum P_i N_i}{D}, \quad D = \frac{\text{base-period market cap}}{\text{base value}}$$

- Stock splits and stock dividends do not change a company's market value, so an ordinary stock split or stock dividend by itself leaves the stock's weight and the divisor unchanged. The divisor is adjusted when constituents are added or removed (reconstitution). The new divisor is the new total market cap divided by the unchanged index value.

- It tracks changes in aggregate investor wealth. A tracking portfolio rebalances itself automatically, and turnover is low.
- The main drawback is that a stock's weight rises as its price rises. Possibly overvalued stocks get more weight and possibly undervalued stocks get less.
- It is the usual benchmark for a passive strategy that aims to earn the market return (e.g. S&P 500).

Float adjustment

The **market float** is the value of shares actually available to the investing public. It excludes shares held by controlling (strategic) shareholders, and often those held by other corporations or governments. **Free float** is narrower still, because it also excludes shares that foreign investors cannot buy. A **float-adjusted market-capitalization-weighted index** weights each firm by float-adjusted capitalization = market cap × float %. Firms with large controlling holdings get less weight than in an unadjusted index. **Float adjustment** belongs to market-cap weighting only.

Fundamental-weighted index

In a **fundamental-weighted index**, weights come from company fundamentals like earnings, dividends, cash flow, sales or book value. These weights are not driven by the share price, although over long periods they are related to it. Such an index has a **value tilt**: compared with market-cap weighting, it overweights stocks with low P/E or high book-to-market ratios. It suits value-oriented and other fundamentals-driven active strategies. Its advantage is that it looks beyond prices and market values. Its drawback is that it may leave out firms without positive earnings or without dividends, which can bias it toward or away from some sectors. Fundamental weights also drift, so the index needs rebalancing.

KEY CONCEPT

Method	Weight driven by	Adjust divisor for splits?	Rebalancing need	Typical bias
Price	Share price	Yes	Automatic	Toward high-priced shares
Equal	$1/N$	No	High	Toward small caps
Market cap / float-adjusted	Market value	No	Automatic	Toward large and recently risen stocks
Fundamental	Earnings, book value, etc.	No	Periodic	Value tilt

4. Example: price return vs. total return

A market-cap-weighted index has a base value of 1,000 when constituent capitalization is \$500 million, so $D = 500{,}000$. One period later the capitalization is \$515 million, and constituents paid \$6 million of dividends.

- Price index value $= 515{,}000{,}000/500{,}000 = 1{,}030$, so the price return is 3.0%.
- Total return index value $= (515 + 6)\ \text{million}/500{,}000 = 1{,}042$, so the total return is 4.2%.
- The capital distribution return is $6/500 = 1.2\%$.

From per-share data, compute each constituent's market value as price × shares at the start and at the end, and its dividends as dividend per share × shares. The market-cap total return is then (ending value + dividends − beginning value) ÷ beginning value.

5. Maintenance: rebalancing vs. reconstitution

- **Rebalancing** means resetting the constituents' weights to target once price moves have shifted them, usually every quarter. It matters mainly for **equal-weighted** and **fundamental-weighted** indexes. When an equal-weighted index is rebalanced, the total value (and the index value) is left unchanged. Each position is simply reset to total ÷ N. For example, with two holdings worth 6,300 and 5,500, each is reset to 5,900: sell 400 of the first and buy 400 of the second. Selling recent winners and buying recent losers runs against a trend-following approach.
- **Reconstitution** means changing which securities are in the index. Securities are deleted when they no longer meet the criteria (for example, they fall out of the size range), mature, are acquired, go bankrupt or are delisted. They are replaced by securities that do meet the criteria. The divisor is adjusted on that date so the index value does not jump.
- Prices react. An added security tends to rise as index-tracking managers buy it, and a deleted security tends to fall as they sell it. The weights of the remaining constituents are also adjusted to fit the weighting scheme.
- Changing the calculation formula or the rules is neither rebalancing nor reconstitution.

COMMON EXAM TRAPS

- Confusing the aggregate dollar value, the divisor and the index value.
- After a split in a price-weighted index, dividing every price by the split ratio or keeping the old divisor. In the example, keeping $D = 1.25$ after the split gives an index of $88.50/1.25 = 70.80$ instead of 100.
- Changing a stock's market-cap weight, or the divisor of a market-cap-weighted index, because of a split.
- Compounding the constituent returns of an equal-weighted index within one period, or adding index returns across periods instead of linking them.
- Leaving income out of a total return, or adding it to a price return.
- Finding a weight after price moves by growing the old weight by the stock's own return. Divide each position's new value by the new total.
- Weighting a float-adjusted index by floating share counts instead of float-adjusted market value (market cap × float %).

EXAM SHORTCUTS

- For a period that starts at equal weights, an equal-weighted index return is the arithmetic mean of the constituent returns, so no share counts or divisor are needed.
- Only a price-weighted index changes its divisor for a stock split or stock dividend; equal-weighted, market-cap-weighted and fundamental-weighted indexes leave it unchanged.

BOTTOM LINE

- A price index uses constituent prices only and gives the price return; a total return index adds income, assumed reinvested, and its total return exceeds the price return when constituents pay distributions.

- A price-weighted index is the sum of prices divided by a divisor that is changed for splits, reverse splits, stock dividends and constituent changes, and its higher-priced stocks carry more weight.
- An equal-weighted index gives each constituent $1/N$ at each rebalancing, overweights small companies relative to their size and needs frequent, costly rebalancing as weights drift.
- A market-capitalization-weighted index weights each stock by its share of total market value, tracks aggregate investor wealth and rebalances automatically, but gives more weight to stocks whose prices have risen; a float-adjusted version uses market cap × float %.
- A fundamental-weighted index weights constituents by fundamentals such as earnings, dividends, cash flow, sales or book value and has a value tilt.
- Rebalancing resets weights to target after price moves and matters mainly for fundamental-weighted and equal-weighted indexes; reconstitution changes the constituents, with the divisor adjusted on that date.

QUICK CHECK

Question 2

Which of the following statements about a total return index is *most accurate*?

A Its value reflects only the price changes of its constituent securities.

B It grows from its beginning value by compounding each period's total return.

C It can rise more slowly than a price return index built from the same securities with the same weights.

Answer: B

The value of a total return index grows by compounding its periodic total returns: $V_n = V_0(1 + R_1)(1 + R_2) \cdots (1 + R_n)$. Each periodic total return includes both price changes and income such as dividends and interest.

Why the other options are wrong

- **A.** An index whose value reflects only price changes is a price return index. A total return index also includes the income (dividends, interest, other distributions) paid by the constituents.
- **C.** Income paid by the constituents can never be negative. A total return index therefore always rises at least as fast as (or falls no faster than) the matching price return index.

Key takeaway Total return index ≥ price return index; values compound by geometric linking.

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Practice questions

Question 3

The prices of the stocks in a price-weighted index add up to $270, and the divisor is 1.80. One constituent, trading at $90, then completes a 3-for-1 stock split. The divisor that should be used immediately after the split is *closest* to:

A 0.60.

B 1.40.

C 1.80.

Question 4

An equal-weighted index contains the three stocks shown in the table below. After the period's price changes, and before any rebalancing, Kestrel Aerospace's index weight is *closest* to:

Equal-weighted index: reference values and price returns for the period

Stock	Reference value at start	Price return for the period
Kestrel Aerospace	$25,000	+12%
Linden Packaging	$25,000	+4%
Oakmere Retail	$25,000	−6%

A 33.3%.

B 34.8%.

C 36.1%.

Question 5

The sponsor of the Harrowgate Midcap 200 Index removes Velden Tools, which was delisted after filing for bankruptcy, and adds Corvin Optics, which now meets the index's size criteria. This change is *best* described as:

A reconstitution.

B rebalancing.

C redefinition.

Answers: 3. B · 4. C · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/benchmarking-returns/. The full bank has 61 questions on this reading.

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QUANTITATIVE METHODS · READING 4

Time Value of Money

What you'll learn

LOS 4.a Calculate and interpret the present value of fixed-income instruments (zero-coupon, coupon, perpetual, amortizing) and equity instruments (preferred stock, constant-growth and multistage DDM) from their expected cash flows.

LOS 4.b Calculate and interpret the implied return of fixed-income instruments and the implied required return and growth rate of equity, given price and cash flows.

LOS 4.c Explain the cash flow additivity principle and the no-arbitrage condition, and use them to derive implied forward interest rates, forward exchange rates and option values.

MODULE 4.1

Discounted Cash Flow Valuation

This reading prices fixed-income and equity instruments as the present value of their expected cash flows and then works the same relationships backward to find implied returns and growth rates. It closes with cash flow additivity and the no-arbitrage principle, which give forward interest rates, forward exchange rates and the value of an option in a one-period binomial model. Most of the calculations use the financial calculator's TVM worksheet.

LOS 4.a — Present value of fixed-income and equity instruments

1. The time value of money

The **time value of money** means a given amount of money today is worth more than the same amount in the future. To give up money now, investors must expect a **rate of return**; that same rate, used as a **discount rate**, converts future cash flows into a **present value (PV)** and present amounts into a **future value (FV)**.

KEY CONCEPT

$$FV = PV(1+r)^t \qquad PV = \frac{FV}{(1+r)^t}$$

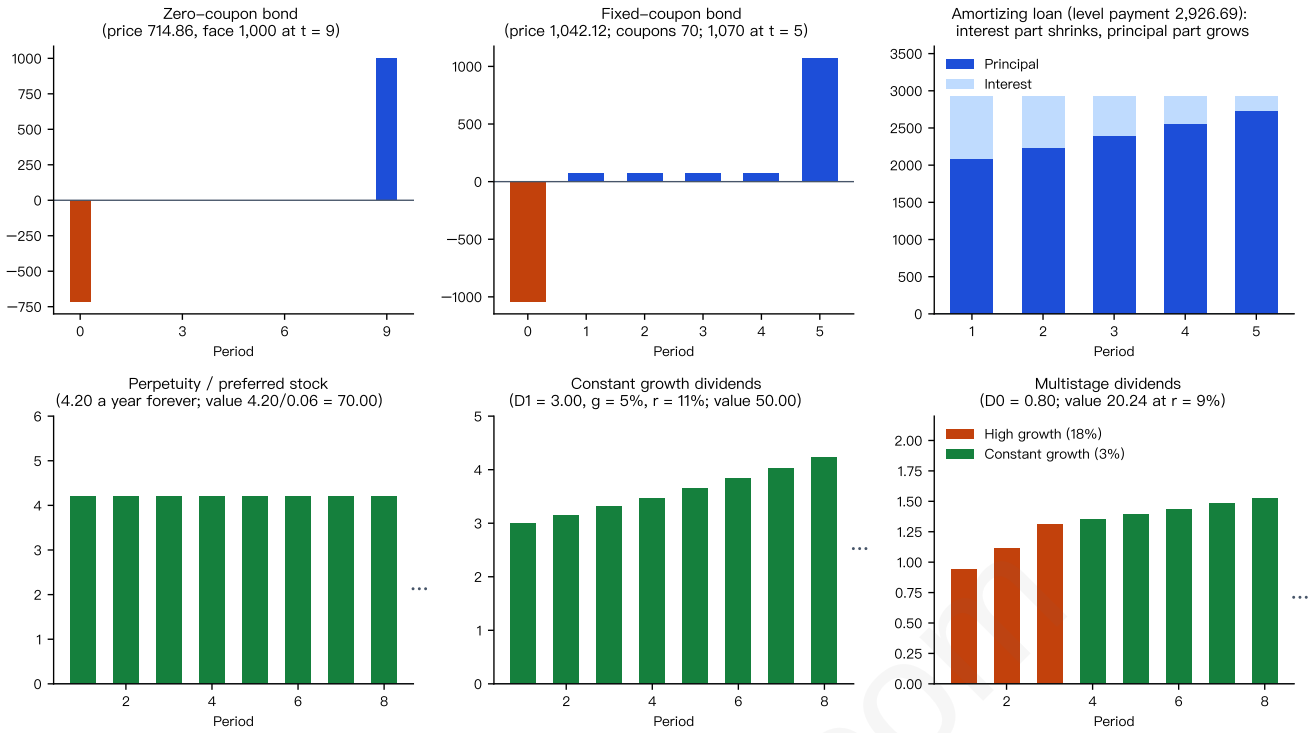
where r is the rate per compounding period and t is the number of periods. With **continuous compounding**: $FV = PVe^{rt}$ and $PV = FVe^{-rt}$.

Financial calculator (TI BA II Plus, P/Y = C/Y = 1 and END mode unless stated) keys: N = number of periods, I/Y = rate per period (in %), PV = present value, PMT = constant periodic payment, FV = future value, CPT = compute. Enter outflows as negative and inflows as positive (e.g., price paid negative, face value received positive). Clear the TVM worksheet between problems.

With m compounding periods a year, use the rate r/m and years $\times\, m$ periods. Unequal cash flows are moved to the target date one at a time and added: at 6% compounded semiannually, 2,000 paid at the end of year 1 is worth $2{,}000 \times 1.03^4 = 2{,}251.02$ at the end of year 3.

The instruments below differ only in the pattern of their cash flows:

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Cash flow patterns of the instruments in this module (numbers from the module's examples)

2. Fixed-income instruments

Pure discount instruments

A **pure discount instrument** (**zero-coupon bond**) is bought for less than its face value, and the investor receives the **face value** at maturity; the difference is the interest earned. Its price is set by two inputs: the time left to maturity and the yield to maturity, which is the rate used to discount the face value.

> *Example.* A $1,000 face value bond with 9 years to maturity and a yield of 3.8% is priced at $1{,}000/1.038^{9} = \$714.86$ (N = 9, I/Y = 3.8, PMT = 0, FV = 1,000, CPT PV = −714.86).

If the yield is negative, the zero-coupon bond is priced at a premium (above face value): at −0.4% the same bond costs $1{,}000/0.996^{9} = \$1{,}036.73$.

Fixed-coupon bonds

For a **fixed-coupon bond**, the **coupon rate** × face value sets the coupon (PMT). The **yield to maturity** (I/Y) is the rate that, used to discount the bond's cash flows, reproduces its price. The two rates generally differ. The price is the PV of the coupons and the face value, all discounted at the YTM:

$$P = \sum_{t=1}^{N} \frac{C}{(1+YTM)^{t}} + \frac{F}{(1+YTM)^{N}}$$

> *Example.* A 5-year, $1,000 bond with a 7% annual coupon and a YTM of 6%: N = 5, I/Y = 6, PMT = 70, FV = 1,000, CPT PV = −1,042.12.

KEY CONCEPT

Relationship	Price vs. par
Coupon rate > YTM	**Premium** (price > par)
Coupon rate = YTM	**Par**
Coupon rate < YTM	**Discount** (price < par)

A **floating-rate bond** resets its coupon to a **market reference rate** plus a fixed margin (e.g., reference 2.5% + 150 basis points = 4.0%); one **basis point** = 0.01%.

Perpetual bonds

A **perpetual bond** (a **perpetuity**) pays a fixed amount forever and has no maturity:

$$PV_{\text{perpetuity}} = \frac{PMT}{r}$$

The first payment is assumed to arrive one period from now.

Amortizing bonds and annuities

An **annuity** is a series of equal payments at equal intervals. The formulas below assume that each payment comes at the end of its period, so the first payment is one period away. An **amortizing bond** (or a fixed-rate mortgage) pays a constant amount that includes both interest and principal. As the outstanding balance falls, the interest portion of each payment declines and the principal portion increases. Whether the early payments are *mostly* interest depends on the rate and the term (with a low rate or a short term, even the first payment can be mostly principal).

$$PV_{\text{annuity}} = PMT \times \frac{1-(1+r)^{-N}}{r} \qquad PMT = \frac{r \times PV}{1-(1+r)^{-N}}$$

Example. A $12,000 loan at 7% repaid with 5 equal year-end payments: N = 5, I/Y = 7, PV = 12,000, FV = 0, CPT PMT = −2,926.69. For monthly payments use N = years × 12 and I/Y = annual rate ÷ 12.

To split a payment into interest and principal:

1. Interest $= r \times$ the balance at the start of the period.
2. Principal repaid = payment − interest.
3. New balance = old balance − principal repaid.

For this loan, the first payment carries interest of $0.07 \times 12{,}000 = \$840.00$ and repays $2{,}926.69 - 840.00 = \$2{,}086.69$ of principal, leaving $9,913.31. The second carries interest of $0.07 \times 9{,}913.31 = \$693.93$ and repays $2,232.76. The interest part falls each year by exactly the amount the principal part rises. The full schedule for the loan:

Amortization schedule: $12,000 at 7%, five year-end payments

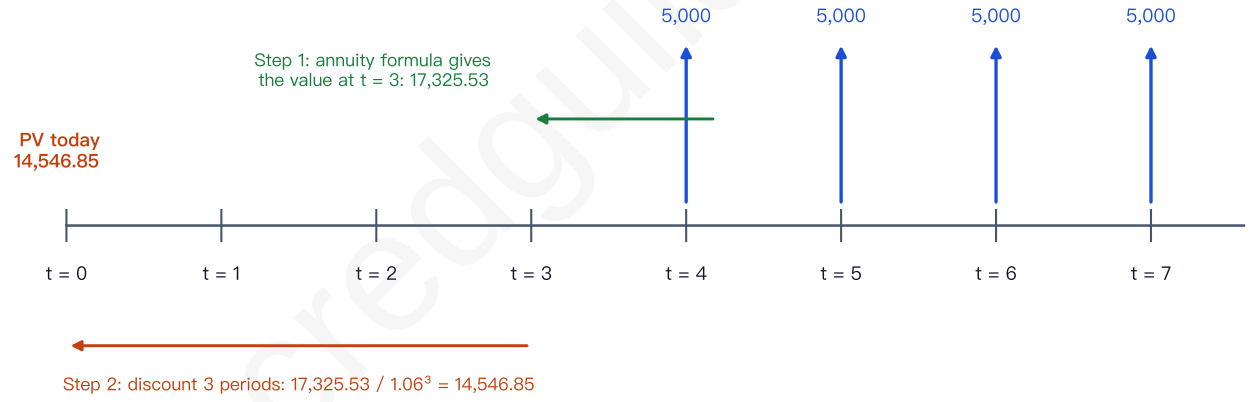
Year	Beginning balance	Payment	Interest (7%)	Principal repaid	Ending balance
1	12,000.00	2,926.69	840.00	2,086.69	9,913.31
2	9,913.31	2,926.69	693.93	2,232.76	7,680.56
3	7,680.56	2,926.69	537.64	2,389.05	5,291.51
4	5,291.51	2,926.69	370.41	2,556.28	2,735.22
5	2,735.22	2,926.69	191.47	2,735.22	0.00

Annuities that start later

The annuity formula gives a value one period before the first payment. For a later first payment:

1. Apply the annuity formula (value one period before the first payment).
2. Discount that value to today as a single sum.

> *Example.* Four payments of 5,000 start at $t = 4$, and the rate is 6%. Step 1 gives $5{,}000 \times [1 - 1.06^{-4}]/0.06 = 17{,}325.53$ at $t = 3$. Step 2 gives $17{,}325.53/1.06^{3} = 14{,}546.85$ today.



Valuing an annuity whose first payment is at t = 4

3. Equity instruments

Equity securities are valued as the PV of expected future cash flows discounted at the investor's **required return**, the return that induces investors to hold the share. Equity usually has no maturity, and its cash flows can vary.

Preferred stock pays a fixed dividend (a stated percentage of **par value**) indefinitely, so it is valued as a perpetuity:

$$V_{\text{preferred}} = \frac{D_p}{r_p}$$

where D_p is the dividend per period and r_p is the market's required return on the preferred stock.

Example. A $4.20 annual dividend and a 6% required return give a value of $70.00. The stated dividend rate sets the cash flow; the required return is the discount rate.

Common stock is a **residual claim** on the company's assets after all other claims are met. Dividends are not promised; management decides whether and when to pay them. The **dividend discount models (DDMs)**:

1. **Constant dividend**: value the share as a perpetuity, as with preferred stock.
2. **Constant growth DDM** (**Gordon growth model**):

$$V_0 = \frac{D_1}{r_e - g_c}, \qquad D_1 = D_0(1 + g_c)$$

Here V_0 is the value of all dividends from D_1 onward, r_e is the required return on common equity and g_c is the constant growth rate of dividends. The model needs the dividend expected next period (D_1), a constant growth rate, and $r_e > g_c$. *Example.* With $D_1 = \$3.00$, $r_e = 11\%$ and $g_c = 5\%$, $V_0 = 3.00/0.06 = \$50.00$.

3. **Multistage DDM**: forecast the dividends of a high-growth phase individually, then apply the Gordon model at the start of the constant-growth phase. The Gordon model gives a value one period before the dividend in its numerator: $P_n = D_{n+1}/(r_e - g_c)$. The high-growth rate may be above r_e, because those dividends are discounted one at a time (18% against 9% in the example below). Only the constant growth rate used in the Gordon step must be below r_e.

Multistage example. $D_0 = \$0.80$, growth 18% for 3 years, then 3% forever, $r_e = 9\%$.
$D_1 = 0.944,\ D_2 = 1.1139,\ D_3 = 1.3144,\ D_4 = 1.3144 \times 1.03 = 1.3539$
$P_3 = 1.3539/(0.09 - 0.03) = 22.564$

$$V_0 = \frac{0.944}{1.09} + \frac{1.1139}{1.09^2} + \frac{1.3144 + 22.564}{1.09^3} = \$20.24$$

WORKED EXAMPLE: PRICING A BOND AND SPLITTING A LOAN PAYMENT

A 5-year bond has a $1,000 face value, a 5.25% annual coupon and a YTM of 4.35%. Separately, a borrower takes a $250,000 loan at 4.8%, repaid with 10 equal year-end payments.

Step 1. The coupon rate (5.25%) is above the YTM (4.35%), so the bond must sell at a premium.

Step 2. Price the bond: N = 5; I/Y = 4.35; PMT = 52.50; FV = 1,000; CPT PV = −1,039.68. The price is $1,039.68, above par as predicted.

Step 3. One year later, with the YTM unchanged: N = 4; I/Y = 4.35; PMT = 52.50; FV = 1,000; CPT PV = −1,032.40. The price is still above par, but closer to it.

Step 4. Loan payment: N = 10; I/Y = 4.8; PV = 250,000; FV = 0; CPT PMT = −32,062.44.

Step 5. First payment: interest $0.048 \times 250{,}000 = \$12{,}000.00$; principal $32{,}062.44 - 12{,}000.00 = \$20{,}062.44$; new balance $229,937.56.

Step 6. Second payment: interest $0.048 \times 229{,}937.56 = \$11{,}037.00$; principal $32{,}062.44 - 11{,}037.00 = \$21{,}025.44$. The interest part fell by $963.00 and the principal part rose by the same amount.

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Result. The bond is priced at $1,039.68. The loan payment is $32,062.44, of which $12,000.00 is interest in year 1 and $11,037.00 in year 2.

COMMON EXAM TRAPS

- Mixing up the coupon rate (which sets PMT) and the YTM (which is I/Y), or, when the bond's own price is given, taking the yield on similar bonds as its YTM.
- Annuities and single sums: confusing a present value with a future value, adding up payments without discounting them, or discounting with simple interest, $FV/(1+rt)$, which ignores compounding.
- Perpetuity value = payment ÷ rate, which is already today's value when the first payment is one period away. Multiplying instead, or discounting that value one more period, gives a wrong value. With the $4.20 dividend and 6% required return above, discounting the $70.00 value one more period gives $66.04.
- Gordon model: using D_0 where D_1 is needed, or applying the model when growth is not constant or not below the required return.
- Multistage DDM: the terminal value P_n uses D_{n+1} and is discounted n periods, not $n+1$. In the multistage example, discounting P_3 four periods gives $18.80 instead of $20.24.
- Annuity that starts later: discounting over the full number of periods to the first payment instead of one period fewer. In the example, discounting 17,325.53 four periods gives 13,723.44 instead of 14,546.85.
- Forgetting that a negative yield gives a price above face value.
- Monthly loans: converting only one of N and I/Y to months.

EXAM SHORTCUTS

- Compare the coupon rate with the YTM before calculating a bond price: a coupon above the YTM means a premium, equal means par, below means a discount, so prices on the wrong side of par can be ruled out at once.
- A zero-coupon bond, a coupon bond and a level loan are all one TVM entry; a zero is the same entry with PMT = 0.
- A level perpetuity or a preferred share needs no worksheet: value = payment ÷ rate.

BOTTOM LINE

- Every instrument here is valued as the present value of its expected cash flows, each discounted with $PV = FV/(1+r)^t$.
- A bond sells at a premium when its coupon rate is above its YTM, at par when they are equal and at a discount when the coupon rate is below the YTM.
- A perpetuity is worth PMT/r one period before its first payment, and preferred stock is valued the same way, D_p/r_p.
- In an amortizing loan each payment's interest is the rate times the opening balance, so the interest part falls and the principal part rises over the life of the loan.
- The Gordon growth model $V_0 = D_1/(r_e - g_c)$ uses next period's dividend and requires constant growth below the required return.
- In a multistage DDM the terminal value $P_n = D_{n+1}/(r_e - g_c)$ is discounted n periods, together with the dividend paid at n.

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QUICK CHECK

Question 1

Brisa Therapeutics expects to receive the licensing fees shown in the table below. Its finance team discounts them at a stated annual rate of 6%, compounded monthly. The present value of the three fees today is *closest* to:

Expected licensing fees

Received at the end of year	Fee
1	$5,000
2	$9,000
3	$12,000

A $22,722.

B $22,802.

C $25,708.

Answer: A

The fees are unequal, so each one is discounted to today on its own and the present values are added. With monthly compounding, the rate per period is 6%/12 = 0.5% and each year has 12 periods, so the fees are discounted over 12, 24 and 36 months.

$$PV = \frac{5{,}000}{1.005^{12}} + \frac{9{,}000}{1.005^{24}} + \frac{12{,}000}{1.005^{36}}$$

$$= 4{,}709.53 + 7{,}984.67 + 10{,}027.74 = \$22{,}721.94$$

Calculator (P/Y = C/Y = 1, END mode), first fee: N = 12; I/Y = 0.5; PMT = 0; FV = 5,000; CPT PV = −4,709.53. Then N = 24; FV = 9,000; CPT PV = −7,984.67, and N = 36; FV = 12,000; CPT PV = −10,027.74.

Why the other options are wrong

- **B.** $22,802 discounts at 6% with annual compounding (1.06, 1.06^2 and 1.06^3). Monthly compounding raises the effective annual rate to $1.005^{12} - 1 = 6.17\%$, so the present value is lower.
- **C.** $25,708 uses the monthly rate of 0.5% but discounts over only 1, 2 and 3 periods, counting years instead of months.

Key takeaway With m compounding periods a year, discount at r/m over $m\times$ years periods, and value unequal cash flows one at a time.

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MODULE 4.2

Implied Returns and Cash Flow Additivity

LOS 4.b — Implied returns and implied growth

The PV relationships from Module 4.1 can be rearranged to solve for the rate of return implied by a known price and known (or assumed) cash flows.

Zero-coupon bond

$$r = \left(\frac{FV}{PV}\right)^{1/N} - 1$$

Example. A $1,000 face value bond maturing in 6 years and priced at $820 yields $(1{,}000/820)^{1/6} - 1 = 3.36\%$ (N = 6, PV = −820, PMT = 0, FV = 1,000, CPT I/Y = 3.36).

Coupon bond

The yield to maturity of a **coupon bond** generally has no closed-form solution, so it is found with the calculator: N, PMT = coupon, FV = face, PV = −price, CPT I/Y. *Example.* A 5-year, $1,000 par bond with a 4% annual coupon priced at $975.00: N = 5, PMT = 40, FV = 1,000, PV = −975, CPT I/Y = 4.57.

Price and yield move in opposite directions: when a bond's price rises, its YTM falls, and when the price falls, its YTM rises. If the bond above rises to $990.00, its YTM falls to 4.23%.

Equity: implied required return

The required return on equity cannot be observed directly. Rearranging the constant growth DDM with the observed price gives:

KEY CONCEPT

$$r_e = \frac{D_1}{P_0} + g_c = \text{dividend yield} + \text{constant growth rate}$$

The **dividend yield** here is the expected dividend divided by the current price.

Equity: implied growth rate

Given a required return, the **implied growth rate** is

$$g_c = r_e - \frac{D_1}{P_0} = \text{required return} - \text{dividend yield}$$

When the required return is the unknown and the dividend given is the one just paid (D_0), first convert it: $D_1 = D_0(1 + g_c)$.

Given	Solve for	Formula
P_0, D_1, g_c	Required return	$D_1/P_0 + g_c$

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Given	Solve for	Formula
P_0, D_1, r_e	Implied growth	$r_e - D_1/P_0$
r_e, g_c, P_0	Dividend yield	$r_e - g_c$

Example. A share priced at $50.00 is expected to pay a $2.00 dividend next year, a dividend yield of 4.0%. With 5% constant growth, the implied required return is 4.0% + 5% = 9.0%. If investors instead require 10%, the implied growth rate is 10% − 4.0% = 6.0%.

LOS 4.c — Cash flow additivity, no arbitrage and their uses

1. Cash flow additivity principle

Under the **cash flow additivity principle**, the value of a stream of cash flows equals the total obtained by discounting each cash flow separately and adding the results. Streams can be added together (combining cash flows that fall on the same date) or split into pieces in any way, and the PVs of the pieces add up to the PV of the whole.

Example. At 8%, payments of $200, $200, $500 and $200 at the ends of years 1–4 are worth the same as a 4-year annuity of $200 (PV = 662.43) plus a single $300 at year 3 (PV = 238.15), a total of $900.58. This is **replication**: an annuity plus a zero-coupon bond recreates the uneven stream. Splitting off an annuity equal to the smallest (or any) level amount leaves a residual stream in the same time order. The order matters, because earlier cash flows are worth more.

To compare two projects, subtract one stream from the other date by date. If the difference is $+X$ earlier and $-X$ later, the project with the earlier cash flow has the higher PV at any positive discount rate, with no NPV calculation needed.

2. No-arbitrage principle (law of one price)

Additivity underpins the **no-arbitrage principle** (the **law of one price**): two assets whose future cash flows match in every possible state must sell for the same price now. When their prices diverge, traders buy the cheaper asset and sell the dearer one, and that trading pushes the two prices back together. Such gaps close fast, so an arbitrageur must act at once.

The principle is applied below to derive forward interest rates and forward exchange rates and to value options.

3. Forward interest rates

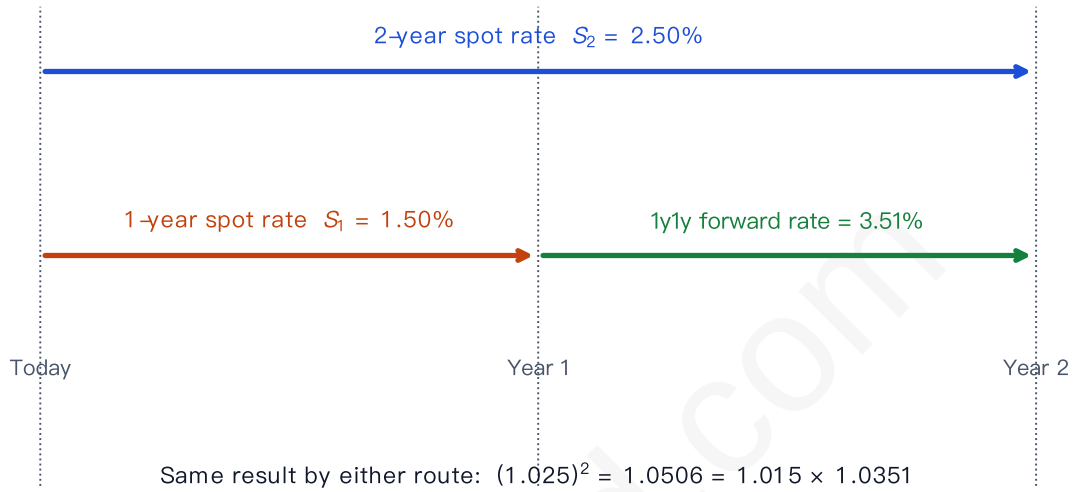
A **spot rate** S_n is the annualized rate for a loan made today for n years. A **forward rate** is for a loan made at a future date: **1y1y** is a 1-year rate one year from now, **2y1y** a 1-year rate two years from now, **3y2y** a 2-year rate three years from now. Borrowing or investing over the same horizon by any route must cost the same:

KEY CONCEPT

$$(1+S_2)^2 = (1+S_1)(1+1y1y) \quad \Rightarrow \quad 1y1y = \frac{(1+S_2)^2}{1+S_1} - 1$$

$$(1+S_3)^3 = (1+S_1)(1+1y1y)(1+2y1y)$$

A forward rate solved from spot rates in this way is an **implied forward rate**. *Example.* With $S_1 = 1.5\%$ and $S_2 = 2.5\%$, $1y1y = 1.025^2/1.015 - 1 = 3.51\%$. With an upward-sloping spot curve, the forward rate lies above both spot rates. An investor is indifferent between the 2-year bond and rolling 1-year bonds at S_1 then 1y1y.



Two ways to invest for two years: a 2-year spot bond, or a 1-year spot bond followed by the 1y1y forward rate

If an investor could instead lock in 4.00% for the second year, the two routes would no longer cost the same. Borrowing \$1,000,000 for two years at S_2 means repaying $1{,}000{,}000 \times 1.025^2 = \$1{,}050{,}625$. Lending the same amount for one year at 1.5% and then for one more year at the locked-in 4.00% returns $1{,}000{,}000 \times 1.015 \times 1.04 = \$1{,}055{,}600$. The \$4,975 gain at year 2 needs none of the investor's own money and carries no risk, so it is an arbitrage profit. Traders exploiting it push the forward rate back to 3.51%.

Longer forward rates follow the same rule. For the 2-year rate one year from now, $(1+S_3)^3 = (1+S_1)(1+1y2y)^2$.

4. Forward exchange rates

An **exchange rate** states how many units of one currency are needed to buy one unit of another. A rate quoted as A/B is the price of one unit of B (the **base currency**) in units of A (the **price currency**). No arbitrage links forward and spot rates to the two riskless interest rates for the forward's horizon. If the forward rate were out of line, a trader could borrow one currency, convert it at the spot rate, invest it at the other currency's riskless rate and sell the proceeds forward, locking in a riskless profit. Use one of two equivalent forms, depending on how the rates are quoted:

- With annual rates and a horizon of T years:

$$\frac{\text{Forward}}{\text{Spot}} = \frac{(1+r_{\text{price}})^T}{(1+r_{\text{base}})^T}$$

- With rates already stated for the forward's period (e.g., a 90-day money-market rate converted as $r \times 90/360$), no exponent is applied:

$$\frac{\text{Forward}}{\text{Spot}} = \frac{1 + r_{\text{price, period}}}{1 + r_{\text{base, period}}}$$

The percentage difference between forward and spot is approximately the interest rate differential. *Example.* For a one-year forward with spot at 1.1000 USD/EUR, a one-year USD rate of 4% and a one-year EUR rate of 2%, the forward rate is $1.1000 \times 1.04/1.02 = 1.1216$, about 1.96% above spot, close to the 2% differential. The currency with the higher interest rate trades at a forward discount.

5. Option values: one-period binomial model

An **option** gives its holder the right to buy (**call option**) or sell (**put option**) an asset at a set **exercise price**, with no obligation to do so. At expiration: call value $= \max(S_T - X, 0)$; put value $= \max(X - S_T, 0)$. At expiration, an option is **out of the money** when exercising would not pay (a call whose exercise price is above the market price, or a put whose exercise price is below it), so the holder lets it expire. An option that is **in the money** at expiration is exercised.

In a **binomial model**, the underlying moves to one of two prices over the period. A one-period model needs the underlying's value today, the exercise price (the option expires at the end of the period), the returns in an up move and a down move, and the risk-free rate for the period. Write one call and buy h shares so that the portfolio's value is the same after an up or down move (risk free):

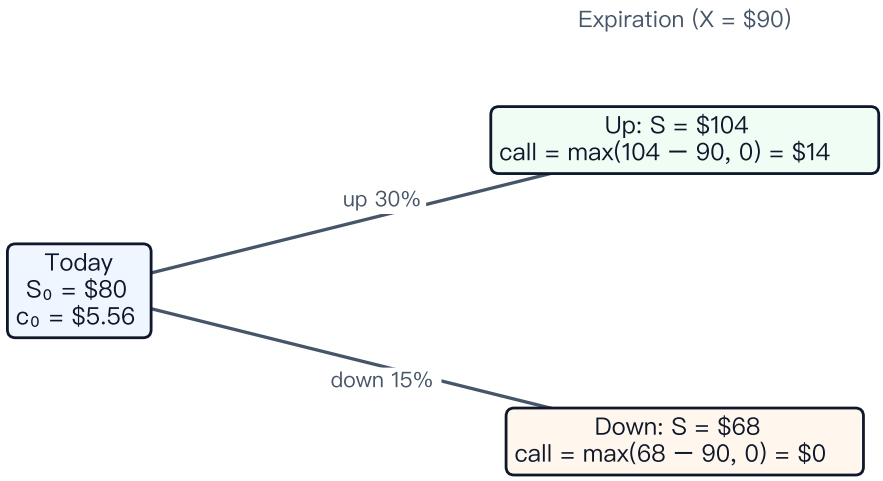
$$h = \frac{c_u - c_d}{S_u - S_d}$$

This h is the **hedge ratio**: the number of shares bought per option written.

The riskless portfolio must earn the **risk-free rate**, so: value today $= \dfrac{hS_d - c_d}{1 + R_f}$, and the option value is $c_0 = hS_0 - \text{portfolio value today}$.

> *Example.* $S_0 = \$80$, up 30% to \$104, down 15% to \$68, $X = \$90$, $R_f = 3.5\%$. Call payoffs: \$14 (up) and \$0 (down). $h = 14/36 = 0.3889$. Portfolio at expiration $= 0.3889 \times 68 = \$26.44$ in both states. PV $= 26.44/1.035 = \$25.55$. Call value $= 0.3889 \times 80 - 25.55 = \$5.56$.

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Hedge ratio h = (14 − 0)/(104 − 68) = 0.3889; riskless portfolio = hS − c = $26.44 in both states

Discounted at 3.5%: $25.55; call value = 0.3889 × 80 − 25.55 = $5.56

One-period binomial tree for the call example: prices, payoffs, hedge ratio and call value

COMMON EXAM TRAPS

- Setting a price limit the wrong way round: for a zero, a return above X% requires a price below the one that gives X%.
- Adding the dividend yield to the required return to get implied growth; the dividend yield is subtracted. With a 10% required return and a 4.0% dividend yield, that error gives 14.0% instead of 6.0%.
- Averaging spot rates to get a forward rate. The forward rate comes from the compounded (geometric) relationship. With $S_1 = 1.5\%$ and $S_2 = 2.5\%$, the average is 2.0%, while 1y1y is 3.51%.
- Forward FX: taking the sum or the average of the two interest rates as the forward premium or discount, or putting the base-currency rate in the numerator of the exact ratio. With spot at 1.1000 USD/EUR, putting the EUR rate on top gives $1.1000 \times 1.02/1.04 = 1.0788$ instead of 1.1216.
- Additivity: writing the residual cash flows in reverse date order, which changes their PV.
- Binomial model: weighting the payoffs 50/50, discounting the hedged portfolio at a rate other than the risk-free rate, or reporting the portfolio's PV as the option value instead of subtracting it from the share position. In the example, weighting the payoffs 50/50 gives $0.5 \times 14/1.035 = \$6.76$ and reporting the portfolio's PV gives $25.55, while the call is worth $5.56.

EXAM SHORTCUTS

- With an upward-sloping spot curve the implied forward rate lies above both spot rates, so a forward rate between the two spot rates can be ruled out.
- Check the direction of a forward exchange rate first: when the price currency has the higher interest rate the forward is above spot, and when it has the lower rate the forward is below spot.
- When two cash flow streams differ only by $+X$ earlier and $-X$ later, the stream with the earlier cash flow has the higher PV at any positive discount rate, with no NPV calculation needed.

BOTTOM LINE

- The implied return on a zero-coupon bond is $(FV/PV)^{1/N} - 1$; a coupon bond's YTM is found with the calculator (CPT I/Y).
- A bond's price and its YTM move in opposite directions.
- Rearranging the Gordon model gives the required return $r_e = D_1/P_0 + g_c$ and the implied growth rate $g_c = r_e - D_1/P_0$.
- Under cash flow additivity the PV of a stream equals the sum of the PVs of its pieces, so an uneven stream can be replicated with annuities and zero-coupon bonds.
- No arbitrage sets the implied forward rate $1y1y = (1 + S_2)^2/(1 + S_1) - 1$ and the forward exchange rate $\text{Forward}/\text{Spot} = (1 + r_{\text{price}})^T/(1 + r_{\text{base}})^T$.
- In a one-period binomial model the hedge ratio is $h = (c_u - c_d)/(S_u - S_d)$, the hedged portfolio earns the risk-free rate, and $c_0 = hS_0$ minus the PV of the hedged portfolio.

QUICK CHECK

Question 2

Chiara Lombardi holds a 7-year corporate bond with a 5.3% annual coupon. Her monthly statement shows that the bond's yield to maturity rose from 5.1% last month to 5.6% this month. Over the month, the bond's price has *most likely*:

A decreased.

B increased.

C remained flat.

Answer: A

Price and yield to maturity are inversely related. The yield to maturity is the discount rate that equates the bond's cash flows with its price. A higher yield means the same fixed cash flows are discounted more heavily, so the price must have fallen.

Why the other options are wrong

- **B.** A price increase would go with a lower yield to maturity, not a higher one.
- **C.** A flat price would leave the yield to maturity almost unchanged (only the passage of one month would move it slightly). It could not produce a 0.5 percentage point rise.

Key takeaway Price and YTM are inversely related: when the yield rises, the price falls.

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Practice questions

Question 3

Brightwater Utilities has preferred stock outstanding that pays a fixed annual dividend of $2.60 per share with no maturity date. An investor who requires a 7.2% rate of return should be willing to pay, per share, a maximum of *closest* to:

A $18.72.

B $33.69.

C $36.11.

Question 4

A small business project is expected to generate the following cash flows:

Expected year-end cash flows

End of year	Cash flow
1	$3,000
2	$5,000
3	$0
4	−$1,500

At a discount rate of 8%, the present value of this cash flow stream is *closest* to:

A $5,962.

B $7,064.

C $8,167.

Question 5

A stock trades at $40. In one year it will either rise by 25% or fall by 20%. A one-year call option on the stock has an exercise price of $44, and the risk-free rate is 4%. Using a one-period binomial model, the value of the call option today is *closest* to:

A $2.88.

B $3.08.

C $3.20.

Answers: 3. C · 4. A · 5. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/time-value-of-money/. The full bank has 44 questions on this reading.

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QUANTITATIVE METHODS · READING 5

Skewness and Kurtosis

What you'll learn

LOS 5.a Calculate, interpret and evaluate measures of dispersion: range, mean absolute deviation, variance and standard deviation (population and sample).

LOS 5.b Describe, interpret and evaluate skewness and kurtosis and their effect on the mean, median, mode and tail risk of a return distribution.

LOS 5.c Calculate, interpret and evaluate covariance and correlation, including the limits of correlation (linear only, no causation, spurious correlation) and the use of scatter plots.

LOS 5.d Calculate, interpret and evaluate semivariance, target downside deviation and the coefficient of variation as measures of downside and relative risk.

MODULE 5.1

Measures of Central Tendency

This reading summarizes a return distribution with measures of central tendency and location (mean, median, mode, trimmed and winsorized means, quantiles) and of dispersion (range, mean absolute deviation, variance and standard deviation), and it interprets skewness and kurtosis. It then calculates the covariance and correlation of two variables and measures downside risk and relative dispersion with the target downside deviation and the coefficient of variation.

LOS 5.a — Central tendency and location

A **measure of central tendency** summarizes where the "center" of a dataset lies, so a single number can stand for the typical or expected value. A **measure of location** (quantiles) describes where a given observation sits within the distribution.

Population vs. sample

A **population** is every element of interest (e.g., all monthly returns a fund has ever earned). A **sample** is a subset of it (e.g., the last 36 months). Sample statistics are used to make inferences about population parameters. Of all the measures of central tendency, analysts rely on the arithmetic mean most often.

$$\text{Sample mean: } \bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \qquad \text{Population mean: } \mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

The three classic measures

KEY CONCEPT

Measure	Definition	Sensitive to outliers?	Notes
Arithmetic mean	Sum of the observations divided by their number (every value weighted equally)	Yes; one extreme value can move it a lot	The **sample mean** is an arithmetic mean

Measure	Definition	Sensitive to outliers?	Notes
Median	Middle value of the data sorted in order; with an even count, the average of the two middle values	No; it is a **robust estimator** (reliable with outliers or when the data come from a different distribution than assumed)	Half of the observations lie above it, half below
Mode	Most frequently occurring value	No	A dataset can have one mode, two (**bimodal**), several, or none

For continuous data such as returns, exact repeats are rare, so outcomes are grouped into intervals and the **modal interval**, the interval containing the most observations, is reported.

The **geometric mean** measures the compound growth rate of returns. Like the arithmetic mean, it uses every observation, so it is also pulled by extreme values.

Example. Six monthly returns: 4%, −1%, 7%, 4%, 2%, 20%.

- Mean $= 36/6 = 6.0\%$.
- Sorted: −1, 2, 4, 4, 7, 20, so the median $= (4 + 4)/2 = 4.0\%$.
- Mode $= 4\%$ (appears twice).
  The single 20% month pulls the mean above both the median and the mode. Replace 20% with 9% and the mean falls to 4.17%, while the median and mode stay at 4%.

Dealing with outliers

Outliers are unusually large or small observations. They can come from measurement, data entry or sampling errors, or simply from the real variability of the data. If they are errors, they distort the mean. If the data are measured correctly, keeping every value may be best, because deleting genuine observations throws away information about the distribution.

- **Trimmed mean**: drop a set percentage of the observations at the extremes, split equally between the two ends, then average what remains. Trimming identifies the outliers objectively but may remove useful information. A 4% trimmed mean drops the lowest 2% and the highest 2%. The denominator is the number of observations *remaining*.
- **Winsorized mean**: keep every observation but *replace* the extremes with percentile values. For a 96% winsorized mean, values below the 2nd percentile are set equal to the 2nd percentile and values above the 98th percentile are set equal to the 98th percentile; then take the mean of all n values.

KEY CONCEPT

	Trimmed mean	Winsorized mean
Extreme values	Removed	Replaced by percentile values
Number of values averaged	Fewer than n	All n
Information kept	Less	More

With only a small number of outliers, the winsorized mean is a better measure of the center than either the arithmetic mean or a trimmed mean. Both adjustments move the estimate *away* from the side where the extremes lie: a few huge positive values make the trimmed/winsorized mean lower than the arithmetic mean; a few huge negative values make it higher.

Quantiles

A **quantile** is a cut-off value: a given share of the observations lies at or below it. The median is the 50th percentile, which is also the second quartile and the fifth decile.

KEY CONCEPT

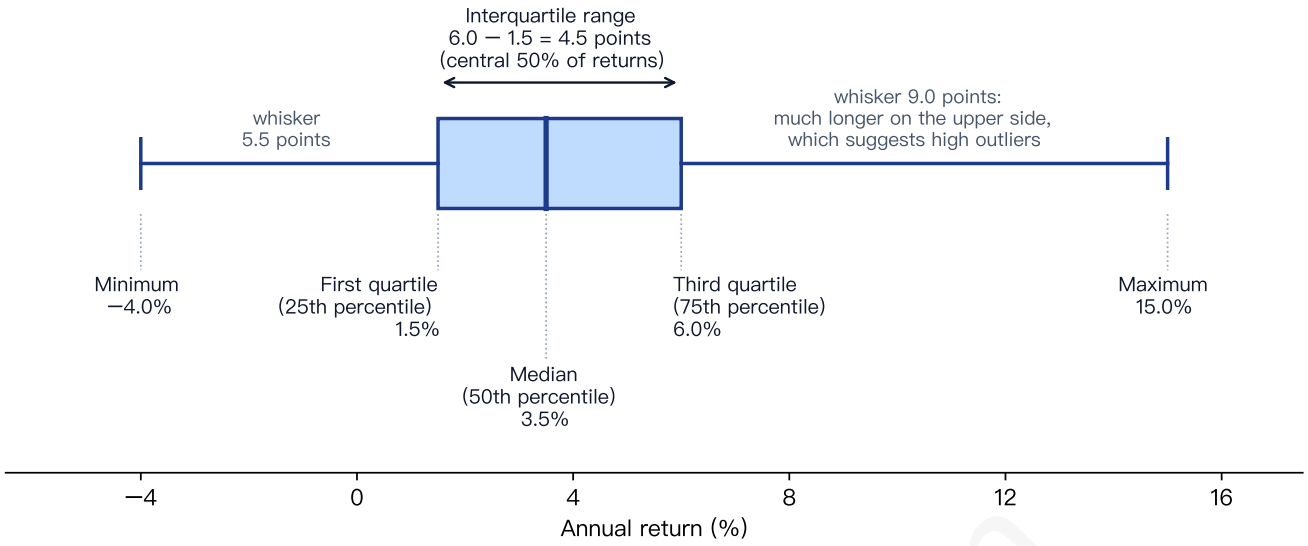
Quantile	Divides the data into	Example
Quartile	4 parts	3rd quartile = 75th percentile
Quintile	5 parts	1st quintile = 20th percentile
Decile	10 parts	9th decile = 90th percentile
Percentile	100 parts	An observation at the 70th percentile has 70% of the observations below it

Because each quantile is named by the share of observations at or below it, the positions always run in the same order: first decile (10th percentile), first quintile (20th), first quartile (25th), median (50th). A lower position can equal but never exceed a higher one.

The **interquartile range** (IQR) runs from the first quartile (25th percentile) to the third quartile (75th percentile), so its width is $Q_3 - Q_1$. It contains the central 50% of the data.

A **box and whisker plot** draws a box that spans the interquartile range, a line inside the box marks the median, and the whiskers extend to the smallest and largest observations (the full range). A whisker much longer on one side hints at outliers on that side.

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Reading a box and whisker plot: annual returns with a long upper whisker

Stationarity

Statistical inference works best when data are **stationary**: the mean and variance of the underlying distribution do not change over time. If they do change, the series is **nonstationary** and forecasts or tests based on it are less reliable. Price or index *levels* that trend upward are typically nonstationary; their periodic percentage *changes* are much more likely to be stationary.

COMMON EXAM TRAPS

- Calling the middle value of ordered data the mean. It is the median; with an even count, average the two middle values.
- Assuming that a value equal to the median also equals the mean. The two may be close without being equal, so compute the mean before choosing.
- Expecting the median to move when only the largest (or smallest) observation changes. As long as that value keeps its rank, only the mean moves.
- Removing X% from each end for an X% trimmed mean. The correct cut is X/2% from *each* end.
- Treating a winsorized mean as if the extremes were dropped (the count stays at n), or reading a percentile as the share of observations *above* the value.
- Linking stationarity to the mean equalling the median or the geometric mean. Stationarity concerns a stable mean and variance over time.

EXAM SHORTCUTS

- Changing only the largest or the smallest observation moves the mean but not the median, as long as that value keeps its rank.
- Quantile positions run in a fixed order (first decile, first quintile, first quartile, median), so an answer that puts a lower quantile above a higher one can be ruled out.
- A few huge positive values make the trimmed and winsorized means lower than the arithmetic mean, and a few huge negative values make them higher.

BOTTOM LINE

- The arithmetic mean weights every observation equally and is sensitive to outliers, while the median, the middle value of sorted data (the average of the two middle values with an even count), is a robust estimator.
- The mode is the most frequent value; a dataset can have one mode, two (bimodal), several or none, and for continuous data the modal interval is reported.
- A trimmed mean drops a set percentage of the extreme observations, split equally between the two ends, and averages the rest; a winsorized mean replaces the extremes with percentile values and averages all n observations.
- When there are only a few outliers, a winsorized mean measures the center better than the arithmetic mean or a trimmed mean does.
- A quantile is a cut-off value with a given share of the observations at or below it, and the interquartile range $Q_3 - Q_1$ contains the central 50% of the data.
- Stationary data have a mean and variance that do not change over time; trending price levels are typically nonstationary, while their periodic percentage changes are much more likely to be stationary.

QUICK CHECK

Question 1

The table below shows one-year returns for seven equity funds.

One-year returns of seven equity funds

Fund	Return (%)
Harbor Growth	5
Juniper Value	11
Lakeview Income	10
Meridian Core	11
Northgate Select	15
Oakmont Small Cap	3
Pinecrest Global	17

Which of the following statements about these returns is *most accurate*?

A The mean is equal to the median.

B The mean is equal to the mode.

C The median is equal to the mode.

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Answer: C

Sorted, the returns are 3, 5, 10, 11, 11, 15, 17. The middle (fourth) value is 11%, and 11% is also the only value that appears twice, so the median and the mode are both 11%. The mean is lower, at about 10.29%.

Sorted: 3, 5, 10, 11, 11, 15, 17. The fourth value is the median, 11%.

Mode $= 11\%$ (appears twice).

$$\bar{X} = \frac{5 + 11 + 10 + 11 + 15 + 3 + 17}{7} = \frac{72}{7} = 10.29\%$$

Why the other options are wrong

- **A.** The mean (10.29%) is below the median (11%) because the low returns of 3% and 5% pull the average down.
- **B.** The mean (10.29%) is not equal to the mode (11%).

Key takeaway Compute all three measures explicitly; the mean is rarely exactly equal to an observed value.

MODULE 5.2

Dispersion, Skewness, and Kurtosis

LOS 5.a — Measures of dispersion

Dispersion is the variability of observations around their central tendency. In investments, central tendency measures reward and dispersion measures risk.

Range and mean absolute deviation

$$\text{range} = \text{maximum value} - \text{minimum value}$$

The **range** is simple, but because it uses only the two most extreme observations it says nothing about how the values between them are spread.

The **mean absolute deviation** (MAD) averages how far each observation lies from the arithmetic mean, ignoring the sign:

$$\text{MAD} = \frac{\sum_{i=1}^{n} |X_i - \bar{X}|}{n}$$

Absolute values are needed because the plain deviations from the mean always sum to zero. A MAD of 5% means that, on average, an observation lies 5 percentage points from the mean. The **median absolute deviation**, $\text{Med}(|X_i - \text{Med}(X)|)$, is a more robust alternative.

Variance and standard deviation

Variance averages the *squared* deviations from the mean; squaring removes the signs but magnifies outliers, so variance is sensitive to them. For asset returns, a higher variance indicates greater risk. Taking the positive square root of the variance gives the **standard deviation**, which is expressed in the same units as the data

(variance is in squared units, e.g. "percent squared").

KEY CONCEPT

	Population	Sample
Variance	$\sigma^2 = \frac{\sum_{i=1}^{N}(X_i - \mu)^2}{N}$	$s^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n-1}$
Standard deviation	$\sigma = \sqrt{\sigma^2}$	$s = \sqrt{s^2}$

Using $n-1$ rather than n in the **sample variance** is called **Bessel's correction**. Dividing by n would systematically underestimate the population variance (especially in small samples), making it a **biased estimator**. Dividing by $n-1$ makes s^2 an unbiased estimator of σ^2. Exam convention: the sample standard deviation s is also described as an unbiased estimator of σ. Current practice: s is the standard estimate of σ, but the square root of an unbiased variance estimate is slightly biased downward; the effect is small except in very small samples.

Example. A sample of four annual returns: 10%, 4%, −2%, 12%.

- Mean $= 24/4 = 6\%$. Deviations: 4, −2, −8, 6 (sum $= 0$).
- MAD $= (4 + 2 + 8 + 6)/4 = 5.0\%$.
- Sum of squared deviations $= 16 + 4 + 64 + 36 = 120$.
- Sample variance $s^2 = 120/3 = 40$ $(\%^2)$; sample standard deviation $s = \sqrt{40} = 6.32\%$.
- Had these been the whole population: $\sigma^2 = 120/4 = 30$, $\sigma = 5.48\%$.
- Range $= 12 - (-2) = 14$ percentage points.

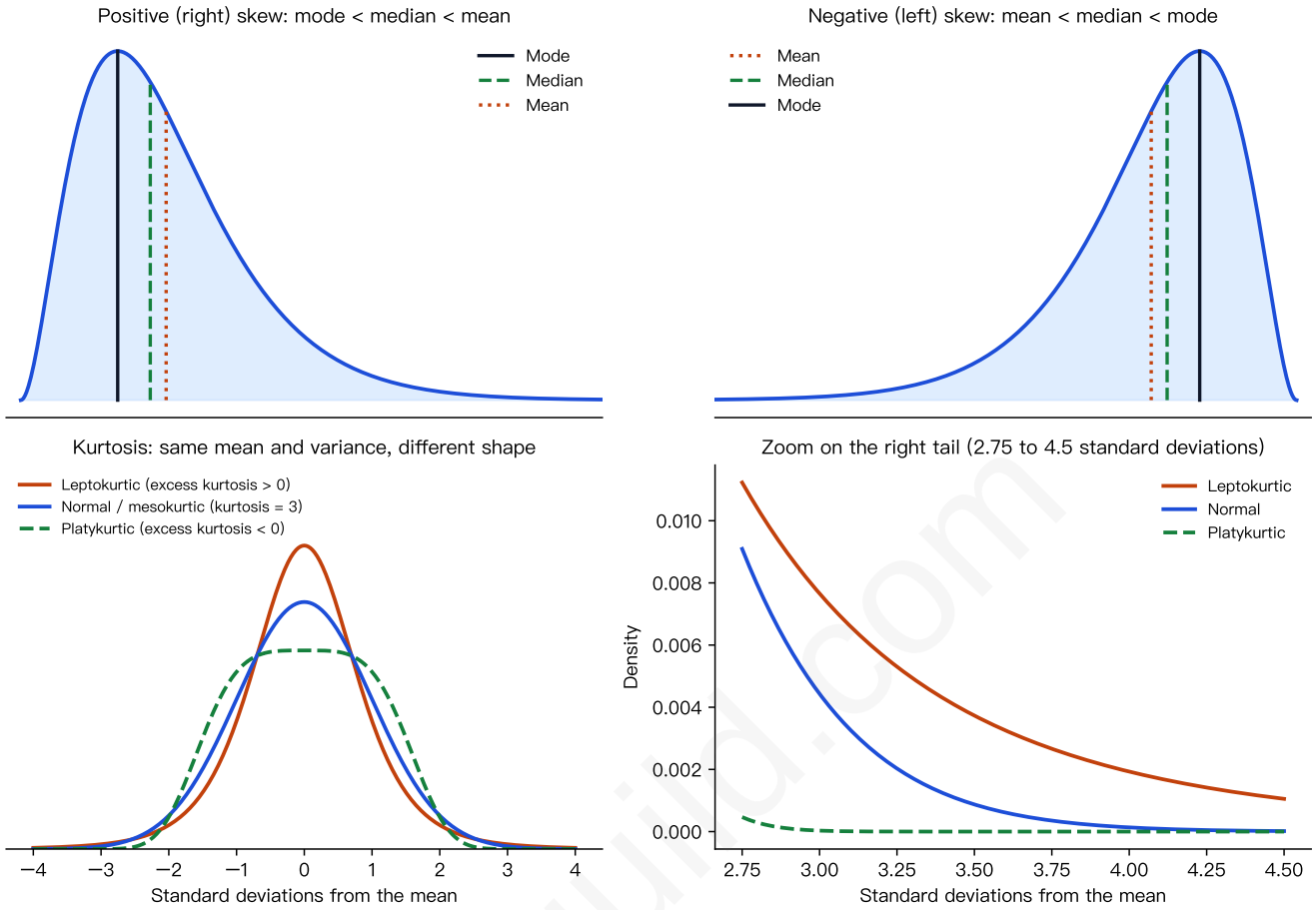
Calculator (TI BA II Plus): [2ND][DATA], enter X01 = 10, X02 = 4, X03 = −2, X04 = 12; [2ND][STAT] (1-V) and scroll: $\bar{x} = 6$, $S_x = 6.32$ (sample), $\sigma_x = 5.48$ (population).

Scaling with time

Under the **root of time law**, variance grows linearly with the length of the period when returns are independent: if daily variance is σ^2, the variance over T days is $T\sigma^2$, so the standard deviation is $\sigma\sqrt{T}$. In practice, long-horizon variance is often lower than this rule predicts.

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LOS 5.b — Skewness and kurtosis



Skewness and kurtosis: how shape shifts the mean, median and mode, and how tails compare with a normal curve

Skewness

A **symmetrical** distribution looks the same on both sides of its mean; its mean, median and mode are equal. With symmetry, a loss interval (e.g., −6% to −4% around a zero mean) occurs as often as the matching gain interval (+4% to +6%). **Skewness** measures the lack of symmetry.

KEY CONCEPT

Shape	Long tail	Order of central measures	Sample skewness
Symmetrical	none	mean = median = mode	0
Positively skewed (skewed right)	right (large positive outliers)	mode < median < mean	> 0
Negatively skewed (skewed left)	left (large negative outliers)	mean < median < mode	< 0

The mean is pulled toward the long tail because it is the measure most affected by outliers. In a skewed unimodal distribution, the median lies between the mode and the mean. For returns, positive skew means that more than half of the periods earn less than the mean (the median is below it), while a few periods bring very large gains. Negative skew is the mirror image: most periods earn more than the mean, and occasional large losses pull the mean down. Sample skewness averages the cubed standardized deviations, where s is the sample standard deviation (the LOS asks for interpretation, not calculation):

$$\text{skewness} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{s}\right)^3$$

Right-skewed data have positive sample skewness, since upside deviations from the mean are bigger on average. A quick way to judge the direction of skew is to compare the distance from the mode (peak) to each end of the range: the longer side is the tail.

Kurtosis

Kurtosis measures how peaked a distribution is and how thick its tails are, relative to a normal distribution (kurtosis $= 3$). **Excess kurtosis** $=$ kurtosis -3. Kurtosis uses the fourth power of the standardized deviations:

$$\text{kurtosis} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{X_i - \bar{X}}{s}\right)^4 \qquad \text{excess kurtosis} = \text{kurtosis} - 3$$

KEY CONCEPT

Term	Kurtosis	Excess kurtosis	Shape vs. normal
Leptokurtic	> 3	> 0 (positive)	More peaked, fatter tails: more small deviations *and* more very large deviations from the mean
Mesokurtic	= 3	= 0	Same as normal
Platykurtic	< 3	< 0 (negative)	Flatter (less peaked), thinner tails

Fat tails matter for risk: a leptokurtic return distribution produces extreme outcomes (both large gains and large losses) more often than a normal model predicts. Actual asset returns tend to show both skewness and excess kurtosis, so risk models that assume normality understate the chance of extreme losses. In general, more negative skew and greater excess kurtosis both mean greater risk.

COMMON EXAM TRAPS

- Dividing by n when the data are a sample, or by $n - 1$ when they are the whole population. For the four sample returns, dividing by 4 gives a standard deviation of 5.48% instead of 6.32%.
- Dividing the MAD by $n - 1$, or averaging the signed deviations, which always sum to zero. In the example, $20/3 = 6.67\%$ instead of 5.0%.
- Stopping at the variance when the standard deviation (its square root) is asked for. In the example, that answer is 40 (%²) instead of 6.32%.

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- Comparing *kurtosis* with 0 instead of 3 (only *excess kurtosis* is compared with 0). A kurtosis of 2.5 is platykurtic even though the number is positive.
- Putting the mean on the wrong side: it is the *largest* of the three measures under positive skew and the *smallest* under negative skew.
- Reading one very low outlier (e.g. a single very poor score) as positive skew. It creates negative skew and pulls the mean below the median.

EXAM SHORTCUTS

- In a skewed unimodal distribution the median lies between the mode and the mean, and the mean lies on the side of the long tail, so the direction of skew fixes the order of all three measures.

BOTTOM LINE

- The range, maximum minus minimum, uses only the two extreme observations, while the mean absolute deviation $\sum|X_i - \bar{X}|/n$ averages the unsigned deviations from the mean.
- Sample variance divides the sum of squared deviations from the mean by $n - 1$ (Bessel's correction), which makes s^2 an unbiased estimator of σ^2; population variance divides by N.
- Taking the positive square root of the variance gives the standard deviation, which is in the same units as the data, while the variance is in squared units.
- Under the root of time law, with independent returns the variance over T periods is $T\sigma^2$ and the standard deviation is $\sigma\sqrt{T}$, although long-horizon variance is often lower in practice.
- A positively skewed distribution has a long right tail with mode < median < mean, a negatively skewed one has a long left tail with mean < median < mode, and a symmetrical one has all three equal.
- A normal distribution has kurtosis 3; excess kurtosis is kurtosis − 3, a leptokurtic distribution (above 3) has fatter tails, and more negative skew and greater excess kurtosis both mean greater risk.

QUICK CHECK

Question 2

A histogram of a biotech fund's daily returns shows a long tail stretching to the right, created by a few very large gains. For this unimodal distribution, which relationship among the measures of central tendency is *most likely*?

A mean < median < mode

B mean = median = mode

C mean > median > mode

Answer: C

A long right tail means the distribution is positively skewed. The large positive outliers pull the mean upward the most, the median less, and the mode (the peak) not at all, so mean > median > mode.

Why the other options are wrong

- **A.** This ordering describes a negatively skewed distribution with a long left tail.
- **B.** Equality of the three measures describes a symmetrical distribution, which has no long tail.

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Key takeaway Positive (right) skew: mode < median < mean.

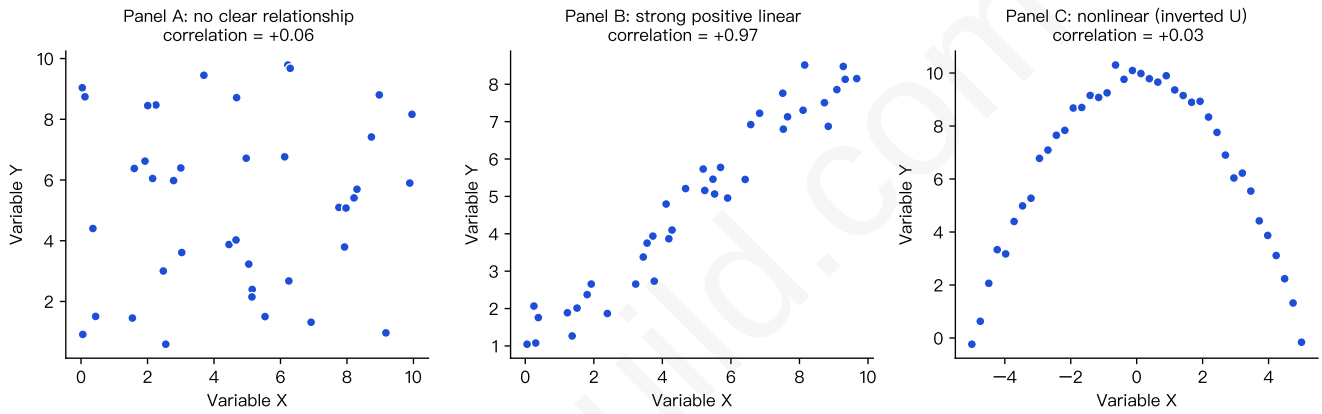
MODULE 5.3

Covariance, Correlation, and Alternative Measures of Dispersion

LOS 5.c — Covariance and correlation

Scatter plots

A **scatter plot** graphs paired observations of two variables, one on each axis. It shows at a glance whether the variables move together, whether the relationship is roughly a straight line, and whether there is a **nonlinear** relationship that a correlation coefficient would miss.



Scatter plots: no relationship, a strong linear relationship, and a nonlinear relationship with near-zero correlation

Covariance

Covariance measures how two variables move together:

$$\text{Cov}_{XY} = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{n-1} \quad \text{(sample)}$$

(The population version uses the population means and divides by N.) A positive covariance means the variables tend to be above (or below) their means at the same time; a negative covariance means they tend to move in opposite directions.

Example. Four annual returns on fund X are 6%, −2%, 10% and 2%, and on fund Y 5%, 1%, 9% and 1%. Both means are 4%.

1. Deviations from the means (percentage points): fund X gives 2, −6, 6, −2; fund Y gives 1, −3, 5, −3.
2. Multiply each pair and add: $2 + 18 + 30 + 6 = 56$.
3. Divide by $n - 1 = 3$: $\text{Cov}_{XY} = 56/3 = 18.67$ (%²).

Every cross-product is positive, so the two funds were above or below their means in the same years.

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Covariance is hard to interpret: its size depends on the units of the data, and its units are the product of the two variables' units (e.g. percent squared). A covariance of 30 shows only the *direction* of co-movement; it says nothing about how strong it is. Covariance has no upper or lower bound. The covariance of a variable with itself is its variance: $\text{Cov}_{XX} = s_X^2$.

Correlation

The **correlation coefficient** (correlation) standardizes covariance by the two standard deviations:

KEY CONCEPT

$$\rho_{XY} = \frac{\text{Cov}_{XY}}{s_X s_Y} \quad \Longleftrightarrow \quad \text{Cov}_{XY} = \rho_{XY}\, s_X\, s_Y$$

Here s_X and s_Y are the sample standard deviations. The population version uses σ_X and σ_Y in the same way.

KEY CONCEPT

Value	Interpretation
$\rho = +1$	**Perfect positive correlation**: deviations from the mean move in exact proportion, same direction; all points lie on one upward-sloping straight line, however steep or flat
$0 < \rho < 1$	Positive linear association; strength grows as ρ approaches 1
$\rho = 0$	**No linear relationship**
$-1 < \rho < 0$	Negative linear association
$\rho = -1$	**Perfect negative correlation**: all points lie on one downward-sloping straight line

Correlation has no units and always lies between −1 and +1. It is *not* a probability: $\rho = -0.6$ does not mean "a 60% chance the variables move in opposite directions." Because the covariance is divided by the two standard deviations, the correlation moves further from zero when the standard deviations shrink and the covariance stays the same.

Example. Returns on two funds have variances of 0.0036 and 0.0100 and a covariance of 0.0021. Standard deviations are $\sqrt{0.0036} = 0.06$ and $\sqrt{0.0100} = 0.10$, so

$$\rho = \frac{0.0021}{0.06 \times 0.10} = 0.35$$

This is a positive but modest linear association. The variances must be converted to standard deviations first.

For the four years of fund returns in the covariance example, $s_X = \sqrt{80/3} = 5.16\%$ and $s_Y = \sqrt{44/3} = 3.83\%$, so $\rho = 18.67/(5.16 \times 3.83) = 0.94$, a strong positive linear association.

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Limits of correlation

- **Correlation is not causation.** A strong correlation does not show that one variable drives the other, or which way any causality runs. Say the variables show a positive (or negative) **association** and investigate cause separately.
- **Outliers** can create or destroy a correlation; check whether removing them changes the result materially and whether they are information or noise.
- **Spurious correlation** is correlation that arises by chance, or because both variables are related to a third variable (e.g., two series that both trend upward with inflation or population), with no causal link between them.

LOS 5.d — Downside risk and relative dispersion

Semivariance and target downside deviation

Variance and standard deviation treat deviations above and below the mean alike. In some situations it is more useful to count only outcomes below the mean or below a chosen target, which measures **downside risk**.

- **Semivariance**: a variance built only from the outcomes that fall *below the mean*. The squared deviations of the below-mean observations are summed and divided by the sample size:

$$\sigma^2_{\text{semi}} = \frac{\sum_{X_i < \bar{X}} (X_i - \bar{X})^2}{n}$$

Its square root is the **semideviation**, a downside counterpart of the standard deviation.

- **Target downside deviation** (also **target semideviation**): like a standard deviation, but only outcomes *below a chosen target* B contribute; the denominator is still the full sample size minus one:

$$s_{\text{target}} = \sqrt{\frac{\sum_{X_i < B} (X_i - B)^2}{n - 1}}$$

Example. Five annual returns: 12%, −4%, 7%, 3%, 10%; target 6%. Only −4% and 3% are below target: squared shortfalls $= 100 + 9 = 109$. $s_{\text{target}} = \sqrt{109/4} = \sqrt{27.25} = 5.22\%$. Raising the target to 8% brings the 7% return into the count and makes the other two shortfalls larger: $\sqrt{(144 + 25 + 1)/4} = \sqrt{42.5} = 6.52\%$. A higher target never lowers the target downside deviation.

Coefficient of variation

Comparing standard deviations of assets with very different mean returns can mislead. A **relative dispersion** measure scales risk by the mean. The **coefficient of variation** (CV) is:

KEY CONCEPT

$$CV = \frac{s}{\bar{X}} \qquad \left(\text{or } \frac{\sigma}{\mu}\right)$$

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It measures **risk per unit of mean return**, so a lower CV is better (less risk per unit of return). Because it is a ratio of two numbers in the same units, it is unit-free and can be quoted as a decimal or a percentage. The Sharpe ratio (Reading 8) turns the CV around: it divides the excess return over the risk-free rate by the standard deviation. With a zero risk-free rate, the CV is simply the inverse of the Sharpe ratio. When a Sharpe ratio and a risk-free rate are given, first recover $s = (\bar{X} - R_f)/\text{Sharpe ratio}$, then divide by the mean.

Example. Fund M has a mean of 5% and a standard deviation of 4%, so $CV = 0.80$. Fund N has a mean of 11% and a standard deviation of 7.7%, so $CV = 0.70$. Fund N has the higher standard deviation but the better (lower) CV.

Measure	Uses	Scale
Standard deviation	All deviations from the mean	Absolute (units of the data)
Target downside deviation	Only shortfalls below a target	Absolute
Coefficient of variation	Standard deviation ÷ mean	Relative (per unit of return)

COMMON EXAM TRAPS

- Inverting the CV or using the variance. CV = standard deviation ÷ mean; when a variance is given, take its square root first. For fund M, inverting gives $5/4 = 1.25$ and using the variance of 16 (%²) gives 3.20, instead of 0.80.
- Subtracting a risk-free rate given in the question. The CV uses the mean itself; excess return over standard deviation is a Sharpe-type ratio, a different measure.
- Picking the asset with the lowest standard deviation as the best risk-return tradeoff, or the one with the highest standard deviation as the highest relative risk. Both are judged by the CV.
- Dividing the covariance by one variance (a slope-type ratio) or by the product of the variances, instead of by the product of the standard deviations. In the two-fund example, $0.0021/(0.0036 \times 0.0100) = 58.33$ and $0.0021/0.0036 = 0.58$, instead of 0.35.
- Reading a correlation near zero as proof that the variables are unrelated. It rules out only a linear relationship.
- Dividing the target downside deviation by the number of below-target outcomes (or that number minus one) instead of the full sample size minus one. In the 6% target example, $\sqrt{109/2} = 7.38\%$ instead of $\sqrt{109/4} = 5.22\%$.

EXAM SHORTCUTS

- With a zero risk-free rate the CV is the inverse of the Sharpe ratio, so one can be read straight from the other.
- A higher target never lowers the target downside deviation, which rules out an answer in which raising the target reduces it.

BOTTOM LINE

- Sample covariance is $\sum(X_i - \bar{X})(Y_i - \bar{Y})/(n-1)$; its sign shows the direction of co-movement, but its size depends on the units and it has no upper or lower bound.

- Correlation $\rho_{XY} = \text{Cov}_{XY}/(s_X s_Y)$ has no units, lies between −1 and +1, and $\rho = 0$ means no linear relationship.
- Correlation is not causation, outliers can create or destroy a correlation, and spurious correlation arises by chance or because both variables are related to a third variable.
- A scatter plot can reveal a nonlinear relationship that a correlation coefficient would miss.
- Semivariance uses only the outcomes below the mean, and the target downside deviation uses only the outcomes below a target B but keeps the full sample size minus one in the denominator.
- The coefficient of variation $CV = s/\bar{X}$ measures risk per unit of mean return, is unit-free, and a lower CV is better.

QUICK CHECK

Question 3

The covariance between the returns on Norland Utilities and Quarry Point Materials is 21.6 (percent squared). The standard deviation of Norland's returns is 8% and that of Quarry Point's returns is 6%. The correlation coefficient of the two return series is *closest* to:

A +0.34.

B +0.45.

C +0.60.

Answer: B

Correlation standardizes covariance by dividing it by the product of the two standard deviations.

$$\rho = \frac{\text{Cov}}{\sigma_N \sigma_Q} = \frac{21.6}{8 \times 6} = \frac{21.6}{48} = 0.45$$

Why the other options are wrong

- **A.** +0.34 divides the covariance by Norland's variance only $(21.6/64)$.
- **C.** +0.60 divides the covariance by Quarry Point's variance only $(21.6/36)$.

Key takeaway $\rho = \text{Cov}/(\sigma_1 \sigma_2)$; dividing by a single variance gives a slope-type ratio.

Practice questions

Question 4

Priya Raman plans to model the monthly percentage changes in a copper price index. Her statistical tests will be most reliable if the series is stationary, which means that:

A its geometric mean and arithmetic mean are the same.

B its mean and median both lie within its interquartile range.

C the mean and variance of its distribution do not change over time.

Question 5

A multi-asset portfolio produced the returns shown below over the last four years.

Annual returns of a multi-asset portfolio (sample)

Year	Return
1	19.0%
2	7.5%
3	−9.6%
4	4.9%

Treating these returns as a sample, their coefficient of variation is *closest* to:

A 0.46.

B 1.87.

C 2.16.

Answers: 4. C · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/skewness-and-kurtosis/. The full bank has 60 questions on this reading.

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QUANTITATIVE METHODS · READING 6

Lognormal Distribution

What you'll learn

LOS 6.a Compute and interpret the unconditional mean, variance, standard deviation and covariance of random variables, including from scenario and joint probability tables.

LOS 6.b Describe the shape, principal moments and typical financial uses of the discrete uniform, binomial, Poisson, continuous uniform, normal, lognormal and logistic distributions.

LOS 6.c Compute and interpret conditional expectations, variances and covariances, including from joint probability tables and probability trees.

LOS 6.d Use Bayes' formula to update a prior probability into a posterior probability when new information arrives.

MODULE 6.1

Probability Distributions and Expected Values

This reading describes random variables through their probability distributions and summarizes them with the expected value, variance and covariance. It then matches the common discrete and continuous distributions to the financial quantities they model, and shows how conditional expectations and Bayes' formula update a forecast when new information arrives.

LOS 6.a — Unconditional mean, variance and covariance of a random variable

Random variables: discrete vs. continuous

A **random variable** is a quantity whose value is not known until an outcome is observed. It is also called a *stochastic variable*.

Feature	Discrete random variable	Continuous random variable
Possible values	A countable set (e.g., the number of issuers in a 50-bond portfolio downgraded this year)	Infinitely many values inside a range (e.g., tomorrow's percentage change in the gold price)
Probability function	**Probability mass function (PMF)**: $p(x) = P(X = x)$ can be positive	**Probability density function (PDF)**: $P(X = x) = 0$ for every single point
Probabilities are assigned to	Individual outcomes	Intervals, e.g. $P(a \leq X \leq b)$
Can outcomes be negative?	Yes, if the countable set includes negatives	Yes, or the range may be restricted to positive values

A **probability distribution** lists the probabilities of all possible outcomes of a random variable, and these probabilities sum to 1. A **probability function** $p(x) = P(X = x)$ gives the probability that X equals a particular value. For a discrete variable, $p(x) = 0$ for a value that cannot occur and $p(x) > 0$ for one that can.

Whether a variable is discrete does not depend on its outcomes being equally likely (the uniform case) or positive. Prices quoted in cents are technically discrete. Because there are so many possible values, analysts usually treat them as continuous and speak of the probability of a range of price changes.

What makes a valid probability function

A function $p(x)$ qualifies as a probability function only if both conditions hold:

KEY CONCEPT

1. $0 \le p(x) \le 1$ for every possible outcome x;
2. $\sum_x p(x) = 1$ over all possible outcomes.

A list such as 0.50, 0.40, 0.20, −0.10 sums to one but fails condition 1. Five values of 0.25 each pass condition 1 but sum to 1.25, so they fail condition 2.

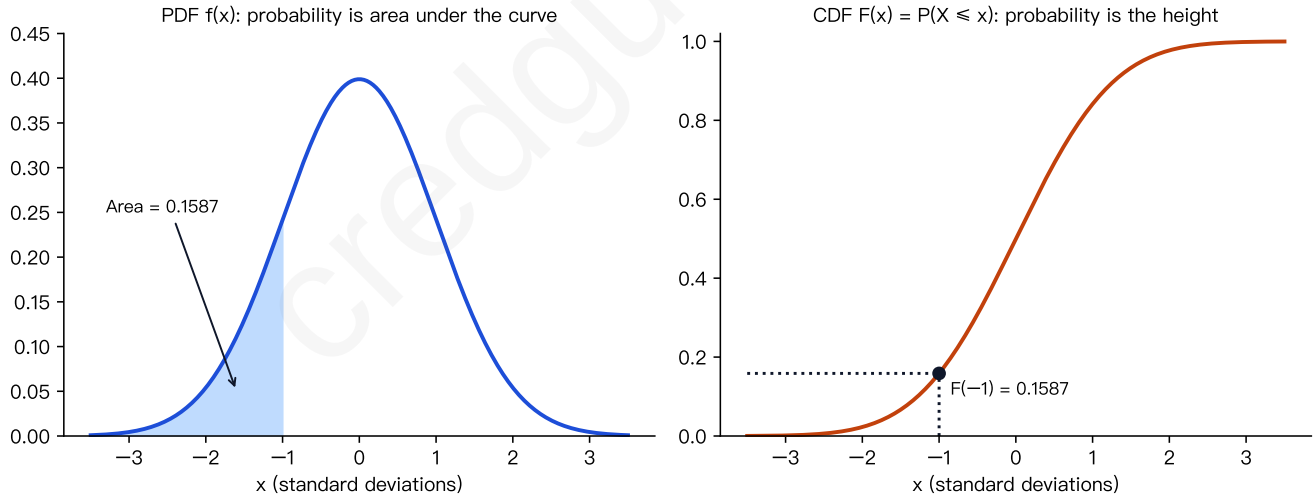
For a continuous variable, a single point carries no probability, so including or excluding the endpoints does not change the probability of an interval: $P(a \le X \le b) = P(a < X < b)$. The height of the PDF at a point is a density. A probability is the area under the PDF over an interval.

Cumulative distribution function (CDF)

The **cumulative distribution function (CDF)**, also called the **distribution function**, gives the probability that X takes a value less than or equal to x:

$$F(x) = P(X \le x) \qquad P(a < X \le b) = F(b) - F(a)$$

For a discrete variable, $F(x)$ is the sum of the probabilities of all outcomes up to and including x. For the standard normal distribution (Module 6.2), $F(-1) \approx 0.1587$: the area under the PDF to the left of −1 equals the height of the CDF at −1.



Standard normal PDF and CDF: the area left of −1 under the PDF equals the height of the CDF at −1

Example. A fund's quarterly return is continuous on $[-5\%, +15\%]$ with $F(x) = (x + 5)/20$. Then $P(0\% < R \le 10\%) = F(10) - F(0) = 15/20 - 5/20 = 0.50$.

Unconditional expected value (mean)

The **unconditional mean** (also the **unconditional expectation** or **expected value**) weights each outcome by its probability:

$$E(X) = \sum_{i=1}^{n} P(x_i)\, x_i$$

The weights are the probabilities, and they must sum to 1, so a probability that is not stated can be found as 1 minus the sum of the others. A simple average of the outcomes weights each by $1/n$, so it is the expected value when every outcome is equally likely; with unequal probabilities it generally gives a different answer. The sample mean of Reading 5 summarizes returns already observed (an ex post measure). An expected value weights possible future outcomes by their probabilities, so it is a forecast (an ex ante measure).

Unconditional variance and standard deviation

The **unconditional variance** is the probability-weighted squared spread around the expected value, and the **unconditional standard deviation** is its square root:

KEY CONCEPT

$$\sigma^2(X) = \sum_{i=1}^{n} P(x_i)\,[x_i - E(X)]^2 \qquad \sigma(X) = \sqrt{\sigma^2(X)}$$

Variance is in *squared* units (e.g., %²), so the standard deviation (back in % units) is the figure compared with returns. An equivalent shortcut is $\sigma^2 = E(X^2) - [E(X)]^2$.

Unconditional covariance

Unconditional covariance shows the direction of the linear relationship between two random variables, that is, whether they tend to be above or below their own means at the same time. Its strength is read from the correlation (last point below). The covariance is computed as:

$$\mathrm{Cov}(X,Y) = \sum_{i=1}^{n} P(x_i, y_i)\,[x_i - E(X)]\,[y_i - E(Y)]$$

The shortcut form is $\mathrm{Cov}(X,Y) = E(XY) - E(X)E(Y)$.

- Positive covariance: the variables tend to move in the same direction; negative: opposite directions.
- For scenario (state) tables, $P(x_i, y_i)$ is simply the probability of the state. In a joint probability table, cells with zero probability contribute nothing.
- Units are the product of the two variables' units (e.g., %² or decimal²), so the size is hard to interpret. Dividing by the two standard deviations gives the unit-free **correlation**: $\rho = \mathrm{Cov}(X,Y)/(\sigma_X \sigma_Y)$, which always lies between −1 and +1.

Measure	Formula core	Units	Range
Variance	$\sum P\,(x-\mu)^2$	squared units	≥ 0
Covariance	$\sum P\,(x-\mu_X)(y-\mu_Y)$	product of units	any sign, unbounded
Correlation	$\mathrm{Cov}/(\sigma_X\sigma_Y)$	none	-1 to $+1$

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WORKED EXAMPLE

State	Probability	Return on P	Return on Q
Expansion	0.20	15%	4%
Normal	0.50	8%	7%
Downturn	0.30	−2%	2%

- $E(R_P) = 0.2(15) + 0.5(8) + 0.3(-2) = 6.4\%$; $E(R_Q) = 0.2(4) + 0.5(7) + 0.3(2) = 4.9\%$.
- $\sigma_P^2 = 0.2(8.6)^2 + 0.5(1.6)^2 + 0.3(-8.4)^2 = 37.24$ (%²), so $\sigma_P = 6.10\%$.
- $\sigma_Q^2 = 0.2(-0.9)^2 + 0.5(2.1)^2 + 0.3(-2.9)^2 = 4.89$ (%²), so $\sigma_Q = 2.21\%$.
- $\text{Cov}(R_P, R_Q) = 0.2(8.6)(-0.9) + 0.5(1.6)(2.1) + 0.3(-8.4)(-2.9) = -1.548 + 1.68 + 7.308 = 7.44$ (%²).
- $\rho = 7.44/(6.10 \times 2.21) \approx 0.55$.

Calculator. The BA II Plus DATA/STAT worksheet computes a probability-weighted mean when each outcome is entered with a frequency proportional to its probability (e.g., 20, 50 and 30). The mean is $\bar{x}$ and the population standard deviation is σ_x.

COMMON EXAM TRAPS

- Averaging outcomes equally instead of weighting by probabilities. For portfolio P in the worked example, the simple average is $(15 + 8 - 2)/3 = 7.0\%$, against an expected return of 6.4%.
- Forgetting the square root: reporting the variance when the standard deviation is asked, or mixing decimals (0.0035) with percent-squared (35.04). For portfolio P in the worked example, the variance is 37.24 (%²) and the standard deviation is 6.10%.
- Using absolute deviations instead of squared deviations, which gives a different measure from the variance and standard deviation.
- Computing covariance without the probabilities, or dividing an unweighted sum of cross-products by the number of states.
- Confusing covariance with correlation; the answer to "covariance" carries squared units and is not bounded by ±1.
- Believing a continuous variable has a positive probability at a single point, or that the PDF height is a probability.
- Reading a CDF value as the probability of that single outcome, or leaving the outcome itself out of $F(x) = P(X \le x)$.

EXAM SHORTCUTS

- When the outcomes listed are mutually exclusive and exhaustive and one probability is missing, it equals 1 minus the sum of the others.
- Variance can be found as $\sigma^2 = E(X^2) - [E(X)]^2$, which avoids computing each deviation.
- For a continuous variable, including or excluding the endpoints does not change the probability of an interval.

BOTTOM LINE

- A discrete random variable has a probability mass function with positive probabilities at individual outcomes; a continuous variable has a density, zero probability at any single point and probabilities assigned to intervals.
- A probability function is valid only if every $p(x)$ lies between 0 and 1 and the probabilities sum to 1.
- The CDF is $F(x) = P(X \leq x)$, and $P(a < X \leq b) = F(b) - F(a)$.
- Expected value is $E(X) = \sum P(x_i)x_i$; variance is $\sum P(x_i)[x_i - E(X)]^2$ in squared units, and the standard deviation is its square root.
- Covariance shows only the direction of co-movement; correlation $\rho = \text{Cov}(X, Y)/(\sigma_X \sigma_Y)$ is unit-free and lies between −1 and +1.

QUICK CHECK

Question 1

A random variable X can take only the values 0, 1, 2 and 3. Which of the following proposed functions is a valid probability function for X?

A $p(0) = 0.40, p(1) = 0.35, p(2) = 0.30, p(3) = -0.05$

B $p(x) = 0.30$ for each of the four values

C $p(x) = (x + 1)/10$

Answer: C

A probability function must satisfy two conditions: every probability lies between 0 and 1 inclusive, and the probabilities over all possible outcomes add up to exactly 1. Only $p(x) = (x + 1)/10$ meets both: it gives 0.1, 0.2, 0.3 and 0.4, each within [0, 1], summing to 1.0.

- Set with $p(3) = -0.05$: sum $= 0.40 + 0.35 + 0.30 - 0.05 = 1.00$, but $p(3) = -0.05 < 0$, so it fails condition 1.
- Constant $p(x) = 0.30$: each value is within [0, 1], but the sum is $4 \times 0.30 = 1.20 \neq 1$, so it fails condition 2.
- $p(x) = (x + 1)/10$: $p = 0.1, 0.2, 0.3, 0.4$; all in [0, 1] and $\sum p = 1.0$, so it is valid.

Why the other options are wrong

- **A.** The probabilities sum to one, but a negative probability is impossible; satisfying only the 'sum to one' condition is not enough.
- **B.** Each probability is between 0 and 1, but together they sum to 1.20. A probability function must exhaust exactly 100% of the probability; an equal-probability function over four outcomes would need 0.25 each.

Key takeaway Check both conditions every time: $0 \leq p(x) \leq 1$ for each x, and $\sum p(x) = 1$.

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MODULE 6.2

Discrete and Continuous Probability Distributions

LOS 6.b — Moments and uses of the key distributions in finance

The four principal moments

Moment	What it describes	Benchmark (normal distribution)
1. Mean	Central location	—
2. Variance	Dispersion around the mean	—
3. Skewness	Asymmetry; positive = long right tail	0 (symmetric)
4. Kurtosis	Weight of the tails relative to the center	3 (excess kurtosis = kurtosis − 3 = 0)

The most useful features of each distribution are what it models and its shape: bounded or unbounded, symmetric or skewed, thin or fat tails. The moment formulas below are for reference.

Distribution	Mean	Variance	Skewness	Excess kurtosis
Discrete uniform (n outcomes, a to b)	$(a+b)/2$	$[(b-a+1)^2-1]/12$	0	$-\frac{6(n^2+1)}{5(n^2-1)}$
Binomial	np	$np(1-p)$	$\frac{1-2p}{\sqrt{np(1-p)}}$	$\frac{1-6p(1-p)}{np(1-p)}$
Poisson	λ	λ	$1/\sqrt{\lambda}$	$1/\lambda$
Continuous uniform	$(a+b)/2$	$(b-a)^2/12$	0	−1.2
Normal	μ	σ^2	0	0
Lognormal	$e^{\mu+\sigma^2/2}$	$(e^{\sigma^2}-1)e^{2\mu+\sigma^2}$	>0	>0
Logistic	μ	$s^2\pi^2/3$	0	1.2

Discrete distributions

Discrete uniform. A finite set of n outcomes, each with probability $1/n$ (e.g., the integers 1 to 8, each with probability 0.125); the probability of k outcomes in a range is k/n, and the CDF at the kth outcome is k/n. For a discrete uniform variable on the consecutive integers $a, a+1, \ldots, b$: mean $= (a+b)/2$ and variance $= [(b-a+1)^2-1]/12$. For a general equally likely set (e.g., $\{1,3,5\}$), compute the arithmetic mean and the population variance directly: here the mean is 3 and the variance is $(4+0+4)/3 = 8/3$. The consecutive-integer formula would wrongly give 2.

Binomial. Counts the number of successes in a fixed number n of independent trials, each with the same success probability p and only two possible results (success/failure). A binomial variable with a single trial is a **Bernoulli distribution**.

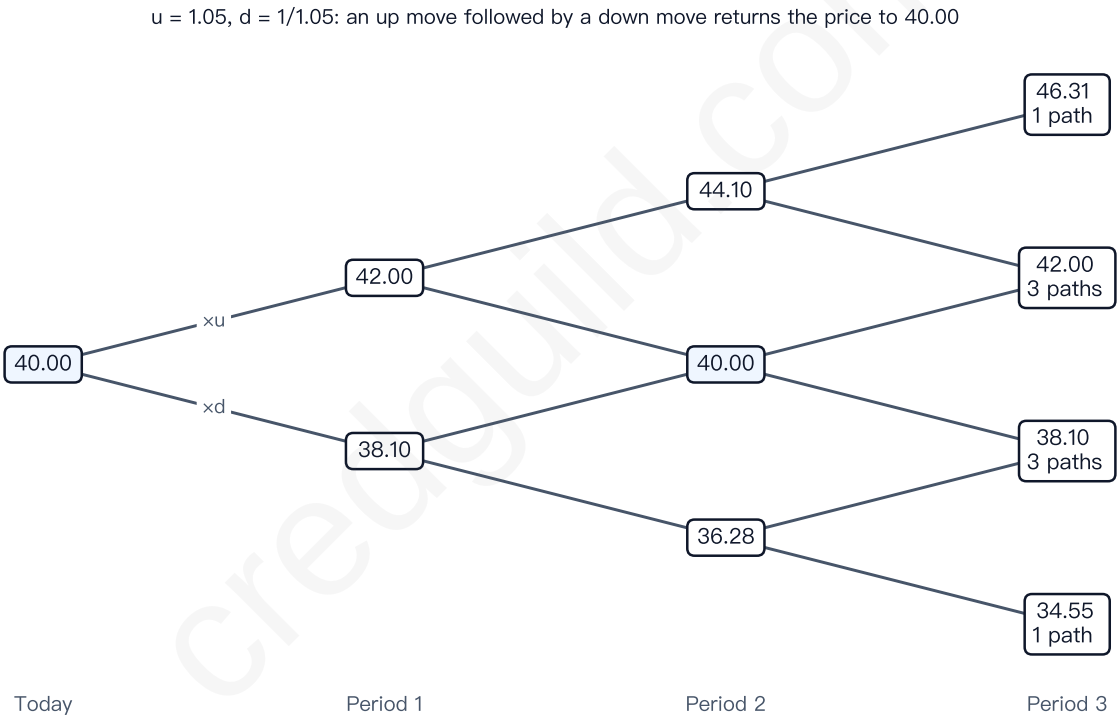
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KEY CONCEPT

$$E(X) = np \qquad \sigma^2(X) = np(1-p)$$

Example. 16 independent new-issue auctions, each with a 0.75 chance of being fully covered: expected fully covered auctions $= 16 \times 0.75 = 12$; variance $= 16 \times 0.75 \times 0.25 = 3$. The expected number of successes need not be a possible outcome (10 trials with $p = 0.43$ give 4.3); it is the average number of successes over many repetitions.

Binomial models are also used for asset prices: at each step the price is multiplied by an **up factor** u or a **down factor** d, so after i up moves and j down moves the price is $S_0 u^i d^j$. The possible paths are drawn as a **probability tree** (or **decision tree**). With $u = 1.05$ and $d = 1/1.05$, a price of 40 after one up and one down move returns to 40, whatever the order.



Three-period binomial price tree with u = 1.05 and d = 1/1.05: the tree recombines

Poisson. Counts how many times an event occurs in a fixed interval when events arrive independently at a known, constant average rate λ. It is the usual model for rare events such as defaults per year, trading halts per month or operational losses per quarter.

$$P(k \text{ events in interval } t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!} \qquad \text{mean} = \text{variance} = \lambda \text{ (for one unit of time, } t = 1)$$

Example. With $\lambda = 2$ outages per year, $P(\text{no outage}) = e^{-2} = 0.135$, so the probability of at least one outage is $1 - e^{-2} = 0.865$. For a pool of n independent credits each with default probability p, $\lambda = np$.

The Poisson is the limit of a binomial as $n \to \infty$ and $p \to 0$ with np fixed.

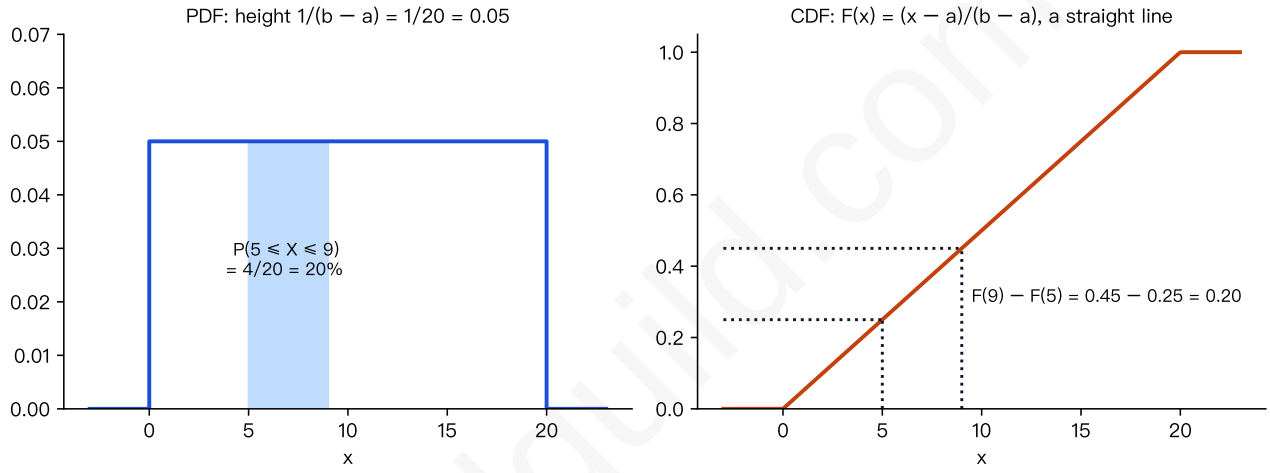
Continuous distributions

Continuous uniform on $[a, b]$: every equal-width sub-interval is equally likely, so the CDF rises in a straight line, $F(x) = (x - a)/(b - a)$ for $a \le x \le b$.

KEY CONCEPT

$$P(x_1 \le X \le x_2) = \frac{x_2 - x_1}{b - a} \qquad \text{mean} = \frac{a + b}{2} \qquad \sigma^2 = \frac{(b - a)^2}{12}$$

Skew 0, kurtosis 1.8 (excess −1.2). Used to generate random draws for Monte Carlo simulation. *Example.* Uniform on [0, 20]: $P(5 \le X \le 9) = 4/20 = 20\%$.



Continuous uniform distribution on [0, 20]: probability as an area, and the straight-line CDF

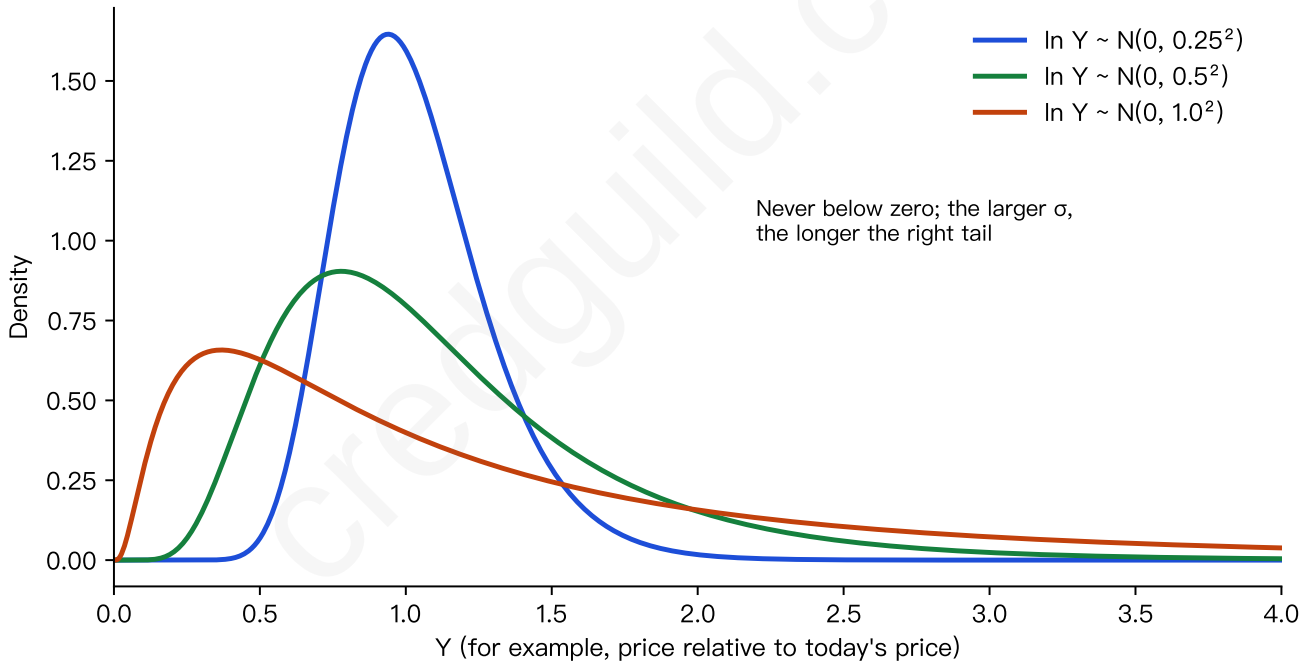
Normal (also called the **Gaussian distribution**), $N(\mu, \sigma^2)$: symmetric bell curve, fully described by its mean and variance, or equivalently its mean and standard deviation (skew 0, excess kurtosis 0). About 68% of outcomes lie within ±1σ and about 95% within ±2σ. Sums of many independent variables tend toward normality, and a binomial converges to a normal as n grows. It is unbounded in both directions, so it can produce negative values. The normal with mean 0 and variance 1, $N(0, 1)$, is the **standard normal distribution**; its horizontal axis measures standard deviations from the mean.

Example. Annual returns are normal with a mean of 7% and a standard deviation of 15%. About 68% of years should fall between $7 - 15 = -8\%$ and $7 + 15 = 22\%$, and about 95% between −23% and 37%. A −23% return lies $(-23 - 7)/15 = -2$ standard deviations from the mean. The curve is symmetric, so the 5% of years outside $\pm 2\sigma$ split evenly: about 2.5% of years should lose more than 23%. Subtracting the two bands gives the share between them: about $(95\% - 68\%)/2 = 13.5\%$ of outcomes lie between one and two standard deviations below the mean, and the same share lies between one and two standard deviations above it.

Lognormal. Y is lognormal when $\ln Y$ is normal; equivalently $Y = e^X$ with X normal.

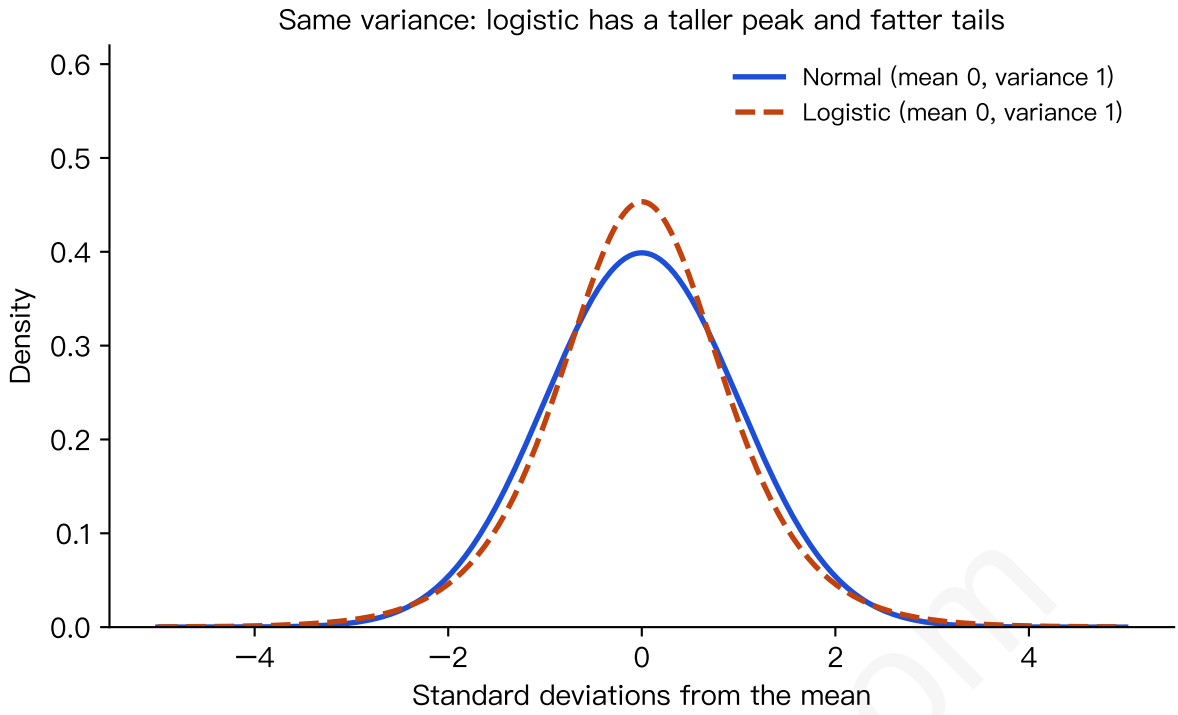
- Because $e^X > 0$ for any X, the lognormal is bounded below by zero and never takes a negative value.

- It is **positively (right) skewed**, with a long right tail.
- That makes it well suited to modeling asset prices. A price following a lognormal cannot fall below zero, so the implied return can never be worse than −100%. A normal price model, by contrast, would allow negative prices (returns below −100%), which is impossible for a limited-liability asset.
- If continuously compounded price returns $\ln(P_1/P_0)$ (no distributions paid) are normally distributed, then prices are lognormal, because $P_T = P_0 e^r$ with r normal. Over several periods, r is the sum of the one-period log returns (Reading 1), and sums of many independent returns tend toward a normal distribution. A lognormal price model therefore fits well even when single-period returns are not exactly normal.
- Its two parameters are the mean and SD of $\ln Y$, called the **location parameter** and the **scale parameter**. They are not the mean and SD of Y itself: the mean of Y is $e^{\mu+\sigma^2/2}$.
- Prices can be projected with a geometric Brownian motion model. Here μ is the expected (arithmetic) growth rate of the price, not the location parameter of $\ln Y$ used above, and $T(\mu - \sigma^2/2)$ is the **drift term**: $\ln S_T = \ln S_0 + T(\mu - \sigma^2/2)$. *Example.* $S_0 = 25$, $\mu = 8\%$, $\sigma = 20\%$, $T = 2$: $\ln S_T = \ln 25 + 2(0.08 - 0.02)$, so $S_T = 25e^{0.12} = 28.19$.
- Exam convention: the price from the drift-term formula is the expected price in T years. Current practice: with μ as the expected growth rate, that figure is the median of the lognormal price distribution; the mean price is $S_0 e^{\mu T}$ (here $25e^{0.16} = 29.34$), which is higher because of the right skew.



Lognormal densities for three standard deviations of ln Y: all start at zero and are skewed to the right (illustrative parameters)

Logistic. Bell-shaped and symmetric like the normal (skew 0), but with **fatter tails**. Its excess kurtosis is +1.2, so extreme outcomes are more likely than under a normal with the same variance. The variance is $\pi^2 s^2/3$ for scale parameter s. The logistic distribution is used in *logistic regression*, where the dependent variable is binary (0/1), and in machine learning.



Normal vs. logistic densities with equal variance.

Matching the distribution to the problem

Situation	Distribution
Number of successes in a fixed number of independent yes/no trials	Binomial
Count of rare events in an interval at a constant rate	Poisson
Equally likely finite outcomes	Discrete uniform
Random draws for simulation over a range	Continuous uniform
Returns, test statistics, sums of many variables	Normal
Asset prices (non-negative, right-skewed)	Lognormal
Binary-outcome regression; symmetric but fat-tailed data	Logistic

COMMON EXAM TRAPS

- Calling the Poisson a "timing" or "waiting-time" distribution: it models the count of events in an interval.
- Thinking a lognormal is negatively skewed or symmetric; it is positively skewed and bounded at zero.
- Reversing the log relation: if Y is lognormal then $\ln Y$ is normal (not lognormal); if X is normal then e^X is lognormal.
- Saying a normal is appropriate for prices "because it only lets prices fall to zero". A normal is unbounded and allows negative prices.
- Attributing skewness to the logistic distribution: its skew is zero, and it differs from the normal in kurtosis (fatter tails).

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- Using the binomial when the number of trials is not fixed or the success probability changes from trial to trial.

EXAM SHORTCUTS

- Identify the distribution from the wording: successes in a fixed number of trials is binomial, a count of events in an interval is Poisson, random draws for simulation are continuous uniform, and asset prices are lognormal.
- Normal bands: about 68% of outcomes lie within ±1σ and 95% within ±2σ, so about 2.5% lie beyond 2σ on each side and about 13.5% lie between 1σ and 2σ on each side.
- For a Poisson variable, the probability of at least one event is $1 - e^{-\lambda}$.

BOTTOM LINE

- A binomial variable counts successes in n independent trials with a constant p: $E(X) = np$ and $\sigma^2 = np(1-p)$; a single trial is a Bernoulli distribution.
- A Poisson variable counts events in an interval at a constant rate λ, and its mean and variance both equal λ.
- For a continuous uniform variable on $[a, b]$, $P(x_1 \leq X \leq x_2) = (x_2 - x_1)/(b - a)$.
- The normal distribution is symmetric, fully described by its mean and variance, and unbounded in both directions.
- If $\ln Y$ is normal, Y is lognormal: bounded below by zero and positively skewed, which suits asset prices.
- The logistic distribution is symmetric like the normal but has fatter tails (excess kurtosis +1.2).

QUICK CHECK

Question 2

Each week, the fixed-income desk at Corrib Securities submits competitive bids in 25 separate government bond auctions. The desk's chance of winning an allocation is 0.36 in every auction, and the result of one auction does not affect any other. For the number of auctions the desk wins in a week, the expected value and the variance are, respectively:

A 16.0 and 5.76.

B 9.0 and 2.40.

C 9.0 and 5.76.

Answer: C

The number of wins is a binomial random variable: there is a fixed number of independent trials (25 auctions), each with only two outcomes (win or lose) and the same probability of success ($p = 0.36$). Its mean is np and its variance is $np(1-p)$.

$$E(X) = np = 25 \times 0.36 = 9.0$$

$$\sigma^2(X) = np(1-p) = 25 \times 0.36 \times 0.64 = 5.76$$

(The standard deviation would be $\sqrt{5.76} = 2.40$.) As with any discrete count, the mean of 9.0 is the long-run average number of wins per week, not a guaranteed result.

Why the other options are wrong

- **A.** 16.0 uses the probability of *losing* an auction, 25×0.64, so it is the expected number of losses. The variance of 5.76 is right, because $np(1-p)$ is the same whichever outcome is called a success.
- **B.** The mean of 9.0 is right, but 2.40 is the *standard deviation*, $\sqrt{5.76}$, which is the square root of the variance.

Key takeaway Binomial: $E(X) = np$ and $\sigma^2 = np(1-p)$. Define "success" as the outcome being counted, and do not stop at the standard deviation when the variance is asked.

MODULE 6.3

Conditional Expectations and Bayesian Updating

LOS 6.c — Joint distributions, conditional expectations, variances and covariances

Joint and marginal probabilities

A **joint probability distribution** gives the probability of every combination of outcomes of two or more variables; for two variables it is *bivariate*. For discrete variables it is often shown as a **contingency table**. Row and column totals are the **marginal distributions**.

For two variables in general, the **joint cumulative distribution function** gives the probability that *both* variables are at or below given values: $F_{12}(x_1, x_2) = P(X_1 \le x_1,\ X_2 \le x_2)$. A bivariate distribution is described by each variable's mean and variance plus how they move together (their covariance or correlation).

400 listed firms	Technology	Non-technology	Total
Return > 10%	64	56	120
Return ≤ 10%	96	184	280
Total	160	240	400

- Marginal: $P(\text{tech}) = 160/400 = 0.40$; $P(R > 10\%) = 120/400 = 0.30$.
- Joint: $P(\text{tech and } R > 10\%) = 64/400 = 0.16$.
- Conditional: $P(R > 10\% \mid \text{tech}) = 64/160 = 0.40$, using only the technology column.

Conditional mean, variance and covariance

A **conditional expectation** $E(X \mid S)$ is the expected value of X given that condition S holds, read "the expected value of X given S". A forecast revised for new information is a conditional expectation; for example, "if the merger is approved, my EPS estimate rises to $4.20".

$$E(X \mid S) = \sum_i P(x_i \mid S)\, x_i$$

The **conditional variance** measures dispersion around the **conditional mean** $E(X \mid S)$ rather than the **unconditional mean** $E(X)$:

$$\mathrm{Var}(X \mid S) = \sum_i P(x_i \mid S)\,[x_i - E(X \mid S)]^2 = E(X^2 \mid S) - [E(X \mid S)]^2$$

The **conditional covariance** is defined the same way for two variables, using conditional probabilities and conditional means. It is a different concept from the conditional variance of one variable.

A **conditional distribution** describes one variable after the other variables in a joint distribution are restricted to certain values. Conditioning changes the relevant probability distribution, so it can change both the expected value and the variance.

Concept	Probabilities used	Deviations measured from
Unconditional mean/variance	$P(x_i)$ over all outcomes	$E(X)$
Conditional mean/variance	$P(x_i \mid S)$ over outcomes consistent with S	$E(X \mid S)$
Joint probability	$P(A \text{ and } B)$	— (not an expectation)

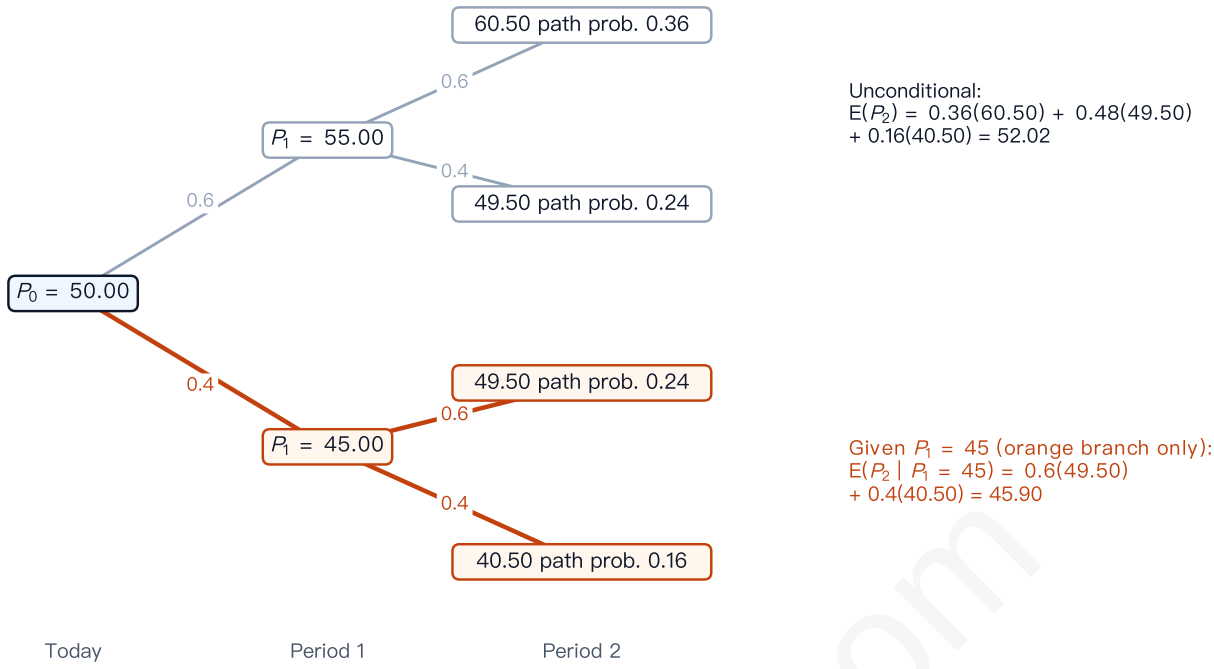
Probability trees

A tree shows sequential outcomes; the probability of a full path is the product of the branch probabilities along it. The unconditional expected value weights every end node by its path probability, the product of the branch probabilities along that path. Imposing a condition (for instance, "the price fell in period 1") prunes the tree. Only branches that start from the observed node remain, and their branch probabilities become the conditional probabilities.

> *Example.* A share at 50 moves up 10% with probability 0.6 or down 10% with probability 0.4 in each of two periods (independent moves).

- End prices: 60.50 (prob. 0.36), 49.50 (prob. 0.48 via two paths), 40.50 (prob. 0.16). Unconditional $E(P_2) = 0.36(60.50) + 0.48(49.50) + 0.16(40.50) = 52.02$.
- Given $P_1 = 45$ (a fall in period 1): $E(P_2 \mid P_1 = 45) = 0.6(49.50) + 0.4(40.50) = 45.90$; $\mathrm{Var}(P_2 \mid P_1 = 45) = 0.6(3.6)^2 + 0.4(-5.4)^2 = 19.44$, so the conditional SD is 4.41.
- Given $P_1 = 55$: $E(P_2 \mid P_1 = 55) = 0.6(60.50) + 0.4(49.50) = 56.10$. Weighting the two conditional expectations by their branch probabilities gives back the unconditional value, $0.6(56.10) + 0.4(45.90) = 52.02$.

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Two-period stock price tree and the branch that remains once the period-1 price is known to be 45

A common error is to weight the remaining outcomes by their *joint* (path) probabilities, e.g. $0.4 \times 0.6 \times 49.50 + 0.4 \times 0.4 \times 40.50$, which gives 18.36. That figure is $P(S)\,E(P_2 \mid S)$ rather than the conditional expectation.

LOS 6.d — Updating probabilities with Bayes' formula

Bayes' formula revises a **prior probability** (also called an **a priori probability**) of an event into a **posterior probability** after new information I is observed. It follows from the **multiplication rule**, $P(I \mid E)P(E) = P(I \text{ and } E)$: writing the joint probability two ways gives $P(E \mid I)P(I) = P(I \mid E)P(E)$, and dividing by $P(I)$ gives

KEY CONCEPT

$$P(E \mid I) = \frac{P(I \mid E)}{P(I)} \times P(E) = \frac{P(I \mid E)\,P(E)}{P(I \mid E)P(E) + P(I \mid E^c)P(E^c)}$$

The denominator is the **unconditional probability of the information**: the sum of its joint probabilities with each state.

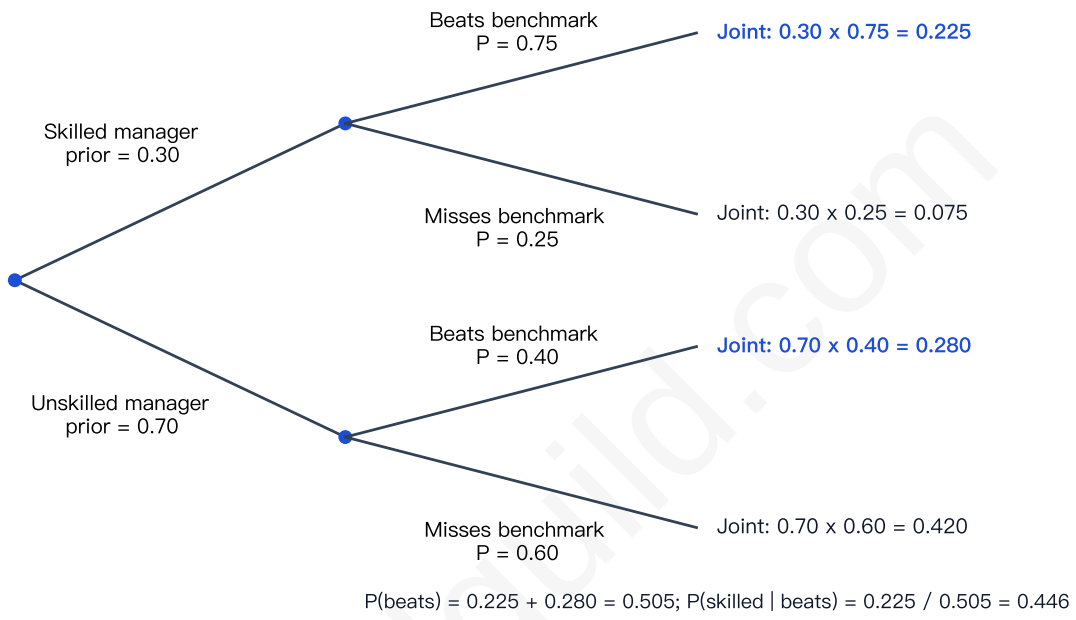
The formula can be checked against the 400-firm table in LOS 6.c. Counting directly, 64 of the 120 firms that returned more than 10% are technology firms, so $P(\text{tech} \mid R > 10\%) = 64/120 = 0.533$. Bayes' formula reaches the same figure from the reverse conditional probability: $P(R > 10\% \mid \text{tech})\,P(\text{tech})/P(R > 10\%) = 0.40 \times 0.40/0.30 = 0.533$.

Applying Bayes' formula:

1. Write down the prior $P(E)$ and its complement $P(E^c) = 1 - P(E)$.

2. Write down the likelihoods $P(I \mid E)$ and $P(I \mid E^c)$; take complements if the information is a "does not happen" event.
3. Multiply to get the joint probabilities of I with each state.
4. Add the joint probabilities to get $P(I)$.
5. Posterior = the joint probability for the state of interest ÷ $P(I)$.

Example. Of the fund managers in a database, 30% are skilled. A skilled manager beats the benchmark in a given year with probability 0.75; an unskilled one with probability 0.40. A manager beat the benchmark. What is the probability that she is skilled?



Bayes' formula as a probability tree: skilled vs. unskilled fund managers

$$P(\text{skilled} \mid \text{beat}) = \frac{0.30 \times 0.75}{0.30 \times 0.75 + 0.70 \times 0.40} = \frac{0.225}{0.505} = 0.446$$

The evidence raises the probability from the 30% prior to about 45%, but because unskilled managers are the majority, a single good year is far from proof of skill.

COMMON EXAM TRAPS (LOS 6.C AND 6.D)

- Reporting the joint probability $P(I \text{ and } E)$ (step 3) as the answer instead of dividing by $P(I)$. In the fund manager example this gives 0.225 instead of 0.446.
- Reporting $P(I)$ (step 4), the unconditional probability of the information, instead of the posterior. In the same example this gives 0.505.
- Confusing $P(I \mid E)$ with $P(E \mid I)$; they are generally very different.
- Comparing likelihoods only, $P(I \mid E)/[P(I \mid E) + P(I \mid E^c)]$, which ignores the priors. There it gives $0.75/1.15 = 0.652$.
- Forgetting to use complements when the information is that something did not happen.
- In trees, using joint path probabilities for a conditional expectation, or averaging all end nodes equally.

- Measuring conditional variance around the unconditional mean.

EXAM SHORTCUTS

- For Bayes' formula, write one row per state with prior × likelihood, add the rows to get $P(I)$, and divide the row for the state of interest by that total.
- Check a conditional expectation answer: the conditional expectations weighted by their branch probabilities must give back the unconditional expectation.

BOTTOM LINE

- A joint probability is $P(A \text{ and } B)$, marginal probabilities are the row and column totals, and a conditional probability uses only the outcomes consistent with the condition.
- The conditional expectation is $E(X \mid S) = \sum P(x_i \mid S)x_i$, and the conditional variance is measured around $E(X \mid S)$.
- In a probability tree a path probability is the product of its branch probabilities.
- Bayes' formula is $P(E \mid I) = P(I \mid E)P(E)/P(I)$, where $P(I)$ adds the joint probabilities of the information with every state.
- $P(I \mid E)$ and $P(E \mid I)$ are generally different, and the posterior still depends on the prior.

QUICK CHECK

Question 3

A high-yield portfolio holds 40% BB-rated bonds and 60% B-rated bonds. Historically, 10% of BB-rated bonds and 30% of B-rated bonds default within five years. If a bond chosen at random from the portfolio defaults within five years, the probability that it was BB-rated is *closest* to:

A 0.100.

B 0.182.

C 0.333.

Answer: B

This is a Bayesian update: the prior probability that a bond is BB-rated (0.40) is revised using the information that the bond defaulted. The posterior is the joint probability of 'BB and default' divided by the unconditional probability of default.

$$P(\text{BB and D}) = P(D \mid BB)P(BB) = 0.10 \times 0.40 = 0.04$$

$$P(\text{B and D}) = P(D \mid B)P(B) = 0.30 \times 0.60 = 0.18$$

$$P(D) = 0.04 + 0.18 = 0.22$$

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$$P(BB \mid D) = \frac{0.04}{0.22} = 0.182$$

Why the other options are wrong

- **A.** 0.100 is $P(\text{default} \mid \text{BB})$, the likelihood. The question asks for the reverse conditional probability, the posterior $P(\text{BB} \mid \text{default})$.
- **C.** 0.333 is the ratio of the two default rates $(0.10/0.30)$; it ignores the portfolio weights (priors) and the unconditional probability of default.

Key takeaway $P(A \mid B) \neq P(B \mid A)$. Build the joint probabilities, add them for the denominator, then divide.

Practice questions

Question 4

A consultant at Northgate Partners has estimated the scenario returns shown in the table below. The standard deviation of returns on Fund Alder is *closest* to:

Scenario forecasts for Fund Alder and Fund Birch

Scenario	Probability	Return on Fund Alder	Return on Fund Birch
Boom	20%	19%	12%
Expansion	25%	15%	11%
Stagnation	35%	9%	10%
Contraction	20%	1%	8%

A 1.34%.

B 6.18%.

C 6.78%.

Question 5

A commodity desk simulates thousands of possible copper price paths and wants to be certain that no simulated price is ever negative. Which distribution for the price level is *best* suited to this goal?

A Normal.

B Logistic.

C Lognormal.

Answers: 4. B · 5. C

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Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/lognormal-distribution/. The full bank has 46 questions on this reading.

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QUANTITATIVE METHODS · READING 7

Null Hypothesis

What you'll learn

LOS 7.a Explain the central limit theorem and use it to build confidence intervals, measure parametric VaR and choose among sampling methods.

LOS 7.b Construct and interpret parametric tests of differences between means, mean differences, a single variance, two variances and a correlation coefficient; recognize p-values and p-hacking.

LOS 7.c Compare parametric and nonparametric tests, identify when a nonparametric test is appropriate, and construct and interpret the Spearman rank correlation test and the chi-square test of independence.

MODULE 7.1

The Central Limit Theorem, Confidence Intervals, and Sampling

This reading moves from a sample to the population: how sample means behave under the central limit theorem, how confidence intervals are built and how samples are drawn. It then sets out the steps of a hypothesis test, the test statistic for each parametric test, the two types of error, and the nonparametric tests used for ranks and categories.

LOS 7.a — The central limit theorem, confidence intervals and sampling

The central limit theorem (CLT)

A **population parameter** such as μ or σ is a fixed number, usually unknown. A **sample statistic** such as $\bar{X}$ or s is computed from the sample actually drawn, so it would take a different value in another sample: it is itself a random variable. The distribution of its values over all possible samples of the same size is the statistic's sampling distribution.

Take many **simple random samples** of size n from a population with mean μ and a *finite* variance σ^2. The **central limit theorem** says that the **sampling distribution of the sample mean** $\bar{X}$ becomes approximately normal as n grows, whatever the shape of the population itself (skewed, flat, lumpy):

KEY CONCEPT

$$\bar{X} \dot{\sim} N\left(\mu,\ \frac{\sigma^2}{n}\right) \quad \text{for large } n \text{ (rule of thumb: } n \geq 30)$$

Three consequences follow:

- The mean of all possible sample means equals the population mean: $E(\bar{X}) = \mu$.
- The variance of the sample mean is the population variance divided by the sample size, σ^2/n, which shrinks as n rises.
- The CLT describes the distribution of the sample mean. Individual observations keep the population's shape, and the CLT says nothing about the distribution of the sample standard deviation.

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Standard error and sampling error

Sampling error is the difference between a sample statistic and the population parameter it estimates (e.g., $\bar{X} - \mu$). Its typical size is the **standard error of the sample mean**:

KEY CONCEPT

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \ (\sigma \text{ known}) \qquad s_{\bar{X}} = \frac{s}{\sqrt{n}} \ (\sigma \text{ unknown, the usual case})$$

When σ is unknown, s replaces it, and the result is an estimate of the standard error. The standard deviation describes how widely individual observations scatter around the mean, and a larger sample does not make it smaller. The standard error describes how precisely $\bar{X}$ estimates μ.

Calculator. From raw data, enter the observations in the DATA worksheet, read the sample standard deviation S_x in the STAT worksheet (1-V), and divide it by $\sqrt{n}$.

Example. A population of quarterly returns has $\sigma = 18\%$. With $n = 36$, $\sigma_{\bar{X}} = 18/6 = 3\%$; quadrupling the sample to 144 halves the standard error to 1.5%.

Desirable properties of an estimator

Property	Meaning
Unbiased	Expected value of the estimator equals the parameter (the sample mean is unbiased for μ).
Efficient	Smallest sampling variance among unbiased estimators. An estimator that is both unbiased and efficient is the **minimum variance unbiased estimator (MVUE)**; the unbiased estimator with the lowest variance among those formed as a linear combination of the sample data is the **best linear unbiased estimator (BLUE)**.
Consistent	Accuracy improves as n increases (standard error falls toward zero).
Robust	Not sensitive to outliers or non-normal data (the sample mean is *not* robust).

Confidence intervals

A **point estimate** is a single value (e.g., $\bar{X}$). A **confidence interval** is a range built around it that contains the true parameter with a stated **confidence level** $1 - \alpha$, where α is the **significance level**:

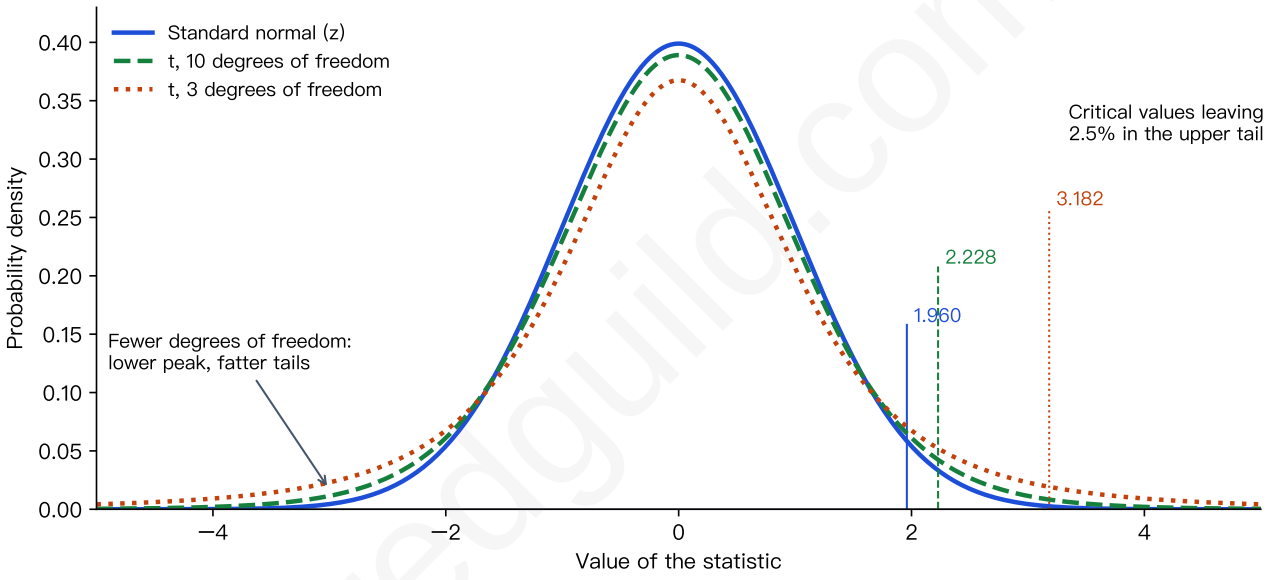
$$\text{point estimate} \pm \underbrace{(\text{reliability factor} \times \text{standard error})}_{\text{margin of error}}$$

Situation	Reliability factor	Interval for μ
Normal population, σ known	$z_{\alpha/2}$	$\bar{X} \pm z_{\alpha/2}\,\sigma/\sqrt{n}$
Normal (or approximately normal) population, σ unknown (use s); required when n is small	$t_{\alpha/2}$ with $n - 1$ df	$\bar{X} \pm t_{\alpha/2}\,s/\sqrt{n}$

Situation	Reliability factor	Interval for μ
σ unknown, large sample ($n \geq 30$), any population shape (CLT)	$t_{\alpha/2}$ preferred; $z_{\alpha/2}$ acceptable as an approximation	$\bar{X} \pm t_{\alpha/2}\, s/\sqrt{n}$, or $\bar{X} \pm z_{\alpha/2}\, s/\sqrt{n}$ as a large-sample approximation

Two-tailed z reliability factors $z_{\alpha/2}$ are 1.65 for 90% (1.645 before rounding), 1.96 for 95% and 2.58 for 99% (2.576 before rounding). Three things set the width of an interval. A higher confidence level (a larger reliability factor) widens it, a more dispersed population widens it, and a larger sample (a smaller standard error) narrows it.

The **Student's t-distribution** is bell-shaped and symmetric but has fatter tails than the normal; its critical values are larger than z-values (wider intervals) and fall toward the z-value as degrees of freedom rise. For a 95% interval, the t reliability factor is 2.228 with 10 df, 2.086 with 20 df and 2.042 with 30 df, against 1.96 for z. Select a t-value from a table using the area in one tail ($\alpha/2$ for a two-sided interval) and $n - 1$ degrees of freedom.



Student's t-distribution with 3 and 10 degrees of freedom compared with the standard normal distribution

Example (σ known). With 49 observations, $\bar{X} = 6.2$ and $\sigma = 14$, the standard error $= 14/7 = 2$; 95% interval $= 6.2 \pm 1.96 \times 2 = 2.28$ to 10.12. *Example (σ unknown, small sample).* Ten observations from a population assumed normal give s. The 95% reliability factor is $t_{9,\,0.025} = 2.262$ rather than 1.96. If a small sample comes from a clearly non-normal population, neither a t- nor a z-interval is reliable.

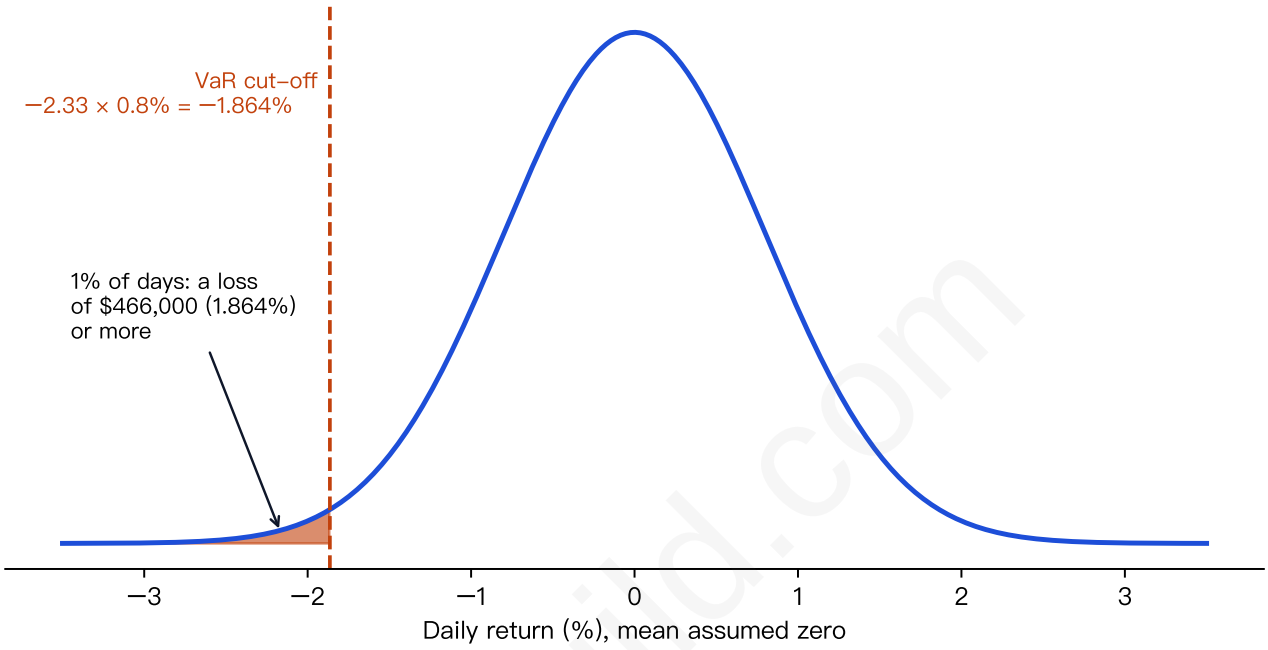
Value at risk (VaR) — a one-tailed interval

Value at risk is the loss that will be equaled or exceeded with a stated (small) probability over a given period. "The daily 1% VaR is $0.47 million" means a 1% chance on any day of losing $0.47 million or more (equivalently, 99% confidence that the loss will be smaller). VaR is the minimum loss at the stated probability; larger losses remain possible, so it is not the worst possible loss.

Under the **parametric method** (normal returns, mean return assumed to be zero):

$$\text{VaR}_\alpha = z_\alpha \times \sigma \times \text{portfolio value}$$

with the whole α in one tail: $z_{10\%} = 1.28$, $z_{5\%} = 1.65$ (1.645 before rounding), $z_{1\%} = 2.33$. With σ in percent and the portfolio value in dollars, VaR is a dollar loss; dropping the portfolio value gives VaR as a percentage of the portfolio. *Example.* For a $25 million portfolio with daily $\sigma = 0.8\%$, the daily 1% VaR $= 2.33 \times 0.008 \times \$25\text{m} = \$466{,}000$, or 1.864% of the portfolio.



Daily 1% VaR as the left-tail cut-off of the return distribution ($25 million portfolio, daily σ = 0.8%)

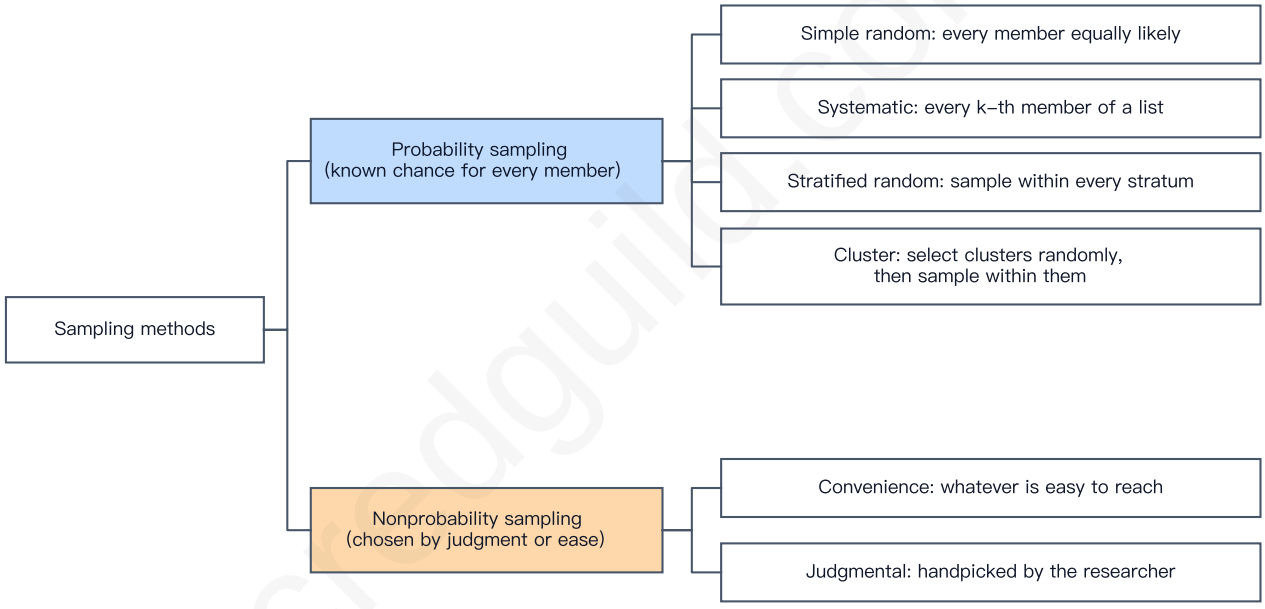
Drawbacks: returns are often skewed/fat-tailed, parameters change over time, history may not reflect new conditions, and it handles nonlinear payoffs (options) poorly.

Sampling methods

Family	Method	How the sample is drawn
Probability sampling (every member has a known, nonzero chance)	**Simple random sampling**	Every member has the *same* chance of selection, through random draws from the full list (so every possible sample of size n is equally likely). Equal individual chances alone are not enough; a systematic sample with a random start also gives every member the same chance.
	Systematic sampling	Select every k-th member of the population list.
	Stratified random sampling	Split the population into **strata** by a characteristic (sector, size, rating); draw a random sample *within each stratum*, usually in proportion to its weight in the population; pool the results.

Family	Method	How the sample is drawn
	Cluster sampling	Split into clusters that each resemble the population; randomly select clusters and sample within them. The CLT applies if the clusters are large enough.
Nonprobability sampling	**Convenience sampling**	Use whatever is easy to reach (e.g., colleagues at one office); cheap but likely unrepresentative.
	Judgmental sampling	The researcher handpicks elements using experience and judgment; may import the researcher's bias, but expert judgment can help on complex research questions.

Probability sampling is generally more accurate; nonprobability sampling is cheaper. Stratified and cluster sampling both split the population into groups. Stratified sampling draws from every group, so the sample keeps the population's mix across groups; cluster sampling uses only some groups, so differences between groups can be lost.



Sampling methods

The CLT does not rescue a non-representative sample.

COMMON EXAM TRAPS

- The CLT requires a finite variance and a large n, where n is the size of each sample; drawing more samples of the same small size does not help. It does not require a normal population, and it describes the sample mean.
- The standard error divides by $\sqrt{n}$ rather than by n. With $\sigma = 18\%$ and $n = 36$, dividing by 36 gives 0.5% instead of 3%.
- Two-sided 95% uses 1.96; one-tailed 5% (VaR) uses 1.645. Always check how many tails. In the VaR example, the two-tailed 1% value 2.58 would give $516,000 instead of $466,000.

- "Random draws, then keep every second one drawn" is still simple random sampling, because keeping fixed positions of an already random sequence leaves the selection purely random. Systematic sampling means every k-th item of a fixed, ordered *population list*.
- Grouping first and sampling within each group is stratified sampling; handpicking is judgmental sampling; choosing what is easiest to reach is convenience sampling.

EXAM SHORTCUTS

- Quadrupling the sample size halves the standard error.
- Reliability factors: two-tailed 90%, 95% and 99% use 1.65, 1.96 and 2.58; one-tailed 10%, 5% and 1% (VaR) use 1.28, 1.65 and 2.33.
- A higher confidence level widens an interval and a larger sample narrows it, which rules out answer choices that move the wrong way.

BOTTOM LINE

- By the central limit theorem, the mean of samples of size $n \geq 30$ from any population with a finite variance is approximately normal with mean μ and variance σ^2/n.
- The sample mean's standard error is $\sigma/\sqrt{n}$, or $s/\sqrt{n}$ when σ is unknown.
- A confidence interval is the point estimate ± reliability factor × standard error, with a t reliability factor and $n-1$ df when σ is unknown.
- A desirable estimator is unbiased, efficient and consistent.
- Parametric VaR is $z_\alpha \times \sigma \times$ portfolio value with the whole α in one tail; it is the minimum loss at the stated probability, not the worst possible loss.
- Simple random, systematic, stratified and cluster sampling are probability methods; convenience and judgmental sampling are nonprobability methods.

QUICK CHECK

Question 1

A fixed-income analyst splits the universe of investment-grade corporate bonds into five credit-rating buckets. From each bucket she randomly draws a number of bonds proportional to that bucket's share of the universe and then combines the draws into one sample of 400 bonds. Her approach is best described as:

A simple random sampling.

B systematic sampling.

C stratified random sampling.

Answer: C

The analyst first divides the population into subgroups (strata) using a classification scheme (here, credit rating) and then draws a random sample within each stratum, pooling the results. That is stratified random sampling. Sampling each stratum in proportion to its weight ensures that the rating mix of the sample mirrors the population.

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Why the other options are wrong

- **A.** In simple random sampling every bond in the universe would have the same chance of selection from a single, undivided list; here the population is first split into rating buckets and each bucket is sampled separately.
- **B.** Systematic sampling selects every k-th member of an ordered population list. The analyst makes random draws inside each rating bucket instead.

Key takeaway Dividing the population into groups first and then sampling randomly within every group is stratified random sampling.

MODULE 7.2

Hypothesis Tests of the Population Mean

LOS 7.b — Hypothesis tests of a population mean

What a hypothesis test does

A **hypothesis** is a claim about what value a population parameter takes (e.g., "the mean daily return of this strategy is zero"). A sample is used to decide whether the data give enough evidence to reject that statement.

- **Parametric tests** make assumptions about the population's distribution and test its parameters (mean, variance).
- **Nonparametric tests** make few distributional assumptions or test things that are not parameters (ranks, category counts); Module 7.4 covers them.

The four steps

1. State the **null hypothesis** H_0 and the **alternative hypothesis** H_a.
2. Identify the **test statistic** and its distribution.
3. Specify the **significance level** α.
4. Establish the **decision rule** (the **critical value**(s)) and compare.

Hypotheses. The null is usually the claim the researcher hopes to reject, and it always contains the equality (=, ≤ or ≥). The alternative is what the researcher wants to find evidence for. The two are mutually exclusive and exhaustive.

Evidence sought	H_0	H_a	Test	Rejection region
Mean differs from μ_0 (either direction)	$\mu = \mu_0$	$\mu \neq \mu_0$	**two-tailed test**	both tails, $\alpha/2$ each
Mean is greater than μ_0	$\mu \leq \mu_0$	$\mu > \mu_0$	**one-tailed test**	right tail, all of α
Mean is less than μ_0	$\mu \geq \mu_0$	$\mu < \mu_0$	**one-tailed test**	left tail, all of α

Test statistic for a mean

The test statistic measures how many standard errors the sample mean lies from the hypothesized value:

KEY CONCEPT

$$\text{test statistic} = \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \quad (\text{use } \sigma/\sqrt{n} \text{ if } \sigma \text{ is known})$$

When s is used and the population is normal (or approximately normal), the statistic follows a **t-distribution** with $n - 1$ degrees of freedom; with a large sample ($n \geq 30$) the CLT makes the t-test approximately valid even for a non-normal population, and the **z-statistic** (standard normal) is an acceptable approximation. A small sample from a clearly non-normal population supports neither test. When the population is normal and its variance is known, the statistic uses σ and follows the standard normal distribution at any sample size; z critical values have no degrees of freedom. The sign matters: a negative statistic means the sample mean is *below* μ_0.

Critical z-values (exam values; three-decimal values in parentheses):

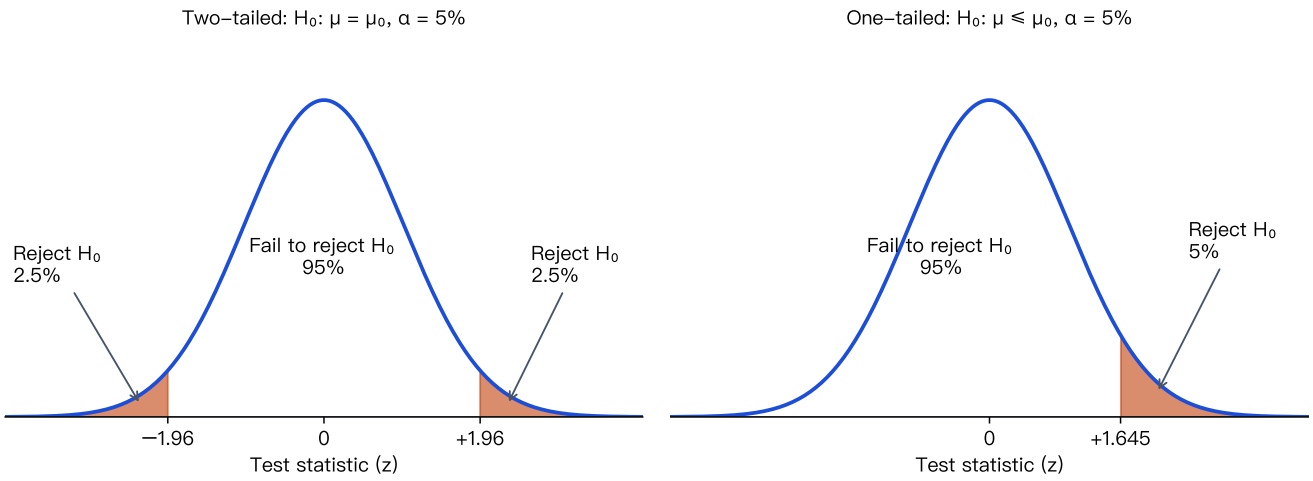
KEY CONCEPT

Significance level	Two-tailed test	One-tailed test
10%	±1.65 (1.645)	1.28 (1.282)
5%	±1.96	1.65 (1.645)
1%	±2.58 (2.576)	2.33 (2.326)

Decision rule. Reject H_0 if the test statistic falls in the rejection region (beyond the critical value); otherwise **fail to reject** H_0. The null is never "accepted"; failing to reject means only that the data were not strong enough to reject it. The **confidence level** is $1 - \alpha$. For a two-tailed test the rule is equivalent to a confidence interval: H_0 is rejected when the sample mean falls outside the interval $\mu_0 \pm$ (critical value $\times$ standard error) built around the hypothesized value.

The critical value depends on three things: the distribution of the test statistic (z, or t with its degrees of freedom), the significance level, and whether the test is one-tailed or two-tailed. At $\alpha = 5\%$, for example, a statistic of 1.75 exceeds the critical value of a right-tailed test (1.65) but not that of a two-tailed test (1.96), so the two tests reach different decisions.

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Rejection regions for z-tests at a 5% significance level: two-tailed (left) and one-tailed, right tail (right).

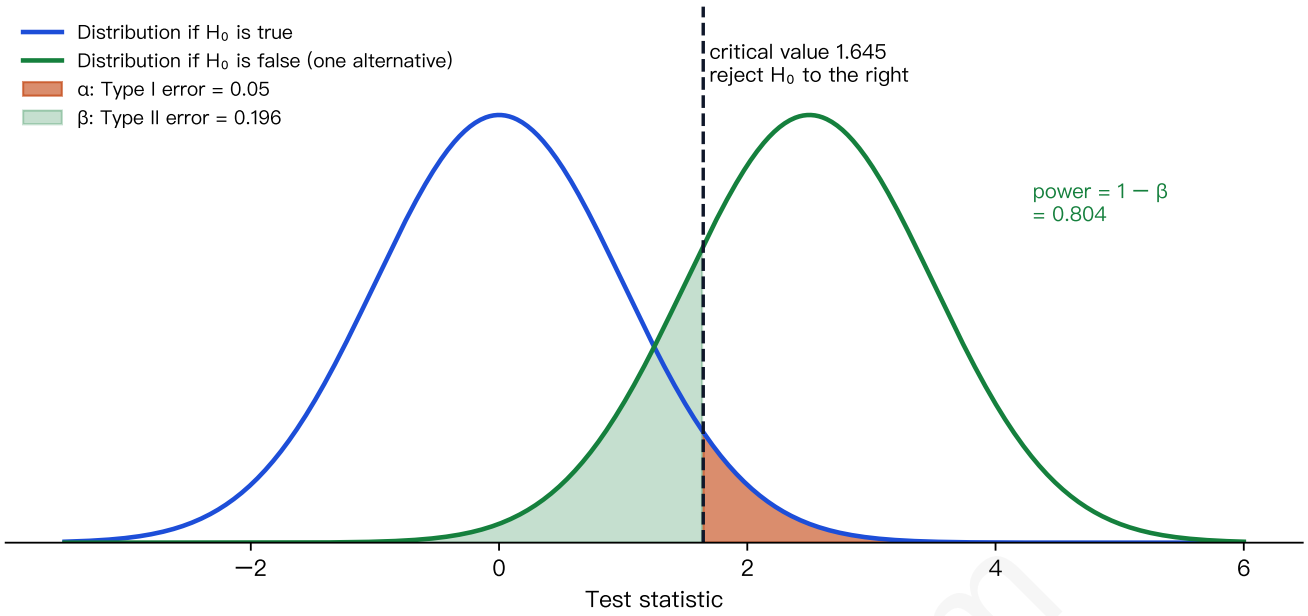
Example. A sample of 81 daily returns has $\bar{X} = 0.42\%$ and $s = 1.8\%$; the test is $H_0 : \mu = 0$, two-tailed, at $\alpha = 5\%$. The standard error is $1.8/9 = 0.2\%$ and the statistic is $0.42/0.2 = 2.10$. Because 2.10 exceeds 1.96, H_0 is rejected. At $\alpha = 1\%$ (critical value ±2.58), H_0 would not be rejected.

What moves the statistic. For a sample mean above μ_0, the statistic (and the chance of rejection) *rises* when the sample mean rises, the standard deviation falls or the sample size rises (smaller standard error).

Type I and Type II errors, power

	H_0 **is true**	H_0 **is false**
Reject H_0	**Type I error** (false positive); probability = α	Correct decision; probability = **power of the test** $= 1 - \beta$
Fail to reject H_0	Correct decision; probability = $1 - \alpha$ (confidence level)	**Type II error** (false negative); probability = β

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Type I and Type II errors for a one-tailed test at α = 5% (illustrative alternative)

- The significance level is the probability of a Type I error, that is, of rejecting a true null by chance.
- The **power of a test** is the probability of correctly rejecting a false null: power $= 1 - P(\text{Type II error})$.
- Only one error type can occur in a given test: a Type I error needs a true null, a Type II error needs a false null.
- Trade-off: with the sample size and the test statistic unchanged, lowering α (e.g., from 5% to 1%) *raises* the probability of a Type II error and *lowers* power; raising power in that setting means accepting a higher α. A lower α also means a larger critical value, so the matching confidence interval is wider.
- Exam convention: for a given α, power can be raised only by increasing the sample size, which lowers the probability of a Type II error while the probability of a Type I error stays at α. Current practice: at a given α, less noisy data (a lower standard deviation) or a more powerful test statistic also raise power.
- The chance of a Type II error depends jointly on α and the sample size, but the relation is not simple and that probability is hard to compute in practice. When several test statistics could be used for the same hypothesis, the most powerful one is normally preferred.

Example. A spam filter tests H_0: "this message is legitimate." Sending a legitimate message to the junk folder rejects a true null (Type I); letting spam through fails to reject a false null (Type II).

WORKED EXAMPLE: A CONFIDENCE INTERVAL AND A ONE-TAILED TEST

A sample of 64 monthly returns has a mean of 0.85% and a standard deviation of 2.4%. The population standard deviation is unknown. Build a 95% confidence interval for the mean, then test at the 5% level whether the mean return is above 0.5%.

Step 1. Standard error $= 2.4/\sqrt{64} = 0.30\%$.

Step 2. 95% confidence interval. With $n = 64$, the z reliability factor is an acceptable approximation: $0.85 \pm 1.96 \times 0.30$, from 0.262% to 1.438%. The t reliability factor with 63 df is 1.998, which gives a slightly wider interval, from 0.251% to 1.449%.

Step 3. Test whether the mean return is above 0.5% at the 5% level: $H_0 : \mu \leq 0.5\%$ and $H_a : \mu > 0.5\%$. The test is one-tailed, so all of the 5% is in the right tail and the critical value is 1.65 (1.669 for t with 63 df).

Step 4. Test statistic $= (0.85 - 0.5)/0.30 = 1.17$.

Step 5. 1.17 is below the critical value, so H_0 is not rejected: the sample does not show a mean return above 0.5%.

Step 6. If the true mean is above 0.5%, this decision is a Type II error. A Type I error cannot have occurred, because H_0 was not rejected.

COMMON EXAM TRAPS

- A 5% significance level corresponds to a 95% confidence level; "95% significance" misreads the two terms.
- The probability of a Type I error in a two-tailed 5% test is 5%; 2.5% is the area in each tail.
- Power is $1 - \beta$; $1 - \alpha$ is the confidence level.
- H_a for a two-tailed test uses "≠"; "<" or ">" signals a one-tailed test.
- Check which tail: for $H_0 : \mu \geq \mu_0$, the null is rejected only when the statistic is *below* the negative critical value.
- A two-tailed 10% test and a one-tailed 5% test share the critical value 1.65, because both place 5% in the tail beyond it.

EXAM SHORTCUTS

- The hypothesis that contains the equality sign is H_0.
- A two-tailed 10% test and a one-tailed 5% test share the critical value 1.65.
- For a two-tailed test, H_0 is rejected when the sample mean falls outside $\mu_0 \pm$ critical value × standard error, so a confidence interval answers the test directly.

BOTTOM LINE

- H_0 always contains the equality and is usually the claim the researcher hopes to reject; H_a is the claim the researcher seeks evidence for.
- The test statistic is (sample mean − μ_0) ÷ standard error, following a t-distribution with $n - 1$ df when s is used.
- H_0 is rejected when the statistic falls beyond the critical value; otherwise the decision is to fail to reject, and the null is never accepted.
- A Type I error rejects a true null with probability α; a Type II error fails to reject a false null with probability β; power is $1 - \beta$.
- With the sample size unchanged, a lower α makes a Type II error more likely and reduces power.

QUICK CHECK

Question 2

A researcher keeps her sample size unchanged but tightens the significance level of her test from 5% to 1%. As a result, the probability of:

A a Type II error increases.

B rejecting a true null hypothesis increases.

C failing to reject a false null hypothesis decreases.

Answer: A

Lowering the significance level lowers the probability of a Type I error. For a fixed sample size, the rejection region shrinks, so a false null is less likely to be rejected: the probability of a Type II error rises and the power of the test falls.

Why the other options are wrong

- **B.** Rejecting a true null is a Type I error, whose probability is the significance level; it falls from 5% to 1%.
- **C.** Failing to reject a false null is a Type II error; its probability *increases* when α is reduced with no change in sample size. A null is never 'accepted'; a test can only fail to reject it.

Key takeaway With n fixed, a lower α means a higher β and a lower power.

MODULE 7.3

Other Parametric Hypothesis Tests

LOS 7.b — Other parametric hypothesis tests

Every test below uses the same four steps as the test of a single mean (hypotheses, test statistic, significance level, decision rule). What changes is the **test statistic** and the distribution its critical values come from.

Summary of the tests

KEY CONCEPT

What is tested	Test statistic	Distribution / degrees of freedom	Key assumptions
Single mean	$t = \frac{\bar{X} - \mu_0}{s/\sqrt{n}}$	t, $n-1$ (z if n is large)	normal population, or a large sample
Difference between means (independent samples)	$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2/n_1 + s_p^2/n_2}}$, with pooled variance $s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$	t, $n_1 + n_2 - 2$	independent samples, normal populations, **equal (unknown) population variances**
Mean difference (paired comparisons test, dependent samples)	$t = \frac{\bar{d} - \mu_{d0}}{s_d/\sqrt{n}}$	t, $n-1$ (n = number of pairs)	paired observations, normal differences

What is tested	Test statistic	Distribution / degrees of freedom	Key assumptions
Single **variance**	$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$	**chi-square**, $n-1$	normal population
Equality of two variances	$F = \frac{s_1^2}{s_2^2}$ (larger variance on top)	**F**, $n_1 - 1$ and $n_2 - 1$	normal populations, independent samples
Correlation equals zero	$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$	t, $n-2$	normally distributed variables

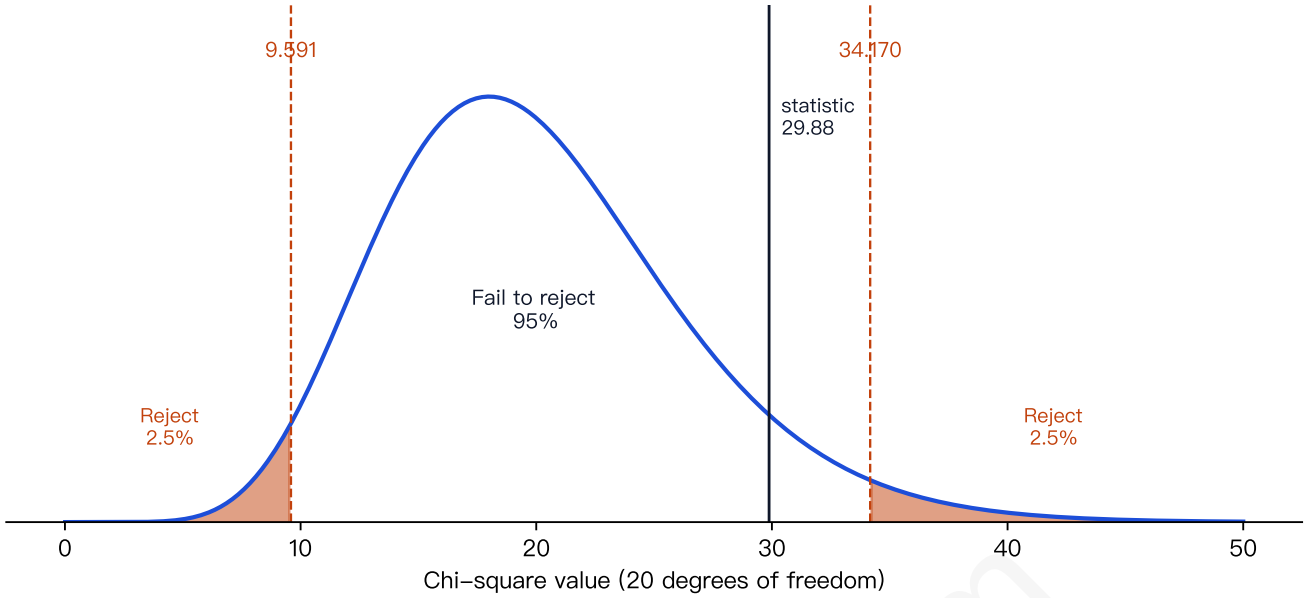
Difference between means vs. paired comparisons

- **Independent samples** (e.g., returns of small-cap funds vs. returns of large-cap funds): test $H_0 : \mu_1 - \mu_2 = 0$ with the difference-in-means t-test, df $= n_1 + n_2 - 2$. These df belong to the **pooled** version, which assumes the two population variances are equal and combines both samples into one variance estimate s_p^2. A test that does not assume equal variances uses the unpooled standard error $\sqrt{s_1^2/n_1 + s_2^2/n_2}$, with df from a separate (Welch) approximation that are usually smaller than $n_1 + n_2 - 2$. When $n_1 = n_2$, the pooled and unpooled standard errors are identical, and when the sample variances are similar the two t-values are close.
  - Exam convention: the difference-in-means statistic may be shown with the unpooled standard error $\sqrt{s_1^2/n_1 + s_2^2/n_2}$ and still be compared with a t critical value for $n_1 + n_2 - 2$ df.
  - Current practice: $n_1 + n_2 - 2$ df go with the pooled standard error (equal variances assumed); the unpooled (Welch) test uses fewer df.
- **Dependent samples** (the *same* members measured twice, e.g., each fund's return before and after a fee cut): compute the difference for each pair, then run a single-mean test on those differences. This is the **paired comparisons test**, with df $= n - 1$.

Example (independent samples). $\bar{X}_1 = 1.9$, $s_1 = 1.5$, $n_1 = 25$; $\bar{X}_2 = 1.2$, $s_2 = 1.3$, $n_2 = 31$. Assuming equal population variances, $s_p^2 = (24 \times 1.5^2 + 30 \times 1.3^2)/54 = 1.939$ and $t = 0.7/\sqrt{1.939/25 + 1.939/31} = 1.87$; df = 54, 5% two-tailed critical value 2.005, so the null of equal means is not rejected. (The unpooled standard error would give $t = 1.84$; the conclusion is the same.)
Example (paired). With 16 pairs, $\bar{d} = 1.2$ and $s_d = 2.0$, $t = 1.2/(2.0/4) = 2.40$. This exceeds $t_{15,\,0.025} = 2.131$, so a zero mean difference is rejected.

Test of a single variance (chi-square)

The **chi-square distribution** is asymmetric (right-skewed) and bounded below by zero. For a two-tailed test at α, find a lower critical value (area $1 - \alpha/2$ to its right) and an upper one (area $\alpha/2$ to its right). *Example.* With 21 observations, $s = 5.5\%$ and $H_0 : \sigma = 4.5\%$, $\chi^2 = 20 \times 5.5^2/4.5^2 = 29.88$. With 20 df and $\alpha = 5\%$ the critical values are 9.591 and 34.170, so the null is not rejected.



Two-tailed chi-square test of a variance at 5% with 20 degrees of freedom

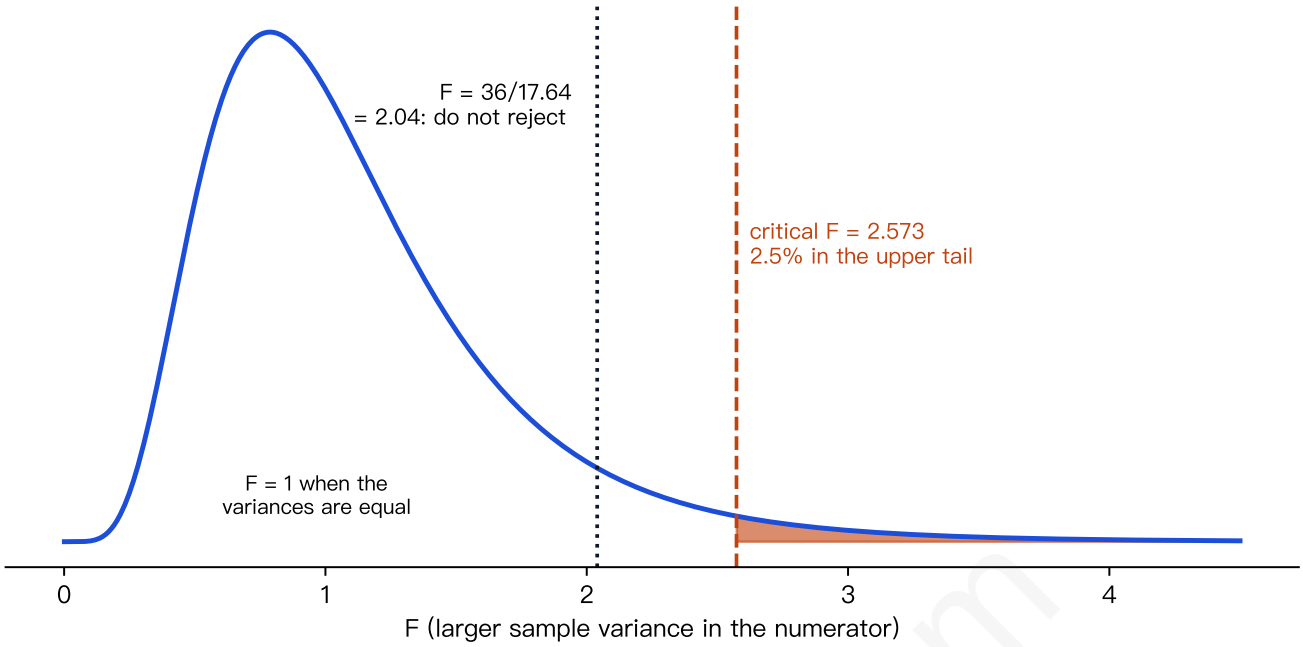
A hypothesis about a standard deviation is written in terms of the variance. With returns in decimals, a claim that σ is below 3% becomes $H_0 : \sigma^2 \geq 0.0009$ versus $H_a : \sigma^2 < 0.0009$, and only the lower critical value is used.

Test of two variances (F-test)

The **F-distribution** is right-skewed, bounded below by zero and defined by two degrees of freedom (numerator $n_1 - 1$, denominator $n_2 - 1$). $F = 1$ when the sample variances are equal. Putting the larger variance in the numerator means only the upper critical value matters (with $\alpha/2$ in that tail for a two-tailed test).

- $H_0 : \sigma_1^2 = \sigma_2^2$ (the variances are *not* different) vs. $H_a : \sigma_1^2 \neq \sigma_2^2$.
- If the F-statistic exceeds the critical F-value, reject H_0 and conclude that the variances are significantly different. *Example.* With $s_1 = 6.0$ ($n_1 = 16$) and $s_2 = 4.2$ ($n_2 = 21$), $F = 36/17.64 = 2.04$. The critical value $F_{15,20}$ with 2.5% in the upper tail is 2.573, so the null is not rejected.

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F-distribution with 15 and 20 degrees of freedom: rejection region above 2.573 and the example statistic 2.04

Test of a correlation coefficient

The **correlation coefficient** (the **Pearson correlation**) gauges how strong a linear relationship is and lies between −1 (perfect negative) and +1 (perfect positive); zero means no linear relationship. $H_0 : \rho = 0$ vs. $H_a : \rho \neq 0$. The statistic needs only the sample correlation r and the sample size n; it follows a t-distribution with $n - 2$ degrees of freedom. It grows with $|r|$ and with n, so a modest correlation can be significant in a large sample: $r = 0.25$ gives $t = 2.56$ with $n = 100$ but only $t = 0.86$ with $n = 13$. *Example.* With $r = 0.30$ and $n = 50$, $t = 0.30\sqrt{48}/\sqrt{0.91} = 2.18$. This exceeds $t_{48,\,0.025} = 2.011$, so $\rho = 0$ is rejected.
When a table is given, select the row for $n - 2$ df and the column for the stated (two-tailed) significance level. The null is rejected at a given level only if $|t|$ exceeds that column's value.

p-values and p-hacking

The **p-value** is the smallest significance level at which the null can be rejected (the probability, if H_0 is true, of a statistic at least as extreme as the one observed). Reject H_0 when p-value < α. **p-hacking** is manipulating data or the testing process to manufacture a statistically significant result, e.g.:

- testing only the best periods, or deleting inconvenient observations as "outliers";
- **data mining**: running many tests until one rejects the null (with $\alpha = 5\%$, about 1 in 20 tests of true nulls rejects by chance);
- using smoothed (e.g., appraisal-based) data to understate risk.
  Because it produces rejections of nulls that are actually true, p-hacking raises the frequency of **Type I errors** (false positives). It is not the same as legitimately choosing a significance level or seeking a more powerful test.

COMMON EXAM TRAPS

- df: correlation test $n - 2$; single mean, paired test and single variance $n - 1$; difference in means $n_1 + n_2 - 2$ (pooled, equal population variances assumed); F-test $n_1 - 1, n_2 - 1$.

- Chi-square tests one variance; F tests two variances; t tests means and correlations.
- The same subjects measured twice call for a paired comparisons test rather than a difference-in-means test.
- Rejecting H_0 in an F-test means concluding the variances are different.

EXAM SHORTCUTS

- Choose the test from what is tested: a mean uses t when the population variance is unknown (z when it is known, or as a large-sample approximation), a test that a correlation is zero uses t when both variables are normally distributed, one variance uses chi-square, and two variances use F.
- With the larger variance in the numerator of the F-statistic, only the upper critical value is needed.
- When a p-value is given, reject H_0 if it is below α; no critical value is needed.

BOTTOM LINE

- Independent samples with equal variances use the pooled difference-in-means t-test with $n_1 + n_2 - 2$ df; the same subjects measured twice use the paired comparisons test with $n - 1$ df.
- A single variance is tested with $\chi^2 = (n-1)s^2/\sigma_0^2$ and $n - 1$ df; the chi-square distribution is right-skewed and bounded below by zero.
- Two variances are tested with $F = s_1^2/s_2^2$ and $n_1 - 1$, $n_2 - 1$ df.
- A correlation is tested with $t = r\sqrt{n-2}/\sqrt{1-r^2}$ and $n - 2$ df; the statistic rises with $|r|$ and with n.
- The p-value is the smallest significance level at which H_0 can be rejected.
- p-hacking raises the frequency of Type I errors.

QUICK CHECK

Question 3

An analyst tests whether a currency strategy's mean daily return is zero and obtains a p-value of 0.021. Which conclusion is *most accurate*?

A The null hypothesis is rejected at the 1% significance level.

B There is a 2.1% probability that the null hypothesis is true.

C The null hypothesis can be rejected at any significance level above 2.1%.

Answer: C

The p-value is the smallest significance level at which the null hypothesis can be rejected. The null is rejected whenever the chosen α exceeds the p-value, so here it is rejected at 5% or at 2.5% but not at 1%.

Why the other options are wrong

- **A.** At 1%, the p-value of 0.021 is above α = 0.01, so the null is not rejected.
- **B.** The p-value is computed assuming the null is true: it is the probability, under H_0, of a test statistic at least as extreme as the one observed. It does not measure the probability that H_0 itself is true.

Key takeaway Reject H_0 when the p-value is below α. A p-value of 0.021 rejects at 5% but not at 1%.

MODULE 7.4

Nonparametric Hypothesis Tests

LOS 7.c — Parametric vs. nonparametric tests

When is a test nonparametric?

	Parametric test	Nonparametric test
What is tested	A parameter of a distribution (mean, variance, correlation)	Something that is not a parameter (ranks, counts in categories), or a parameter when assumptions fail
Assumptions	Specific distribution (e.g., normal population)	Few or no distributional assumptions
Typical data	Values measured on a scale	Ranks or categorical data
Accuracy	Tends to be more accurate when its assumptions hold	More flexible, but tends to be less accurate when the parametric assumptions hold

Use a nonparametric test when (1) the data are ranks or categories, (2) the distributional assumptions of a parametric test do not hold (e.g., a small sample from a clearly non-normal population, so the CLT cannot be relied on), or (3) the question is not about a parameter at all, for example whether two sets of ranks are related or whether a set of observations is consistent with a particular distribution.

Spearman rank correlation test

The **Spearman rank correlation** measures the strength of a *monotonic* relation between two sets of ranks (e.g., a fund's performance rank this year vs. next year). With integer ranks and d_i = difference between the two ranks of item i:

KEY CONCEPT

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)}$$

Example. For six funds with rank differences of 1, −1, 0, 2, −1 and −1, $\sum d_i^2 = 8$ and $r_s = 1 - 48/210 = 0.77$.
Significance is tested with the same statistic as a Pearson correlation, $t = r_s\sqrt{n-2}/\sqrt{1-r_s^2}$, which for large samples ($n > 30$) follows a t-distribution with $n-2$ df. A test of whether performance ranks persist is therefore a **nonparametric test**.

Test of independence using a contingency table

A **contingency table** counts observations classified by two categorical characteristics (e.g., issuer sector × credit-rating change). The **chi-square test of independence** asks whether the two characteristics are related.

- H_0: the two characteristics are independent. H_a: they are not independent.
- **Expected frequency** of cell (i, j) if independent. Under independence an observation's row says nothing about its column, so each row total is split across the columns in the same proportions as the column totals:

$$E_{i,j} = \frac{(\text{total for row } i) \times (\text{total for column } j)}{\text{total number of observations}}$$

- **Test statistic** (sum over all cells of squared deviations, each scaled by the expected count):

$$\chi^2 = \sum_i \sum_j \frac{(O_{i,j} - E_{i,j})^2}{E_{i,j}}, \qquad \text{df} = (r-1)(c-1)$$

- Reject independence if χ^2 exceeds the critical **chi-square** value (right tail only).
- The **standardized residual** (Pearson residual) of a cell is $(O_{i,j} - E_{i,j})/\sqrt{E_{i,j}}$; its square is that cell's contribution to χ^2.
  - Exam convention: each standardized residual is approximately standard normal, and cells with values beyond ±2 are flagged as contributing significantly to the lack of independence.
  - Current practice: because the row and column totals are estimated from the data, the raw residual has a variance below 1, so ±2 works as a screening rule of thumb. The *adjusted* residual, which divides $O_{i,j} - E_{i,j}$ by $\sqrt{E_{i,j}(1 - \text{row share})(1 - \text{column share})}$, is the one that is approximately standard normal.

Example. A sample of 220 bonds is classified by issuer size and rating action over a year:

Issuer size	Upgraded	Unchanged	Downgraded	Total
Small	42	38	20	100
Large	18	62	40	120
Total	60	100	60	220

Expected (small, upgraded) $= 100 \times 60/220 = 27.27$. Summing $(O - E)^2/E$ over the six cells gives $\chi^2 = 20.38$ with $(2-1)(3-1) = 2$ df. The 5% critical value is 5.991 (1%: 9.210), so independence is rejected. The standardized residual for (small, upgraded) is $(42 - 27.27)/\sqrt{27.27} = 2.82 > 2$: by the ±2 rule of thumb, small issuers were upgraded notably more often than independence implies.

COMMON EXAM TRAPS

- The df for a contingency table are $(r-1)(c-1)$; $r + c - 2$ and $r \times c$ are common errors. Count only the category rows and columns and leave out the "Total" row and column. For the 2 × 3 bond table the df are 2; $r + c - 2$ gives 3 and $r \times c$ gives 6.
- Expected counts come from the row and column totals rather than from the observed cell.
- The statistic divides each squared deviation by the expected count; the plain sum of deviations is always zero.
- Rank data (e.g., performance rankings) call for a nonparametric Spearman test; category counts call for a chi-square test on a contingency table.
- Standardized residuals are screened against the ±2 rule of thumb cell by cell, while the overall statistic is compared with a chi-square critical value.

EXAM SHORTCUTS

- Rank data point to the Spearman test; category counts point to a chi-square test on a contingency table.
- Contingency table df are $(r-1)(c-1)$, counting only category rows and columns and leaving out the totals.

BOTTOM LINE

- A nonparametric test is used for ranks or categories, when the assumptions of a parametric test do not hold, or when the question is not about a parameter.
- The Spearman rank correlation is $r_s = 1 - 6\sum d_i^2/[n(n^2-1)]$, tested in large samples ($n > 30$) with the same t-statistic as a Pearson correlation.
- In a chi-square test of independence the expected count is row total × column total ÷ total, $\chi^2 = \sum(O-E)^2/E$ with $(r-1)(c-1)$ df, and only the right tail is used.
- A standardized residual is $(O-E)/\sqrt{E}$, and cells beyond ±2 are flagged by the rule of thumb.

QUICK CHECK

Question 4

A credit researcher compiles the following default data for bond issuers in three sectors:

Bond issuers by sector and default status

Sector	No default	Technical default	Actual default	Total
Energy	94	16	15	125
Healthcare	108	9	13	130
Utilities	126	8	16	150
Total	328	33	44	405

Assuming sector and default status are independent, the expected number of energy issuers in technical default is *closest* to:

A

B

C

Answer: A

Under independence, the expected count for a cell is (row total × column total) / total number of observations. For energy issuers in technical default, use the Energy row total (125) and the Technical default column total (33).

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$$E_{i,j} = \frac{\text{total for row } i \times \text{total for column } j}{\text{total number of observations}} = \frac{125 \times 33}{405} = \frac{4{,}125}{405} = 10.2$$

Why the other options are wrong

- **B.** About 14 (13.6) uses the Actual default column total (44) instead of the Technical default column total: $125 \times 44/405 = 13.6$.
- **C.** 16 is the *observed* number of energy issuers in technical default; the question asks for the count expected under independence.

Key takeaway Expected count = row total × column total / grand total.

Practice questions

Question 5

A multi-asset portfolio is worth €40 million, and its daily returns have a standard deviation of 1.2%. Using the parametric method and assuming a daily mean return of zero, the portfolio's daily 5% value at risk (VaR) is *closest* to:

A €0.79 million.

B €0.94 million.

C €1.12 million.

Answers: 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/null-hypothesis/. The full bank has 67 questions on this reading.

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QUANTITATIVE METHODS · READING 8

Portfolio Standard Deviation

What you'll learn

LOS 8.a Calculate and interpret the expected return, variance, standard deviation, covariance and correlation of portfolio returns, including multiperiod returns and the effects of portfolio drift and diversification.

LOS 8.b Describe, calculate and interpret the minimum-variance portfolio, the efficient frontier and portfolios that combine a risky portfolio with the risk-free asset.

LOS 8.c Explain optimal portfolio selection given an investor's risk aversion and the capital allocation line, the extension to the market portfolio and the capital market line, and the CAPM.

MODULE 8.1

Portfolio Expected Return

This reading computes a portfolio's expected return, variance and standard deviation from its weights, standard deviations and covariances or correlations, and shows how lower correlation produces a diversification benefit. It then derives the minimum-variance portfolio and the efficient frontier, combines a risky portfolio with a risk-free asset along the capital allocation line, selects an optimal portfolio from an investor's risk aversion and indifference curves, and extends the analysis to the capital market line, beta and the CAPM.

LOS 8.a — Portfolio expected return, historical returns and portfolio drift

Why portfolio statistics matter

Managing a portfolio means working with a small set of statistics: the **expected returns**, **standard deviations** and **correlations** of the holdings. These can be historical (built from realized past returns) or forward-looking (built from forecasts or models). Historical data are a sensible anchor, but a short history can give biased estimates; forecasts can blend history with economic views. Either way, the estimates must be revisited regularly.

Expected return of a portfolio

The **expected return** of a portfolio is the **weighted average** of the expected returns of its assets, where each weight is the asset's share of total portfolio value:

KEY CONCEPT

$$E(R_P) = \sum_{i=1}^{N} w_i \, E(R_i), \qquad \sum_{i=1}^{N} w_i = 1$$

Example. A portfolio holds 30% in a fund expected to earn 5% and 70% in a fund expected to earn 9%, so $E(R_P) = 0.30(5\%) + 0.70(9\%) = 7.8\%$.

The **historical return** of a portfolio for one period is the same weighted average applied to realized returns:

$$R_{P,t} = \sum_{i=1}^{N} w_{i,t-1}\, R_{i,t}$$

where $w_{i,t-1}$ is the weight at the *start* of period t.

Multiperiod returns: arithmetic vs. geometric

KEY CONCEPT

Measure	Formula	Comment
Sum of period returns (cumulative arithmetic return)	$\sum_{t=1}^{T} R_{P,t}$	Adds the period returns; ignores compounding
Arithmetic average	$\bar{R}_P = \frac{1}{T}\sum_{t=1}^{T} R_{P,t}$	Ignores compounding
Cumulative (compound) return	$R_{P,T} = \prod_{t=1}^{T}(1 + R_{P,t}) - 1$	Assumes income is reinvested and no capital is added or withdrawn
Geometric average	$\bar{R}_{g,P} = \left[\prod_{t=1}^{T}(1 + R_{P,t})\right]^{1/T} - 1$	Compound growth rate per period

If the portfolio is not rebalanced, the cumulative return over the whole period equals the weighted average of each asset's cumulative return using the initial weights: $R_{P,T} = \sum_i w_{i,0}\left[\prod_{t=1}^{T}(1 + R_{i,t}) - 1\right]$. With or without drift, the formulas in the table are applied to the period-by-period portfolio returns $R_{P,t}$.

Portfolio drift and style drift

Market moves change the weights: winners grow and losers shrink. This is **portfolio drift**. With no rebalancing, each period's return must be computed with the new (drifted) weights, which come from the start-of-period market values.

Style drift is a portfolio moving away from its stated strategy. It can come from portfolio drift (a strong equity market pushes a balanced portfolio overweight in stocks) or from changes in the holdings themselves (a small-cap stock that grows into the mid-cap category). Portfolio drift can sometimes offset style drift, but managers still monitor and rebalance to keep a portfolio aligned with its goals.

KEY CONCEPT

How drift changes a portfolio depends on the starting weights.

- Exam convention: in an equal-weighted portfolio, drift raises the weights of the high-return securities, so performance tilts toward higher-growth or higher-volatility assets; in a portfolio that is not equally weighted, drift tends to enlarge the already large allocations, so the portfolio behaves like a **momentum** strategy and its **concentration risk** may rise.
- Current practice: in any unrebalanced portfolio, whichever assets outperform gain weight and underperformers lose weight. Whether concentration or overall risk rises depends on *which* assets outperform: if the largest positions (or the more volatile assets) do best, concentration and risk

increase; if smaller positions outperform, the weights can move closer together (e.g., an 80/20 portfolio whose assets return 0% and 100% drifts to about 67/33). A high realized return does not by itself mean the asset is high-volatility.

WORKED EXAMPLE — RETURNS WITH CHANGING WEIGHTS

A client invests $50,000 in Asset A and $50,000 in Asset B and never rebalances. Returns: A earns 10% in Period 1 and −4% in Period 2; B earns 2% and then 8%.

1. Period 1 return $= 0.50(10\%) + 0.50(2\%) = 6.00\%$. End values: A = $55,000, B = $51,000 (total $106,000).
2. New weights: A $= 55{,}000/106{,}000 = 51.89\%$; B $= 48.11\%$.
3. Period 2 return $= 0.5189(-4\%) + 0.4811(8\%) = 1.77\%$.
4. Cumulative return $= (1.06)(1.0177) - 1 = 7.88\%$ (check: end value $52,800 + $55,080 = $107,880). Arithmetic average $= (6.00\% + 1.77\%)/2 = 3.89\%$; the simple sum of the two period returns is 7.77%, below the compound 7.88%.

COMMON EXAM TRAPS

- Expected return is a weighted average of expected returns only; do not plug in variances or correlations.
- Applying the original weights to a later period of an unrebalanced portfolio gives the wrong return: 2.00% instead of 1.77% for Period 2 in the example.
- A holding that grows from small cap into the mid-cap category causes style drift; portfolio drift refers only to weights shifting with market values.

EXAM SHORTCUTS

- For a portfolio that is not rebalanced, the cumulative return equals the initial-weight average of each asset's cumulative return, so the cumulative figure needs no period-by-period weights.

BOTTOM LINE

- A portfolio's expected return is the weighted average $E(R_P) = \sum w_i E(R_i)$, with each weight equal to the asset's share of total portfolio value and the weights summing to 1.
- A portfolio's historical return for one period is $\sum w_{i,t-1} R_{i,t}$, using the weights at the start of the period.
- The cumulative (compound) return $\prod(1 + R_{P,t}) - 1$ and the geometric average include compounding, while the sum and the arithmetic average of the period returns ignore it.
- Without rebalancing, market moves shift the weights (portfolio drift), so each period's return must use the drifted weights from start-of-period market values.
- Style drift is a portfolio moving away from its stated strategy, either through portfolio drift or through changes in the holdings themselves, such as a small-cap stock growing into the mid-cap category.

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- Exam convention: drift raises the weights of high-return securities in an equal-weighted portfolio and enlarges already large allocations in a portfolio that is not equally weighted, so it behaves like a momentum strategy with possibly higher concentration risk; current practice: in any unrebalanced portfolio the outperformers gain weight, and whether concentration or risk rises depends on which assets outperform.

QUICK CHECK

Question 1

Nadia Okafor builds the two-fund portfolio shown in the table below and does not rebalance it at any point. Okafor's portfolio return for Year 2 is *closest* to:

Okafor portfolio: starting values and annual returns

Holding	Value at start of Year 1	Return in Year 1	Return in Year 2
Equity fund	$60,000	20%	−10%
Bond fund	$40,000	2%	6%

A −2.00%.

B −3.60%.

C −4.21%.

Answer: C

Without rebalancing, the weights drift with market values (portfolio drift). Each period's portfolio return must use the weights at the start of that period, so the Year 2 return is computed with the weights at the end of Year 1, not the original 60/40 split.

1. Values at the end of Year 1

Equity fund: $\$60{,}000 \times 1.20 = \$72{,}000$; bond fund: $\$40{,}000 \times 1.02 = \$40{,}800$; total $= \$112{,}800$.

2. Weights at the start of Year 2

$$w_{\text{equity}} = \frac{72{,}000}{112{,}800} = 0.6383, \qquad w_{\text{bond}} = \frac{40{,}800}{112{,}800} = 0.3617$$

3. Year 2 portfolio return

$$R_{P,2} = 0.6383(-10\%) + 0.3617(6\%) = -6.383\% + 2.170\% = -4.21\%$$

Check: end values $\$64{,}800 + \$43{,}248 = \$108{,}048$; $108{,}048/112{,}800 - 1 = -4.21\%$.

Why the other options are wrong

- **A.** −2.00% is the equal-weighted average of the two Year 2 returns, $(-10\% + 6\%)/2$; it ignores both the original and the drifted weights.

- **B.** −3.60% uses the original weights, $0.60(-10\%) + 0.40(6\%)$. That would be right only if the portfolio had been rebalanced back to 60/40 at the end of Year 1.

Key takeaway No rebalancing means the weights drift: recompute each weight from start-of-period market values before weighting the period's returns.

MODULE 8.2

Portfolio Risk Measures

LOS 8.a — Variance and standard deviation of a portfolio

Portfolio variance

Risk is measured by how far returns scatter around their mean: the **variance** and its square root, the **standard deviation**. Because the standard deviation is in the same units as returns, it is the usual headline risk number.

For N assets, portfolio variance depends on the weights, each asset's variance and every pair's **covariance**:

$$\sigma_P^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \operatorname{Cov}(R_i, R_j)$$

For two assets the formula becomes:

KEY CONCEPT

$$\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\operatorname{Cov}_{1,2}$$

$$\sigma_P = \sqrt{\sigma_P^2}$$

Covariance and correlation

Covariance is often given indirectly through the **correlation** (correlation coefficient) $\rho_{1,2}$, which is bounded between −1 and +1:

KEY CONCEPT

$$\operatorname{Cov}_{1,2} = \rho_{1,2}\,\sigma_1\,\sigma_2 \quad \Longrightarrow \quad \sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

For forward-looking (expected) values the identity does not hold mechanically, because joint variability and nonlinear interactions among the three inputs also matter. Exam calculations use the identity above.

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Special cases

KEY CONCEPT

Correlation	Portfolio standard deviation (two assets, positive weights)	Diversification benefit
$\rho = +1$	$\sigma_P = w_1\sigma_1 + w_2\sigma_2$ (weighted average)	None
$-1 < \rho < +1$	Below the weighted average	Some; larger as ρ falls
$\rho = -1$	$\sigma_P = \lvert w_1\sigma_1 - w_2\sigma_2 \rvert$	Maximum; zero risk only when $w_1\sigma_1 = w_2\sigma_2$

WORKED EXAMPLE

A portfolio is 20% in Asset J (standard deviation 25%) and 80% in Asset K (standard deviation 10%); the correlation is 0.20.

$$\sigma_P^2 = (0.20)^2(0.25)^2 + (0.80)^2(0.10)^2 + 2(0.20)(0.80)(0.20)(0.25)(0.10)$$

$$= 0.0025 + 0.0064 + 0.0016 = 0.0105 \qquad \sigma_P = \sqrt{0.0105} = 10.25\%$$

The weighted average of the two standard deviations is $0.20(25\%) + 0.80(10\%) = 13\%$; the gap (13% vs. 10.25%) is the diversification benefit.

WORKED EXAMPLE — COVARIANCE GIVEN DIRECTLY, THREE-ASSET EXTENSION

If the covariance is given, use it as is. Two assets with $\sigma_1^2 = 0.0400$, $\sigma_2^2 = 0.0225$ and $\text{Cov}_{1,2} = -0.0030$, held 60/40:

$$\sigma_P^2 = 0.36(0.0400) + 0.16(0.0225) + 2(0.60)(0.40)(-0.0030) = 0.0144 + 0.0036 - 0.00144 = 0.01656,$$

$$\sigma_P = 12.87\%$$

The negative covariance makes the third term negative, so the portfolio is far less risky than a 60/40 average of the standard deviations ($0.6 \times 20\% + 0.4 \times 15\% = 18\%$) would suggest. With three assets, the variance has three weighted-variance terms and three covariance terms, each doubled:

$$\sigma_P^2 = \sum_{i=1}^{3} w_i^2\sigma_i^2 + 2w_1w_2\text{Cov}_{1,2} + 2w_1w_3\text{Cov}_{1,3} + 2w_2w_3\text{Cov}_{2,3}$$

Calculator tip

Compute each of the three terms separately, add them, then take the square root last. Keep returns in decimals throughout (0.25 rather than 25) or in percent throughout; mixing the two gives wrong answers.

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COMMON EXAM TRAPS

- Forgetting the factor 2 in the covariance term. In the J/K example, this gives $\sqrt{0.0097} = 9.85\%$ instead of 10.25%.
- Using variances instead of standard deviations inside $\rho\sigma_1\sigma_2$. In the J/K example, $0.20 \times 0.0625 \times 0.0100$ as the covariance gives a variance of 0.00894 and a standard deviation of 9.46%, instead of 10.25%.
- Reporting the variance as if it were the standard deviation (e.g., 0.0105 read as "1.05%").
- Averaging the standard deviations when $\rho < 1$: that is correct only for perfect positive correlation. In the J/K example, this gives 13% instead of 10.25%.
- Assuming $\rho = -1$ always means zero risk: it does only at one specific set of weights.

EXAM SHORTCUTS

- For two assets with positive weights, the portfolio standard deviation equals the weighted average of the two standard deviations only when $\rho = +1$ and is below it for any lower correlation, so with $\rho < +1$ an answer at or above the weighted average can be ruled out.

BOTTOM LINE

- Two-asset portfolio variance is $\sigma_P^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\mathrm{Cov}_{1,2}$, and the portfolio standard deviation is its square root.
- Covariance can be written $\mathrm{Cov}_{1,2} = \rho_{1,2}\sigma_1\sigma_2$, which exam calculations use, although for forward-looking expected values the identity does not hold mechanically.
- With positive weights, a correlation of +1 gives a portfolio standard deviation equal to the weighted average of the standard deviations (no diversification benefit), and any lower correlation gives less, with a larger benefit as ρ falls.
- With $\rho = -1$, $\sigma_P = |w_1\sigma_1 - w_2\sigma_2|$, which is zero only when $w_1\sigma_1 = w_2\sigma_2$.
- A negative covariance makes the cross term negative, so the portfolio is much less risky than the weighted average of the standard deviations suggests.

MODULE 8.3

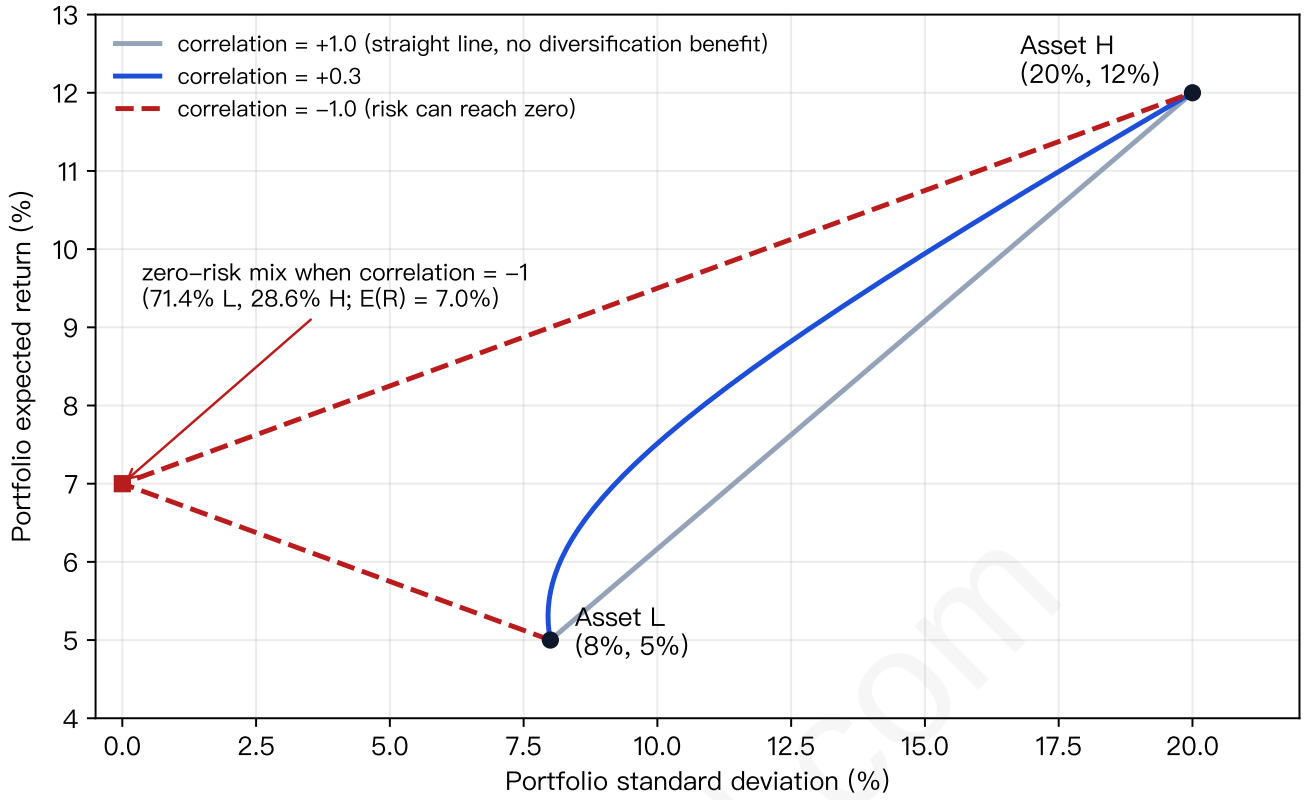
Correlation and Diversification Benefits

LOS 8.a — Correlation, diversification and portfolios of many assets

Correlation drives the diversification benefit

Changing the weights of two assets traces out a curve of attainable risk–return combinations. When the assets are less than perfectly correlated, the curve bows to the left: moving from the riskier asset toward the safer one cuts risk faster than it cuts return. This is the **diversification benefit**, and it grows as correlation falls. At $\rho = +1$ the combinations lie on a straight line; at $\rho = -1$ one mix has zero risk.

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Two-asset portfolios under different return correlations

Of all the feasible mixes, the efficient ones are those with the highest return for their level of risk, which lie on the upper part of the curve.

More assets: the variance–covariance matrix

With N assets, the variance-covariance matrix has N variances on its diagonal and

KEY CONCEPT

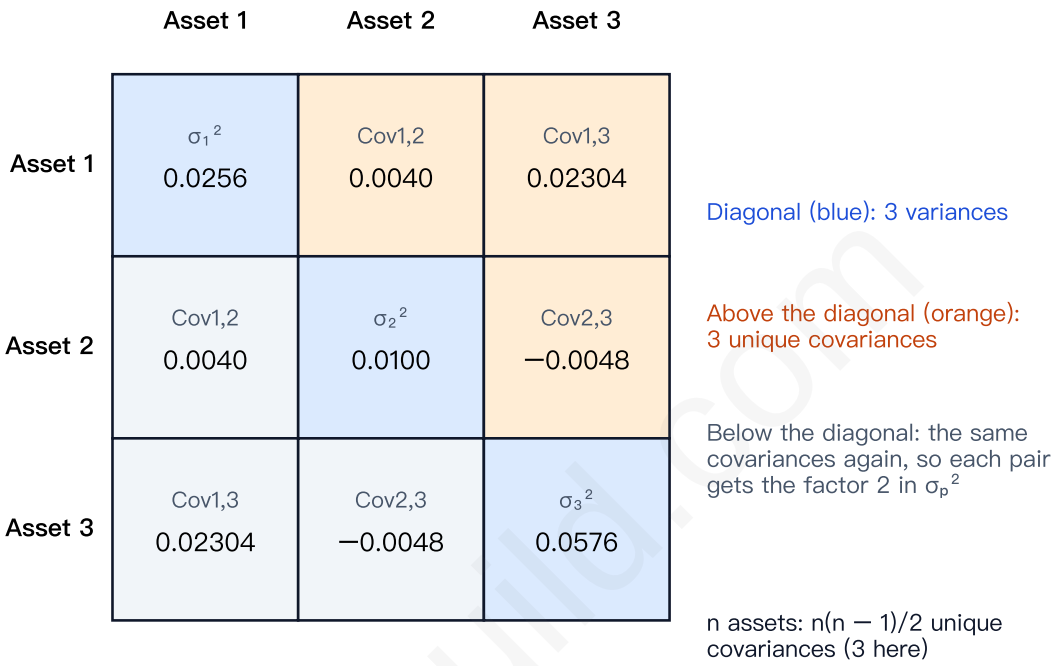
$$\text{number of unique covariances} = \frac{N^2 - N}{2}$$

So 20 assets require $(400 - 20)/2 = 190$ covariances. Normalizing the covariance matrix gives a **correlation matrix**, which helps an investor spot the pairs with the lowest correlations (the most useful diversifiers). Using correlations, the general formula is

$$\sigma_P^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \rho_{i,j} \sigma_i \sigma_j, \qquad \rho_{i,i} = 1$$

Example. A portfolio holds 40% in Asset 1, 35% in Asset 2 and 25% in Asset 3, with standard deviations of 16%, 10% and 24%. The correlations are $\rho_{1,2} = 0.25$, $\rho_{1,3} = 0.60$ and $\rho_{2,3} = -0.20$. Each covariance is $\rho_{i,j}\sigma_i\sigma_j$ (for example, $0.25 \times 0.16 \times 0.10 = 0.0040$), and the diagonal holds the variances:

Covariance matrix	Asset 1	Asset 2	Asset 3
Asset 1	0.0256	0.0040	0.02304
Asset 2	0.0040	0.0100	−0.0048
Asset 3	0.02304	−0.0048	0.0576



The three-asset covariance matrix: variances on the diagonal, each covariance appearing twice

Each pair appears twice in the matrix, which is where the factor 2 in the cross terms comes from:

$$\sigma_P^2 = (0.004096 + 0.001225 + 0.0036) + (0.00112 + 0.004608 - 0.00084) = 0.013809, \qquad \sigma_P = 11.75\%$$

The first bracket holds the weighted variances $w_i^2\sigma_i^2$ and the second the terms $2w_iw_j\mathrm{Cov}_{i,j}$. The result is well below the 15.9% weighted average of the three standard deviations because every correlation is below +1.

Expected portfolio variance from scenarios

A forward-looking variance can be built from economic scenarios m with probabilities p_m that sum to 1:

1. Portfolio return in each scenario: $R_{P,m} = \sum_i w_i R_{i,m}$.
2. Expected portfolio return: $E(R_P) = \sum_m p_m R_{P,m}$, which equals the weighted average of the assets' expected returns, $\sum_i w_i E(R_i)$.
3. Expected variance: $\sigma_P^2 = \sum_m p_m \left[R_{P,m} - E(R_P)\right]^2$, then take the square root.

Example. Two equally likely scenarios give portfolio returns of 12% and 2%. Then $E(R_P) = 7\%$ and $\sigma_P^2 = 0.5(5)^2 + 0.5(-5)^2 = 25$ (in %²), so $\sigma_P = 5\%$.

Large portfolios: variance approaches average covariance

For an **equally weighted** portfolio of N assets, the exact variance is

KEY CONCEPT

$$\sigma_P^2 = \frac{\bar{\sigma}^2}{N} + \frac{N-1}{N}\overline{\text{Cov}}$$

where $\bar{\sigma}^2$ is the average variance and $\overline{\text{Cov}}$ the average pairwise covariance. As N grows, each asset's own variance matters less and less and the portfolio variance approaches the **average covariance**. For a large portfolio, the approximation writes this as the **average correlation** times the **average variance**:

$$\sigma_P^2 \approx \bar{\rho}\,\bar{\sigma}^2$$

- Exam convention: for an equally weighted portfolio, a more exact variance is $\sigma_P^2 = \bar{\sigma}^2/N + (1 - 1/N)\,\bar{\rho}\,\bar{\sigma}^2$, and $\bar{\rho}\,\bar{\sigma}^2$ is its large-N approximation.
- Current practice: that more exact form is exact only when all assets have the same variance, because only then does $\bar{\rho}\,\bar{\sigma}^2$ equal the average covariance. In general the exact variance is $\bar{\sigma}^2/N + (N-1)/N$ times the average covariance, as above.

With unequal variances the gap can be large. For example, two assets with standard deviations of 10% and 30% and a correlation of 0.5 have a covariance of 0.015, but $\bar{\rho}\,\bar{\sigma}^2 = 0.5 \times 0.05 = 0.025$. *Example.* With 40 assets, an average variance of 0.05 and an average correlation of 0.25, $\sigma_P^2 \approx 0.25 \times 0.05 = 0.0125$, $\sigma_P \approx 11.2\%$. If all 40 assets had the same variance of 0.05, the finite-N term would add $0.75 \times 0.05/40 = 0.0009$, giving 0.0134 (11.6%).

Systematic vs. unsystematic risk

KEY CONCEPT

	Unsystematic risk	Systematic risk
Other names	Unique, diversifiable, firm-specific	Market, nondiversifiable
Removed by diversification?	Yes	No
Rewarded with higher expected return?	No	Yes

A portfolio of roughly 20 to 30 well-chosen assets captures most of the diversification benefit; beyond about 30 assets the extra benefit of each new holding is close to zero, and what remains is systematic risk.

COMMON EXAM TRAPS

- The approximation gives a variance; take the square root for standard deviation. In the 40-asset example, reading 0.0125 as 1.25% instead of taking the square root, about 11.2%.
- Number of covariances is $(N^2 - N)/2$, not N^2. For 20 assets, N^2 gives 400 instead of 190.

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- Adding assets reduces risk only to the extent that their correlations are below +1; with correlations of +1, more assets bring no diversification benefit.

EXAM SHORTCUTS

- The expected portfolio return from scenarios equals $\sum w_i E(R_i)$, the assets' expected returns weighted by portfolio weight, so it can be found without the scenario portfolio returns; the variance still needs them.

BOTTOM LINE

- The diversification benefit grows as correlation falls: two-asset combinations lie on a straight line at $\rho = +1$, bow to the left when correlation is below +1, and include one zero-risk mix at $\rho = -1$.
- With N assets the variance-covariance matrix holds N variances and $(N^2 - N)/2$ unique covariances.
- Expected portfolio variance from scenarios is $\sum_m p_m[R_{P,m} - E(R_P)]^2$, with scenario probabilities that sum to 1.
- An equally weighted portfolio of N assets has variance $\bar{\sigma}^2/N + [(N-1)/N]\,\overline{\text{Cov}}$, which approaches the average covariance as N grows and is approximated for a large portfolio by $\bar{\rho}\,\bar{\sigma}^2$.
- Unsystematic risk is removed by diversification and is not rewarded with higher expected return; systematic risk is not removed and is rewarded.
- Roughly 20 to 30 well-chosen assets capture most of the diversification benefit, and beyond about 30 assets each new holding adds close to none.

QUICK CHECK

Question 2

Brookline Endowment holds an equally weighted portfolio of 50 stocks. The average variance of returns of the individual stocks is 0.09, and the average correlation between pairs of stocks is 0.30. Using the approximation for a portfolio with a large number of assets, the standard deviation of the portfolio's returns is *closest* to:

A 2.7%.

B 9.0%.

C 16.4%.

Answer: C

For an equally weighted portfolio of many assets, the individual variances matter little and portfolio variance is approximately the average correlation times the average variance. The standard deviation is the square root of that variance.

$$\sigma_P^2 \approx \bar{\rho}\,\bar{\sigma}^2 = 0.30 \times 0.09 = 0.027$$

$$\sigma_P \approx \sqrt{0.027} = 0.164 = 16.4\%$$

The approximation ignores the finite-N term. If every stock had the same variance of 0.09, the exact equal-weight formula would add $(1-\bar{\rho})\sigma^2/N = 0.70(0.09)/50 = 0.00126$, giving $\sigma_P^2 = 0.02826$ and $\sigma_P = 16.8\%$, still closest to 16.4%. With unequal variances, the exact result depends on the average covariance, which the given averages do not pin down, so the question asks for the approximation. The average single stock has a standard deviation of $\sqrt{0.09} = 30\%$; diversification cuts this to about 16%, and the remainder is systematic risk.

Why the other options are wrong

- **A.** 2.7% is the approximate portfolio *variance* (0.027) quoted as if it were the standard deviation.
- **B.** 9.0% multiplies the average correlation by the average standard deviation ($0.30 \times 30\%$). The approximation multiplies the correlation by the average *variance*, and the square root is taken afterward.

Key takeaway For a large portfolio, $\sigma_P^2 \approx \bar{\rho}\bar{\sigma}^2$. Diversification removes unsystematic risk, but the average covariance (systematic risk) remains.

MODULE 8.4

The Minimum-Variance Portfolio and the Efficient Frontier

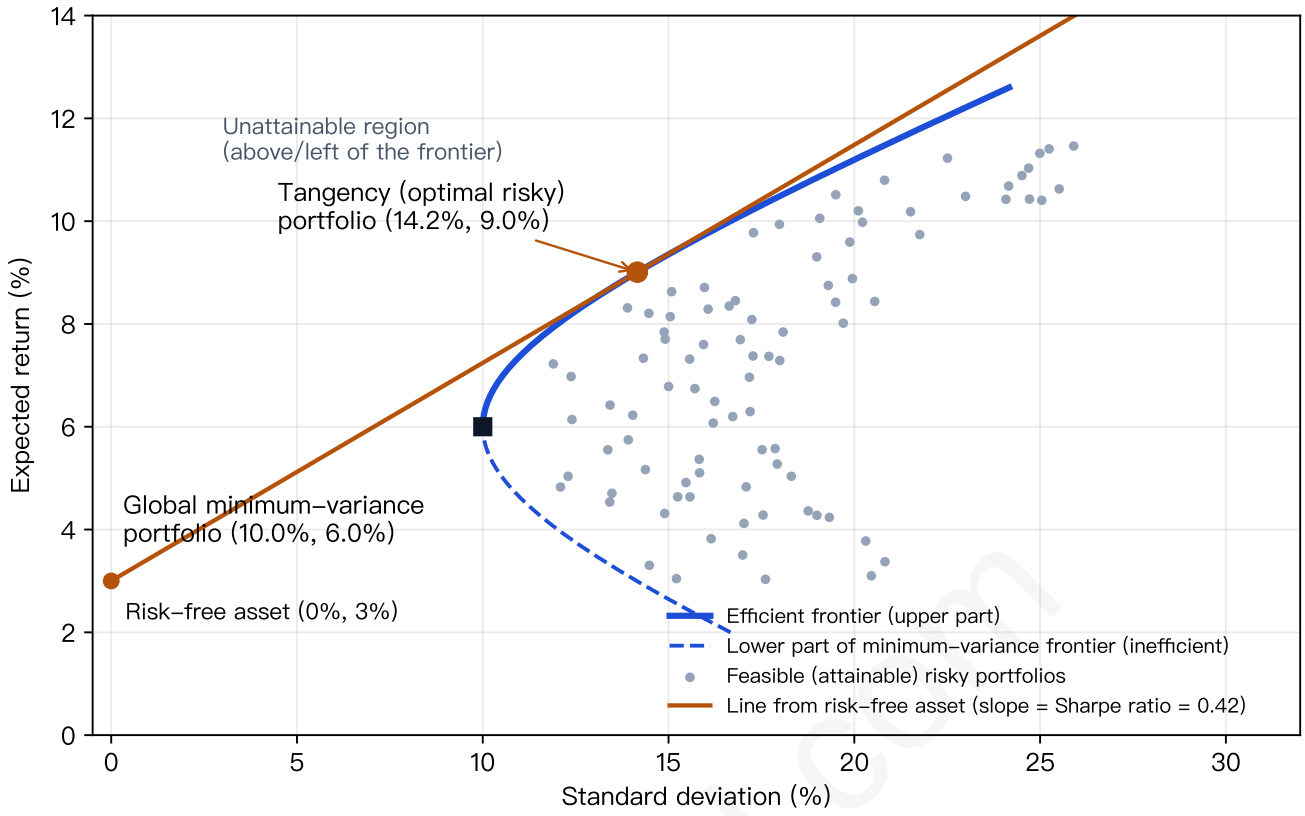
LOS 8.b — The minimum-variance portfolio and the efficient frontier

Feasible set, minimum-variance frontier and efficient frontier

Every mix of the available risky assets is a **risky portfolio**; together they form the set of feasible (attainable) portfolios.

- The **minimum-variance frontier** is the set of portfolios with the lowest risk for each level of expected return. It is the left boundary of the feasible set.
- The **global minimum-variance portfolio** is the point on that frontier with the lowest risk of all.
- The **efficient frontier** is the set of portfolios that offer the most expected return at any given level of risk. On a graph it is the upper part of the minimum-variance frontier, beginning at the global minimum-variance portfolio and rising from there. The lower part is inefficient: another portfolio offers more return for the same risk.
- Points to the left of the minimum-variance frontier, including every point above the efficient frontier, are **unattainable** with the available assets. Attainable points below the efficient frontier are **inefficient**.

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Feasible portfolios, minimum-variance frontier, efficient frontier and the tangency portfolio

In a theoretical universe of all investable assets, the upper segment is called the **Markowitz efficient frontier**.

Minimum-variance portfolio of two risky assets

KEY CONCEPT

$$w_A = \frac{\sigma_B^2 - \mathrm{Cov}_{A,B}}{\sigma_A^2 + \sigma_B^2 - 2\,\mathrm{Cov}_{A,B}}, \qquad w_B = 1 - w_A$$

When both weights are positive and the other inputs are held constant, a higher variance of an asset lowers its weight in the minimum-variance portfolio, and a lower variance raises it.

Example. With $\sigma_A = 15\%$, $\sigma_B = 25\%$ and $\rho = 0.20$, $\mathrm{Cov} = 0.20(0.15)(0.25) = 0.0075$.

$$w_A = \frac{0.0625 - 0.0075}{0.0225 + 0.0625 - 0.0150} = \frac{0.0550}{0.0700} = 78.6\%, \qquad w_B = 21.4\%$$

The less risky asset gets the larger weight. Expected returns do not enter the formula.

Adding a risk-free asset

A **risk-free asset** has a return with zero variance. Exam convention: the risk-free asset has zero correlation with any risky asset. Current practice: because its return never varies, its covariance with every asset is zero and the correlation coefficient is, strictly, undefined. Either way the covariance term drops out of the portfolio variance.

The straight line from the risk-free rate that just touches (is tangent to) the efficient frontier identifies the **optimal risky portfolio**, also called the **tangency portfolio**: the efficient portfolio with the highest **Sharpe ratio**,

KEY CONCEPT

$$\text{Sharpe ratio} = \frac{E(R_P) - R_f}{\sigma_P}$$

Combining the risk-free asset with a risky portfolio P:

$$E(R_C) = w_P E(R_P) + (1 - w_P) R_f, \qquad \sigma_C = w_P \, \sigma_P$$

To reach a target return, solve for the risky weight:

$$w_P = \frac{r_{\text{target}} - R_f}{E(R_P) - R_f}$$

Example. With $R_f = 4\%$, a risky portfolio with a 12% expected return and a 20% standard deviation, and a 10% target, $w_P = 6/8 = 0.75$ and $\sigma_C = 0.75 \times 20\% = 15\%$. Weights between 0 and 100% mean lending at the risk-free rate; a risky weight above 100% means borrowing at the risk-free rate.

Portfolio optimization

KEY CONCEPT

Approach	What it does
Return-constrained optimization	Minimizes risk for a given level of return
Risk-constrained optimization	Maximizes expected return for a given level of risk (finds points on the efficient frontier)
Risk-adjusted return optimization	Maximizes the Sharpe ratio

The key inputs are **expected returns**, **volatilities** (standard deviations) and **asset correlations**. Investor-specific factors (risk tolerance, time horizon, cash needs) and constraints such as no short selling or position limits can move weights away from the unconstrained optimum.

COMMON EXAM TRAPS

- In the minimum-variance formula, use variances (σ^2) rather than standard deviations, and keep the covariance term. In the example, standard deviations give $(0.25 - 0.0075)/(0.15 + 0.25 - 0.015) = 63.0\%$ instead of 78.6%.
- With a risk-free asset, combined risk is just $w_P \sigma_P$; do not add a covariance term.

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- The risky weight for a target return is $(r_{\text{target}} - R_f)/(E(R_P) - R_f)$; dividing the target by $E(R_P)$ ignores the return on the risk-free asset. In the example, $10/12 = 0.833$ instead of 0.75.

EXAM SHORTCUTS

- Expected returns do not enter the two-asset minimum-variance weights, so any expected returns given in such a question can be set aside.

BOTTOM LINE

- The minimum-variance frontier holds the lowest-risk portfolio for each level of expected return, and the global minimum-variance portfolio is its point of lowest risk.
- The efficient frontier is the upper part of the minimum-variance frontier from the global minimum-variance portfolio upward; points to the left of the frontier are unattainable and attainable points below the efficient frontier are inefficient.
- For two risky assets, $w_A = (\sigma_B^2 - \text{Cov}_{A,B})/(\sigma_A^2 + \sigma_B^2 - 2\text{Cov}_{A,B})$ and $w_B = 1 - w_A$ give the minimum-variance portfolio.
- The optimal risky (tangency) portfolio is where a line from the risk-free rate touches the efficient frontier, the efficient portfolio with the highest Sharpe ratio $(E(R_P) - R_f)/\sigma_P$.
- Combining the risk-free asset with portfolio P gives $E(R_C) = w_P E(R_P) + (1 - w_P)R_f$ and $\sigma_C = w_P\sigma_P$, with $w_P = (r_{\text{target}} - R_f)/(E(R_P) - R_f)$ for a target return and a weight above 100% meaning borrowing at the risk-free rate.
- Return-constrained optimization minimizes risk for a given return, risk-constrained optimization maximizes expected return for a given risk, and risk-adjusted return optimization seeks the highest Sharpe ratio.

MODULE 8.5

Risk Aversion

LOS 8.c — Risk aversion, utility and indifference curves

Assumptions behind optimal portfolio choice

- Investors are rational and **risk averse**; they maximize satisfaction (**utility**) and prefer more return and less risk.
- A single-period time horizon.
- No taxes or transaction costs.
- Investors can borrow and lend at a constant **risk-free rate**.

Three attitudes toward risk

KEY CONCEPT

Attitude	Behavior	Coefficient A	Indifference curves
Risk seeking	Prefers an uncertain outcome to a certain one with the same expected value; may accept a lower expected return (e.g., casino gambling)	$A < 0$	Slope downward
Risk neutral	Indifferent between a gamble and a sure amount with the same expected value; cares only about return	$A = 0$	Flat (horizontal)
Risk averse	Prefers the certain outcome; needs extra return to bear extra risk	$A > 0$	Slope upward

A **fair gamble** (expected gain of zero) raises the expected utility of a risk-seeking investor, leaves a risk-neutral investor indifferent, and lowers the utility of a risk-averse investor. Most real-world investors are risk averse; risk neutrality is a useful theoretical case (e.g., large institutions making many small trades whose variance averages out).

Utility function

A common **utility function** is

KEY CONCEPT

$$U = E(r) - \tfrac{1}{2}A\sigma^2$$

where A is the investor's **coefficient of risk aversion** and σ^2 the variance of returns (use decimals). A higher A means a bigger utility penalty for risk. Portfolio theory assumes every investor is risk averse ($A > 0$). Research suggests A typically lies between 2 and 3 and is rarely below 1. **Mean-variance utility**, $U_P = U[E(R_P), \sigma_P]$, depends only on expected return and risk and ignores other factors such as total wealth.

Example. With $A = 4$, an expected return of 10% and a standard deviation of 15%, $U = 0.10 - 0.5(4)(0.0225) = 0.055$, i.e., 5.5%. The investor would be equally happy with a certain 5.5% return.

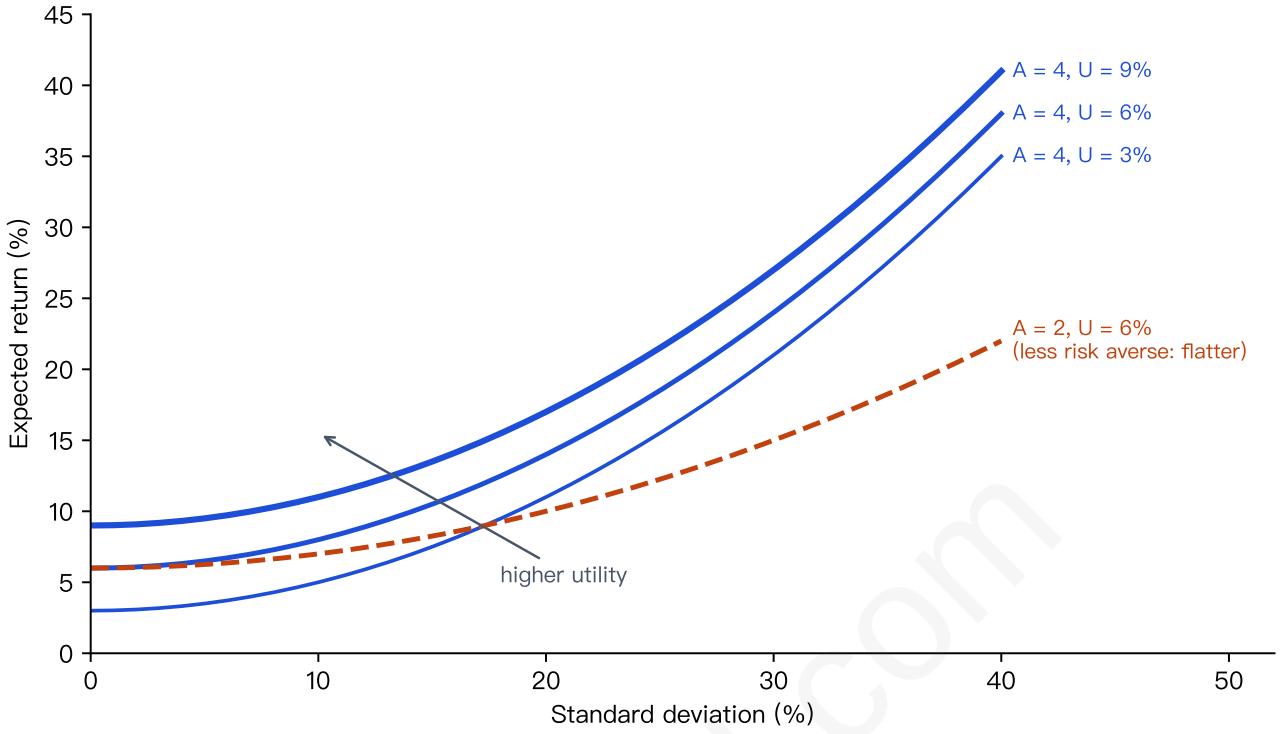
Risk tolerance is the willingness to accept risk for higher expected return; it is the inverse of risk aversion and can change with age, wealth and economic conditions, and differs across countries and cultures. **Absolute risk aversion** looks at how utility changes with extra risk holding wealth constant; **relative risk aversion** looks at how the *share* of wealth put at risk changes as wealth changes.

Indifference curves

An **indifference curve** plots the risk–return combinations that give an investor the same expected utility. For a risk-averse investor the curves slope upward and are convex; curves higher and to the left represent higher utility. The slope measures how much extra return is required per unit of extra risk:

- A more risk-averse (less risk-tolerant) investor has steeper curves.

- A less risk-averse (more risk-tolerant) investor has flatter curves.



Indifference curves: higher curves give higher utility, and a less risk-averse investor has flatter curves (illustrative utility levels)

COMMON EXAM TRAPS

- The utility function uses the variance rather than the standard deviation, together with the factor $\frac{1}{2}$. In the example, using the standard deviation gives $0.10 - 0.5(4)(0.15) = -0.20$ and dropping the $\frac{1}{2}$ gives 0.010, instead of 0.055.
- The highest expected return or the lowest risk is not necessarily the highest-utility choice; compute U for each.

BOTTOM LINE

- Optimal portfolio theory assumes rational, risk-averse investors who maximize utility, a single-period horizon, no taxes or transaction costs, and borrowing and lending at a constant risk-free rate.
- A risk-seeking investor has $A < 0$ and downward-sloping indifference curves, a risk-neutral investor has $A = 0$ and flat curves, and a risk-averse investor has $A > 0$ and upward-sloping curves.
- Utility is $U = E(r) - \frac{1}{2}A\sigma^2$ with the variance in decimals, so a higher coefficient of risk aversion A means a bigger penalty for risk.
- Risk tolerance, the inverse of risk aversion, can change with age, wealth and economic conditions.
- A risk-averse investor's indifference curves slope upward and are convex, curves higher and to the left give higher utility, and a more risk-averse investor has steeper curves.

QUICK CHECK

Question 3

An adviser measures a client's utility with the function $U = E(r) - \frac{1}{2}A\sigma^2$ and estimates the client's coefficient of risk aversion, A, at 3. Based on the table below, which fund gives this client the highest utility?

Funds under consideration

Fund	Expected return	Standard deviation of returns
Northgate Growth Fund	11%	20%
Westbury Balanced Fund	8%	12%
Eastholm Income Fund	6%	5%

A Northgate Growth Fund.

B Westbury Balanced Fund.

C Eastholm Income Fund.

Answer: B

Utility rewards expected return and penalizes variance in proportion to the coefficient of risk aversion. Computing the utility of each fund (with returns and standard deviations in decimals and the standard deviation squared) shows that the balanced fund offers the best trade-off for this client.

$$U = E(r) - \tfrac{1}{2}(3)\sigma^2 = E(r) - 1.5\sigma^2$$

Fund	$E(r)$	σ^2	U
Northgate Growth	0.11	0.0400	$0.11 - 0.0600 = 0.0500$
Westbury Balanced	0.08	0.0144	$0.08 - 0.0216 = 0.0584$
Eastholm Income	0.06	0.0025	$0.06 - 0.00375 = 0.05625$

Westbury Balanced has the highest utility (5.84%).

Why the other options are wrong

- **A.** Northgate Growth has the highest expected return, but its large variance creates the biggest utility penalty ($1.5 \times 0.04 = 0.06$), leaving a utility of only 5.00%.
- **C.** Eastholm Income has the lowest risk and is the choice that results if the standard deviation is used instead of the variance in the penalty term. With the variance, its utility (5.625%) is slightly below that of the balanced fund.

Key takeaway Use σ^2 (in decimals) and the $\frac{1}{2}$ in the utility function; neither the highest return nor the lowest risk is automatically the best choice.

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MODULE 8.6

Capital Allocation Line and Capital Market Line

LOS 8.c — Capital allocation line, optimal portfolio and the capital market line

The capital allocation line (CAL)

Combining a risky portfolio with the risk-free asset (assuming zero risk for the risk-free asset, a risky portfolio whose return and risk exceed the risk-free rate, only these two assets available, and unlimited borrowing and lending at the risk-free rate) gives a straight line in risk–return space:

KEY CONCEPT

$$E(R_C) = R_f + \left[\frac{E(R_P) - R_f}{\sigma_P}\right]\sigma_C$$

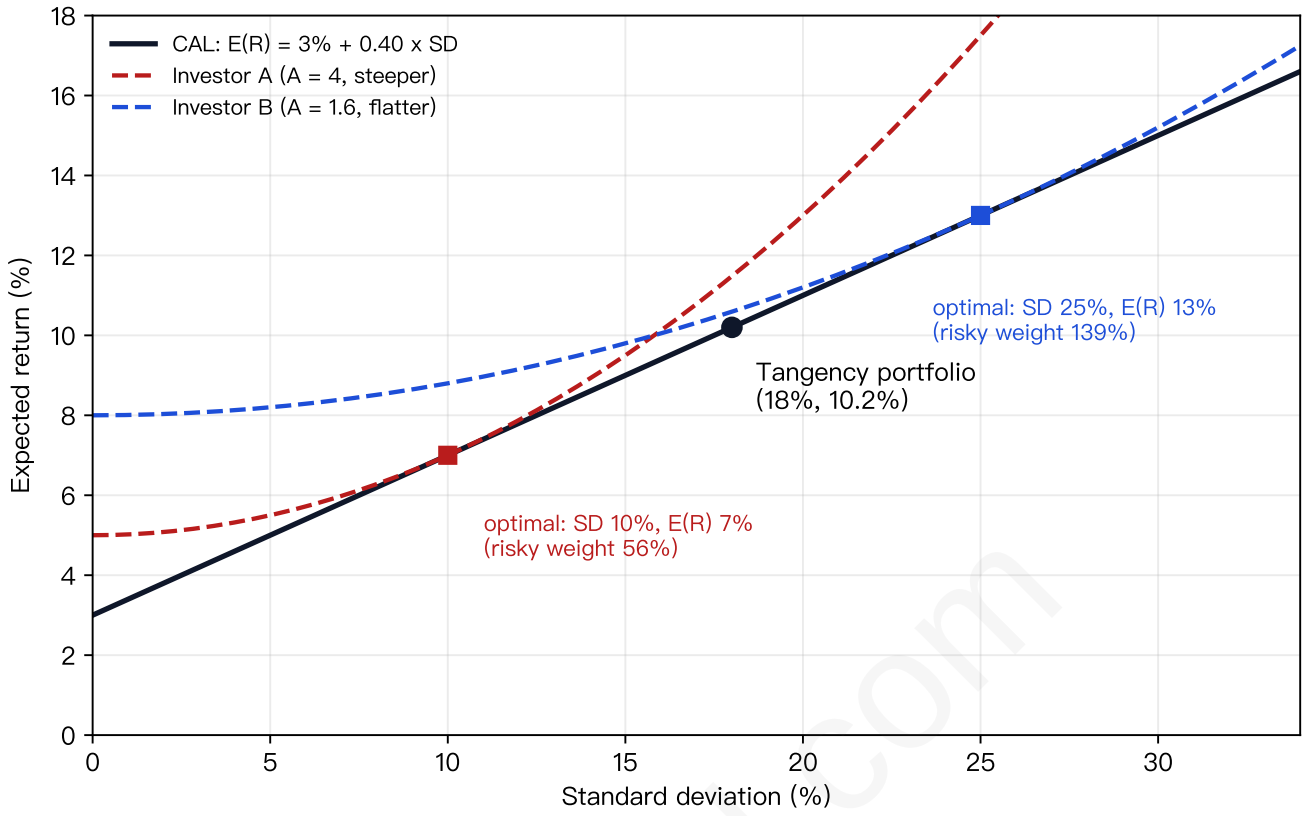
This is the risky portfolio's **capital allocation line (CAL)**. It starts at the risk-free rate on the vertical axis, and its slope is the risky portfolio's Sharpe ratio. Points between the intercept and the risky portfolio involve lending (a positive weight in the risk-free asset); points beyond it involve borrowing.

Example. Let $R_f = 3\%$; the tangency portfolio has an expected return of 10.2% and a standard deviation of 18% (Sharpe ratio $= 7.2/18 = 0.40$). A client with 40% in the risky portfolio has $E(R) = 0.40(10.2\%) + 0.60(3\%) = 5.88\%$ and $\sigma = 0.40(18\%) = 7.2\%$; check with the CAL: $3\% + 0.40(7.2\%) = 5.88\%$. A client who borrows 30% of wealth at 3% and puts 130% in the risky portfolio has $E(R) = 1.30(10.2\%) - 0.30(3\%) = 12.36\%$ and $\sigma = 1.30(18\%) = 23.4\%$; check: $3\% + 0.40(23.4\%) = 12.36\%$. Both portfolios lie on the same line.

Choosing the optimal portfolio

The investor's **optimal portfolio** is the point where the CAL touches the highest attainable indifference curve. Higher curves would be better but cannot be reached; lower curves are dominated.

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Optimal portfolios on a CAL for a more and a less risk-averse investor

In the figure, the more risk-averse Investor A (steeper indifference curves) reaches the highest attainable curve at a standard deviation of 10% (about 56% in the risky portfolio, the rest lent at the risk-free rate). The less risk-averse Investor B (flatter curves) chooses 25% (about 139% in the risky portfolio, financed partly by borrowing). Both points lie on the same CAL: risk aversion changes where on the line an investor sits, while the slope of the line stays the same.

The capital market line (CML)

The **capital market line (CML)** is the special CAL whose risky portfolio is the **market portfolio**, which holds all risky assets weighted by market value:

$$E(R_P) = R_f + \left[\frac{E(R_M) - R_f}{\sigma_M}\right]\sigma_P$$

The market portfolio sits on the efficient frontier and is **mean-variance efficient** (the highest Sharpe ratio possible). The **mutual fund theorem** states that every investor can obtain an efficient portfolio by combining the **risk-free asset** with the **market portfolio**; investors differ only in how much of each they hold.

KEY CONCEPT

	CAL	CML
Risky portfolio	Any risky portfolio (e.g., the investor's own tangency portfolio)	The market portfolio

	CAL	CML
Expectations	**Heterogeneous** — each investor can have a different CAL	**Homogeneous** — one CML for all investors
Slope	Sharpe ratio of that risky portfolio	Sharpe ratio of the market portfolio

In practice a portfolio of "all risky assets" cannot be held, so a broad market index is used as the proxy.

COMMON EXAM TRAPS

- Every portfolio on a CAL has the same Sharpe ratio as that CAL's risky portfolio, however much is lent or borrowed.
- The mutual fund theorem combines the risk-free asset with the market portfolio specifically; another efficient portfolio or the minimum-variance portfolio does not qualify.
- Owning "about 30 stocks" diversifies away most unsystematic risk; that rule of thumb is a different idea from the mutual fund theorem.

EXAM SHORTCUTS

- Every portfolio on a CAL with a positive allocation to the risky portfolio has the Sharpe ratio of that risky portfolio, however much is lent or borrowed, so its Sharpe ratio can be read from the risky portfolio directly.

BOTTOM LINE

- Under its assumptions (a riskless risk-free asset, a risky portfolio with return and risk above the risk-free rate, only these two assets, and unlimited borrowing and lending at the risk-free rate), the CAL is $E(R_C) = R_f + [(E(R_P) - R_f)/\sigma_P]\sigma_C$, with intercept R_f and slope equal to the risky portfolio's Sharpe ratio.
- Points on the CAL between the intercept and the risky portfolio involve lending at the risk-free rate, and points beyond the risky portfolio involve borrowing.
- An investor's optimal portfolio is where the CAL touches the highest attainable indifference curve; a more risk-averse investor chooses a lower-risk point on the same line.
- The CML is the CAL whose risky portfolio is the market portfolio, which holds all risky assets weighted by market value and is mean-variance efficient.
- Under the mutual fund theorem, every investor can obtain an efficient portfolio by combining the risk-free asset with the market portfolio.
- A CAL allows heterogeneous expectations, so each investor can have a different CAL, while the CML assumes homogeneous expectations and is the same for all investors.

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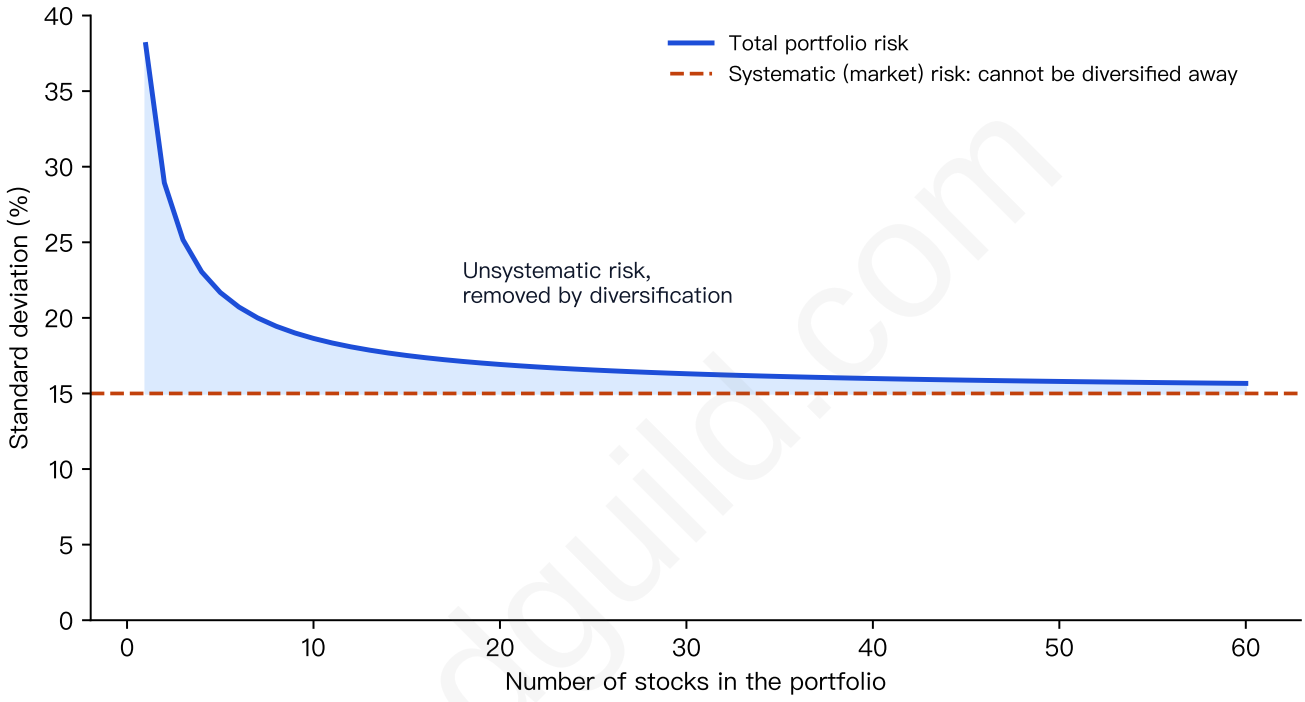
MODULE 8.7

The Capital Asset Pricing Model

LOS 8.c — Systematic risk, beta and the capital asset pricing model (CAPM)

Only systematic risk is priced

The market portfolio holds every risky asset, so all unsystematic risk has been diversified away within it and only systematic risk remains (Module 8.3). Since unsystematic risk can be removed at no cost, the market rewards investors only for bearing systematic risk, and the CAPM prices assets on that basis.



Diversification: portfolio risk falls toward systematic risk as stocks are added (illustrative figures)

Return-generating models and the CAPM

Return-generating models estimate expected returns from one or more factors (macroeconomic factors such as GDP growth or inflation, or fundamental factors such as earnings growth or firm size). The **capital asset pricing model (CAPM)** is a single-factor model: expected return depends only on exposure to systematic risk.

The CAPM assumes **homogeneous expectations**; investors hold efficient portfolios that maximize their expected utility; investors are **price takers**; and markets are in **equilibrium** (no mispriced assets).

KEY CONCEPT

$$E(R_i) = R_f + \beta_i [E(R_M) - R_f]$$

$E(R_M) - R_f$ is the **market risk premium**, the extra return expected for holding the market instead of the risk-free asset.

Beta

Beta measures an asset's sensitivity to market (systematic) risk:

KEY CONCEPT

$$\beta_i = \frac{\mathrm{Cov}(R_i, R_M)}{\sigma_M^2} = \frac{\rho_{i,M}\,\sigma_i\,\sigma_M}{\sigma_M^2} = \frac{\rho_{i,M}\,\sigma_i}{\sigma_M}$$

The market portfolio has a beta of 1.

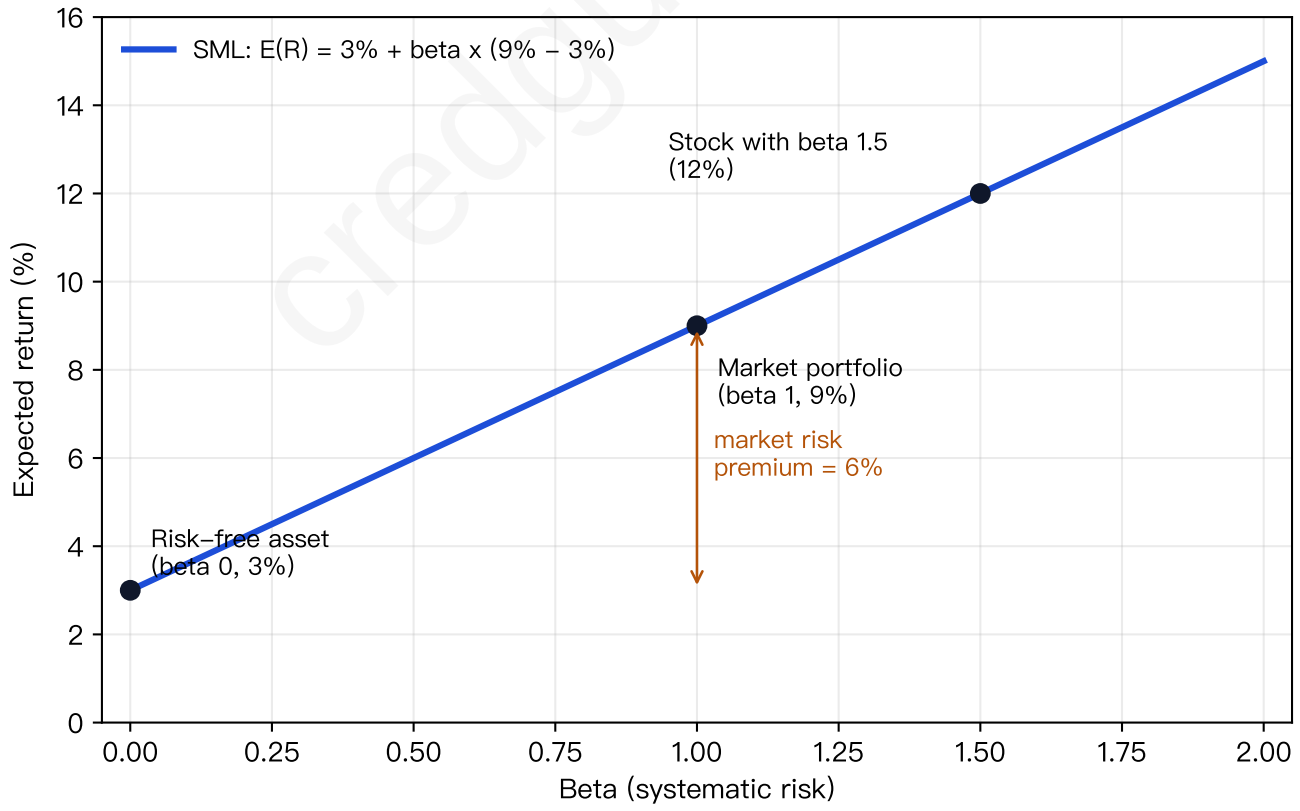
- Exam convention: a beta above 1 indicates an expected return above the market's, and a beta below 1 an expected return below it.
- Current practice: this holds only when the market risk premium is positive; with a negative premium the ordering reverses.

Beta can be estimated by regression (see the reading on simple linear regression).

Example. With $\rho_{i,M} = 0.5$, $\sigma_i = 36\%$ and $\sigma_M = 20\%$, $\beta = 0.5 \times 36/20 = 0.90$. A stock can be almost twice as volatile as the market, as here, and still have a beta below 1 if its correlation with the market is low.

The security market line (SML)

The **security market line (SML)** is the graph of the CAPM: expected return on the vertical axis, **beta** on the horizontal axis, intercept R_f, slope equal to the market risk premium.



The security market line

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Example (from the figure). With $R_f = 3\%$, $E(R_M) = 9\%$ and $\beta = 1.5$, $E(R) = 3\% + 1.5(6\%) = 12\%$.

KEY CONCEPT

	CML	SML
Horizontal axis	Total risk (standard deviation)	Systematic risk (beta)
Applies to	Efficient portfolios only	Any asset or portfolio
Slope	Market Sharpe ratio	Market risk premium

Limitations of portfolio theory

Asset returns are not normally distributed (skewness, kurtosis); correlations are unstable, especially in market stress; markets are less than perfectly efficient; models are very sensitive to estimation error; the future may not look like the past. Practical constraints ignored by the theory include transaction costs from rebalancing and taxes. Taxes on distributions and on capital appreciation differ across portfolios and can change the optimal portfolio, and realizing capital losses to offset capital gains changes after-tax, risk-adjusted realized returns.

COMMON EXAM TRAPS

- If the question gives the market risk premium, multiply it by beta directly; if it gives the market return, subtract R_f first. In the SML example, using the 9% market return as the premium gives $3\% + 1.5(9\%) = 16.5\%$ instead of 12%.
- Do not forget to add the risk-free rate back. In the SML example, $1.5 \times 6\% = 9\%$ instead of 12%.
- Beta uses the correlation and both standard deviations: $\beta = \rho\sigma_i/\sigma_M$ rather than σ_i/σ_M. In the example, $36/20 = 1.80$ instead of 0.90.

BOTTOM LINE

- Because unsystematic risk can be diversified away at no cost, the market rewards investors only for bearing systematic risk.
- The CAPM is a single-factor model, $E(R_i) = R_f + \beta_i[E(R_M) - R_f]$, that assumes homogeneous expectations, utility-maximizing investors holding efficient portfolios, price takers and markets in equilibrium.
- Beta is $\mathrm{Cov}(R_i, R_M)/\sigma_M^2 = \rho_{i,M}\sigma_i/\sigma_M$, and the beta of the market portfolio is 1.
- Exam convention: a beta above 1 indicates an expected return above the market's and a beta below 1 one below it; current practice: this ordering holds only when the market risk premium is positive.
- The SML plots expected return against beta with intercept R_f and slope equal to the market risk premium and applies to any asset, while the CML plots against total risk and applies only to efficient portfolios.
- Portfolio theory is limited by non-normal returns, unstable correlations, imperfectly efficient markets, sensitivity to estimation error and a future that may differ from the past, and it ignores transaction costs and taxes.

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QUICK CHECK

Question 4

Using the capital asset pricing model (CAPM) and the data in the table below, the expected return on Gorham Textiles shares is *closest* to:

Inputs for Gorham Textiles

Input	Value
Beta of Gorham Textiles	0.8
Market risk premium	6%
Risk-free rate	3%

A 7.8%.

B 5.4%.

C 4.8%.

Answer: A

Under the CAPM, an asset's expected return equals the risk-free rate plus beta times the market risk premium. The market risk premium is given directly, so it is multiplied by beta without subtracting the risk-free rate again.

$$E(R_i) = R_f + \beta_i \left[E(R_M) - R_f\right] = 3\% + 0.8(6\%) = 3\% + 4.8\% = 7.8\%$$

Why the other options are wrong

- **B.** 5.4% treats 6% as the market *return* and subtracts the risk-free rate again: $3\% + 0.8(6\% - 3\%)$. The 6% is already the market risk premium.
- **C.** 4.8% is only the asset's risk premium, $0.8 \times 6\%$; the risk-free rate must be added.

Key takeaway Check whether the question gives the market return or the market risk premium before plugging into the CAPM.

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/portfolio-standard-deviation/. The full bank has 23 questions on this reading.

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QUANTITATIVE METHODS · READING 9

Simulation of Financial Asset Prices and Returns

What you'll learn

LOS 9.a Describe historical simulation, its assumptions, strengths and weaknesses, and how it is used in investment applications such as VaR and bond portfolios.

LOS 9.b Describe bootstrap resampling (sampling with replacement), its strengths and weaknesses, and its investment applications.

LOS 9.c Describe Monte Carlo simulation, including multivariate simulation with the Cholesky decomposition, its strengths and weaknesses, and its investment applications.

MODULE 9.1

Historical Simulation

This reading describes three ways to simulate asset prices and returns: historical simulation, which applies actual past changes to current positions; bootstrap resampling, which draws from the observed sample with replacement; and Monte Carlo simulation, which draws from distributions the analyst specifies. For each method it covers the assumptions, investment applications such as VaR and option valuation, and the main strengths and weaknesses.

LOS 9.a — Historical simulation

What a simulation does

KEY CONCEPT

A **simulation** is a predictive modeling technique that estimates the range of possible outcomes of a complex system (for example, a portfolio's return distribution, its risk, or future cash flows and liquidity) by running it through many scenarios. Every simulation follows the same four steps:

1. **Set parameters**: state the objective, choose the output variables, and define the calculations linking inputs to outputs.
2. **Generate scenarios**: create the input values for each trial (simple or drawn from complex distributions).
3. **Evaluate scenarios**: run each scenario through the calculation engine to produce outcome paths.
4. **Compile and interpret results**: aggregate the outcomes (e.g., into a distribution) and draw conclusions.

Historical simulation (this module), bootstrap resampling (Module 9.2) and Monte Carlo simulation (Module 9.3) differ mainly in step 2, where the scenarios come from.

Historical simulation

Historical simulation applies actual past changes in prices, returns or risk factors to the current portfolio positions. It assumes that **past data represent the future** (past performance is a reliable guide to future outcomes and risks, and the historical record stands in for the population of outcomes). When the parameters are set, the chosen dates and risk factors may need simplifying assumptions so that all returns cover the same time frame. Equity inputs are usually logarithmic (continuously compounded) price returns, and percentage returns work as well; for bonds, the simulated yields serve as discount rates for the future cash flows.

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KEY CONCEPT

Strengths	Weaknesses
Easy to explain — narratives are built around real past events	Assumes the past repeats; weak when there are **regime changes** or unprecedented events
Nonparametric — no need to force the data into a distribution such as the normal; handles **excess kurtosis**, **skewness** and **fat tails**, and captures extreme events and drift better than a parametric model	Can only produce outcomes that actually occurred in the lookback period
Captures **volatility clustering** (calm and turbulent periods tend to persist)	Results depend on the quality and completeness of the data

Missing data and proxies

Gaps in the historical record arise from thinly traded instruments, asynchronous global markets (holidays, outages, trading halts), and securities that did not exist during part of the period (e.g., a recent IPO). The fix is a **proxy**: a substitute instrument, or a weighted combination of instruments with data available, that stands in for the missing observations.

Application: value at risk (VaR)

KEY CONCEPT

Value at risk (VaR) is the minimum loss expected over a specified period in the worst 5% (or 1%) of outcomes. Equivalently, it is the maximum potential loss at the matching level of confidence (95% or 99%); larger losses still occur in the tail. Using historical simulation for a long-only portfolio:

1. *Set parameters*: compute daily (usually logarithmic) returns of each asset over the lookback period.
2. *Generate scenarios*: apply each day's returns to today's positions to get one profit-or-loss (P&L) figure per historical day.
3. *Evaluate scenarios*: build the P&L distribution (e.g., a histogram).
4. *Compile and interpret*: rank the P&L from worst to best; the 95% VaR is the fifth percentile of the distribution. If the percentile falls between two observations, use either neighbor or a weighted average.

Example (historical vs. parametric VaR). A $4 million equity portfolio is revalued under each of the last 500 daily return scenarios. Five percent of 500 is 25 outcomes, so the fifth percentile lies between the 25th-worst P&L (−$118,000) and the 26th-worst (−$114,000). The 1-day 95% historical VaR is therefore about $116,000 (the average of the two neighbors; either neighbor is also acceptable). A parametric VaR that assumes normal returns, a zero daily mean and a daily standard deviation of 1.5% gives $1.645 \times 0.015 \times \$4{,}000{,}000 = \$98{,}700$. The historical figure is larger because the actual return distribution has a fatter left tail than the normal distribution allows.

Because VaR focuses on the loss tail, a few large gains barely affect it. For daily VaR, analysts usually assume a mean return of zero, because a daily drift is hard to distinguish statistically from zero (the null hypothesis of no directional move) and daily volatility dwarfs any trend.

Application: bonds

A bond's historical prices are misleading because its interest-rate sensitivity (duration) shrinks as it approaches maturity: a 10-year zero-coupon bond bought five years ago now behaves like a 5-year bond. Historical simulation for bonds therefore uses historical changes in yields (for example, benchmark yields for the same remaining maturity) and applies them to today's cash flows. To simulate a bond with three years to maturity, use yield changes that were observed on bonds that had three years left to maturity at the time.

COMMON EXAM TRAPS

- Historical simulation assumes past data represent the future. Fitting a statistical model to the population describes Monte Carlo simulation, and resampling with replacement describes bootstrapping.
- Historical 95% VaR is the fifth percentile of the simulated P&L distribution (about the 25th-worst of 500 outcomes) rather than the single worst outcome.
- A proxy fills missing data; it is not a risk factor or a hypothetical scenario.

EXAM SHORTCUTS

- With N historical scenarios, the 95% VaR lies between the $(0.05N)$th-worst P&L and the next-worst one, so only those two outcomes need to be located (either one or a weighted average of the two is acceptable).

BOTTOM LINE

- Every simulation sets parameters, generates scenarios, evaluates them and compiles the results; historical simulation, bootstrapping and Monte Carlo simulation differ mainly in where the scenarios come from.
- Historical simulation applies actual past changes in prices, returns or risk factors to the current portfolio positions and assumes that past data represent the future.
- Historical simulation is nonparametric and captures skewness, excess kurtosis, fat tails and volatility clustering, but it can only produce outcomes from the lookback period and is weak when there are regime changes or unprecedented events.
- Missing historical data are filled with a proxy, a substitute instrument or a weighted combination of instruments with data available.
- VaR is the minimum loss over a specified period in the worst 5% (or 1%) of outcomes; the 95% historical VaR is the fifth percentile of the simulated P&L distribution, and daily VaR usually assumes a mean return of zero.
- Because a bond's duration shrinks as it approaches maturity, historical simulation for bonds applies past yield changes for the same remaining maturity to today's cash flows instead of using the bond's own past prices.

QUICK CHECK

Question 1

Which of the following is *most likely* an advantage of historical simulation compared with a parametric approach that assumes normally distributed returns?

A It remains reliable after a regime change that makes the future unlike the past.

B It captures fat tails and volatility clustering without imposing a distributional assumption.

C It can generate outcomes more extreme than any observed during the lookback period.

Answer: B

Historical simulation is nonparametric: it uses actual past changes, so the scenarios automatically carry the fat tails, skewness and volatility clustering present in the data. A normal-distribution model tends to understate extreme outcomes.

Why the other options are wrong

- **A.** Reliance on the past is the main weakness of historical simulation; after a regime change, past data may no longer represent the future.
- **C.** Historical simulation can only reproduce outcomes that actually occurred in the lookback period; simulating conditions never seen before is an advantage of Monte Carlo simulation.

Key takeaway Historical simulation is nonparametric, captures fat tails and volatility clustering and is easy to explain; its limit is that it can only replay what actually happened.

MODULE 9.2

Bootstrap Resampling

LOS 9.b — Bootstrap resampling

How bootstrapping works

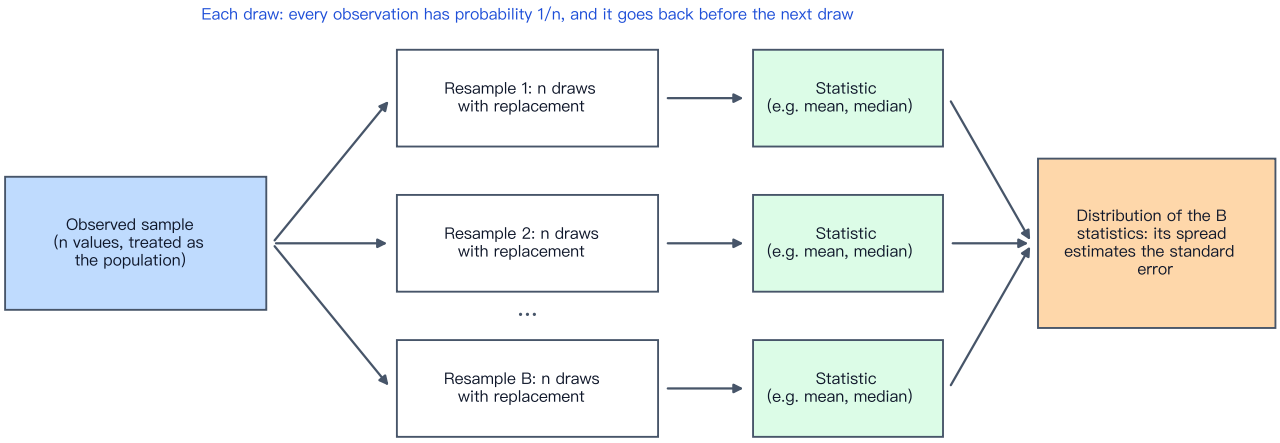
KEY CONCEPT

Bootstrap resampling (bootstrapping) is a **nonparametric** simulation method. It treats the observed historical **sample as a proxy for the population** and creates many new datasets by drawing observations from it at random **with replacement**: each observation is drawn, used, and put back so that it can be drawn again. Drawing a random value in this way is called **pulling**.

- With a dataset of n observations, each observation has a probability of $1/n$ of being selected on every draw.
- Within one scenario, the same observation can be drawn as many times as there are draws in that scenario.
- Across scenarios, the same observation can appear in many samples (or in none).

Example. To simulate a 6-month path from 60 historical monthly returns, draw six random integers from 1 to 60 and look up the corresponding returns. Any single month has a 1/60 chance on each draw and could, in principle, appear up to six times in the same path (though that is very unlikely). Repeat thousands of times.

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How bootstrap resampling builds a sampling distribution

Bootstrapping vs. historical simulation

The two methods treat the data differently. Historical simulation replays the past dataset directly, as if it were the population, so each scenario is a sequence of events that actually happened. Bootstrapping treats the observed sample as a proxy for the population and builds new combinations of past observations by resampling with replacement. Module 9.3 compares all three simulation methods side by side.

Bootstrapping needs no theoretical model. Each resample gives one value of the statistic of interest (a mean, say), so thousands of resamples trace out an approximate sampling distribution of that statistic even when the original dataset is small. The spread of that distribution makes the stability of an estimate easy to assess. The repeated resampling takes somewhat more computation than historical simulation. Because it preserves the empirical distribution of the data while reshuffling it, it reduces the risk of overfitting to one particular historical sequence. It suits finance problems in which the population distribution is unknown but can be inferred from the sample.

Applications

- **European-style call option values at maturity**. A European call can be exercised only at maturity and is valuable only if the underlying price ends above the exercise price, so the objective is the option's value at maturity. Resample historical returns to build many price paths over the investment period, compute the option payoff at the end of each path, and study the resulting distribution and its mean. This avoids the constant-volatility and normality assumptions of theoretical option-pricing models. Because of the option's leverage and limited downside, the distribution of option values can be more skewed than the distribution of share prices. The result is the distribution of terminal payoffs under historical (real-world) returns. It is not by itself the option's current no-arbitrage price: discounting a real-world average payoff at the risk-free rate would not give that price (Module 9.3 sets out the conditions for Monte Carlo pricing).
- **Estimating the volatility of target date funds** (which shift toward less risky assets as the retirement date approaches): bootstrap monthly returns, compute the standard deviation of each simulated series, and compare ranges across funds to judge which is more aggressive.

Strengths and weaknesses

KEY CONCEPT

Strengths	Weaknesses
Minimal assumptions — no need to specify a distribution such as the normal	**Sensitive to sample quality** — any bias in the original sample is carried into every resample
Simple to implement	**Computationally intensive** for thousands of resamples on large datasets
Statistically robust — reliable estimates of the variability of a statistic, even from small samples	**No analytical insight** — estimates, but no theoretical explanation
Reliable standard errors and confidence intervals, less influenced by outliers	**Limited extrapolation** — cannot generate outcomes outside the range of observed data (weak for extreme tail events)
Adaptable to many statistics (mean, median, correlation)	**Not suited to autocorrelated time series** — random resampling breaks the time-sequence dependence
Works with the sample at hand; no large external datasets needed	

Bootstrapping is useful when the data show time-varying volatility, skewness or fat tails that are hard to model parametrically, and when historical patterns are expected to persist. It complements analytic methods, which explain the underlying relationships in more detail.

COMMON EXAM TRAPS

- Bootstrapping makes no distributional assumption; Monte Carlo is the method that requires one.

BOTTOM LINE

- Bootstrapping is a nonparametric method that uses the sample at hand as a proxy for the population and builds new datasets by drawing from it at random with replacement, so each of n observations has a $1/n$ chance on every draw.
- In bootstrapping the same observation can be drawn several times within one scenario and can appear in many samples or in none.
- Historical simulation replays the past dataset as if it were the population, while bootstrapping builds new combinations of past observations, which reduces the risk of overfitting to one historical sequence.
- Bootstrapping can give the distribution of a European call's payoff at maturity under historical returns, which is not by itself the option's current no-arbitrage price, and can estimate the volatility of target date funds.
- Bootstrapping needs minimal assumptions and works with small samples, but it carries any bias in the original sample into every resample, cannot generate outcomes outside the observed range and is not suited to autocorrelated time series.

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QUICK CHECK

Question 2

An analyst builds each bootstrap scenario for an eight-quarter horizon by drawing from a dataset of 40 historical quarterly returns. Within a single scenario, any one historical observation can be drawn:

A up to eight times.

B only once.

C up to 40 times.

Answer: A

Bootstrapping samples with replacement: after each draw the observation is returned to the dataset. An eight-quarter scenario needs eight draws, so the same observation could in principle be pulled on every one of them, although with a 1/40 chance per draw this is extremely unlikely.

Each scenario consists of 8 draws of a random integer from 1 to 40, each mapped to one historical quarterly return. The probability that a given observation is chosen on any single draw is $1/40$, and the maximum number of appearances in one scenario equals the number of draws, 8.

Why the other options are wrong

- **B.** "Only once" describes sampling *without* replacement. Bootstrap resampling replaces each observation, so repeats are possible.
- **C.** Forty is the size of the dataset; a scenario of eight quarters contains only eight draws.

Key takeaway Sampling with replacement: repeats are possible, capped by the number of draws in the scenario.

MODULE 9.3

Monte Carlo Simulation

LOS 9.c — Monte Carlo simulation

The method

Monte Carlo simulation generates scenarios by random sampling from probability distributions specified by the analyst (normal, lognormal, or any other shape with any parameters) rather than from raw historical observations. Running thousands of trials produces a **simulated frequency distribution** of prices or returns, from which risk and performance measures are estimated. The analyst should run as many trials as practical: more trials give more precise estimates, and the number of trials is limited essentially by computational power and time. Because the analyst controls the input assumptions, the sensitivity of the output to each assumption can be tested ("what if" analysis).

Applications

- **Portfolio performance** over a future horizon: expected return and standard deviation to a chosen level of confidence.

- **Valuing securities without an analytic (closed-form) pricing formula**, such as path-dependent options (e.g., **Asian-style options**, whose payoff depends on an average price), mortgage-backed securities (the borrowers' prepayment option acts like an embedded call) and convertible bonds.
- **European call options**.
  - Exam convention: a Monte Carlo value for a European call should match the Black-Scholes-Merton value, and it gets closer as more trials are run.
  - Current practice: this holds only when the simulated price dynamics match the Black-Scholes-Merton assumptions: a lognormal price with constant volatility and, for pricing, an expected return equal to the risk-free rate less any dividend yield (risk-neutral valuation). The average payoff discounted at the risk-free rate then converges to the Black-Scholes-Merton value as the number of trials grows. A lognormal assumption alone is not enough: simulating with a real-world expected return and then discounting at the risk-free rate would not reproduce that value.

Steps for an option: (1) set the horizon, risk-free rate, the price distribution (e.g., lognormal), volatility and expected return; (2) draw random numbers to create price paths; (3) compute the payoff on each path over thousands of trials; (4) average the payoffs and discount at the risk-free rate.

Multivariate simulation and the Cholesky decomposition

When several variables move together (a portfolio, or an option on several assets), the simulated inputs must have the right correlation. Under the usual lognormal price model, the assets' log returns are assumed to be jointly normal, and the inputs are each asset's volatility and the correlations between returns. The **Cholesky decomposition** factorizes the covariance (or correlation) matrix and maps independent standard normal draws into correlated normal variables, which are then used to generate the simulated paths. For two variables with independent draws Z_1 and Z_2 (mean 0, standard deviation 1):

KEY CONCEPT

$$X_1 = \sigma_1 Z_1, \qquad X_2 = \sigma_2 \left(\rho Z_1 + \sqrt{1-\rho^2}\, Z_2\right)$$

Example. With $\sigma_1 = 10\%$, $\sigma_2 = 15\%$, $\rho = 0.8$ and draws $Z_1 = 0.5$ and $Z_2 = 1.0$, $X_1 = 5\%$; $X_2 = 15\%(0.8 \times 0.5 + 0.6 \times 1.0) = 15\%$.

Strengths and weaknesses

KEY CONCEPT

Strengths	Weaknesses
Wide applicability — assets, risk factors, portfolios; nonlinear relationships, time-varying volatility, fat tails	**Complexity** — needs modeling expertise and a time-consuming set-up
Flexibility for nonnormal distributions (skewness, excess kurtosis)	**Model dependency** — results are only as good as the assumed distributions and parameters

Strengths	Weaknesses
Precision given enough trials	**Computational intensity** — slow and costly for real-time use
Correlation handling among many assets	**Data requirements** — extensive data to estimate parameters such as volatilities and correlations
Forward-looking — can simulate hypothetical or extreme conditions never seen in history	Gives a **statistical estimate**, not an exact answer, and cannot provide the cause-and-effect **insight of analytic methods**

Monte Carlo is usually run alongside analytic methods. Where no analytic method can be applied, it can serve as the valuation tool itself, and its robustness is checked by changing the assumptions and rerunning.

Comparing the three methods

KEY CONCEPT

Feature	Historical simulation	Bootstrapping	Monte Carlo simulation
Data source	Historical dataset	Observed sample	Assumed (modeled) distribution
Sampling	Directly from history	Resampling with replacement	Random draws from a specified distribution
Assumption	Past data represent the future	Sample represents the population	Process generating outcomes can be modeled statistically
Typical uses	Risk management, stress tests	Confidence intervals, standard errors	Complex systems, long-term outcomes
Flexibility	Limited to historical patterns	Limited by sample size	Highly flexible, hypothetical scenarios
Uncertainty captured	Only what was observed in history	Variability from resampling	Hypothetical uncertainty from the modeled inputs
Computational intensity	Low to moderate	Moderate	High

COMMON EXAM TRAPS

- The Cholesky decomposition generates correlated paths from an assumed correlation; it does not estimate correlations or covariances.

EXAM SHORTCUTS

- Identify the method from the source of the scenarios: actual past changes mean historical simulation, resampling with replacement from the observed sample means bootstrapping, and random draws from

an assumed distribution mean Monte Carlo simulation.

BOTTOM LINE

- Monte Carlo simulation draws scenarios from probability distributions specified by the analyst and runs thousands of trials to build a simulated frequency distribution, with more trials giving more precise estimates.
- Monte Carlo simulation is used to estimate portfolio performance and to value securities without a closed-form pricing formula, such as Asian-style options, mortgage-backed securities and convertible bonds.
- Exam convention: a Monte Carlo value for a European call should match the Black-Scholes-Merton value, getting closer as more trials are run; current practice: this holds only when the simulated prices are lognormal with constant volatility and a risk-neutral expected return, with the average payoff discounted at the risk-free rate.
- The Cholesky decomposition maps independent standard normal draws into correlated normal variables using an assumed correlation; it does not estimate correlations or covariances.
- Monte Carlo simulation is flexible, forward-looking and handles nonnormal distributions and correlations, but it is complex, model-dependent, computationally intensive and data-hungry, and it gives a statistical estimate without the cause-and-effect insight of analytic methods.
- Historical simulation assumes past data represent the future, bootstrapping assumes the sample represents the population, and Monte Carlo simulation assumes the process generating outcomes can be modeled statistically.

QUICK CHECK

Question 3

Compared with historical simulation and bootstrap resampling, an advantage of Monte Carlo simulation is that it:

A has lower data requirements.

B produces results that are easier to explain.

C can model hypothetical scenarios.

Answer: C

Monte Carlo scenarios are generated from distributions chosen by the analyst, so the method can simulate hypothetical or extreme conditions that never appeared in the historical record. Historical simulation and bootstrapping can only reuse what actually happened.

Why the other options are wrong

- **A.** Monte Carlo usually has heavy data requirements, because volatilities, correlations and other parameters must be estimated.
- **B.** Monte Carlo models are more complex and harder to communicate than methods that replay or resample actual history.

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> **Key takeaway** Monte Carlo is forward-looking: it can model scenarios with no historical precedent.

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/simulation-of-financial-asset-prices-and-returns/. The full bank has 16 questions on this reading.

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QUANTITATIVE METHODS · READING 10

Least Squares Regression

What you'll learn

LOS 10.a Describe and interpret simple linear regression and estimate its coefficients using the least squares criterion.

LOS 10.b Describe the assumptions of simple linear regression, detect violations from residuals, and evaluate goodness of fit, coefficient tests and ANOVA results.

LOS 10.c Calculate and interpret predicted values, the standard error of estimate and prediction intervals, and describe log-lin, lin-log and log-log functional forms.

LOS 10.d Estimate and interpret CAPM beta and related statistics with a simple linear regression of excess returns on the market risk premium.

MODULE 10.1

Linear Regression Basics

This reading fits a simple linear regression by least squares, interprets its slope and intercept, and checks the model's assumptions with residual plots. It then measures fit with ANOVA, the standard error of estimate, R^2 and the F-test, tests the coefficients, builds prediction intervals, chooses among log-transformed functional forms, and estimates a stock's CAPM beta by regression.

LOS 10.a — Simple linear regression and the least squares criterion

Purpose and vocabulary

Simple linear regression explains the **variation** of one **dependent variable** with the variation of a single **independent variable**. *Variation* is the total squared deviation from the mean, $\sum_{i=1}^{n}(Y_i - \bar{Y})^2$; it is related to, but not the same as, the variance (the variance divides the variation by $n - 1$).

	Dependent variable (Y)	Independent variable (X)
Role	The variable being explained	The variable doing the explaining
Other names	Explained, endogenous, predicted variable	Explanatory, exogenous, predicting variable
Example: using GDP growth to forecast stock returns	Stock returns	GDP growth

A model with one dependent variable and one independent variable is a *simple* regression; a model with one dependent variable and more than one independent variable is a **multiple regression** (Level II).

The model and the regression line

KEY CONCEPT

$$Y_i = b_0 + b_1 X_i + \varepsilon_i \qquad \text{(population model)}$$

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$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i \qquad \text{(estimated regression line, the line of best fit)}$$

The hat marks an estimate. The **residual** for observation i, $Y_i - \hat{Y}_i$, is the gap between the actual value and the value the fitted line predicts for that X.

- Exam convention: the residual, the **error term** and the **disturbance term** are treated as the same thing, written ε_i.
- Current practice: the error term ε_i is the unobservable deviation in the population model, and the residual is its sample estimate.

Least squares criterion

Of all the lines that could be drawn through the scatter plot, **ordinary least squares (OLS)** regression picks the one that minimizes the **sum of squared errors (SSE)**, found by squaring each vertical gap between an observed value of the dependent variable and its fitted value, then adding the squares:

$$\text{SSE} = \sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$$

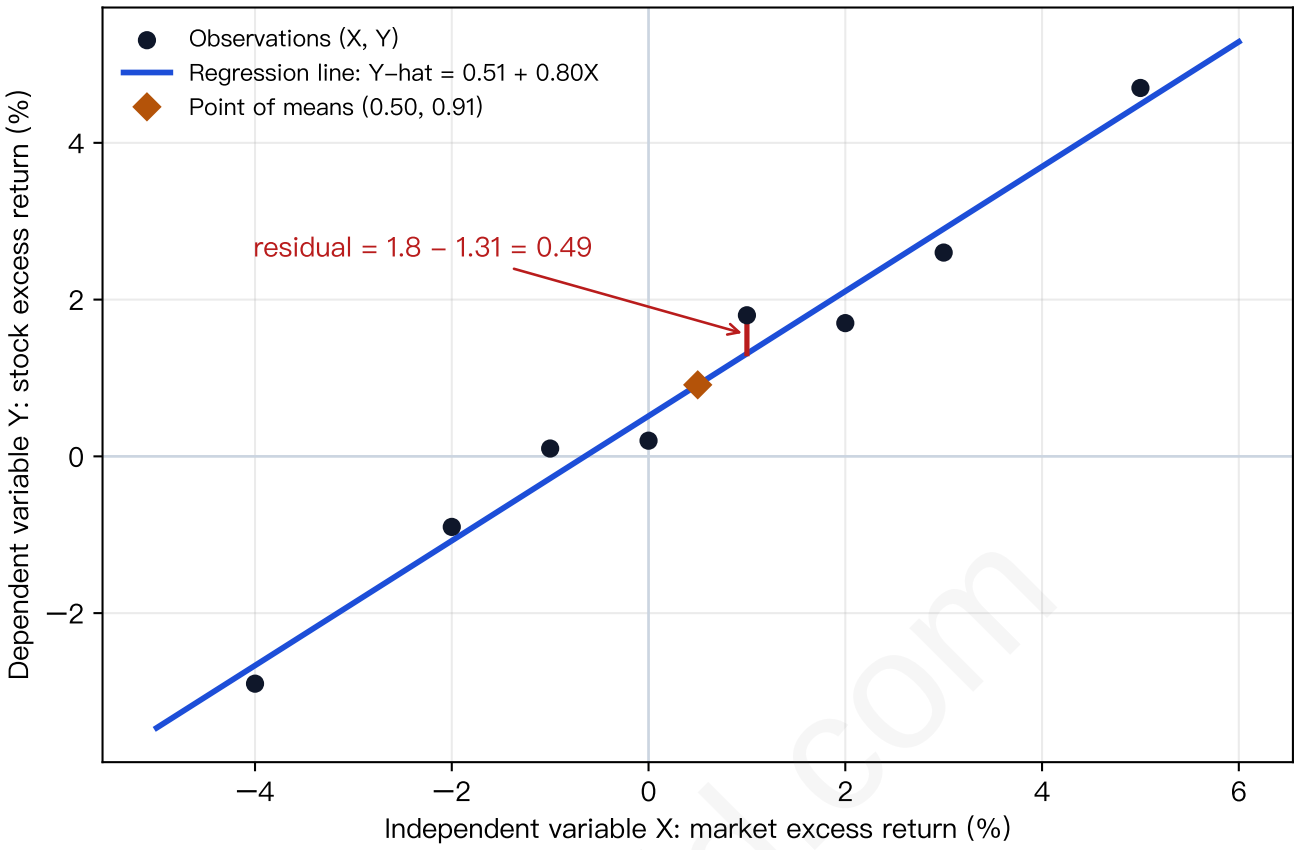
The resulting estimates are:

KEY CONCEPT

$$\hat{b}_1 = \frac{\text{Cov}(X, Y)}{\sigma_X^2}, \qquad \hat{b}_0 = \bar{Y} - \hat{b}_1 \bar{X}$$

The intercept formula shows that the regression line always passes through the point of means $(\bar{X}, \bar{Y})$. The covariance and the variance share the divisor $n - 1$, which cancels, so from raw data the slope is $\hat{b}_1 = \sum (X_i - \bar{X})(Y_i - \bar{Y}) / \sum (X_i - \bar{X})^2$.

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Scatter plot with the least-squares regression line

WORKED EXAMPLE (DATA IN THE FIGURE)

Eight monthly observations give $\bar{X} = 0.50\%$, $\bar{Y} = 0.9125\%$, $\text{Cov}(X, Y) = 6.593$ and $\sigma_X^2 = 8.286$ (both in %²). The underlying sums are $\sum(X_i - \bar{X})(Y_i - \bar{Y}) = 46.15$ and $\sum(X_i - \bar{X})^2 = 58.0$, and $46.15/58.0$ gives the same slope.

$$\hat{b}_1 = 6.593/8.286 = 0.7957 \approx 0.80, \qquad \hat{b}_0 = 0.9125 - 0.7957(0.50) = 0.51\%$$

For the observation at $X = 1\%$ with $Y = 1.8\%$: $\hat{Y} = 0.51 + 0.80(1) = 1.31\%$, so the residual is $1.8 - 1.31 = 0.49\%$.

Interpreting the coefficients

- **Slope coefficient** $\hat{b}_1$: how much the dependent variable is expected to move when the independent variable rises by one unit. In the example, a market excess return 1 percentage point higher goes with a stock excess return 0.80 point higher. When Y is a stock's excess return and X the market's, the slope is the stock's **beta**.
- **Intercept** $\hat{b}_0$: the predicted value of Y when X equals zero (where the line crosses the vertical axis).
- A large slope does not by itself mean a strong relationship; significance must be tested (t-test or confidence interval).

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Types of data

KEY CONCEPT

Type	Definition	Example
Time series	Observations of one variable at regular intervals over time	A company's quarterly EPS for five years
Cross-sectional data	Observations of many subjects at one point in time	Today's closing prices of 30 stocks in an index
Panel data	Cross-sectional observations repeated over time	Quarterly EPS of 10 companies over five years

COMMON EXAM TRAPS

- The slope divides the covariance by the variance of X, the independent variable. Dividing by the variance of Y is a common error.
- OLS minimizes the squared differences between actual and predicted values of Y. It does not compare coefficients or X values.
- The slope is the change in Y per unit change in X. Reading it as the change in X per unit of Y reverses the roles.

EXAM SHORTCUTS

- The regression line passes through $(\bar{X}, \bar{Y})$, so once the slope is known the intercept is $\bar{Y} - \hat{b}_1\bar{X}$ with no further sums.

BOTTOM LINE

- Simple linear regression explains the variation of a dependent variable Y, $\sum(Y_i - \bar{Y})^2$, with the variation of a single independent variable X.
- Ordinary least squares chooses the line that minimizes the sum of squared errors, $\text{SSE} = \sum(Y_i - \hat{Y}_i)^2$.
- The OLS estimates are $\hat{b}_1 = \text{Cov}(X, Y)/\sigma_X^2$ and $\hat{b}_0 = \bar{Y} - \hat{b}_1\bar{X}$, so the fitted line passes through the point of means.
- The slope is the expected change in Y for a one-unit rise in X, and the intercept is the predicted value of Y when X equals zero.
- The residual $Y_i - \hat{Y}_i$ is the gap between the actual and the fitted value; exam convention treats the residual, error term and disturbance term as the same thing, while current practice treats the residual as the sample estimate of the unobservable error term.
- Time series data follow one variable over time, cross-sectional data cover many subjects at one point in time, and panel data repeat cross-sectional observations over time.

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QUICK CHECK

Question 1

A credit analyst gathers data on 80 bond issuers at a single date to see whether an issuer's interest coverage ratio helps explain the yield spread on its bonds. In her simple linear regression, the dependent variable is:

A the interest coverage ratio, because it is the explanatory variable.

B the yield spread, because it is the variable being explained.

C neither, because data from a single date cannot be used in a regression.

Answer: B

The dependent variable is the one whose variation the model explains, here the yield spread. The interest coverage ratio does the explaining, so it is the independent (explanatory) variable. Observations of many issuers at one date are cross-sectional data, which a simple linear regression can use as readily as time-series data.

Why the other options are wrong

- **A.** An explanatory variable is the independent variable. Making the coverage ratio the dependent variable reverses the roles set by the analyst's question.
- **C.** Data on many subjects at one point in time are cross-sectional data, and a regression can be run on them. Time-series data are not required.

Key takeaway Dependent variable (Y): explained, endogenous, predicted. Independent variable (X): explanatory, exogenous, predicting. The research question decides which is which.

MODULE 10.2

Analysis of Variance (ANOVA) and Goodness of Fit

LOS 10.b — Assumptions, residual analysis, ANOVA and goodness of fit

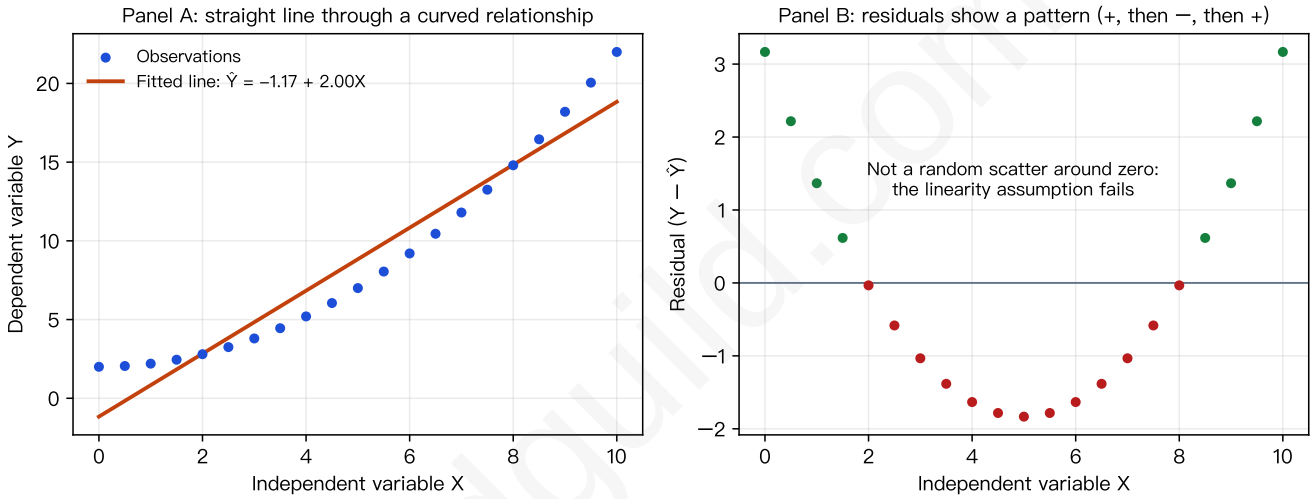
Assumptions of simple linear regression

KEY CONCEPT

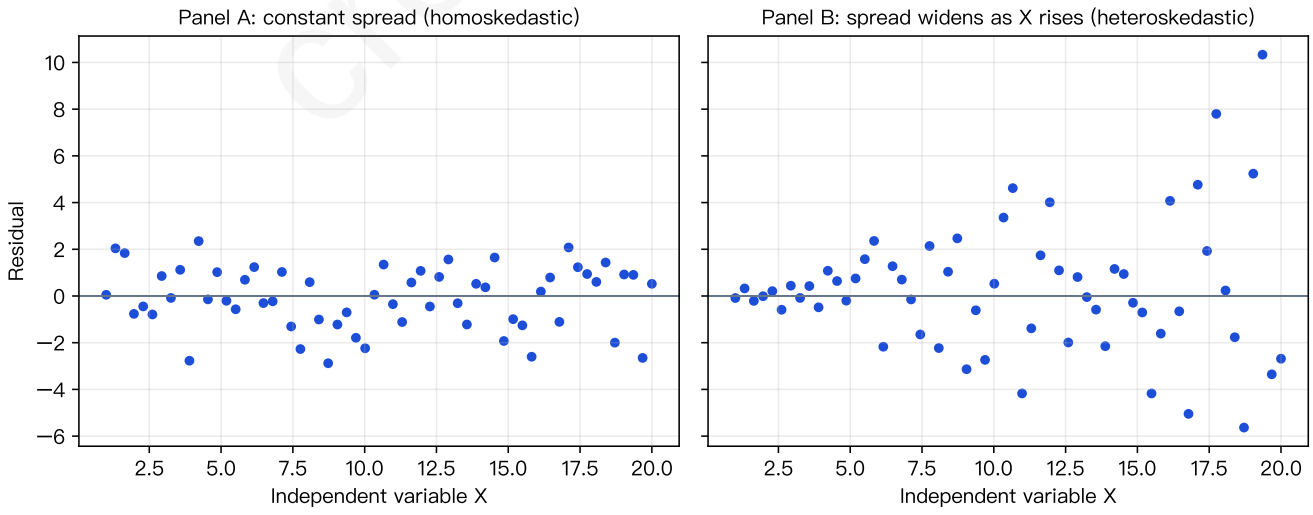
1. **Linearity**: the relationship between X and Y is linear.
2. **Homoskedasticity**: the residuals have the same variance for every observation.
3. **Independence**: residuals are uncorrelated with one another (the X–Y pairs are independent).
4. **Normality**: the residual term is normally distributed.

Violations show up in plots of the residuals against X (or against time). In a well-specified model the residuals scatter randomly around zero with no pattern.

Violation	What the residual plot shows	Name
Nonlinear relationship	A systematic pattern, e.g., residuals positive, then negative, then positive as X rises	—
Residual variance not constant	Spread of residuals widens (or narrows) with X, or changes over time	**Heteroskedasticity**
Residuals correlated	Recurring pattern, e.g., large errors every December (seasonality); the estimated variances of the coefficients are then wrong	**Autocorrelation**
Residuals not normal	Skewed or fat-tailed residuals; hypothesis tests may be unreliable in small samples (with a large sample, the central limit theorem means the estimates may still be valid)	—



A straight line fitted to a curved relationship, and the residual pattern it leaves



Residual plots: homoskedastic vs. heteroskedastic

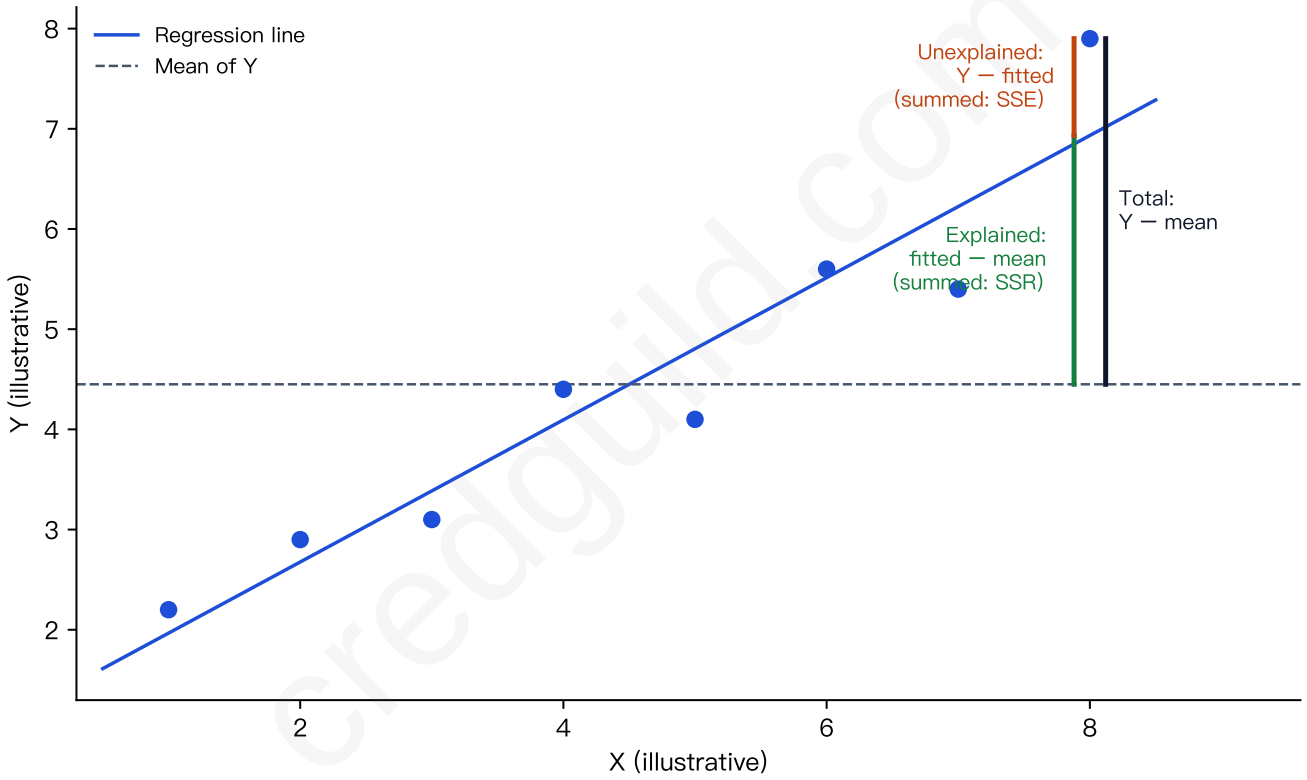
Outliers are observations far from the line. They can pull the estimated coefficients so that the line fits the other points poorly.

Analysis of variance (ANOVA)

Analysis of variance (ANOVA) splits the total variation of the dependent variable into an explained and an unexplained part:

- **Total sum of squares (SST)** $= \sum(Y_i - \bar{Y})^2$: total variation.
- **Sum of squares regression (SSR)** $= \sum(\hat{Y}_i - \bar{Y})^2$: variation explained by the regression.
- **Sum of squared errors (SSE)** $= \sum(Y_i - \hat{Y}_i)^2$: unexplained variation (also called the residual sum of squares).

$$\text{SST} = \text{SSR} + \text{SSE}$$



Total variation split into explained and unexplained parts (illustrative data)

The sample variance of Y is its total variation divided by its degrees of freedom, $\text{SST}/(n-1)$.

- **Mean square regression (MSR)** $= \text{SSR}/k$; with one independent variable ($k = 1$), MSR = SSR.
- **Mean squared error (MSE)** $= \text{SSE}/(n-k-1) = \text{SSE}/(n-2)$ in a simple regression.

KEY CONCEPT

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares
Regression (explained)	$k = 1$	SSR	MSR = SSR/1
Error (unexplained)	$n - 2$	SSE	MSE = SSE/$(n-2)$

Source of variation	Degrees of freedom	Sum of squares	Mean sum of squares
Total	$n-1$	SST	

Goodness-of-fit measures

$$\text{SEE} = \sqrt{\text{MSE}} = \sqrt{\frac{\text{SSE}}{n-2}}$$

The **standard error of estimate (SEE)**, also called the **root mean squared error**, is the standard deviation of the residuals; the lower it is, the better the fit.

KEY CONCEPT

$$R^2 = \frac{\text{SSR}}{\text{SST}} = 1 - \frac{\text{SSE}}{\text{SST}}$$

The **coefficient of determination** (R^2) measures what fraction of the dependent variable's total variation the independent variable accounts for. In a simple regression $R^2 = r^2$, so the **correlation coefficient** is $r = \pm\sqrt{R^2}$, with the sign of the slope. When only R^2 and SSR are known, SST $=$ SSR$/R^2$ and SSE $=$ SST $-$ SSR, which then gives the MSE and the SEE.

F-test

The F-test compares the variation the regression explains, per degree of freedom, with the variation it leaves unexplained, per degree of freedom. A large ratio means the independent variable explains a lot relative to the noise.

$$F = \frac{\text{MSR}}{\text{MSE}} \qquad df = 1 \text{ (numerator)},\ n-2 \text{ (denominator)}$$

The **F-test** asks whether the independent variables, as a group, explain the variation of Y; with several independent variables its null is that all slope coefficients are zero. With one independent variable, it tests H_0 : $b_1 = 0$ vs. H_a : $b_1 \neq 0$, the same hypothesis as the t-test of the slope. Reject H_0 if $F > F_c$ (a one-tailed test using the upper tail of the F-distribution). Rejecting H_0 means X and Y have a significant linear relationship.

WORKED EXAMPLE

$n = 30$; SSR = 64 and SSE = 112, so SST = 176. MSE $= 112/28 = 4$; SEE $= \sqrt{4} = 2$; $R^2 = 64/176 = 0.364$; $r = \sqrt{0.364} = 0.60$ (if the slope is positive); $F = 64/4 = 16$, well above the 5% critical value of about 4.2 for 1 and 28 degrees of freedom, so the slope is significant.

t-test of a regression coefficient

$$t = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}} \qquad df = n-2$$

To test significance, set $H_0 : b_1 = 0$ (two-tailed) and reject if $|t| > t_c$. The test of the slope is equivalent to testing the correlation, $t = r\sqrt{n-2}/\sqrt{1-r^2}$. With one independent variable, the F-statistic equals the square of this t-statistic, so the F-test and the t-test of the slope always reach the same decision (in the worked example above, $F = 16$ corresponds to $t = 4$). The intercept can be tested the same way ($H_0 : b_0 = 0$).

Indicator variables

An **indicator variable** (**dummy variable**) takes the value **1** when a condition holds and **0** otherwise (e.g., 1 in recession quarters). Regressing Y on the indicator tests whether the condition matters: the slope equals the **difference in the mean of Y** between periods with and without the condition, and testing $b_1 = 0$ is equivalent to a difference-in-means test. The intercept is the mean of Y in periods without the condition (indicator = 0), and intercept plus slope is the mean in periods with it. *Example.* Regressing a company's quarterly earnings on a recession indicator tests whether earnings are cyclical; for cyclical earnings the slope is expected to be negative (lower earnings in recession quarters). If quarterly EPS averaged 1.40 in expansion quarters and 0.95 in recession quarters, the fitted line is $\widehat{\text{EPS}} = 1.40 - 0.45D$, where $D = 1$ in a recession quarter.

COMMON EXAM TRAPS

- $R^2 = \text{SSR}/\text{SST}$. SSE/SST is the unexplained share ($1 - R^2$), and SSR/SSE is not a standard measure. In the worked example, $112/176 = 0.636$ is the unexplained share; R^2 is 0.364.
- F is a ratio of mean squares, MSR/MSE. Dividing SSR by SSE ignores the degrees of freedom. In the worked example, $64/112 = 0.571$ instead of $F = 16$.
- SEE divides SSE by $n - 2$, the degrees of freedom, before taking the square root. Dividing by n understates it. In the worked example, $\sqrt{112/30} = 1.93$ instead of 2.
- A negative slope gives a negative r even though R^2 is positive. The standard error of the slope is a different statistic and is not a correlation.

EXAM SHORTCUTS

- With one independent variable, $F = t^2$ for the test of $H_0 : b_1 = 0$ against $H_a : b_1 \neq 0$, so at the same significance level the F-test and the two-tailed slope t-test reach the same decision and only one needs computing.
- When only R^2 and SSR are given, SST $= \text{SSR}/R^2$ and SSE $=$ SST $-$ SSR, which lead straight to the MSE and the SEE.
- In a simple regression $r = \pm\sqrt{R^2}$, taking the sign of the slope.

BOTTOM LINE

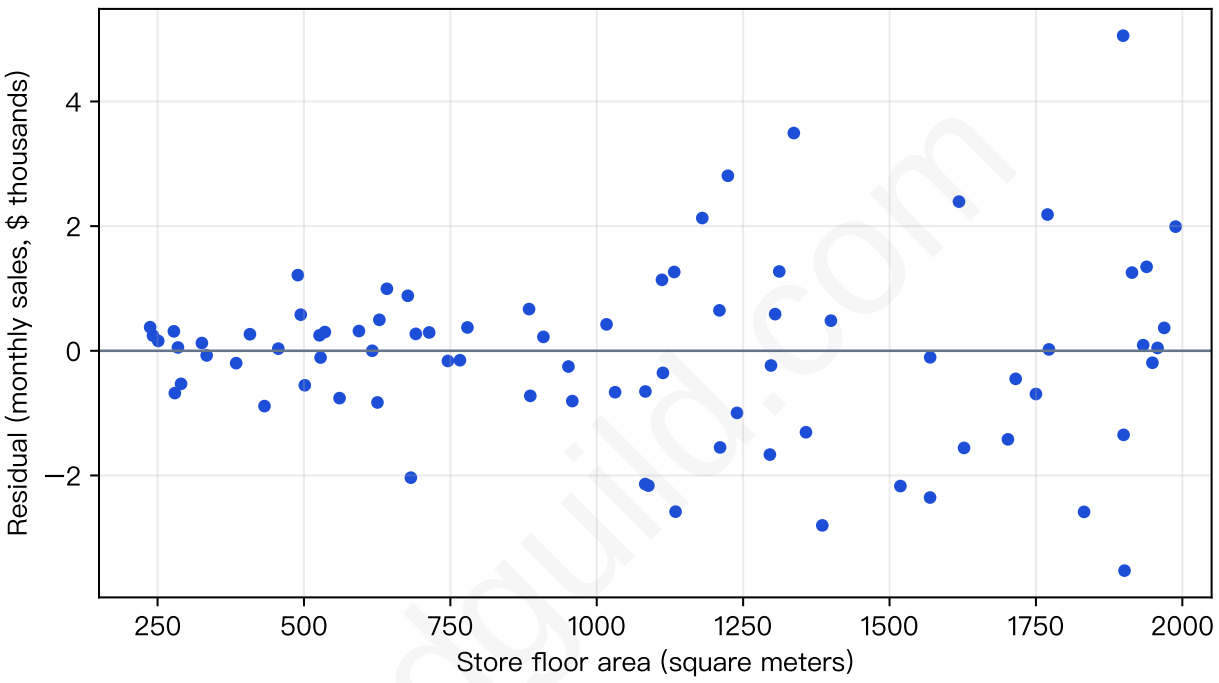
- Simple linear regression assumes a linear relationship, homoskedastic residuals, residuals uncorrelated with one another, and normally distributed residuals.
- Residual variance that widens or narrows with X or over time is heteroskedasticity, and residuals correlated with one another, such as a recurring seasonal pattern, show autocorrelation.
- ANOVA splits total variation into explained and unexplained parts, $\text{SST} = \text{SSR} + \text{SSE}$.
- In a simple regression $\text{MSE} = \text{SSE}/(n-2)$, $\text{SEE} = \sqrt{\text{MSE}}$, and $R^2 = \text{SSR}/\text{SST} = 1 - \text{SSE}/\text{SST}$, which equals r^2.
- $F = \text{MSR}/\text{MSE}$ with 1 and $n - 2$ degrees of freedom is a one-tailed test, and with one independent variable it tests $H_0 : b_1 = 0$, the same hypothesis as the slope's t-test.

- Regressing Y on an indicator variable gives a slope equal to the difference in the mean of Y between periods with and without the condition, and an intercept equal to the mean when the indicator is 0.

QUICK CHECK

Question 2

An analyst regresses the monthly sales of 80 stores on each store's floor area. The figure below plots the regression residuals against floor area. The pattern *most likely* indicates that the regression's residual term:



Residuals plotted against store floor area

A does not have a constant variance.

B is not normally distributed.

C is not independently distributed.

Answer: A

The residuals fan out as floor area increases: their spread (variance) grows with the independent variable. A residual variance that is not constant across observations is heteroskedasticity, a violation of the homoskedasticity assumption.

Why the other options are wrong

- **B.** The plot shows residuals centered on zero with a changing spread; nothing in it points specifically to a nonnormal shape, and normality is judged from the shape of the residuals' distribution. How their spread changes with X is a separate question.
- **C.** Non-independence (autocorrelation) shows up as residuals that are correlated with one another, such as a recurring seasonal pattern over time. A widening spread with X is a variance problem.

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Key takeaway Residual spread widening with X = heteroskedasticity (non-constant residual variance).

MODULE 10.3

Predicted Values and Functional Forms of Regression

LOS 10.c — Predicted values, prediction intervals and functional forms

Predicted values

A **predicted value** of the dependent variable plugs a forecast of the independent variable, X_p, into the estimated equation:

$$\hat{Y} = \hat{b}_0 + \hat{b}_1 X_p$$

Example: $\hat{Y} = 1.2 + 0.9X$ and $X_p = 4$: $\hat{Y} = 1.2 + 3.6 = 4.8$. Include the intercept and keep the sign of each term.

Confidence (prediction) interval for a predicted value

KEY CONCEPT

$$\hat{Y} \pm t_c \, s_f \qquad df = n - 2$$

where s_f is the **standard error of the forecast**. At Level I it is usually given; if not:

KEY CONCEPT

$$s_f^2 = \text{SEE}^2 \left[1 + \frac{1}{n} + \frac{(X_p - \bar{X})^2}{(n-1)s_X^2} \right]$$

The forecast is more precise (smaller s_f) when the SEE is smaller, the sample is larger, X_p is closer to $\bar{X}$, and the independent variable has a greater variance.

Example (continued): $s_f = 1.5$, $n = 27$, so $df = 25$ and the two-tailed 5% critical value is 2.060. The 95% interval is $4.8 \pm 2.060(1.5) = 4.8 \pm 3.09$, i.e., 1.71 to 7.89.

Computing s_f when it is not given (a separate regression): SEE = 2.0, $n = 27$, $\bar{X} = 3$, $s_X^2 = 4$, $X_p = 4$:

$$s_f^2 = 2.0^2 \left[1 + \frac{1}{27} + \frac{(4-3)^2}{26 \times 4} \right] = 4(1 + 0.0370 + 0.0096) = 4.187, \qquad s_f = 2.05$$

The standard error of the forecast is always larger than the SEE: the forecast carries both the residual uncertainty and the uncertainty in the estimated coefficients.

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Interval	Centered on	Width driven by
Confidence interval for a coefficient	$\hat{b}_1$ (or $\hat{b}_0$)	Standard error of that coefficient
Prediction interval for Y	$\hat{Y}$ at X_p	Standard error of the forecast s_f

Functional forms

If the relationship between X and Y is not linear, taking natural logarithms of the dependent variable, the independent variable or both can make it linear. For example, EPS growing at a roughly constant percentage rate over time traces an upward curve.

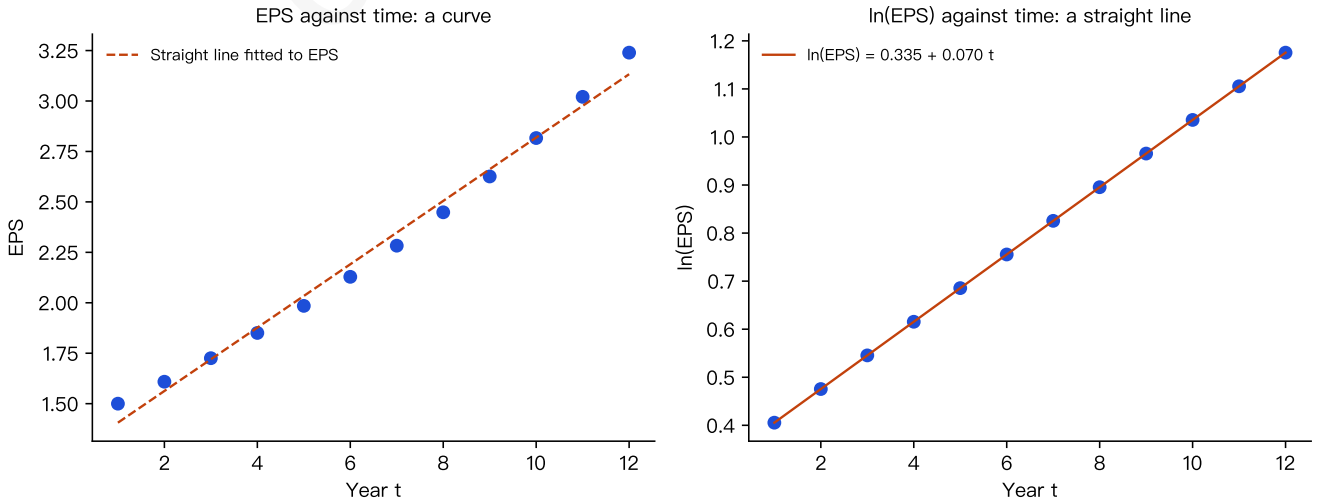
KEY CONCEPT

Model	Equation	Slope interpretation	Typical use
Log-lin model	$\ln Y_i = b_0 + b_1 X_i + \varepsilon_i$	Relative (%) change in Y for an absolute change in X	Constant growth rate over time
Lin-log model	$Y_i = b_0 + b_1 \ln X_i + \varepsilon_i$	Absolute change in Y for a relative (%) change in X	Y responds to proportional changes in X
Log-log model	$\ln Y_i = b_0 + b_1 \ln X_i + \varepsilon_i$	Relative change in Y for a relative change in X (an elasticity)	Both variables in proportional terms

Example. In a log-lin regression of a company's annual EPS on a time index (1, 2, 3, ...), a slope of 0.07 means EPS grows by roughly 7% a year (exactly $e^{0.07} - 1 = 7.25\%$). It does not mean EPS rises by 0.07 currency units a year.

Choose the form by the nature of the variables and by comparing goodness-of-fit measures and residual plots.

- Exam convention: a higher R^2, a higher F-statistic and a lower SEE indicate the better-fitting functional form.
- Current practice: R^2 and SEE are directly comparable only when the dependent variable is the same; a model of $\ln Y$ and a model of Y measure fit on different scales.



Log-lin model: EPS growing about 7% a year (illustrative)

COMMON EXAM TRAPS

- Prediction intervals use $df = n - 2$ in a simple regression (two estimated parameters), not $n - 1$. In the example, 26 degrees of freedom give a critical value of 2.056 and an interval of 1.72 to 7.88 instead of 1.71 to 7.89.
- A prediction interval uses the standard error of the forecast s_f. The standard error of the slope belongs to the interval for the slope coefficient.
- "Log" is attached to the variable that is logged: log-lin means Y is logged and X is linear.

EXAM SHORTCUTS

- The standard error of the forecast is always larger than the SEE, so a prediction interval narrower than $\hat{Y} \pm t_c \times$ SEE can be ruled out.

BOTTOM LINE

- A predicted value is $\hat{Y} = \hat{b}_0 + \hat{b}_1 X_p$, using the intercept and the sign of each term.
- The prediction interval for Y is $\hat{Y} \pm t_c s_f$ with $n - 2$ degrees of freedom, where s_f is the standard error of the forecast.
- The forecast is more precise when the SEE is smaller, the sample is larger, X_p is closer to $\bar{X}$ and the independent variable has a greater variance.
- A log-lin model ($\ln Y$ on X) has a slope giving the relative change in Y for an absolute change in X, a lin-log model the absolute change in Y for a relative change in X, and a log-log model an elasticity.
- Exam convention: a higher R^2, a higher F-statistic and a lower SEE identify the better-fitting functional form; current practice: R^2 and SEE are directly comparable only when the dependent variable is the same.

QUICK CHECK

Question 3

An analyst regresses a fertilizer producer's monthly stock returns on monthly changes in a crop price index, using 32 observations. The estimated coefficients are in the table below. She expects next month's change in the index to be −0.015 and computes a standard error of the forecast of 0.028. Critical t-values with 30 degrees of freedom are 1.310 with 10% in one tail and 1.697 with 5% in one tail. The 90% prediction interval for next month's return is *closest* to:

Regression of monthly stock returns on crop price changes

	Coefficient	Standard error
Intercept	0.0040	0.0035
Crop price change	0.6500	0.2100

A −0.0533 to 0.0418.

B −0.0424 to 0.0309.

C −0.0338 to 0.0613.

Answer: A

A prediction interval is the predicted value plus or minus the critical t-value times the standard error of the forecast. A two-sided 90% interval leaves 5% in each tail, so with n − 2 = 30 degrees of freedom the critical value is 1.697.

- Predicted return: $\hat{Y} = 0.0040 + 0.6500(-0.015) = 0.0040 - 0.00975 = -0.00575$
- Half-width: $t_c \times s_f = 1.697 \times 0.028 = 0.0475$
- Interval: -0.00575 ± 0.0475, that is, -0.0533 to 0.0418

Calculator: `0.65` [×] `0.015` [=] gives 0.00975; `1.697` [×] `0.028` [=] gives 0.047516.

Why the other options are wrong

- **B.** This interval uses 1.310, the critical value with 10% in one tail. A two-sided 90% interval splits the 10% between the two tails, so each tail holds 5%.
- **C.** This interval is centered on $0.0040 + 0.00975 = 0.01375$, which drops the minus sign of the forecast change in the index.

Key takeaway Prediction interval $= \hat{Y} \pm t_c s_f$, with n − 2 degrees of freedom and α/2 in each tail. Keep the sign of the forecast X when computing $\hat{Y}$.

MODULE 10.4

Estimating CAPM Values With Simple Linear Regression

LOS 10.d — Estimating the CAPM with simple linear regression

The CAPM as a one-factor regression

The capital asset pricing model states

$$E(R_i) = R_f + \beta_i [E(R_M) - R_f]$$

Because the only explanatory variable is the **market risk premium**, the CAPM is a **one-factor risk model** and can be estimated with simple linear regression, using past observations as proxies for expected values. Subtracting R_f from both sides gives the estimable form:

KEY CONCEPT

$$Z_{i,t} = b_0 + b_1 Z_{M,t} + \varepsilon_t, \qquad Z_{i,t} = R_{i,t} - R_{f,t}, \quad Z_{M,t} = R_{M,t} - R_{f,t}$$

KEY CONCEPT

Regression element	CAPM meaning
Dependent variable	The asset's **excess return** $(R_i - R_f)$
Independent variable	The **market risk premium** $(R_M - R_f)$
Slope $\hat{b}_1$	The asset's estimated **beta**
Intercept $\hat{b}_0$	Return not explained by market exposure (zero if the CAPM holds exactly)

Steps

1. **Collect data**: the asset's returns; a **security market index** as the proxy for the **market return**; yields on **short-term government debt** as the proxy for the **risk-free rate**. At least 30 observations is the usual minimum (e.g., 1–3 years of weekly data or 3–5 years of monthly data).
2. Compute the asset's excess returns and the market risk premium for each period.
3. Run the least-squares regression of excess returns on the market risk premium.
4. Use $\hat{\beta}$ and an expected market risk premium to estimate the expected return: $E(R_i) = R_f + \hat{\beta}[E(R_M) - R_f]$.

Testing and interpreting beta

Example: the slope is 1.25 with a standard error of 0.10 (large sample, so use z-values).

- Beta above 1: the stock's excess returns amplify the market's; below 1: they are dampened.
- Test $H_0 : \beta = 0$: $t = 1.25/0.10 = 12.5 > 1.96$, so reject; beta is significantly different from zero.
- 95% confidence interval: $1.25 \pm 1.96(0.10) = 1.054$ to 1.446.
- Test $H_0 : \beta = 1$ (does the stock move one-for-one with the market?): $t = (1.25 - 1)/0.10 = 2.5 > 1.96$, so reject; beta is significantly different from 1, and the estimate lies above it. This matches the interval above, which does not contain 1.
- A one-sided question, such as whether beta exceeds 1, uses $H_0 : \beta \leq 1$ versus $H_a : \beta > 1$, the same statistic $(\hat{b}_1 - 1)/s_{\hat{b}_1}$ and a one-tailed critical value.
- With $R_f = 3\%$ and an expected market risk premium of 5%: $E(R_i) = 3\% + 1.25(5\%) = 9.25\%$.

Changing inputs: $\Delta E(R_i) = \Delta R_f + \hat{\beta}\,\Delta[E(R_M) - R_f]$. With $\hat{\beta} = 0.8$, a 0.4% rise in R_f combined with a 0.4% fall in the market risk premium changes expected return by $0.4\% - 0.8(0.4\%) = +0.08\%$; for a beta above 1 the same changes would lower it.

Reading a typical output

	Coefficient	Standard error	t-statistic
Intercept	0.05	0.08	0.63
Market risk premium (slope)	1.25	0.10	12.50

The slope row carries beta and its test; the intercept row is not significant here (t = 0.63), consistent with the CAPM's prediction of a zero intercept. The R^2 of such a regression tells how much of the asset's excess-return variation is systematic (market-driven); $1 - R^2$ is the share that is unsystematic.

Choosing the sample

Longer samples give more observations and more precise estimates but risk including periods when the company's business mix, leverage and therefore beta were different. More frequent data (daily or weekly) add observations quickly but can be noisy for thinly traded assets. The choice depends on the purpose: a short-term speculator favors recent high-frequency data; a buy-and-hold investor favors monthly or quarterly data spanning at least one full business cycle.

WORKED EXAMPLE: ESTIMATING AND TESTING A CAPM BETA

Sixty monthly observations of a stock's excess return and the market risk premium give a covariance of 0.00168 and a variance of the market risk premium of 0.00140. The regression reports a standard error of 0.16 for the slope, and with 60 observations the sample is large, so the two-tailed 5% critical value is 1.96. Estimate beta, test at the 5% level whether beta differs from 1, build the matching confidence interval for beta, and estimate the stock's expected return with a risk-free rate of 2.8% and an expected market risk premium of 5.5%.

Step 1. Beta is the slope coefficient: $\hat{b}_1 = 0.00168/0.00140 = 1.20$.

Step 2. Test $H_0 : \beta = 1$ against $H_a : \beta \neq 1$: $t = (1.20 - 1)/0.16 = 1.25$. Because 1.25 is below 1.96, H_0 is not rejected, so beta is not significantly different from 1.

Step 3. The 95% confidence interval is $1.20 \pm 1.96(0.16)$, or 0.886 to 1.514. It contains 1, which agrees with Step 2.

Step 4. Expected return: $E(R_i) = 2.8\% + 1.20(5.5\%) = 9.4\%$.

Result. The estimated beta is 1.20, not significantly different from 1 at the 5% level, and the CAPM expected return is 9.4%.

COMMON EXAM TRAPS

- The dependent variable is the asset's excess return and the independent variable is the market risk premium. A regression of raw returns on raw returns is not the CAPM specification.
- Beta is the slope coefficient. The intercept says nothing about beta, so a negative intercept does not make beta negative.
- Confidence interval for beta: slope $\pm$ critical value $\times$ standard error of the slope. The intercept's standard error plays no part. With the output above, using the intercept's 0.08 gives 1.093 to 1.407 instead of 1.054 to 1.446.

EXAM SHORTCUTS

- A two-tailed 95% confidence interval for beta that excludes 1 means $H_0 : \beta = 1$ is rejected at the 5% level, and one that includes 1 means it is not, so either result gives the other.

BOTTOM LINE

- The CAPM is estimated by regressing the asset's excess return $R_i - R_f$ on the market risk premium $R_M - R_f$; the slope is the asset's beta and the intercept is zero if the CAPM holds exactly.
- A security market index proxies the market return, short-term government debt yields proxy the risk-free rate, and at least 30 observations is the usual minimum.

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- Beta is tested with $t = (\hat{b}_1 - \text{hypothesized value})/s_{\hat{b}_1}$, and its confidence interval is the slope $\pm$ critical value $\times$ the slope's standard error.
- The expected return is $E(R_i) = R_f + \hat{\beta}[E(R_M) - R_f]$, and a change in inputs gives $\Delta E(R_i) = \Delta R_f + \hat{\beta}\,\Delta[E(R_M) - R_f]$.
- The R^2 of the CAPM regression is the share of the asset's excess-return variation that is systematic, and $1 - R^2$ is the unsystematic share.
- Longer samples give more precise estimates but may include periods with a different beta, and more frequent data can be noisy for thinly traded assets.

QUICK CHECK

Question 4

To estimate a stock's beta, an analyst regresses the stock's excess returns on the market risk premium and obtains an intercept of −0.15 and a slope of 0.72. Based on this model, the stock's beta is:

A negative.

B between 0 and 1.

C greater than 1.

Answer: B

In the regression $(R_i - R_f) = b_0 + b_1(R_M - R_f) + \varepsilon$, the slope coefficient is the estimate of the asset's beta. The slope is 0.72, so beta lies between 0 and 1: the stock's excess returns tend to move less than one-for-one with the market risk premium.

$$(R_i - R_f) = -0.15 + 0.72\,(R_M - R_f) \;\Rightarrow\; \hat{\beta} = \hat{b}_1 = 0.72$$

Why the other options are wrong

- **A.** The negative number is the intercept, which is not beta. Beta is the slope, 0.72.
- **C.** A beta above 1 would require a slope above 1; the estimated slope is 0.72.

Key takeaway Beta = slope of the excess-return regression; the intercept is irrelevant to beta.

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/least-squares-regression/. The full bank has 29 questions on this reading.

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QUANTITATIVE METHODS · READING 11

Machine Learning in Finance

What you'll learn

LOS 11.a Describe how big data, machine learning and artificial intelligence are used in financial data science, fintech and investment management.

MODULE 11.1

Introduction to Financial Data Science

This reading describes how financial data science draws on big data: its types, sources and characteristics, and how the data are processed and visualized. It then explains data mining and machine learning (supervised, unsupervised and reinforcement), overfitting and underfitting, the split into training, validation and test datasets, neural networks, and natural language processing and large language models.

LOS 11.a — Big data, machine learning and AI in finance

What financial data science is for

Financial data science applies both quantitative and qualitative analysis to generate insight into a specific financial question, for example how customers behave or where market prices may move next. Among its main uses in finance are (1) evaluating investment opportunities, (2) optimizing portfolios and (3) mitigating risk. **Fintech** is where finance and technology meet. Of most interest in investment management are the fintech developments that aid quantitative analysis. The word is also used more broadly for technology developments applied to financial services and for the companies that build them.

The objective is the insight itself. Data-quality work, described below under the characteristics of big data and under curation, supports that objective but is not the objective.

Big data: types and sources

Big data is the broad term for all the potentially useful information being generated. By format it is:

Type	What it looks like	Examples
Structured data	Rows and columns in a table or database	Financial statement line items, daily closing prices
Semi-structured data	Not a table, but tagged so it can be stored in an organized way	HTML code of a web page
Unstructured data	No predefined organization; usually needs ML/AI or special code to use	Social media posts, voice recordings, pictures, sensor output

Data come from **traditional sources** (markets, company financial reports, regulatory filings, government statistics) or **alternative data** sources:

- **Individuals**: social media activity, clicks, searches, time spent on web pages (mostly unstructured and growing fast).
- **Business processes**: transaction and corporate records such as sales data or banking records (usually structured). They can act as leading or real-time indicators, whereas quarterly reports give a lagging view.

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- **Sensors**: smartphones, connected cars, smart buildings, satellites. The network of these devices is the **Internet of Things (IoT)**; it produces the largest volumes, mostly unstructured.

Buying alternative data or collecting it by **web scraping** (automated extraction from websites) raises legal and ethical issues: some of what is captured may be personal information protected by privacy rules.

The "Vs" of big data

KEY CONCEPT

Characteristic	Question it answers	Vocabulary to recognize
Volume	How much data?	megabytes → gigabytes → **terabytes** → **petabytes**
Velocity	How fast are data communicated?	**low latency** = real time; **high latency** = periodic or delayed
Variety	In what structures do the data exist?	structured, semi-structured, unstructured
Veracity	Are the data reliable and credible?	fake posts, errors, doubtful sources

A terabyte is 1,000 gigabytes and a petabyte is 1,000 terabytes. Faster data generate more of it: volumes rise from batch collection (megabytes) through periodic (gigabytes) and near-real-time (terabytes) to real-time feeds (petabytes). More complex structures also need more storage.

The first three are the classic **3 Vs** (volume, velocity, variety); **veracity** is the fourth V that matters especially for financial data. Other data-quality concerns are outliers, bad or missing data, sampling biases, and whether the amount collected is sufficient and appropriate for the intended use.

Data science: processing and visualization

Data science is the field concerned with extracting information from big data. Its processing methods:

Method	What happens	Do not confuse with
Capture	Collecting data and converting them into a usable form	Transfer
Curation	Ensuring data quality: fixing errors, adjusting for bad or missing data	Capture
Storage	Archiving data and accessing them later; choice depends on structure and latency	Search
Search	Querying stored data to find what is needed	Storage
Transfer	Moving data from the source or storage medium to the analytical tool	Capture

A **data lake** is a central repository that keeps structured, semi-structured and unstructured data in their raw, original format. **Distributed computing frameworks** let data be processed and analyzed on many machines at once.

Visualization: charts and graphs suit structured data; less structured data call for **word clouds** (show how often words occur in a text sample), **heat maps**, 3D graphics and **mind maps** (show logical relations between concepts).

Typical inputs include share prices and trading volumes, asset returns, interest rates, exchange rates, accounting figures and macroeconomic series. The work calls for expertise in finance as well as in data handling, because much of the data is sequential, time dependent and sometimes cyclical. Financial data bring special problems: noise, nonstationarity, interdependencies, nonlinearity, **fat tails** (extreme events more likely), very **high-frequency data** in trading, and too little data in some markets (e.g., emerging markets). Regulated firms must meet strict rules on data management, risk assessment and model validation; firms outside financial regulation still have to comply with business law and their contracts, although they have more freedom in how they handle data. Data privacy, thorough model testing and human review of model decisions matter for every firm.

Data mining, AI and machine learning

Data mining searches large datasets (usually structured) for patterns that can be used to predict outputs; pattern identification rests on conditional probabilities (e.g., Bayes' theorem). Its key risk is **overfitting**.

Artificial intelligence (AI) refers to computer systems that simulate human thinking. **Machine learning (ML)** is a major subcategory of AI: an algorithm receives input data, without any assumption about how they are distributed, possibly with target outputs, and learns without human help to model the outputs or to find patterns.

KEY CONCEPT

Approach	Data given	What the machine does	Typical finance use
Supervised learning	Labeled inputs and outputs	Learns to model outputs from inputs, then predicts on new data	Forecasting company or stock performance, default prediction
Unsupervised learning	Unlabeled inputs, no outputs	Describes the structure of the data and finds patterns; reduces dimensionality	Customer segmentation, anomaly detection
Reinforcement learning	Feedback on its own actions	Learns by trial and error from **rewards and penalties**	Portfolio optimization (sequential decisions)

A supervised model trained on a large, reliable dataset should predict well but may not adapt to new situations. Reinforcement learning is costly: designing a reward system that actually improves the model can take much training and many resources. Whatever the method, people with subject expertise are needed to choose the model, clean the inputs, limit bias and interpret the output.

Deep learning uses many layers of neural networks to find patterns of increasing complexity and may use supervised, unsupervised or semi-supervised learning (image and speech recognition, natural language processing).

Overfitting vs. underfitting

KEY CONCEPT

	Overfitting	Underfitting
Model is	Too complex	Not complex enough
Error	Treats **noise** as a true pattern	Treats **true parameters** as noise
Result	Spurious relationships, poor generalization to new data	Misses actual patterns and relationships

Other ML issues: results can be a "**black box**" (hard to explain), and **data leakage** occurs when information from outside the training dataset influences what the model learns.

Training, validation and test datasets

- **Training dataset**: the model looks for relationships here; it is the largest set, about 60%–80% of the data (upper end for large datasets).
- **Validation dataset**: used to refine (tune) the relationships found in training.
- **Test dataset**: used last, to judge the model's predictive ability.
  The validation and test sets share the remaining 20%–40% equally. Observations are normally assigned at random, which reduces bias, overfitting and data leakage. **Time series** data must keep their chronological order instead:
- **Time-based split**: the oldest 60%–80% of observations go to training, the next 10%–20% to validation and the most recent 10%–20% to testing.
- **Rolling-window validation**: train on one window, validate on a later period, then shift the window forward and repeat.
  Either way, the span covered by validation plus test data should match the intended forecast horizon.

Example. A credit model has 48,000 randomly ordered loan records and uses a 75% / 12.5% / 12.5% split. Training gets 36,000 records, validation 6,000 and test 6,000. If instead the data were 12 years of monthly returns (144 months) and a time-based split with the same proportions were used, months 1–108 would train the model, months 109–126 would validate it and months 127–144 would test it.

Neural networks, NLP and LLMs

Neural networks suit nonlinear relationships with many interdependencies: an input layer, several **hidden layers** of weighted nodes (**neurons**), and an output layer. They can learn in a supervised or unsupervised way, and each neuron's weight reflects its importance. They are often trained by **backpropagation** (a form of supervised learning): a **forward pass** produces a prediction that is compared with the actual outcome, the loss is computed, the **loss gradient** shows how weight changes affect the error, and **gradient descent** adjusts the weights step by step to minimize the loss.

- **Generative adversarial networks (GANs)** pit a generator network against a discriminator network to create realistic synthetic data (e.g., plausible market scenarios for stress tests, trained on historical market data). Because GANs can also produce convincing fake material, their output needs careful review.

- **Variational autoencoders (VAEs)** compress data and then reconstruct them using encoder and decoder networks. They are probabilistic generative models, used to reduce dimensionality and spot anomalies.
- **Copulas** (e.g., the Gaussian copula) are a more traditional mathematical tool for nonlinear relationships that uses **marginal probability distributions** to capture dependencies between random variables. Exam convention: the Cholesky decomposition is given as an example of a copula. Current practice: the Cholesky decomposition is a matrix factorization, used to generate the correlated normal draws in a Gaussian-copula simulation.

Natural language processing (NLP) lets computers interpret human language: speech recognition, translation, scanning employee communications for compliance, and detecting sentiment shifts in research reports or earnings-call transcripts. **Text analytics** works mainly with unstructured material, such as filings and research reports or recordings of earnings calls and surveys. NLP converts that material into usable data much more quickly than a person could and searches it for patterns; **lexical analysis**, for example, measures how often each word appears. **Large language models (LLMs)** are generative AI that produce human-like text, using neural networks to read context and meaning in qualitative inputs (nonfinancial inputs such as election polls can matter too). Their output is text such as a summary or a forecast based on tone and sentiment. They are typically trained on a very large dataset through **self-supervised learning**, an unsupervised approach in which the model generates the labels itself from raw data. They are not built to catch errors in financial data, so outputs need expert review and should be used alongside traditional data-driven models.

Common tools are **Python** (fintech apps), **R** (statistics), **Java** (runs across platforms) and **C/C++** (high-speed algorithmic trading). Common databases are **SQL** (structured data, on a server), **SQLite** (structured data, not on a server; common in mobile apps) and **NoSQL** (unstructured data).

COMMON EXAM TRAPS

- Business-process records such as sales data are usually structured, yet they count as alternative data. Structure does not decide whether a source is traditional.

EXAM SHORTCUTS

- Classify a data source by where it comes from, not by its structure: records from individuals, business processes and sensors are alternative data even when they are structured.
- Match the type of machine learning to what the algorithm is given: labeled inputs and outputs mean supervised learning, unlabeled inputs alone mean unsupervised learning, and rewards and penalties for its own actions mean reinforcement learning.

BOTTOM LINE

- Financial data science applies quantitative and qualitative analysis to gain insight into a specific financial question, with main uses in evaluating investment opportunities, optimizing portfolios and mitigating risk.
- Big data can be structured (rows and columns), semi-structured (tagged, such as HTML code) or unstructured (social media posts, voice recordings, pictures, sensor output), and comes from traditional sources or from alternative sources: individuals, business processes and sensors.
- The characteristics of big data are volume, velocity (low latency means real time), variety of structures and, for financial data especially, veracity, meaning whether the data are reliable and credible.

- Data science processes data through capture, curation (ensuring data quality), storage, search and transfer, and a data lake keeps structured, semi-structured and unstructured data in their raw format.
- Data mining searches large datasets for patterns to predict outputs, while machine learning, a subcategory of AI, learns from input data without assumptions about their distribution and without human help.
- Supervised learning uses labeled inputs and outputs, unsupervised learning finds structure in unlabeled inputs, and reinforcement learning learns by trial and error from rewards and penalties.
- Overfitting comes from a model that is too complex and treats noise as a true pattern; underfitting comes from one that is not complex enough and treats true parameters as noise.
- The training dataset is the largest, about 60%–80% of the data, with validation and test sets splitting the rest equally; observations are normally assigned at random, but time series data keep their chronological order through a time-based split or rolling-window validation.

QUICK CHECK

Question 1

A new analyst at Ottery Capital drafts three glossary entries for the firm's research team. Which entry about fintech is *most accurate*?

A Fintech covers only payment apps and online banking services sold to consumers.

B Fintech is another name for financial data science, the analysis of financial data.

C Fintech is the meeting of finance and technology, such as tools that aid quantitative analysis.

Answer: C

Fintech refers to the meeting of the finance and technology worlds. In investment management the developments of most interest are those that aid quantitative analysis, such as big data tools and machine learning. The term is also used more broadly for technology applied to financial services and for the firms that build it.

Why the other options are wrong

- **A.** Consumer payment and banking apps are one use of fintech. Limiting the term to them leaves out the investment tools, such as machine learning and big data processing, that matter most to analysts.
- **B.** Financial data science is the quantitative and qualitative analysis of financial information to gain insight into a financial question. Fintech is the technology that supports such work and other financial services, so the two terms are related but different.

Key takeaway Fintech is where finance meets technology; financial data science is the analysis that some of that technology makes possible.

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Practice questions

Question 2

Voice recordings of earnings calls, satellite images of shipping ports and posts on social media platforms are all examples of:

A unstructured data.

B structured data.

C semi-structured data.

Question 3

A data team at Larchmont Analytics is documenting the steps it uses to process data. Which of the following statements about these data processing methods is *most accurate*?

A Search is the process of archiving data so that they can be retrieved later.

B Curation involves cleaning the data, for example by correcting errors and dealing with missing values, so that the data are of good quality.

C Capture is the process of moving data from a storage medium to the analytical tool that will use them.

Question 4

A lender at Brackenridge Credit trains a model on records of past borrowers, each tagged with the borrower's financial ratios and whether the borrower eventually defaulted. The model learns to map the ratios to the default outcome and is then applied to new loan applicants. This machine learning process is best described as:

A unsupervised learning.

B supervised learning.

C reinforcement learning.

Answers: 2. A · 3. B · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/quantitative-methods/machine-learning-in-finance/. The full bank has 25 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Economics

Exam weight 6–9% · 8 readings

How firms behave in different market structures, why economies move through business cycles, and what fiscal and monetary policy can do. The second half covers geopolitics, international trade, capital flows and exchange-rate arithmetic.

Reading 12	Oligopoly
Reading 13	Business Cycle
Reading 14	Fiscal Policy
Reading 15	Quantitative Easing
Reading 16	Introduction to Geopolitics
Reading 17	Comparative Advantage
Reading 18	Capital Flows and the FX Market
Reading 19	Exchange Rate Calculations

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ECONOMICS · READING 12

Oligopoly

What you'll learn

LOS 12.a Determine and interpret breakeven and shutdown points (short and long run) and how economies and diseconomies of scale affect long-run average cost.

LOS 12.b Describe the characteristics of perfect competition, monopolistic competition, oligopoly and pure monopoly.

LOS 12.c Explain demand, optimal price and output, and pricing strategy for firms under monopolistic competition, in the short run and the long run.

LOS 12.d Explain demand, optimal price and output, and pricing strategy under oligopoly: kinked demand, Cournot, Stackelberg, Nash equilibrium, collusion and the dominant firm model.

LOS 12.e Identify the type of market structure a firm operates in and describe the use and limitations of concentration measures (N-firm concentration ratio and HHI).

MODULE 12.1

Breakeven, Shutdown, and Scale

This reading explains how a firm decides whether to operate, shut down or change its scale, and how price and output are set under perfect competition, monopolistic competition, oligopoly and monopoly. It ends with the concentration ratio and the Herfindahl-Hirschman Index, the measures used to identify a market structure.

LOS 12.a — Breakeven, shutdown and scale

Short run versus long run

The **short run** is the period over which at least one factor of production (usually capital: plant and equipment) is fixed, so the firm cannot change its scale. In the **long run** every input is variable: leases can lapse and equipment can be sold, so no cost is fixed.

Revenue and cost measures

The breakeven and shutdown rules compare revenue with cost, either in totals or per unit of output (Q):

Measure	Definition
Total revenue (TR)	$TR = P \times Q$
Average revenue (AR)	$AR = TR/Q$, equal to the price when every unit sells at one price
Marginal revenue (MR)	Extra revenue from selling one more unit, $\Delta TR/\Delta Q$
Total fixed cost (TFC)	Costs that do not change with output in the short run, such as rent under a lease
Total variable cost (TVC)	Costs that rise with output, such as materials or merchandise
Total cost (TC)	$TC = TFC + TVC$

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Measure	Definition
Average variable cost (AVC)	$AVC = TVC/Q$
Average total cost (ATC)	$ATC = TC/Q$
Marginal cost (MC)	Extra cost of producing one more unit, $\Delta TC/\Delta Q$

Breakeven and shutdown for a price taker

A firm in perfect competition is a **price taker**: it can sell any quantity at the market price, so price = AR = MR.

KEY CONCEPT

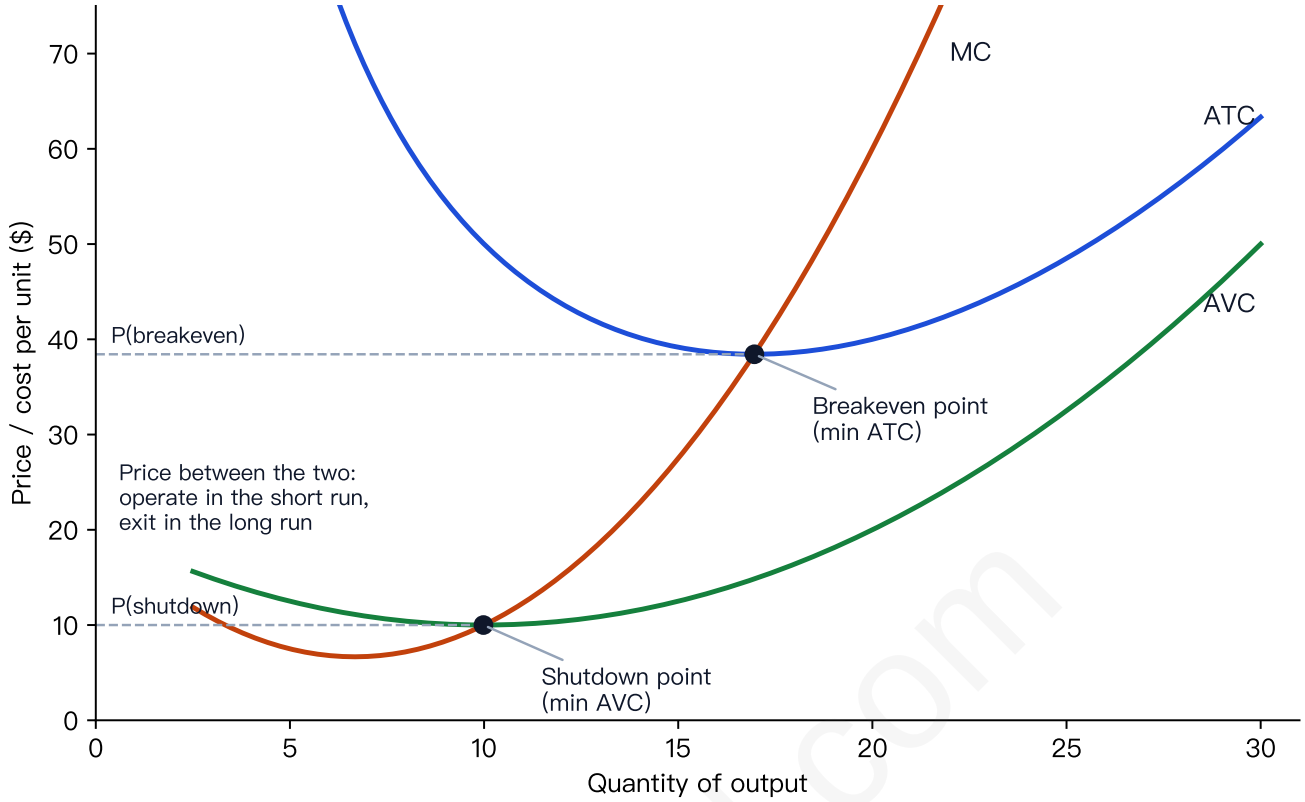
Condition	Short run	Long run
$AR \geq ATC$	Operate (zero or positive economic profit)	Stay in the market
$AVC \leq AR < ATC$	Operate: revenue covers variable cost, so the loss is no larger than fixed cost	Exit (shut down) unless price is expected to rise
$AR < AVC$	Shut down: operating loses more than fixed cost	Exit

- **Breakeven point:** $P = AR = ATC$ (minimum of ATC for a price taker), where $TR = TC$ and economic profit is zero.
- **Short-run shutdown point:** minimum of AVC; below it the firm shuts down.
- **Long-run shutdown point:** minimum of ATC, the same price as the breakeven point; a firm that expects price to stay below ATC leaves the market.

Both shutdown rules ask one question: does revenue cover the costs the firm could avoid by closing? In the short run only variable costs are avoidable, because fixed costs such as a lease must be paid anyway, so price is compared with AVC. In the long run every cost is avoidable, so price is compared with ATC.

MC cuts both AVC and ATC at their lowest points. When the next unit costs less than the current average, producing it pulls the average down; when it costs more, it pulls the average up. The averages therefore fall while MC lies below them and rise once MC is above them. Minimum AVC comes at a smaller output than minimum ATC. The shutdown point therefore sits below and to the left of the breakeven point, both on the MC curve.

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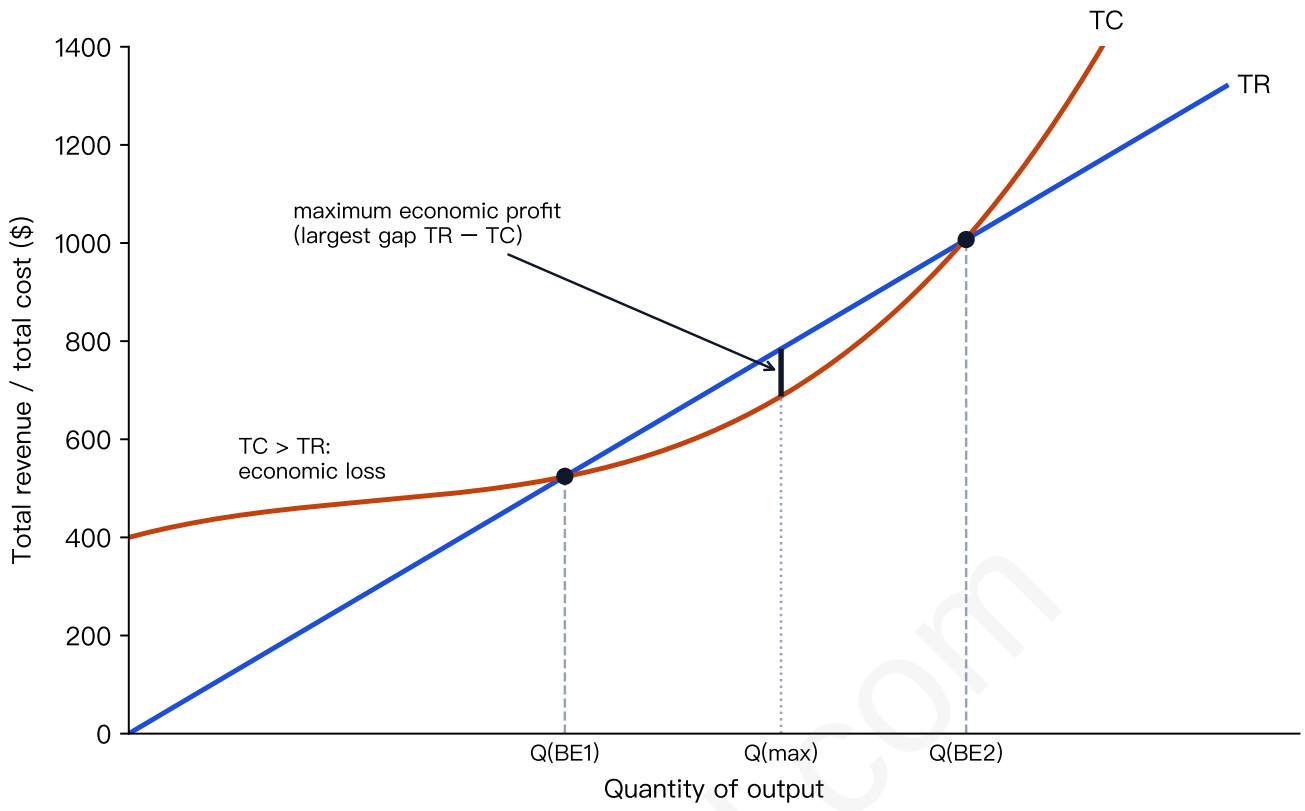
Short-run cost curves of a price taker: breakeven and shutdown points

A price taker is too small relative to the market to raise the market price by cutting its own output. Reducing output to push the price up is therefore never a remedy under perfect competition.

Breakeven and shutdown in total terms (all market structures)

For **price searchers** (downward-sloping demand), price no longer equals MR, so total revenue and total cost are the cleaner test. The same test works for price takers:

1. Compare TR with TC. If $TR \geq TC$, the firm stays in the market in both the short run and the long run; $TR = TC$ is the breakeven point.
2. If $TR < TC$, compare TR with TVC. If $TR \geq TVC$, keep operating in the short run and exit in the long run unless conditions are expected to improve.
3. If $TR < TVC$, shut down now and exit in the long run as well.



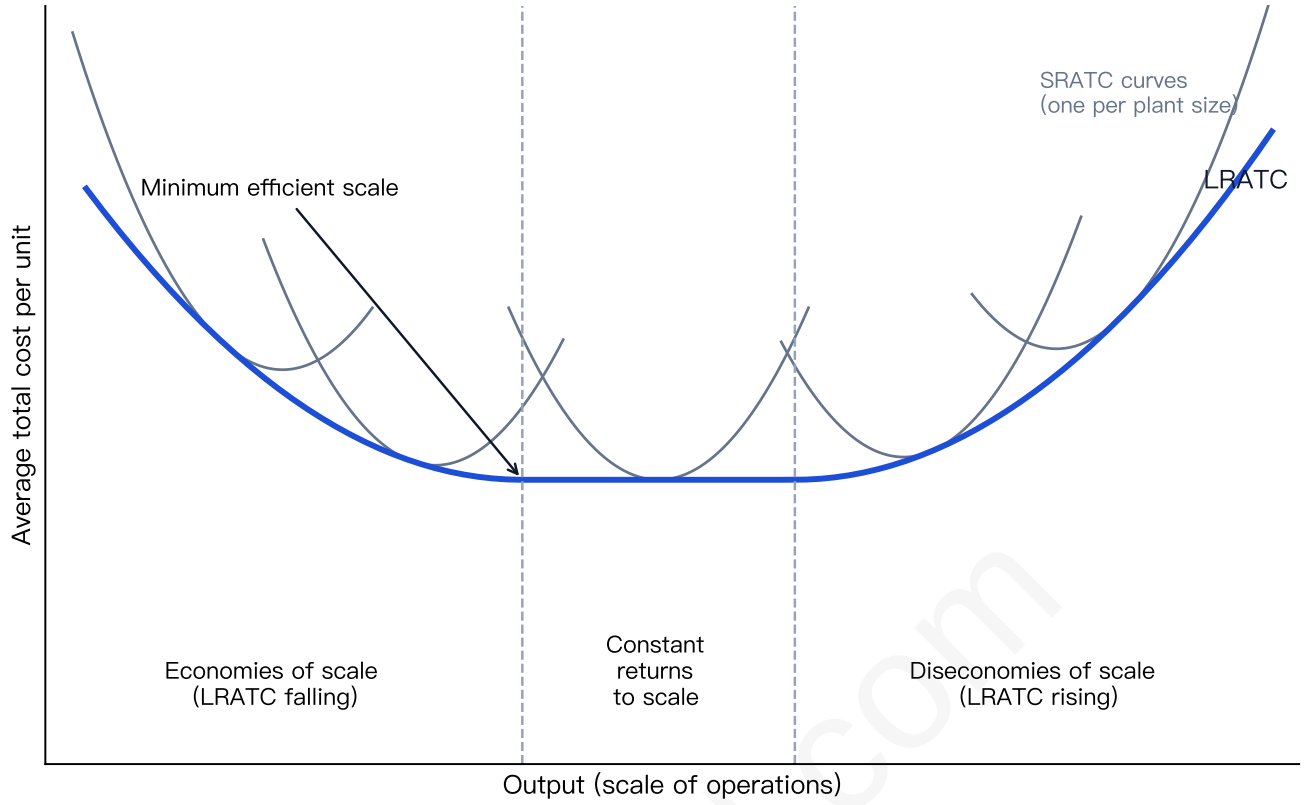
Total revenue and total cost of a price taker

With a straight-line TR (price taker) and a cubic-shaped TC there can be two **breakeven quantities**. Economic profit is largest where the vertical gap $TR - TC$ is greatest, which is where the slope of TC (marginal cost) equals the slope of TR (marginal revenue, here the price). At the lower breakeven quantity MC is below MR, so producing more adds profit; at the upper breakeven quantity MC is above MR, so producing less adds profit. If TC lies above TR everywhere, the firm operates (if at all) where the loss $TC - TR$ is smallest, which is the same as where $TR - TC$ is largest (least negative).

Worked example. Fernhill Printing expects annual total revenue of $1.8 million, total variable cost of $1.5 million and total fixed cost of $0.5 million, so total cost is $2.0 million. Step 1: $TR < TC$, so the firm is not breaking even. Step 2: $TR > TVC$, so operating gives $1.8 - 2.0 = -\$0.2$ million, against $-\$0.5$ million (the whole fixed cost) if it closes. It should keep operating in the short run and, unless conditions improve, exit in the long run. If revenue instead fell to $1.3 million, step 3 would apply: $TR < TVC$, and operating would give $1.3 - 2.0 = -\$0.7$ million, a loss larger than the fixed cost, so closing at once is the better choice.

Economies and diseconomies of scale

The **long-run average total cost (LRATC)** curve shows the lowest ATC achievable at each output level when plant size can be chosen freely; it is the envelope of the **short-run average total cost (SRATC)** curves for the different plant sizes.



Long-run average total cost as the envelope of short-run average total cost curves

LRATC segment	Name	Causes / meaning
Downward sloping	**Economies of scale** (increasing returns to scale)	Specialization of labor, mass production, better equipment and technology, volume discounts on inputs; output rises proportionally more than total input cost
Flat	**Constant returns to scale**	ATC roughly unchanged across a range of plant sizes
Upward sloping	**Diseconomies of scale**	Bureaucracy, harder-to-motivate workforce, barriers to innovation; unit cost rises as scale rises

The plant size at which LRATC first reaches its lowest level is the **minimum efficient scale**. Under perfect competition firms must operate there in long-run equilibrium, where price = minimum LRATC and economic profit is zero; a firm that has chosen a scale with higher ATC earns economic losses and must either move to the minimum efficient scale or leave the industry. A firm still in the economies-of-scale range can become more competitive by expanding output and lowering its unit cost. A firm experiencing diseconomies of scale should reduce its plant size (scale) toward the minimum efficient scale. This is a long-run decision, because plant size cannot change in the short run. The US auto industry is a commonly cited case of diseconomies of scale. Economies of scale are a long-run concept: they describe the LRATC curve. The cost curve of a single existing plant cannot show them.

COMMON EXAM TRAPS

- The minimum efficient scale is the plant size at the lowest point of LRATC. It is not the largest plant the firm could build.

- The output with the lowest ATC is the cheapest output per unit. It is not necessarily the profit-maximizing output, which is where MR = MC.
- Shutting down "temporarily until price exceeds ATC" is wrong when price is above AVC: it would turn a loss smaller than fixed cost into a loss equal to fixed cost.

EXAM SHORTCUTS

- Decide breakeven and shutdown with two comparisons in order: TR against TC, then TR against TVC.
- Short-run shutdown compares price with AVC; long-run exit compares price with ATC.

BOTTOM LINE

- In the short run at least one input is fixed; in the long run every input is variable.
- A price taker breaks even at a price equal to minimum ATC and shuts down in the short run when price falls below minimum AVC.
- When price is between AVC and ATC the firm keeps operating in the short run, because its loss is smaller than its fixed cost, and exits in the long run unless price is expected to rise.
- In any market structure: $TR \geq TC$ means stay; $TVC \leq TR < TC$ means operate in the short run and exit in the long run if the shortfall is expected to persist; $TR < TVC$ means shut down.
- Marginal cost crosses AVC and ATC at their minimum points.
- The minimum efficient scale is the lowest point of LRATC; economies of scale make LRATC fall and diseconomies of scale make it rise.

QUICK CHECK

Question 1

Hartwell Joinery employs 35 carpenters and makes furniture in a factory it leases on a five-year contract, using machinery it owns. Over which inputs does the firm have a choice in the long run?

A The carpenters only.

B The factory and the machinery only.

C The carpenters, the factory and the machinery.

Answer: C

The long run is the period over which every factor of production is variable. The firm can let the lease expire or sign a new one, sell or buy machinery and change its workforce, so all three inputs can be changed. In the short run the factory and machinery are fixed and only labor (and materials) can vary.

Why the other options are wrong

- **A.** Labor alone describes the short run, when the plant and equipment are fixed. In the long run the plant and equipment can be changed as well.
- **B.** This reverses the usual assumption: labor is variable even in the short run, so it is certainly variable in the long run.

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Key takeaway In the long run all costs are variable; the short run is defined by at least one fixed input, usually capital.

MODULE 12.2

Characteristics of Market Structures

LOS 12.b — The four market structures

Every firm maximizes profit by producing the quantity at which **marginal revenue (MR) equals marginal cost (MC)**. What differs is the demand curve the firm faces. Demand is **elastic** when buyers react strongly to a price change (a small price rise loses many sales) and **inelastic** when they react little:

KEY CONCEPT

- Under **perfect competition** the firm is a **price taker**: its demand curve is horizontal (**perfectly elastic**) at the market price, so $P = AR = MR$.
- Under **monopolistic competition**, **oligopoly** and **monopoly** the firm is a **price searcher**: its demand curve slopes downward, so $MR < P$ and, at the profit-maximizing output, $P > MC$. As a rough rule, monopolistic competitors face relatively elastic demand, a monopolist the least elastic demand, and oligopolists something in between.

Why $MR < P$ for a price searcher: to sell one more unit the firm must cut its price, and the lower price applies to every unit it sells, not only to the extra one. MR is the price of the extra unit minus the revenue given up on the units that could have been sold at the higher price.

Revenue schedule of a price searcher (illustrative)

Quantity	Price ($)	Total revenue ($)	Marginal revenue ($)
1	50	50	50
2	46	92	42
3	42	126	34
4	38	152	26
5	34	170	18
6	30	180	10

Example. In the schedule above, moving from 3 to 4 units lowers the price from \$42 to \$38. The fourth unit brings in \$38, but the first three units now sell for \$4 less each, a loss of \$12. So $MR = 38 - 12 = \$26$, the same as $\Delta TR/\Delta Q = (152 - 126)/1$. A price taker faces no such loss: every extra unit sells at the market price, so $MR = P$, and selling 10% more units raises its total revenue by 10%.

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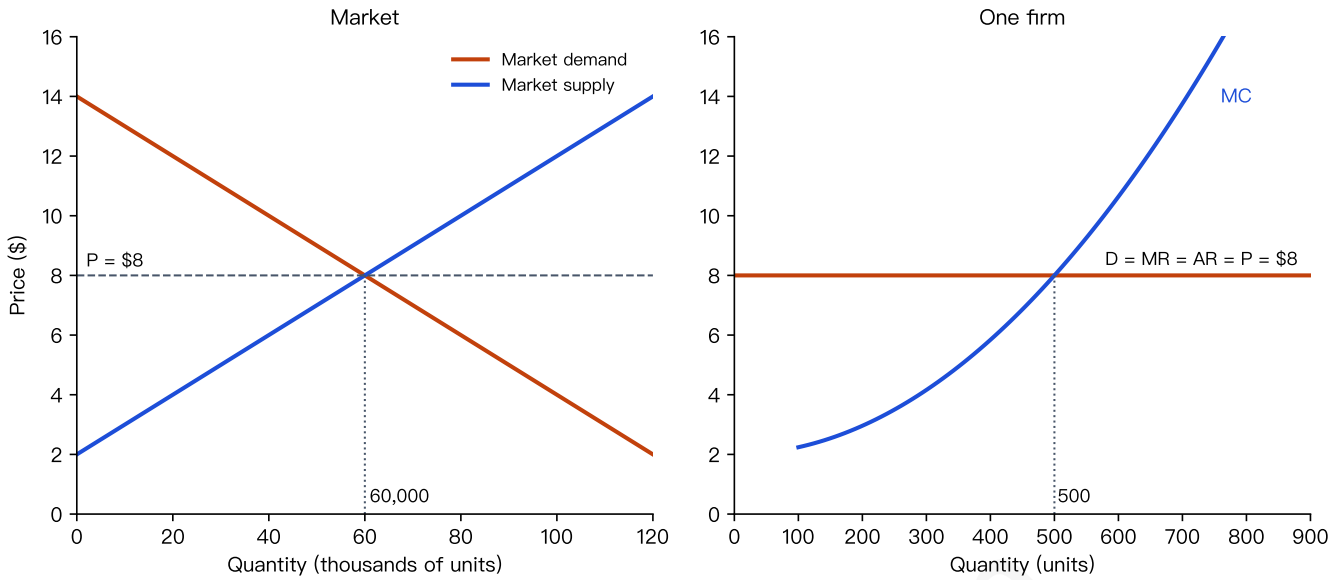
Where a market sits on the spectrum depends on five factors: the number and relative size of firms, the degree of product differentiation, the firms' pricing power, **barriers to entry** and exit, and how much firms compete on things other than price.

Characteristics of the four market structures

Feature	Perfect competition	Monopolistic competition	Oligopoly	Monopoly
Number of sellers	Many	Many	Few	One
Barriers to entry	Very low	Low	High	Very high
Substitute products	Very good substitutes (identical)	Good substitutes, but differentiated	Good substitutes or differentiated	No good substitutes
Nature of competition	Price only	Price, marketing, features	Price, marketing, features	Advertising
Pricing power	None	Some	Some to significant	Significant
Firm demand curve	Horizontal (perfectly elastic)	Downward sloping, relatively elastic	Downward sloping	Downward sloping (market demand)
Long-run economic profit	Zero	Zero	May be positive	May be positive

Perfect competition. Many small firms sell identical (homogeneous) products, barriers to entry and exit are very low, and firms compete only on price. No firm can affect the market price; each can sell all it wants at that price, so charging less than the market price simply lowers its revenue. Example: a regional market for a standard grade of grain. The market price comes from market supply and demand, and the market demand curve slopes downward as usual. Each firm takes that price as given, so the demand curve facing the individual firm is horizontal at it.

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Perfect competition: the market sets the price and each firm takes it as given (illustrative)

Monopolistic competition. Many independent sellers, each a price searcher, with **differentiated products** (quality, features, branding, marketing) and low barriers to entry. Example: toothpaste or restaurants. Heavy advertising is typical.

Oligopoly. A few sellers, each with a large market share; products may be similar or differentiated; high barriers to entry, often due to large **economies of scale**. The defining feature is **interdependence**: each firm's best price and output depend on how its rivals will react. Firm demand slopes downward and can be more or less elastic than under monopolistic competition. Examples: automobiles, commercial aircraft, oil.

Monopoly. One firm is the only seller, buyers have no good substitutes for its product, and very high barriers to entry keep rivals out. The barriers can be patents and copyrights, control of a key resource, or government licensing and regulation (e.g., a local utility). The firm faces the market demand curve and has significant pricing power.

COMMON EXAM TRAPS

- "Maximizes profit" and "produces where MR = MC" apply to all structures, so they never distinguish one structure from another.
- A vertical demand curve is perfectly inelastic, the opposite of the perfectly competitive firm's horizontal curve.
- Barriers to entry under monopolistic competition are low: neither absent altogether nor high.
- Advertising and product differentiation separate monopolistic competition from perfect competition; low barriers and zero long-run economic profit are common to both.

LOS 12.c — Monopolistic competition: price, output and strategy

Features that drive behavior:

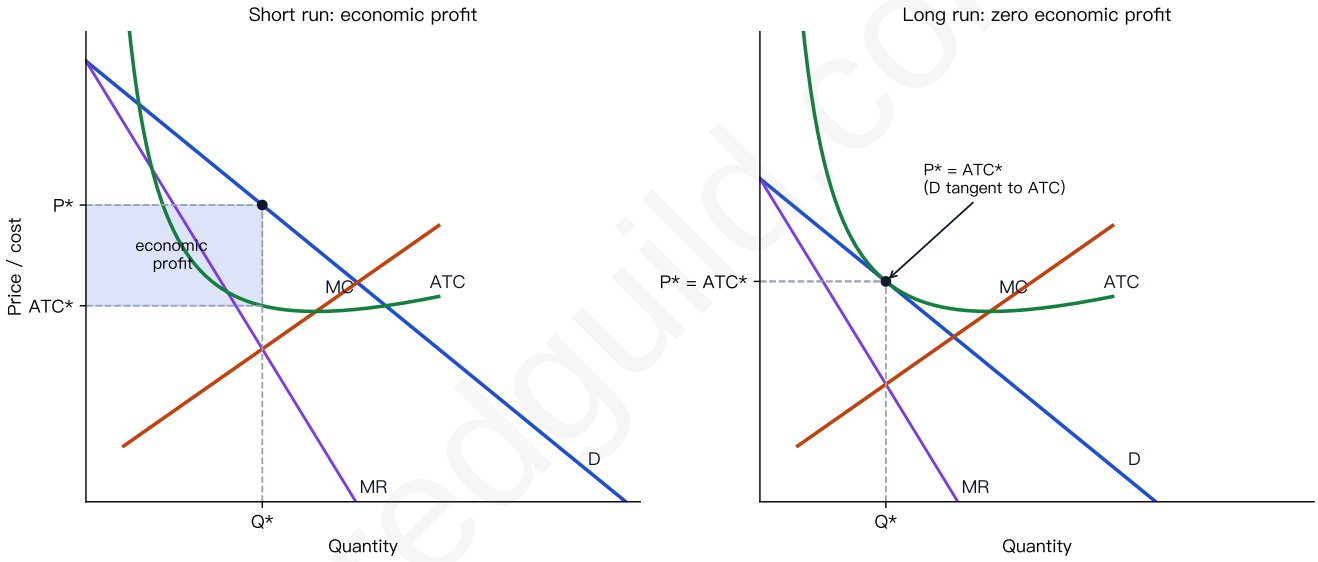
1. **Many independent sellers.** Each firm's share is small, so no firm has significant power over price; firms watch the *average* market price rather than individual rivals' prices; there are too many firms for collusion (price-fixing) to work.

2. **Differentiated products** that are close substitutes. Firms compete more on quality, features and marketing than on price. Because close substitutes exist, firm demand is relatively **elastic**: a firm that raises its price loses many customers.
3. **Low barriers to entry**: because entering and exiting cost little, new firms arrive whenever existing firms earn economic profit.

Short run. The firm sets its price and output in three steps (left panel below):

1. Find the output Q^* at which $MR = MC$.
2. Go up from Q^* to the demand curve and read off the price P^* that buyers will pay for that quantity.
3. Compare P^* with ATC at Q^*. Economic profit is $(P^* - ATC^*) \times Q^*$, positive when price exceeds ATC and negative when it falls short.

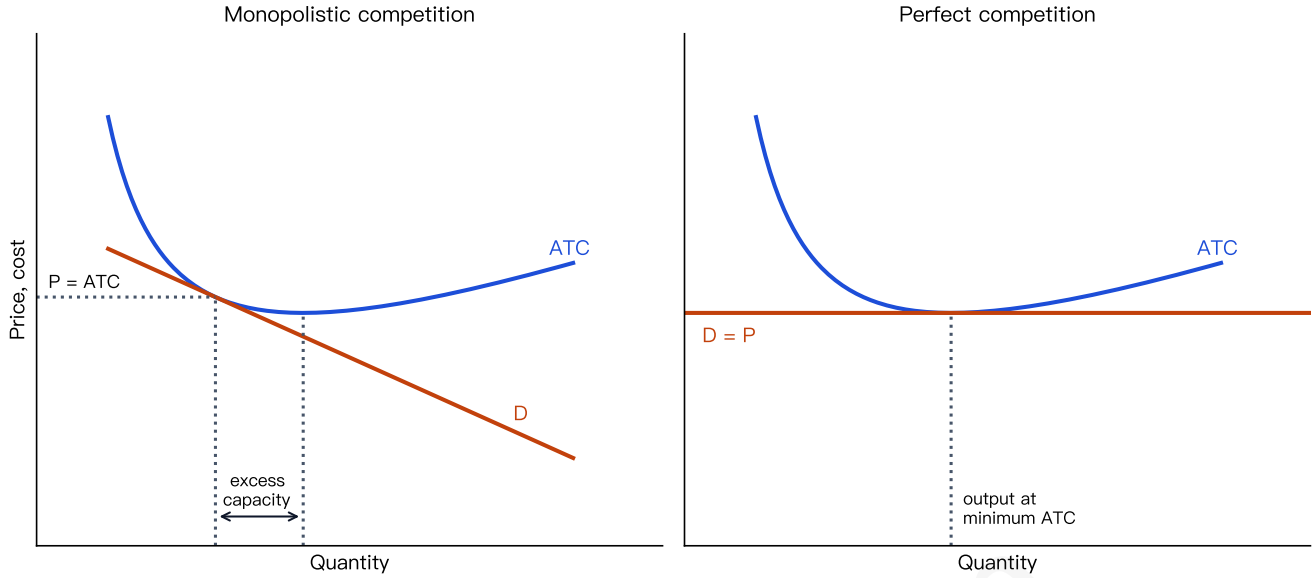
Long run. Low barriers attract entrants, and each existing firm's demand shifts left until it is just tangent to ATC: $P = ATC$ and **economic profit is zero** (right panel). The same outcome can arise without entry if firms keep raising marketing spending (part of ATC) to defend market share. Accounting profit can stay positive even while economic profit is driven to zero.



Monopolistic competition: short-run economic profit and long-run equilibrium

Long-run comparison with perfect competition: in monopolistic competition $P > MR = MC$, ATC is *not* at its minimum (excess capacity), price is higher and quantity lower. The offset is product variety, which consumers value.

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Long-run equilibrium: monopolistic competition versus perfect competition (illustrative)

Product differentiation and brands. The fewer close substitutes buyers see for a product, the steeper (less elastic) its demand curve. A medicine that some patients tolerate better than its rivals faces steeper demand than one brand of shampoo among many, and a new product with no close rival is in the same position. **Brand names** can signal quality and help consumers choose. There is no well-defined firm supply curve.

Worked example. A craft-coffee roaster sells 5,000 bags a month at \$14 while its ATC is \$11: economic profit $(14 - 11) \times 5{,}000 = \$15{,}000$. New roasters enter; the roaster's demand shifts left until, at its new MR = MC output, price equals ATC (for example, both at \$12.50) and economic profit is zero.

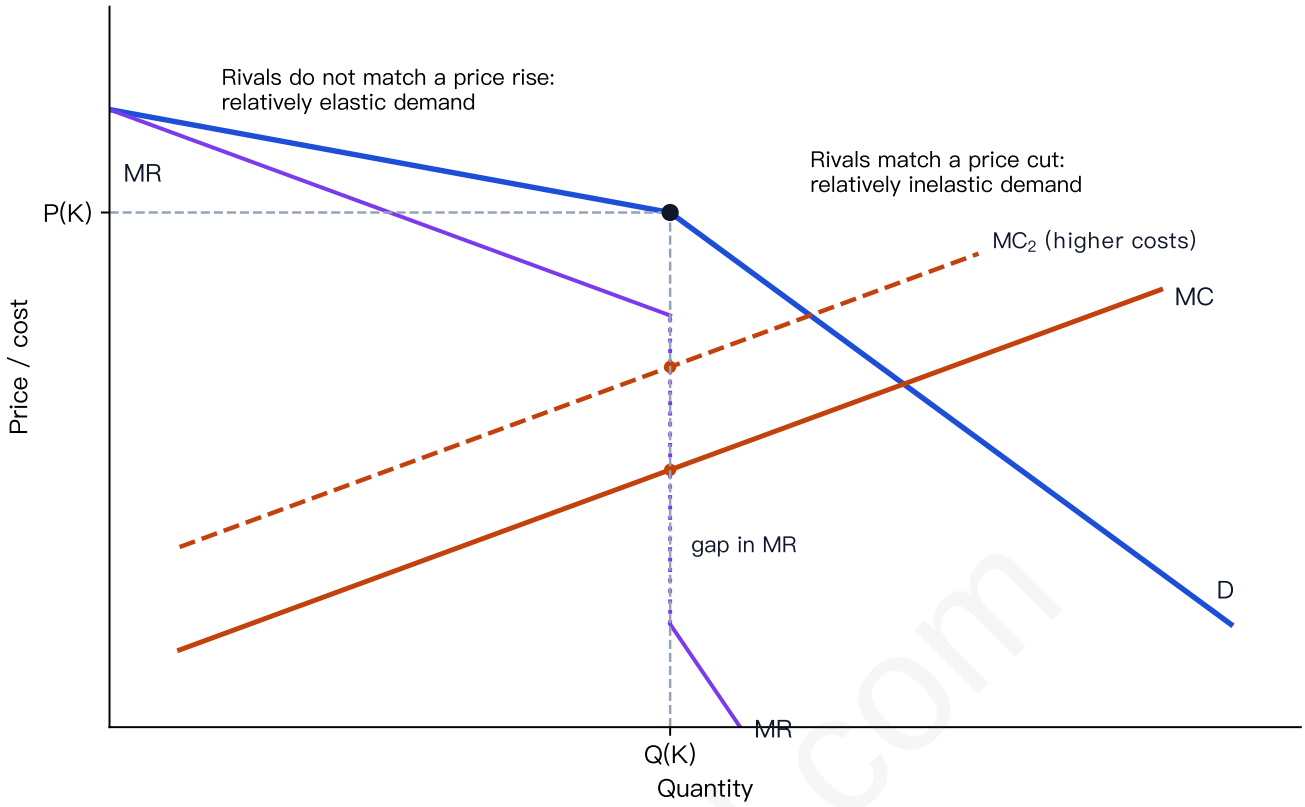
COMMON EXAM TRAPS

- Reading the price off the MR or MC curve at Q^* understates it: both lie below the demand curve there, because $P > MR$ for a price searcher.

LOS 12.d — Oligopoly: interdependence and pricing models

Because oligopolists are **interdependent**, a price change by one firm is likely to trigger a response that shifts the others' demand curves. The optimal price therefore depends on what a firm assumes about its rivals' reactions. Four models:

1. Kinked demand curve model. Rivals are assumed *not* to match a price increase but *to* match a price cut. The firm's demand curve is therefore relatively elastic above the current price P_K and relatively inelastic below it, creating a kink at P_K. Each segment of demand has its own MR curve, and the MR of the steep lower segment starts well below the MR of the flat upper segment, so MR drops straight down at Q_K and leaves a gap. Any MC curve passing through the gap leaves the profit-maximizing price at P_K and output at Q_K. A rise in costs that keeps MC inside the gap (from MC to MC_2 in the figure) therefore leaves price and output unchanged, so prices tend to be sticky. Only a cost change that pushes MC above or below the gap moves them.



Kinked demand curve model: any MC through the MR gap gives the same price and output

2. Cournot duopoly model. Two firms with identical MC curves selling a homogeneous product; each chooses its **output (quantity)** assuming the competitor's output will not change, and the firms choose **simultaneously** each period. The market price is whatever single price clears the combined output. In the long-run equilibrium both firms sell the same quantity and split the market equally, at a price below the monopoly price but above the perfectly competitive price. With more firms, the price moves toward the competitive price.

3. Stackelberg model (the Stackelberg dominant firm model). Decisions are **sequential**: the leader chooses its output first and the follower then chooses its own output in response. The leader exploits this **first-mover advantage** to produce the larger quantity and so gains the larger market share and the larger share of total profits, even in the long run. Both firms still sell the homogeneous product at one common market price; the leader does not charge a different price from the follower.

Exam convention: both models are described with each firm choosing its selling price; a Cournot firm assumes the rival keeps last period's price, and the Stackelberg leader sets its price first, charges a higher price and earns the larger share of total profit. Current practice: both are the quantity-setting models described above, with one market price. Both are rules-based models and fall under **strategic games**, decision models in which a firm's best choice depends on rivals' expected actions.

4. Nash equilibrium. A **Nash equilibrium** is reached when no firm can improve its profit by *unilaterally* changing its choice, given the others' choices. The Cournot outcome is a Nash equilibrium. Absent collusion, the long-run oligopoly equilibrium is a Nash equilibrium.

To find a Nash equilibrium in a payoff table:

1. For each choice the rival could make, mark the firm's best reply, meaning the choice that earns the firm more.
2. Repeat for the rival, taking each of the firm's choices in turn.

3. A cell in which both choices are marked is a Nash equilibrium.

> *Worked example.* Kestrel and Osprey each choose a high or low price; payoffs are shown below. Step 1: if Osprey prices high, Kestrel's best reply is low (850 > 700); if Osprey prices low, it is low again (400 > 250). Step 2: the payoffs are symmetric, so Osprey's best reply is also low whatever Kestrel does. Step 3: only (low, low) has both choices marked, so it is the Nash equilibrium, with 400 each, even though (high, high) would give both 700.

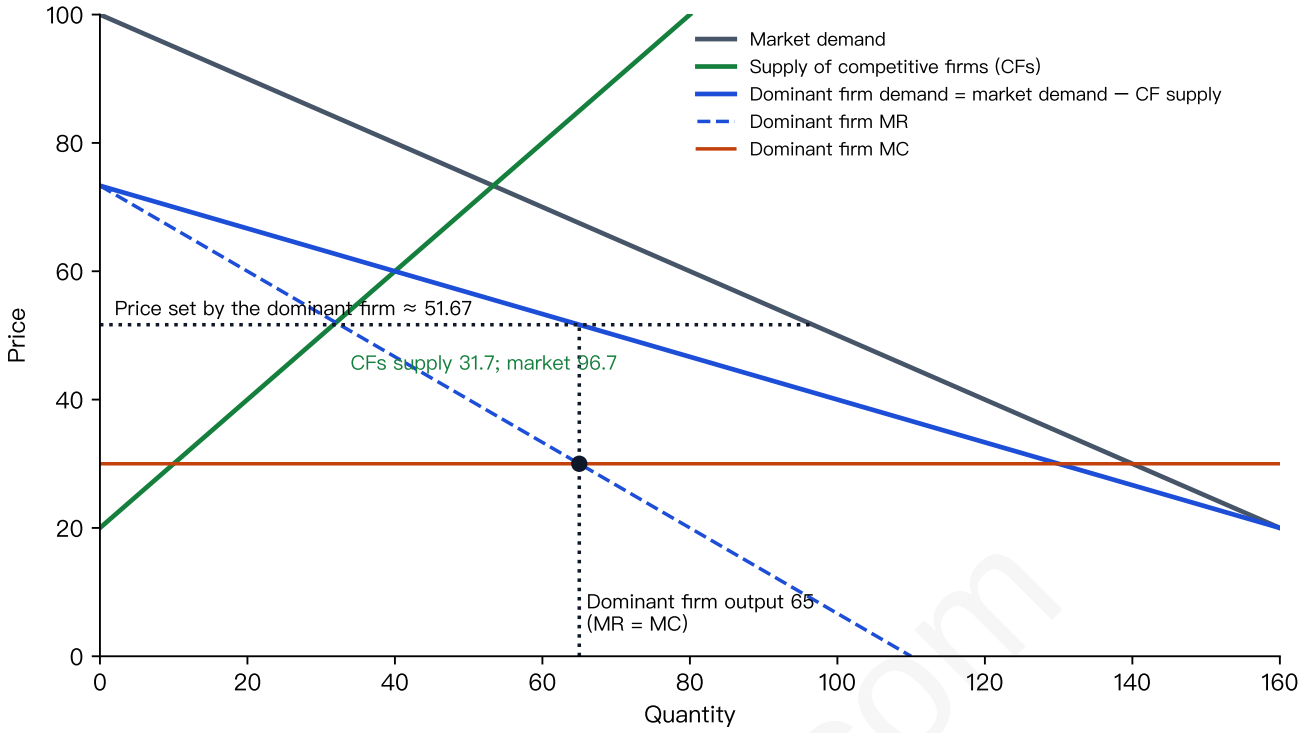
Profits (Kestrel, Osprey) for each pricing combination

	Osprey: high price	**Osprey: low price**
Kestrel: high price	Kestrel 700, Osprey 700	Kestrel 250, Osprey 850
Kestrel: low price	Kestrel 850, Osprey 250	Kestrel 400, Osprey 400

Collusion. Collusion is an agreement among competitors to charge a given price or to limit output to agreed levels. If the firms could agree on (and enforce) high prices, both would earn more; side payments can make the collusive outcome better for each firm than the Nash equilibrium. That is the incentive behind a **cartel** such as OPEC, whose members agree to restrict output to raise price. Members have an incentive to cheat by producing more. Collusion is more likely to hold when there are fewer firms, products are more similar, cost structures are more similar, purchases are small and frequent, retaliation against cheaters is more certain and severe, and there is little competition from outside the cartel. Such agreements are illegal in many countries.

Dominant firm model. One firm with a large market share, owing to its greater scale and lower cost structure, effectively sets the market price; the smaller **competitive firms (CFs)** take that price as given and produce where their MC equals it. The dominant firm's demand curve is market demand minus the CFs' supply (the CFs supply less at lower prices); the dominant firm sets price where its own MR equals its MC. If CFs cut price, the dominant firm follows; in the long run the CFs reduce output or exit, and the dominant firm's market share rises.

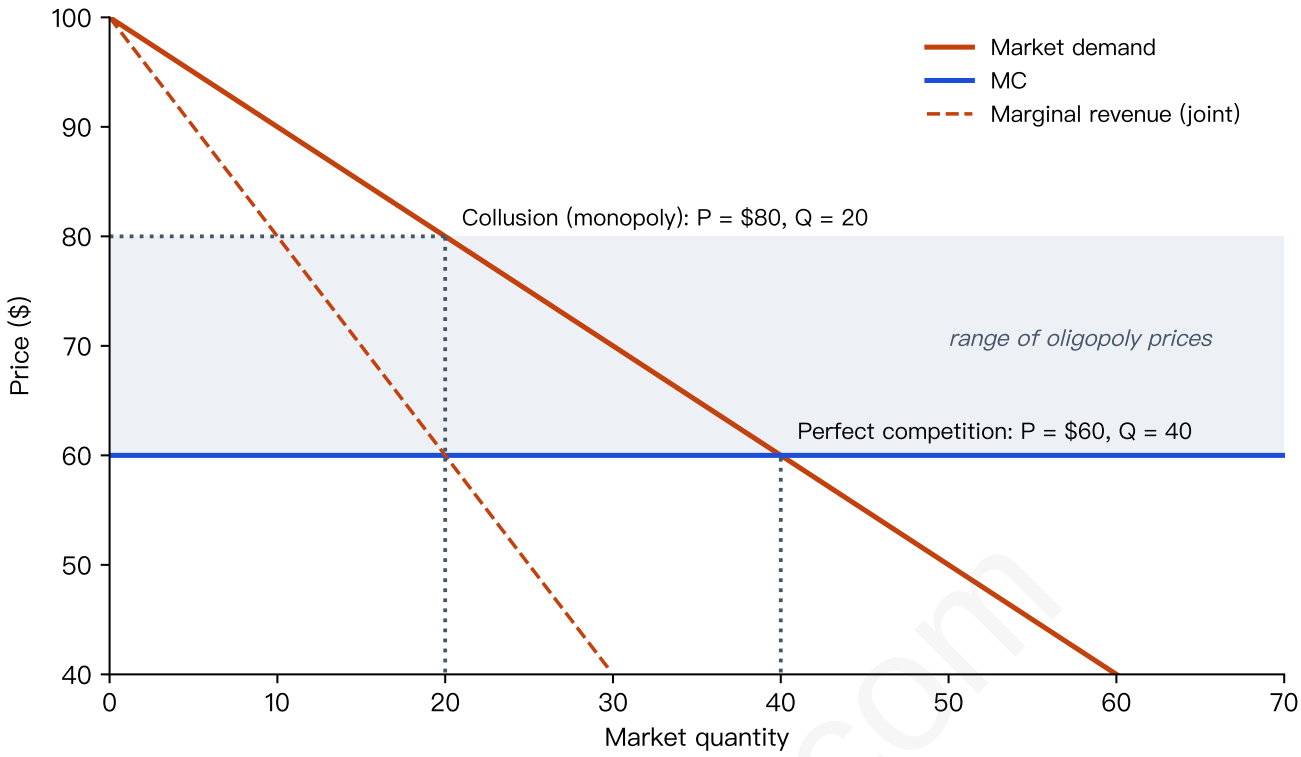
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Dominant firm model (illustrative linear curves): the dominant firm sets price where its MR equals its MC

Range of outcomes. Oligopoly price lies between two limits: the **collusion (monopoly) price** that maximizes joint profit, and the **perfectly competitive price** that yields zero economic profit. Example: if the joint-profit-maximizing price is $80 and the competitive price is $60, the oligopoly price should be somewhere in between, e.g. about $70, and neither above $80 nor below $60.

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Oligopoly outcomes lie between the collusive and the competitive price (illustrative)

As in monopolistic competition, $P > MR = MC$ in equilibrium, there is no well-defined supply curve, and economic profit may be positive in the long run, though it may erode over time.

COMMON EXAM TRAPS

- Cournot and Stackelberg differ in timing (simultaneous versus sequential choices). Both are strategic-game (rules-based) models, so that is not a difference between them.
- An option that cites many sellers or low barriers to entry does not describe an oligopoly.
- A Nash equilibrium need not maximize joint profit; the collusive outcome usually earns more.

EXAM SHORTCUTS

- Classify a market from two facts first: the number of firms and the barriers to entry. Many firms with identical products is perfect competition; many with differentiated products and low barriers is monopolistic competition; a few interdependent firms behind high barriers is oligopoly; one firm with no close substitutes is monopoly.
- To find a Nash equilibrium, mark each player's best reply to every choice of the rival; a cell with both choices marked is an equilibrium.
- Under the kinked demand curve model, a cost change that keeps MC inside the gap in MR leaves price and output unchanged.

BOTTOM LINE

- Every firm maximizes profit where MR = MC; a price taker has $P = MR$, while a price searcher has $MR < P$.

- Monopolistic competition has many firms, differentiated products and low barriers, and earns zero economic profit in the long run with excess capacity.
- Oligopoly is defined by interdependence among a few large firms protected by high barriers to entry.
- Cournot firms decide simultaneously and the Stackelberg leader decides first; exam convention: they choose prices, the Cournot firm assuming its rival keeps last period's price and the Stackelberg leader charging a higher price and earning the larger share of total profit; current practice: they choose quantities and sell at one market price.
- In a Nash equilibrium no firm can gain by changing its choice alone, and the outcome need not maximize joint profit.
- The oligopoly price lies between the collusive (monopoly) price and the perfectly competitive price.

QUICK CHECK

Question 2

An industry study reports the following:

- many sellers act independently of one another;
- product features differ from firm to firm;
- entry into the industry is easy;
- each firm's demand curve slopes downward but is quite elastic.

The industry is best characterized as:

A an oligopoly.

B monopolistic competition.

C a monopoly.

Answer: B

Many independent sellers, differentiated products, low barriers to entry and downward-sloping, highly elastic firm demand are the conditions of monopolistic competition.

Why the other options are wrong

- **A.** An oligopoly has a small number of interdependent sellers and high barriers to entry.
- **C.** A monopoly has a single seller protected by high barriers to entry.

Key takeaway Checklist for monopolistic competition: many sellers, differentiation, easy entry, elastic downward-sloping demand.

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MODULE 12.3

Identifying Market Structures

LOS 12.e — Identifying the market structure and measuring concentration

Identifying the structure

To classify the market a firm operates in, compare the industry with the characteristics of each structure (see the comparison table in Module 12.2): the **number of firms** and their relative sizes, the **barriers to entry**, the nature of **substitute products** (identical, differentiated, none), and the nature of competition (price only versus marketing and features). For an analyst trying to gauge **pricing power**, these are the factors that matter. Significant interdependence in pricing and output is present in every oligopoly, and some interdependence can appear even under monopolistic competition.

Classifying a market does not, however, show how much pricing power each firm has. The ideal would be to measure the **elasticity of firm demand** directly, but that is statistically difficult and imprecise. Regulators and analysts therefore use measures based on market shares, called **concentration measures**, as a proxy for market power and to help identify the market structure. Competition authorities often block mergers when the combined firm's share would be too high.

The two concentration measures

Measure	Calculation	Strength	Weakness
N-firm concentration ratio	Sum of the percentage market shares of the largest N firms	Simple to compute and interpret	Relatively insensitive to mergers among the largest firms; does not measure market power or elasticity directly
Herfindahl-Hirschman Index (HHI)	Sum of the *squared* market shares of the largest N firms	Squaring gives big firms more weight, so it is more sensitive to mergers	Still does not measure market power directly

KEY CONCEPT

$$\text{N-firm concentration ratio} = \sum_{i=1}^{N} s_i \qquad \text{HHI} = \sum_{i=1}^{N} s_i^2$$

where s_i is firm i's market share as a decimal. With shares in percent, the HHI is expressed on a 0 to 10,000 scale. A market served by one firm has a concentration ratio of 100% and an HHI of 1 (10,000 in percent terms), the highest possible values.

To compute either measure:

1. Rank the firms by market share and keep the N largest. If the data are sales rather than shares, first divide each firm's sales by total industry sales, including the sales of all the smaller firms.
2. For the concentration ratio, add their shares. For the HHI, square each share and add the squares.

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3. To assess a merger, combine the merging firms' shares into one firm, re-rank all the firms and repeat steps 1 and 2. The list of the N largest can change: the merged firm may move up the ranking, and if two of the top N firms merge, the next-largest firm moves into the top N.

Worked example. Market shares are shown below. Two of the top five firms, Dunmore and Easton, plan to merge.

Market shares before a merger of Dunmore and Easton

Firm	Market share
Ashby	28%
Brennan	22%
Corlett	16%
Dunmore	10%
Easton	8%
All others (each below 5%)	16%

Before the merger: 4-firm ratio $= 28 + 22 + 16 + 10 = 76\%$; 4-firm HHI $= 0.28^2 + 0.22^2 + 0.16^2 + 0.10^2 = 0.0784 + 0.0484 + 0.0256 + 0.0100 = 0.1624$.

After the merger the combined firm has 18%, and the top four are 28%, 22%, 18%, 16%: 4-firm ratio $= 84\%$; 4-firm HHI $= 0.0784 + 0.0484 + 0.0324 + 0.0256 = 0.1848$.

Limitations common to both measures

- Neither considers **barriers to entry** or **potential competition**. Firms with large market shares may still have little pricing power if entry is easy: would-be entrants stand ready to come in if incumbents raise prices, so firm demand can be quite elastic despite high concentration. A few firms making an easily copied product with no brand or patent protection may therefore behave much like perfectly competitive firms.
- Neither measures the elasticity of firm demand (market power) directly.

COMMON EXAM TRAPS

- Mixing scales: an HHI of 0.1624 (shares as decimals) is the same as 1,624 (shares in percent).
- Reading a small rise in the N-firm ratio after a merger as a small rise in concentration. When two of the top N firms combine, the ratio grows only by the share of the firm that enters the top N, while the HHI can rise sharply.

EXAM SHORTCUTS

- With shares in whole percentages, square and add them to get the HHI on the 0 to 10,000 scale without converting to decimals.
- When two of the top N firms merge, the N-firm concentration ratio rises only by the share of the firm that enters the top N.

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BOTTOM LINE

- A market structure is identified by the number of firms, the barriers to entry, the nature of substitutes and the nature of competition.
- The N-firm concentration ratio is the sum of the N largest market shares; the HHI is the sum of their squared shares.
- The HHI is more sensitive than the concentration ratio to mergers among the largest firms.
- Neither measure accounts for barriers to entry or potential competition, and neither measures the elasticity of firm demand directly.

QUICK CHECK

Question 3

An analyst has annual sales for the producers of industrial fasteners in a region:

Annual sales of industrial fastener producers (€ millions)

Firm	Sales (€ millions)
Arlo	240
Brisk	180
Calder	150
Dunmark	120
Eske	60
All other firms	50

The industry's three-firm concentration ratio and three-firm Herfindahl-Hirschman Index (HHI) are *closest* to:

A 71.25% and 0.1758.

B 71.25% and 0.1983.

C 76.00% and 0.2000.

Answer: A

Both measures use market shares, so each firm's sales must first be divided by total industry sales, including the sales of all the smaller firms. The three-firm concentration ratio adds the shares of the three largest firms; the three-firm HHI adds the squares of those shares.

Total industry sales $= 240 + 180 + 150 + 120 + 60 + 50 = 800$.

Shares of the three largest firms: $240/800 = 30\%$, $180/800 = 22.5\%$, $150/800 = 18.75\%$.

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$$\text{3-firm ratio} = 30 + 22.5 + 18.75 = 71.25\%$$

$$\text{HHI} = 0.30^2 + 0.225^2 + 0.1875^2 = 0.0900 + 0.0506 + 0.0352 = 0.1758$$

Why the other options are wrong

- **B.** 0.1983 also squares Dunmark's 15% share ($0.1758 + 0.0225$), which is a four-firm HHI. The question asks for the three largest firms only.
- **C.** These values leave the other firms' sales of 50 out of the total, so shares are computed on 750: $32\% + 24\% + 20\% = 76\%$ and $0.1024 + 0.0576 + 0.0400 = 0.2000$.

Key takeaway Convert sales to shares using total industry sales, then use the same N largest firms for both measures.

Practice questions

Question 4

Lindqvist Ferries operates in a market in which it faces a downward-sloping demand curve. For the coming year it forecasts total revenue of $2.10 million, total variable costs of $1.65 million and total fixed costs of $0.72 million. If these conditions are expected to persist, the firm should *most appropriately*:

A shut down in the short run, because total revenue is less than total cost.

B continue operating in the short run but exit the market in the long run.

C continue operating in both the short run and the long run, because total revenue exceeds total variable cost.

Question 5

Norden Cement and Solvik Cement are the only two suppliers in a regional market. Each firm independently chooses a high or a low price, and the resulting annual profits are shown below.

Annual profits (in $ thousands) of Norden Cement and Solvik Cement under each pricing strategy

	Solvik: high price	**Solvik: low price**
Norden: high price	Norden 800, Solvik 700	Norden 300, Solvik 900
Norden: low price	Norden 950, Solvik 250	Norden 500, Solvik 450

Based on the concept of a Nash equilibrium, the *most likely* outcome is that:

A both firms charge a high price.

B both firms charge a low price.

C Norden charges a high price and Solvik charges a low price.

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Answers: 4. B · 5. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/oligopoly/. The full bank has 65 questions on this reading.

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ECONOMICS · READING 13

Business Cycle

What you'll learn

LOS 13.a Describe the business cycle, its four phases (trough, expansion, peak, contraction) and the classical, growth and growth rate cycle measures.

LOS 13.b Describe credit cycles and how they interact with and can amplify business cycles.

LOS 13.c Describe how inventories, labor and capital use, consumer, housing and external trade activity vary over the cycle, and classify leading, coincident and lagging indicators.

MODULE 13.1

Business Cycles

This reading describes the four phases of the business cycle, how cycles are drawn and dated, and how credit cycles interact with them. It then explains how inventories, the use of labor and capital, consumer spending, housing and external trade change over the cycle, and how leading, coincident and lagging indicators help identify its turning points.

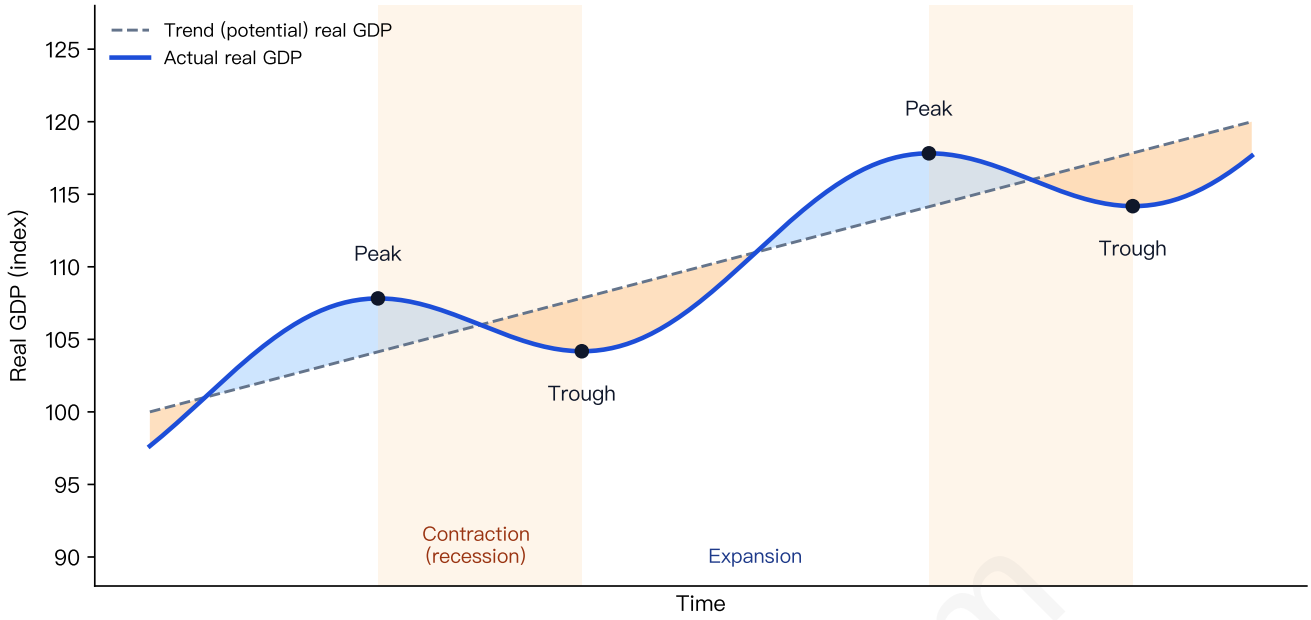
LOS 13.a — The business cycle and its four phases

A **business cycle** is the recurring (but irregular) swing of aggregate economic activity, usually tracked through **real GDP**. Most sectors of the economy grow together in an expansion and shrink together in a contraction. Every cycle passes through four phases:

KEY CONCEPT

Phase	What real GDP is doing	Typical conditions
Trough	Stops falling and starts rising; growth turns from negative to positive	Unemployment high; firms start using more overtime and temporary staff; durable goods and housing spending may pick up; inflation moderate or still falling; employment growth may not pick up until the recovery is well established
Expansion	Rising	GDP growth picks up; unemployment falls as hiring speeds up; investment in equipment and new homes rises; inflation may rise; imports rise with domestic incomes
Peak	Reaches its highest level for the cycle, then starts to fall	Growth slows; unemployment is still falling, but hiring slows; consumer and business spending grow more slowly; inflation is rising
Contraction (recession)	Falling	Hours worked cut, unemployment rises; consumer spending, home building and business investment fall; inflation eases, but with a lag; imports fall

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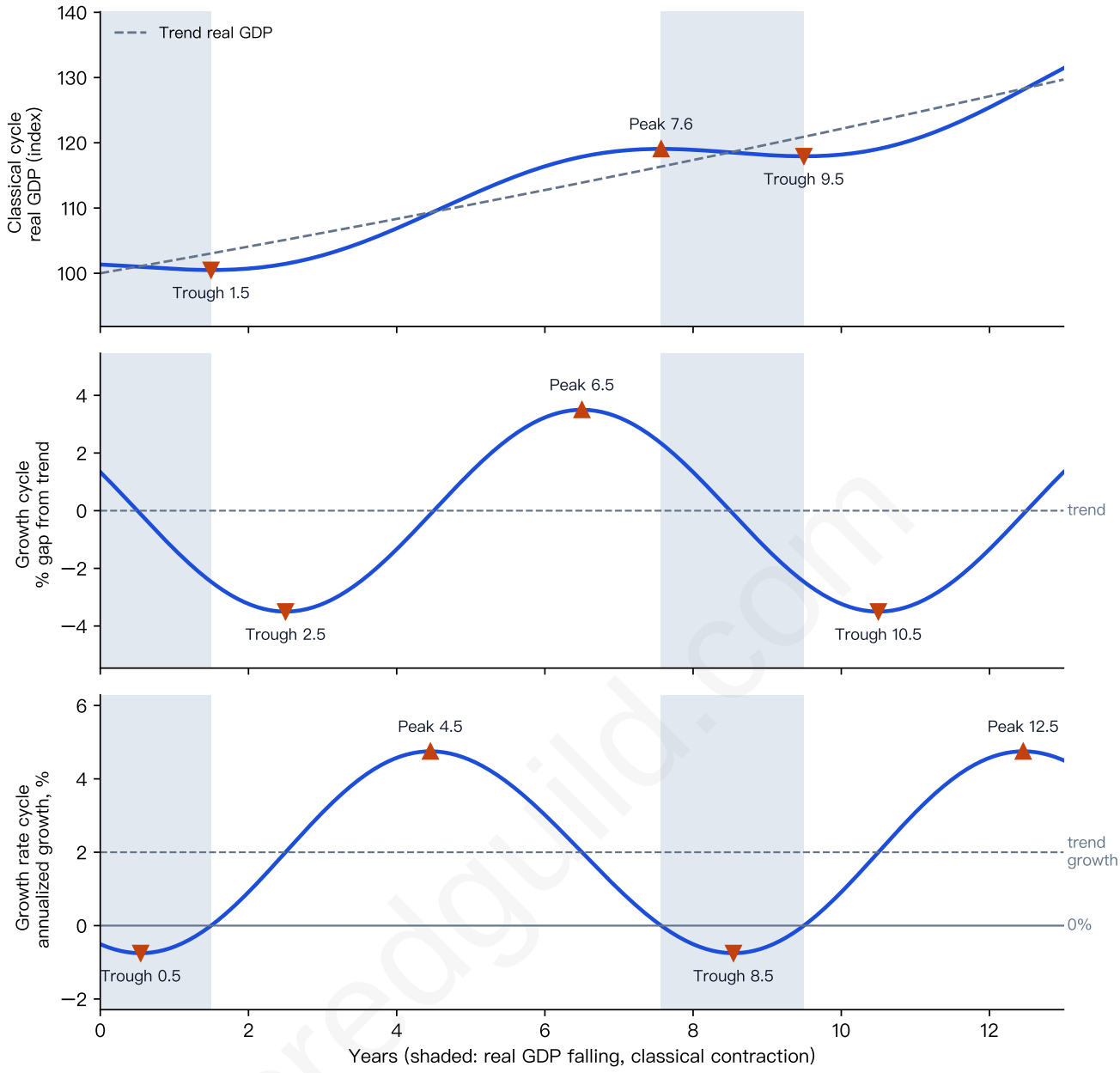
Business cycle phases: actual real GDP fluctuating around its rising trend

Three ways to draw a cycle.

- The **classical cycle** plots the level of real GDP against a starting value; turning points are highs and lows in the *level*.
- The **growth cycle** plots the percentage gap between real GDP and its long-term trend (potential) value.
- The **growth rate cycle** plots the period-to-period (annualized) growth rate of real GDP. It tends to turn *earlier* than the other two measures, at both peaks and troughs. Growth starts to slow while the level of GDP is still rising, and starts to recover while the level is still falling, so the growth rate reaches its high and low points first. Exam convention: the growth rate cycle is the measure economists and practitioners prefer, and it shows GDP relative to a trend rate, like the growth cycle. Current practice: the gap between GDP and its trend is what the growth cycle measures; the growth rate cycle tracks the growth rate itself, which can then be compared with the trend growth rate.

The figure below draws one real GDP path all three ways, with the classical contraction shaded.

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One real GDP path measured three ways: classical, growth and growth rate cycles

Dating a cycle. A common rule of thumb treats two consecutive quarters of falling real GDP as the start of a contraction, and two consecutive quarters of rising real GDP as the start of an expansion. Official dating bodies (in the US, the National Bureau of Economic Research) also weigh unemployment, industrial production, inflation and other data.

Worked example. Real GDP starts at an index of 100.0 and grows as follows over nine quarters:

Quarter	1	2	3	4	5	6	7	8	9
Growth, quarter on quarter (%)	0.6	1.0	0.6	0.3	-0.1	-0.5	-0.2	0.3	0.8
Real GDP (index)	100.60	101.61	102.22	102.52	102.42	101.91	101.70	102.01	102.83

1. Classical cycle: the level of real GDP peaks in quarter 4 (102.52) and bottoms in quarter 7 (101.70).
2. Rule of thumb: real GDP falls in quarters 5 and 6, two quarters in a row, so a contraction began after quarter 4. The rises in quarters 8 and 9 mark the start of the next expansion.
3. Growth rate cycle: growth is highest in quarter 2 and lowest in quarter 6, two quarters before the level peaks and one quarter before it bottoms.

Cycles are not regular. They recur, but their length and severity vary widely, from about a year to more than a decade. The concept applies to economies made up mainly of private businesses. Subsistence and centrally planned economies do not have business cycles in this sense.

Common exam traps

- "Growth turns from negative to positive" describes the trough. The expansion is the phase that follows it.
- Easing inflation belongs to the contraction, not to the expansion or the peak.
- In a contraction, real GDP growth is low or negative, never above its sustainable long-run rate.

LOS 13.b — Credit cycles

Credit cycles are cyclical swings in the availability of loans and in interest rates. In expansions lenders are eager to lend and charge lower rates. When the economy slows, they tighten standards and demand higher rates.

- Credit cycles tend to amplify business cycles. Expansions that coincide with easy credit tend to be stronger, and contractions that coincide with tight credit tend to be deeper and longer.
- Loose credit can feed **asset price bubbles**, meaning prices built on unrealistic expectations. Housing and mortgage credit before 2007–2009 is the classic case.
- The two cycles do not always line up. On average, credit cycles have lasted longer than business cycles.
- Credit-sensitive sectors such as construction and housing move most closely with the credit cycle.

Common exam traps: an option saying that credit cycles dampen business cycles, or that they are shorter than business cycles, reverses the relation.

LOS 13.c — How activity varies over the cycle, and economic indicators

Inventories and the inventory-sales ratio. Firms aim for a normal **inventory-sales ratio** during steady growth.

- *Late in an expansion (approaching the peak):* sales growth slows while production still reflects optimistic plans, so unsold goods pile up and the ratio rises above normal. Firms then cut production, which helps bring on the contraction. Unplanned inventory growth counts as output in GDP, so GDP alone may look strong just as weakness begins.
- *Near the trough:* firms have already cut output. When sales pick up, inventories run down quickly and the ratio falls below normal, so firms raise production to restock.
- *Inventories falling faster than planned* mean sales are running ahead of production, so firms raise output to rebuild stock. If the economy is already at full employment, the extra spending also tends to push inflation up, as it does when an expansion nears its peak.

Worked example. A distributor normally holds inventory of 1.5 times monthly sales, for example 330 against sales of 220. Late in an expansion, sales slip to 210 while deliveries planned earlier lift inventory to 357.

The ratio is now $357/210 = 1.70$, well above normal, which is a signal to cut orders and production.

Labor and physical capital. Hiring and firing are costly and hurt morale, so firms adjust existing resources first:

KEY CONCEPT

	Early response (change intensity)	Later response (change capacity), once the trend looks durable
Labor	Change overtime and output per hour	Hire (expansion) or lay off workers (contraction)
Physical capital	Run existing plant and equipment more or less intensively	Expansion: invest in new capacity. Contraction: defer maintenance and delay replacing worn-out equipment rather than sell assets

Consumer sector. Consumer spending is the largest part of GDP and rises and falls with current and expected income.

- **Durable goods** (cars, appliances, furniture) are the most cyclical. Buyers postpone them in contractions and sometimes into early recovery.
- **Services** are moderately cyclical: travel and restaurant meals are discretionary, while health care and telecom are not.
- **Nondurable goods** (groceries, household supplies) are the least cyclical.

Housing sector. Housing is small relative to consumer spending but swings sharply. Its main drivers are:

- **Mortgage rates**: low rates stimulate buying and building.
- **Housing costs relative to income**: activity can fall late in an expansion if prices outrun incomes.
- **Speculative activity**: rising prices attract buyers who expect further gains, which leads to overbuilding and then a slump.
- **Demographics**: a larger share of people aged 25–40 means more household formation, and urbanization raises construction needs.

External trade sector.

- **Imports** rise with domestic GDP growth: when home incomes rise, residents buy more foreign goods.
- **Exports** depend mainly on the GDP growth of trading partners.
- An appreciating domestic currency tends to raise imports and reduce exports; a depreciating currency does the reverse. Currency effects work through persistent trends and can point in a different direction from GDP effects. GDP effects are more direct and immediate.

Economic indicators.

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KEY CONCEPT

Type	Turning points and use	US examples
Leading indicators	Before peaks and troughs; used to anticipate a turn	Average weekly hours in manufacturing; average weekly initial claims for unemployment insurance; manufacturers' new orders; building permits for private housing; the S&P 500 index; the 10-year Treasury minus fed funds spread; average consumer expectations for business conditions (index of consumer expectations); ISM new orders index; Leading Credit Index
Coincident indicators	At about the same time; show the current phase	Employees on nonagricultural payrolls; personal income less transfer payments; **manufacturing and trade sales**; **industrial production**
Lagging indicators	After the turn is under way; confirm a turn that has already happened	**Average duration of unemployment**; ratio of manufacturing and trade **inventories to sales**; change in unit labor costs; average bank prime rate; commercial and industrial loans outstanding; ratio of consumer installment credit to personal income; change in CPI for services

Other countries build their own composite indexes from different components.

Most leading series turn early for a plain reason: they record decisions that come before output changes. Firms cut overtime (average weekly hours) before they lay workers off, new orders and building permits come before production and construction, and share prices reflect expected future earnings.

Common exam traps

- The inventory-sales ratio is sometimes used in forecasting, but it is classified as a lagging indicator because it peaks after the economy does.
- *Initial* jobless claims lead; the *duration* of unemployment lags.
- Layoffs and asset sales are not a firm's first response to a slowdown.
- Domestic expansion raises imports. It does not directly raise or cut exports.

EXAM SHORTCUTS

- The growth rate cycle reaches its high and low points before the level of real GDP does, because growth slows while the level is still rising and recovers while the level is still falling.
- Series that record decisions made before output changes, such as average weekly hours, new orders, building permits and share prices, are leading indicators.

BOTTOM LINE

- The business cycle has four phases: trough (real GDP stops falling and growth turns positive), expansion (real GDP rising), peak (real GDP at its high for the cycle, then starting to fall) and contraction or recession (real GDP falling).

- The classical cycle plots the level of real GDP and the growth cycle its percentage gap from trend; the growth rate cycle turns earlier at peaks and troughs and, under the exam convention, is the preferred measure that shows GDP relative to a trend rate, while in current practice it tracks the growth rate itself, which can be compared with trend growth.
- A common rule of thumb dates a contraction from two consecutive quarters of falling real GDP and an expansion from two consecutive quarters of rising real GDP, while official dating bodies also weigh unemployment, industrial production and inflation.
- Credit cycles, swings in loan availability and interest rates, tend to amplify business cycles, can feed asset price bubbles and have on average lasted longer than business cycles.
- The inventory-sales ratio rises above normal late in an expansion as sales growth slows, leading firms to cut production, and falls below normal near the trough as sales pick up, leading firms to raise output.
- Firms first change how intensively they use existing workers and equipment, and only once the trend looks durable do they hire or lay off workers, invest in new capacity, or defer maintenance and delay replacing equipment.
- Durable goods spending is the most cyclical, services spending moderately cyclical and nondurable goods spending the least; imports rise with domestic GDP growth, while exports depend mainly on trading partners' growth.
- Leading indicators turn before peaks and troughs, coincident indicators turn at about the same time, and lagging indicators turn after a new phase is under way and confirm it.

QUICK CHECK

Question 1

An economy that is in the contraction phase of the business cycle would *most typically* display:

A a falling unemployment rate.

B real GDP growth above its sustainable long-term rate.

C decreasing inflationary pressure.

Answer: C

A contraction (recession) is a period of declining real GDP. Weaker demand and rising unemployment reduce pressure on prices, so inflation pressure typically decreases, often with a lag.

Why the other options are wrong

- **A.** Unemployment usually rises in a contraction because hours are cut and, if the downturn persists, workers are laid off.
- **B.** In a contraction real GDP growth is low or negative. Growth above the sustainable rate is associated with an expansion, especially one nearing its peak.

Key takeaway Contraction: output falls, unemployment rises, inflation eases.

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Practice questions

Question 2

A bank economist makes three claims about the lending (credit) cycle and its link to the business cycle. Which claim is *least accurate*?

A Easy credit conditions during an expansion can contribute to asset price bubbles.

B Credit cycles tend to amplify business cycles.

C On average, credit cycles have been shorter than business cycles.

Question 3

Orders at Brightwater Castings, a maker of pump housings, have started to fall as the economy begins to contract. Management is not yet convinced the downturn will last and wants to keep its workforce and its productive capacity intact for now. Its *first* adjustment is *most likely* to be:

A laying off part of its workforce.

B postponing scheduled maintenance on its machinery.

C cutting back the overtime hours its employees work.

Question 4

An analyst wants a US data series that tends to change direction *before* the overall economy does. Which of the following should she choose?

A The index of consumer expectations.

B The average duration of unemployment.

C The index of industrial production.

Answers: 2. C · 3. C · 4. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/business-cycle/. The full bank has 21 questions on this reading.

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ECONOMICS · READING 14

Fiscal Policy

What you'll learn

LOS 14.a Compare fiscal policy (taxes and spending, set by government) with monetary policy (money and credit, set by the central bank).

LOS 14.b Describe the objectives of fiscal policy, discretionary policy versus automatic stabilizers, and the arguments for and against concern about deficits and the debt-to-GDP ratio.

LOS 14.c Describe spending and revenue tools of fiscal policy, their advantages and disadvantages, the fiscal multiplier, the balanced budget multiplier and Ricardian equivalence.

LOS 14.d Explain how fiscal policy is implemented, the recognition, action and impact lags and other difficulties, and how to judge whether policy is expansionary or contractionary.

MODULE 14.1

Fiscal Policy Objectives

This reading compares fiscal and monetary policy, sets out the objectives of fiscal policy and the arguments over whether the size of the national debt relative to GDP matters. It then covers spending and revenue tools, the fiscal and balanced budget multipliers, the lags and difficulties of implementation, and how to judge whether fiscal policy is expansionary or contractionary.

LOS 14.a — Fiscal policy versus monetary policy

Fiscal policy is the government's use of **taxation** and **government spending** to influence economic activity. **Monetary policy** is the central bank's management of the **quantity of money and credit** (and so of short-term interest rates) in the economy.

Budget vocabulary: the budget is **balanced** when tax revenues equal expenditures. A **budget surplus** means revenues exceed spending, and a **budget deficit** means spending exceeds revenues.

KEY CONCEPT

	Fiscal policy	Monetary policy
Who decides	Government (legislature / finance ministry)	Central bank
Tools	Tax rates, government spending, transfer payments	Money supply, policy (short-term) interest rates, credit availability
Expansionary form	Higher spending or lower taxes, so a larger deficit or smaller surplus	More money and credit, lower short-term rates: **expansionary** (**accommodative**, **easy**)
Contractionary form	Lower spending or higher taxes, so a smaller deficit or larger surplus	Less money and credit, higher short-term rates: **contractionary** (**restrictive**, **tight**)

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	Fiscal policy	Monetary policy
Goals	**Price stability** and **economic growth**, plus **redistribution of income and wealth**	Price stability and economic growth

Key points:

- Only fiscal policy is a direct tool for **redistribution** of income and wealth, through who pays taxes and who receives transfers.

Common exam traps: "fiscal policy aims at growth, monetary policy at price stability" is false because both aim at both. "Accommodative" is another name for expansionary monetary policy, not a neutral stance.

LOS 14.b — Roles and objectives of fiscal policy; does the debt-to-GDP ratio matter?

Objectives of fiscal policy

1. Influencing the level of economic activity and **aggregate demand**.
2. Shifting income and wealth between different groups in the population.
3. Allocating resources among economic agents and sectors.

Lower taxes and higher spending raise aggregate demand, growth and employment; higher taxes and lower spending do the opposite.

Schools of thought. Keynesian economists hold that fiscal policy can strongly affect output when the economy is below full employment. **Monetarists** see fiscal stimulus as only temporary and prefer monetary policy for managing inflation over time. They also oppose using monetary policy to fine-tune aggregate demand over the cycle.

Discretionary fiscal policy vs. automatic stabilizers

KEY CONCEPT

	Discretionary fiscal policy	Automatic stabilizers
What it is	Active, deliberate decisions on spending and taxes to steer the economy	Built-in features of the tax and transfer system that respond to the state of the economy
New legislation needed?	Yes, so it is slowed by the policy lags described under LOS 14.d	No, so they work without the delay of legislative action
Examples	A stimulus package, a temporary tax cut, a new infrastructure program	**Unemployment compensation**, the **progressive income tax**, the corporate profits tax
Effect in a recession	Whatever lawmakers choose	Transfers rise and tax receipts fall, so the budget moves toward **deficit** (expansionary)

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	Discretionary fiscal policy	Automatic stabilizers
Effect in a boom	Whatever lawmakers choose	Tax receipts rise and transfers fall, so the budget moves toward **surplus** (contractionary)

Automatic stabilizers therefore deliver countercyclical fiscal policy and dampen the cycle.

Debt dynamics. Persistent deficits build up debt, which carries interest costs. Analysts judge deficits, debt and interest expense relative to GDP. When these ratios climb past certain levels, markets may start to doubt the country's solvency. The **debt ratio** is aggregate government debt divided by GDP. Because tax revenue is linked to GDP, it grows as real GDP grows. Exam convention: with tax rates held constant,

- if the real interest rate on government debt is above the real growth rate of GDP, the debt ratio rises over time;
- if the real interest rate is below real growth, the debt ratio falls.

Current practice: the rule holds when the **primary budget** (the budget balance before interest payments) is balanced and other changes in the debt, such as valuation changes, are ignored. A primary deficit can raise the ratio even when the real interest rate is below real growth, and a primary surplus can lower it even when the real interest rate is above real growth.

Worked example. Debt is 70% of GDP, the real interest rate on the debt is 4%, real GDP growth is 2%, and the primary budget is balanced. The debt grows only by the interest on it (4%) while GDP grows 2%, so next year the ratio is about $0.70 \times 1.04/1.02 = 0.714$. It creeps up to about 71.4%.

With a primary deficit, next year's ratio $\approx d \times (1+r)/(1+g)+$ primary deficit as a share of GDP. For example, with $d = 60\%$, $r = 1\%$, $g = 3\%$ and a primary deficit of 4% of GDP, the ratio becomes $0.60 \times 1.01/1.03 + 0.04 \approx 62.8\%$.

Arguments for and against concern about the size of a fiscal deficit

KEY CONCEPT

Reasons for concern	Reasons against concern
Higher deficits imply **higher future taxes**, which weaken incentives to work and start businesses and so reduce long-term economic growth	If the debt is held mostly by the country's **own citizens**, the problem is overstated
Markets may lose confidence and refuse to refinance. This can lead to default on foreign-currency debt, or to money printing and inflation for local-currency debt	Debt that finances **productive capital investment** can pay for itself through future growth
The **crowding-out effect**: government borrowing displaces private borrowing and investment (steps below)	Deficits may prompt needed **tax reform**

Reasons for concern	Reasons against concern
	Ricardian equivalence: households raise saving to meet the expected future taxes, which just offsets the deficit's effect on demand
	If the economy is below full capacity, deficits do not divert capital from productive uses and can raise GDP and employment

Crowding out, step by step: a larger deficit is financed by borrowing in the loanable funds market. Demand for loanable funds rises, the real interest rate rises, and some private investment projects become uneconomic, which offsets part of the deficit's boost to aggregate demand.

Common exam traps

- Listing Ricardian equivalence as a reason to worry about deficits reverses it.
- Crowding out is not a multiplier effect, and it does not work through higher private saving; the saving response is Ricardian equivalence.
- Unemployment compensation is an automatic fiscal stabilizer. It is neither discretionary nor a monetary policy tool.

EXAM SHORTCUTS

- Classify a policy by who acts: taxes, government spending and transfers set by the government are fiscal policy, while money, credit and short-term rates managed by the central bank are monetary policy.

BOTTOM LINE

- Fiscal policy is the government's use of taxes and government spending to steer economic activity, and monetary policy is the central bank's management of the quantity of money and credit.
- Both policies aim at price stability and economic growth, and only fiscal policy is a direct tool for redistributing income and wealth.
- Expansionary fiscal policy means higher spending or lower taxes, giving a larger deficit or smaller surplus; expansionary monetary policy is also called accommodative or easy, and contractionary monetary policy restrictive or tight.
- Automatic stabilizers such as unemployment compensation and the progressive income tax push the budget toward deficit in a recession and toward surplus in a boom without new legislation, while discretionary fiscal policy needs legislation and is slowed by policy lags.
- Exam convention: with tax rates held constant, the debt ratio rises when the real interest rate on government debt exceeds real GDP growth and falls when it is below; current practice: this holds when the primary budget is balanced, since a primary deficit or surplus can reverse the result.
- Concerns about deficits are higher future taxes, a loss of market confidence and crowding out of private investment; arguments against concern include debt held by citizens, debt that finances productive investment, needed tax reform, Ricardian equivalence and an economy below full capacity.

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QUICK CHECK

Question 1

To support a slowing economy, the finance ministry of Norvessa announces a cut in personal income tax rates together with a larger budget for road and rail projects. These measures are *best* described as:

A fiscal policy.

B monetary policy.

C a combination of fiscal and monetary policy.

Answer: A

Fiscal policy is the government's use of taxation and spending to influence economic activity. Cutting tax rates and raising infrastructure spending are both fiscal tools, and they are chosen here to influence growth.

Why the other options are wrong

- **B.** Monetary policy is conducted by the central bank through the quantity of money and credit (and short-term interest rates). Neither tax rates nor government spending are monetary tools.
- **C.** No central bank action (money supply or policy rate change) is involved, so nothing in the package is monetary policy.

Key takeaway Taxes and government spending mean fiscal policy; money, credit and policy rates mean monetary policy.

MODULE 14.2

Fiscal Policy Tools and Implementation

LOS 14.c — Fiscal policy tools: spending and revenue, advantages and disadvantages

Spending tools

- **Transfer payments** (entitlement programs) such as government pensions and unemployment benefits redistribute income. They are *not* counted in GDP, because they move income from taxpayers to recipients without paying for any newly produced good or service.
- **Current spending** is routine government purchases of goods and services.
- **Capital spending** pays for infrastructure (for example, roads, bridges, schools and hospitals) and is expected to raise future productivity.

Reasons for government spending include providing public goods (e.g., national defense), building infrastructure, supporting growth and employment through aggregate demand, guaranteeing a minimum standard of living, and subsidizing high-risk research (e.g., green technology).

Revenue tools

- **Direct taxes** are levied on income or wealth: income, wealth, estate, corporate, capital gains and social security taxes. Progressive direct taxes help redistribute income.

- **Indirect taxes** are levied on goods and services: sales taxes, **value-added taxes (VATs)** and excise taxes. They can discourage specific consumption (tobacco, alcohol, gambling).

Desirable attributes of tax policy: simplicity (easy to comply with and to enforce); **efficiency** (little interference with market forces or work incentives); **fairness**, which includes **horizontal equality** (similar people pay similar taxes) and **vertical equality** (richer people pay more); and **sufficiency** (enough revenue to meet spending needs).

Advantages	Disadvantages
Indirect taxes can be changed quickly to implement social policy	Direct taxes and transfer payments take time to change, which delays the effect
Indirect taxes can raise revenue quickly at little extra cost	Capital spending takes a long time; the economy may have recovered before it bites

Announcing a fiscal change can itself move expectations. A pledge to raise taxes next year may cut consumption at once, so the drop in aggregate demand arrives well before the tax does.

Which tools pack the biggest punch? Spending has the largest effect on aggregate demand because every unit is spent. Tax cuts are weaker because households save part of the tax cut instead of spending it. The **marginal propensity to consume (MPC)** is the share of an extra unit of disposable income that households spend. The rest is saved, so the **marginal propensity to save (MPS)** is $1 - MPC$. Tax cuts aimed at low-income households work better because those households have a higher MPC.

The fiscal multiplier. Extra government spending becomes someone's income, and part of that income is spent again. The rounds build up as follows:

1. Spending of ΔG raises incomes by ΔG.
2. Taxes take the fraction t, so disposable income rises by $\Delta G(1 - t)$.
3. Households spend the fraction MPC of that, so the next round of income is $\Delta G \times MPC(1 - t)$.
4. Every later round is $MPC(1 - t)$ times the one before, so the rounds shrink and their total converges:

KEY CONCEPT

$$\text{fiscal multiplier} = \frac{1}{1 - MPC(1 - t)}$$

The multiplier rises with the MPC and falls with the tax rate.

Worked example. With $MPC = 0.85$ and $t = 0.20$, each round of income is $0.85 \times 0.80 = 0.68$ times the one before. A \$40 billion rise in spending adds \$40 billion of income, then $40 \times 0.68 = \$27.2$ billion, then $27.2 \times 0.68 \approx \$18.5$ billion, and so on. The multiplier is $1/(1 - 0.68) = 1/0.32 = 3.125$, so aggregate demand can rise by up to $40 \times 3.125 = \$125$ billion.

Balanced budget multiplier. A tax increase lowers disposable income, and the first-round cut in spending is $\Delta T \times MPC$, which is then multiplied:

$$\Delta AD_{\text{tax}} = -\Delta T \times MPC \times \text{fiscal multiplier}$$

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Continuing the example, a \$40 billion tax increase reduces AD by $40 \times 0.85 \times 3.125 = \$106.25$ billion. Exam convention: an equal rise of \$40 billion in government spending and (autonomous) taxes leaves the budget balanced and raises AD by $125 - 106.25 = \$18.75$ billion, so the **balanced budget multiplier is positive**. Current practice: with a 20% income tax, the higher output also brings in extra tax receipts of about $0.20 \times 18.75 = \$3.75$ billion, so the budget ends slightly in surplus; the direction of the output effect is unchanged. The multiplier is positive because all of the extra spending enters the economy, while part of the tax increase comes out of saving (the MPS share) rather than consumption. To leave AD unchanged, taxes would have to rise by $40/0.85 \approx \$47.06$ billion.

Tax cuts and Ricardian equivalence. Ricardian equivalence (LOS 14.b) also limits what a debt-financed tax cut can do. If taxpayers save the whole cut to meet the future tax bill, aggregate demand does not change. If they underestimate that liability, the equivalence fails. Whether it holds in practice is an open question.

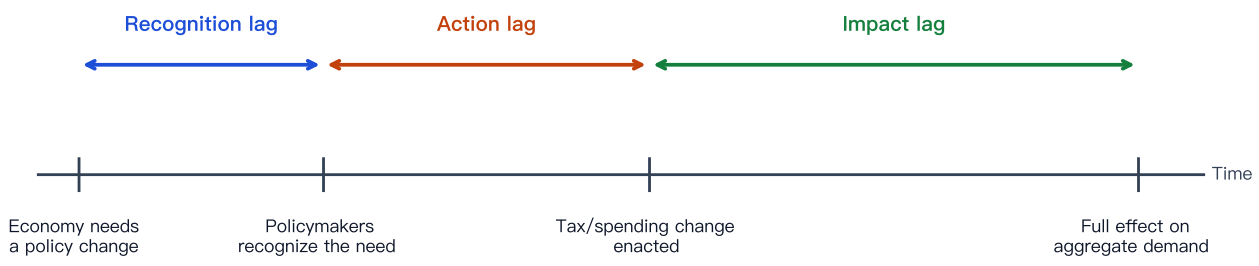
Common exam traps

- Equal increases in spending and taxes are not neutral: real GDP rises. In the worked example, AD rises by \$18.75 billion instead of staying unchanged.
- When consumption is given as a share of *pre-tax* income, that share already equals $MPC(1-t)$. Do not multiply it by $(1-t)$ again.
- Use $(1-t)$ in the multiplier, not t, and don't forget the tax rate. $1/(1-MPC)$ ignores taxes and overstates the effect. In the worked example, $1/(1-0.85) = 6.67$ and $1/(1-0.85 \times 0.20) = 1.20$, instead of 3.125.
- Cutting the fed funds target rate or changing the money supply is monetary policy, even when the goal is the same.

LOS 14.d — Implementing fiscal policy: lags, difficulties, and judging the stance

Discretionary fiscal policy should be expansionary (higher spending, lower taxes) when the economy is below full employment, and contractionary (lower spending, higher taxes) during an inflationary boom. The aim is to stabilize aggregate demand by changing how much households and firms have to spend and invest.

Why timing is hard. Forecasts can be wrong, and recessions and expansions cannot be predicted accurately. On top of that, three lags separate the need for policy from its effect:



The three lags of discretionary fiscal policy

KEY CONCEPT

Lag	Time between …	Example
Recognition lag	the economy needing a change and policymakers realizing it (the data describe where the economy was a few months ago, not where it is now)	Data arrive late and are revised, so the slowdown is confirmed only months later
Action lag	recognizing the need and enacting the change	Drafting, debating and voting on a budget bill
Impact lag	enactment and the change actually affecting the economy	Households and firms respond gradually, and multiplier rounds take time

Because of these lags, a badly timed policy can *destabilize* the economy. Stimulus may arrive after a private-sector recovery is already under way. Proper timing is what lets discretionary policy exert a stabilizing influence.

Other difficulties

- **Misreading economic statistics:** full employment cannot be measured precisely. Stimulus applied at full capacity only raises inflation.
- **Crowding-out effect** (LOS 14.b): it can shrink the demand boost from expansionary policy, and opinions differ on how large it is.
- **Supply shortages:** if output is held back by scarce labor or resources rather than weak demand, stimulus mainly produces inflation.
- **Limits to deficits:** if markets consider the deficit already too high relative to GDP, financing becomes hard and interest rates rise.
- **Multiple targets:** fiscal policy cannot fight high unemployment and high inflation at the same time.

Is fiscal policy expansionary or contractionary?

KEY CONCEPT

Change	Stance
Deficit increases, or surplus decreases	**Expansionary**
Deficit decreases, or surplus increases	**Contractionary**
Higher spending item (e.g., highway construction, transfers)	Expansionary
Higher revenue item (e.g., sales tax, income tax rates)	Contractionary

Structural vs. actual deficit. In a recession the actual deficit widens automatically because tax receipts fall and transfers rise, even if lawmakers change nothing. To judge the *policy stance*, economists use the **structural budget deficit** (**cyclically adjusted budget deficit**): the deficit current policies would produce if the economy were at full employment. The difference between the actual and the structural deficit is the part caused by the cycle. To read the stance from budget data:

1. Use the change in the budget balance over the period; the level of the deficit alone does not reveal the stance.

2. Remove the effect of the cycle by using the structural deficit. A change in the actual deficit that leaves the structural deficit unchanged reflects the cycle rather than a change in policy.
3. A rise in the structural deficit (or a fall in a structural surplus) means expansionary policy; the reverse means contractionary policy.

Worked example. Last year the actual deficit was 6% of GDP and the structural deficit 2.5%, so 3.5 percentage points came from the weak economy. This year the economy recovers and the actual deficit narrows to 4.5%, but a tax cut lifts the structural deficit to 3.5%. The smaller actual deficit reflects the recovery; the larger structural deficit shows that fiscal policy has turned expansionary.

Common exam traps

- Higher government spending is not expansionary if taxes rise by more. Judge the stance by the change in the budget balance.
- An actual deficit above the structural deficit does not show the stance either way: the gap reflects the cycle, while the change in the structural deficit shows the discretionary stance.

EXAM SHORTCUTS

- To leave aggregate demand unchanged when government spending rises by ΔG, taxes must rise by $\Delta G/MPC$, more than the spending increase.

BOTTOM LINE

- Fiscal spending tools are transfer payments (not counted in GDP), current spending and capital spending, and revenue tools are direct taxes on income or wealth and indirect taxes on goods and services; indirect taxes can be changed quickly, while direct taxes, transfers and capital spending take time.
- Government spending raises aggregate demand more than an equal tax cut, because households save part of a tax cut, and tax cuts aimed at low-income households, who have a higher MPC, work better.
- The fiscal multiplier is $1/[1 - MPC(1 - t)]$, which rises with the MPC and falls with the tax rate.
- Exam convention: equal increases in government spending and taxes leave the budget balanced and raise aggregate demand, so the balanced budget multiplier is positive; current practice: with an income tax the higher output also raises tax receipts, but the direction of the output effect is unchanged.
- Recognition, action and impact lags separate the need for discretionary fiscal policy from its effect, so a badly timed policy can destabilize the economy.
- The stance is judged from the change in the structural (cyclically adjusted) deficit, the deficit current policies would produce at full employment: a rise means expansionary policy, while a change in the actual deficit that leaves it unchanged reflects the cycle.

QUICK CHECK

Question 2

Inflation in Brennholt is running well above target. Which of the following is an appropriate *discretionary fiscal policy* response?

A Scaling back government spending on large public construction projects.

B Raising the central bank's target for the overnight interbank rate.

C Reducing the money supply.

Answer: A

Discretionary fiscal policy consists of the government's decisions about spending and taxes. Cutting spending on major public construction lowers aggregate demand and eases pressure on prices, which is the appropriate direction when inflation is high.

Why the other options are wrong

- **B.** Raising a policy interest rate target (like the federal funds target rate in the US) is a monetary policy action by the central bank. It fights inflation, but it is not fiscal policy.
- **C.** Reducing the money supply is also a monetary policy action.

Key takeaway Right direction is not enough; check the tool. Spending and taxes are fiscal, while interest rates and the money supply are monetary.

Practice questions

Question 3

The governments of Tarsia and Morvenna both keep their tax rates unchanged and both run a balanced primary budget, meaning that tax revenue exactly covers all government spending other than interest on the debt. Tarsia's government debt equals 85% of GDP, the real interest rate on that debt is 3.1%, and Tarsia's real GDP grows by 1.3% a year. Morvenna's government debt equals 50% of GDP, the real interest rate on that debt is 0.9%, and Morvenna's real GDP grows by 2.7% a year. If these conditions persist, the ratio of government debt to GDP will *most likely*:

A rise in Tarsia and fall in Morvenna.

B rise in both Tarsia and Morvenna.

C fall in both Tarsia and Morvenna.

Question 4

The government of Kestrel Isles increases its spending by $60 million. Households have a marginal propensity to consume of 90%, and the tax rate is 30%. The potential increase in aggregate demand is *closest* to:

A $600 million.

B $162 million.

C $82 million.

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Question 5

Over the past year, new tax and spending decisions (not the business cycle) have widened the budget deficit of Estavia's government and shrunk the budget surplus of Marquessa's government. How would the fiscal policies of the two countries *best* be described?

A Both are expansionary.

B Both are contractionary.

C One is expansionary and the other is contractionary.

Answers: 3. A · 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/fiscal-policy/. The full bank has 37 questions on this reading.

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ECONOMICS · READING 15

Quantitative Easing

What you'll learn

LOS 15.a Describe the roles of central banks (currency supplier, bankers' bank, regulator, lender of last resort, reserve holder, monetary policy) and their objectives, chiefly price stability.

LOS 15.b Describe the policy rate, reserve requirements and open market operations, the monetary transmission mechanism, and how monetary policy affects growth, inflation, interest rates and exchange rates.

LOS 15.c Describe the qualities of effective central banks, compare inflation, interest rate and exchange rate targeting, judge the policy stance against the neutral rate, and describe the limitations of monetary policy.

LOS 15.d Explain how combinations of monetary and fiscal policy affect interest rates, output and the private and public sectors.

MODULE 15.1

Central Bank Objectives and Tools

This reading covers the roles and objectives of central banks, the tools of monetary policy and the transmission mechanism through which a policy-rate change reaches inflation. It then describes the qualities of an effective central bank, inflation, interest rate and exchange rate targeting, the neutral interest rate, the limits of monetary policy, and how monetary and fiscal policy interact.

LOS 15.a — What central banks do and what they aim for

Roles of a central bank. A modern **central bank** typically performs six roles:

KEY CONCEPT

Role	What it means in practice
Sole supplier of currency	Only the central bank can issue the country's **currency** (notes and coins) and bank reserves. Exam convention: the central bank is described as the sole supplier of money. Current practice: its monopoly covers currency and bank reserves, while most deposit money is created by commercial banks when they lend. Today currency is **fiat money** — money with no backing in gold or any other tangible asset, accepted because the law makes it **legal tender** and because it keeps its value.
Banker to the government and other banks	Holds accounts for the government and for commercial banks and provides them with banking services.
Regulator and supervisor of the payments system	Sets risk-taking standards and **reserve requirements** for banks and oversees domestic and cross-border clearing.
Lender of last resort	Because it can create money, it can lend to banks that run short of funds. Knowing this backstop exists reassures depositors and helps prevent **bank runs**.

Role	What it means in practice
Holder of gold and foreign exchange reserves	Keeps the nation's official gold and foreign currency reserves.
Conductor of monetary policy	Controls or influences the quantity of money and its growth over time.

Collecting taxes is not a central bank function; a separate government agency does it. Being banker to the government means providing it with banking services, such as holding its accounts; making loans to government agencies is not among the central bank roles listed here.

KEY CONCEPT

Objectives. The **primary objective** is to **control inflation** so as to promote **price stability**. High inflation imposes **menu costs** (firms repeatedly re-pricing their goods) and **shoe leather costs** (people making extra trips to the bank to avoid holding cash that loses value). Some central banks have additional goals:

- stable exchange rates with foreign currencies;
- **full employment**;
- sustainable positive economic growth;
- moderate long-term interest rates.

Inflation targets. Most developed-country central banks aim for inflation in a range around **2% to 3%**. They avoid a **zero** target because random swings around zero would often mean **deflation**, which is disruptive. Exam convention: the US Federal Reserve and the Bank of Japan are the developed-country exceptions without an explicit inflation target. The Fed's goals also include maximum employment and moderate long-term rates, and Japan has struggled with persistent deflation. Current practice: both now have explicit 2% goals. The Fed adopted a longer-run goal of 2% inflation, measured by the PCE price index, in January 2012, alongside its **dual mandate** (maximum employment as well as stable prices). The Bank of Japan adopted a 2% CPI price stability target in January 2013 while fighting deflation.

Pegging. Some countries instead **peg** their currency to another currency, most often the US dollar, and run monetary policy to hold that exchange rate. How a peg is defended, and what it does to the money supply, interest rates and inflation, is covered under exchange rate targeting (LOS 15.c).

Common exam traps

- "Minimize long-term interest rates" is not a goal. The goal is *moderate* long-term rates.
- A mandate focused only on price stability means the bank fights above-target inflation even during a recession.

LOS 15.b — Policy tools and the monetary transmission mechanism

The three main tools

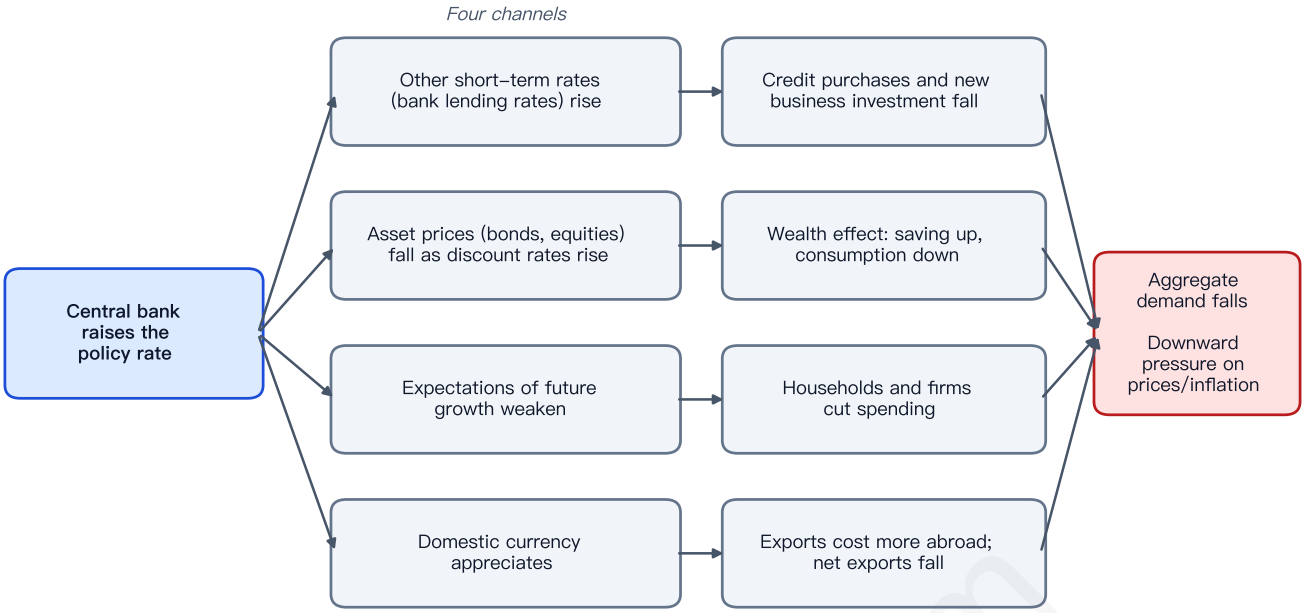
KEY CONCEPT

Tool	Expansionary use	Contractionary use
Policy rate — the rate at which the central bank lends reserves to banks: the **discount rate** (US), the **refinancing rate** (ECB), the **two-week repo rate** (Bank of England, lending through a repurchase agreement)	Lower it	Raise it
Reserve requirements — percentage of deposits banks must hold as reserves	Lower them (more funds to lend)	Raise them
Open market operations — buying and selling (usually government) securities	**Buy** securities	**Sell** securities

- In a **repurchase agreement** (repo), the central bank buys securities from a bank that agrees to buy them back later at a slightly higher price. The percentage difference between the two prices is the interest rate on what is, in effect, a short-term loan to the bank.
- **Open market purchases**: cash replaces securities in investors' accounts, banks get **excess reserves**, more funds are available for lending, the **money supply** rises and interest rates fall. **Open market sales** do the reverse and push rates up.
- In the US, the **federal funds rate** is the market rate banks charge *each other* for overnight loans of reserves. The Federal Reserve sets a **target** for it and uses open market operations (its most-used tool) to steer the actual rate there. Do not confuse it with the **discount rate**, which is the rate a bank pays when it borrows *from the Federal Reserve*.
- The central bank does not set exchange rates, the prices of government securities or the yield on existing bonds directly. It also cannot *compel* banks to tighten or loosen their credit standards. It can only encourage them.
- Lowering reserve requirements has an effect only when banks want to extend more loans and borrowers want to take them.
- Exam convention: the policy rate, reserve requirements and open market operations are the three working tools of monetary policy. Current practice: the Federal Reserve set its reserve requirement ratios to zero in March 2020, and with reserves plentiful since the 2008 financial crisis it steers the federal funds rate mainly through the interest rate it pays on banks' reserve balances rather than through day-to-day open market operations.

The monetary transmission mechanism is how a change in the policy rate reaches the price level and inflation. It works through **four channels**: other short-term rates, **asset prices**, **expectations**, and **exchange rates**. The figure traces a policy-rate *increase*:

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Monetary transmission mechanism of a policy-rate increase

1. Banks raise their short-term lending rates, so consumers cut credit purchases and firms cut investment.
2. Higher rates for discounting future cash flows lower bond, equity and other asset prices. The resulting **wealth effect** raises saving and lowers consumption.
3. Expectations of future growth weaken, so spending falls.
4. Higher rates attract foreign capital, the **domestic currency appreciates**, exports become more expensive for foreigners and **net exports** fall. Through this channel the **trade balance** worsens.

Together these reduce **aggregate demand** and push the price level down. The policy rate acts directly on the first links in the chain: other short-term rates, asset prices, expectations and the exchange rate. Inflation is the end point, reached through these channels and only after a lag. A policy-rate cut works through the same channels in the opposite direction.

Expansionary policy step by step. The central bank buys securities, so bank reserves rise and the interbank rate falls. Other short-term rates and then longer-term rates fall, and the currency depreciates. Business investment, consumer purchases of houses, autos and durable goods, and net exports all rise. Aggregate demand increases, which raises inflation, employment and real GDP in the short run. If **money neutrality** holds, monetary policy has no effect on real output in the long run.

Example. The central bank of a small economy raises its policy rate from 3.00% to 3.50% and sells government bonds to make the higher rate stick. Expected effects: bank reserves and excess reserves fall; short- and long-term market rates rise; bond and share prices fall; the currency appreciates; investment, durable-goods spending and exports fall. Aggregate demand falls, so in the short run both real GDP and the price level fall, and the unemployment rate rises.

Common exam traps

- Tightening raises the currency's foreign exchange value, so a "decrease in the foreign exchange value" is not a channel through which tightening cuts spending.

- Tightening raises *real* interest rates in the short run and increases unemployment.
- If inflation is below target, the central bank eases: it buys securities or cuts the policy rate or reserve requirement.

EXAM SHORTCUTS

- Monetary tightening makes the domestic currency appreciate, so an answer in which tightening cuts spending through a weaker currency can be ruled out.

BOTTOM LINE

- A central bank supplies currency, acts as banker to the government and other banks, regulates and supervises the payments system, is lender of last resort, holds gold and foreign exchange reserves and conducts monetary policy; under the exam convention it is the sole supplier of money, while in current practice its monopoly covers currency and bank reserves and commercial banks create most deposit money.
- Controlling inflation to promote price stability is a central bank's primary objective, and some central banks also aim for stable exchange rates, full employment, sustainable growth and moderate long-term interest rates.
- Most developed-country central banks aim for inflation in a range around 2% to 3%, not zero, because swings around a zero target would often mean deflation.
- Exam convention: open market operations, the policy rate and reserve requirements are the three working tools, used expansionarily by cutting the rate or requirements and buying securities; current practice: the Federal Reserve has set reserve requirements to zero and steers the federal funds rate mainly through the rate it pays on reserve balances.
- The federal funds rate is the market rate US banks charge each other for overnight loans of reserves, which the Federal Reserve targets, while the discount rate is what a bank pays to borrow from the Federal Reserve.
- A policy-rate change reaches the price level through other short-term rates, asset prices, expectations and exchange rates; a rate increase lowers aggregate demand and appreciates the currency, and if money neutrality holds, monetary policy has no long-run effect on real output.

QUICK CHECK

Question 1

Rumors about loan losses lead depositors at several banks in Harwick to start withdrawing their money. Harwick's central bank announces that it will provide whatever funds are needed to any sound bank facing a cash shortage, and the withdrawals soon stop. This announcement reflects the central bank's role as:

A lender of last resort.

B holder of gold and foreign exchange reserves.

C sole supplier of currency.

Answer: A

Because the central bank can create central bank money (reserves and currency) at will, it can lend to banks that run short of funds. Knowing this backstop exists assures depositors their money is safe and helps prevent bank runs. That is the lender-of-last-resort role.

Why the other options are wrong

- **B.** Holding the nation's gold and foreign currency reserves is a separate role. It concerns official reserves rather than emergency lending to domestic banks.
- **C.** Being the sole supplier of currency is what *enables* emergency lending, but the role being performed here is lending to banks in distress, i.e., lender of last resort.

Key takeaway Preventing bank runs by standing ready to lend to banks is the lender-of-last-resort role.

MODULE 15.2

Monetary Policy Effects and Limitations

LOS 15.c — Effective central banks, targeting regimes and limits of policy

Three essential qualities of an effective central bank

KEY CONCEPT

Quality	Meaning	Clue in a question
Independence	Free from political interference, which matters because politicians may want to boost growth before elections at the cost of higher inflation	Who decides the policy rate or the target?
Credibility	The bank follows through on what it says, so its targets become self-fulfilling: wages and contracts are set on the expected (target) inflation rate. A target set by a heavily indebted government instead would lack credibility, because that government gains from letting inflation overshoot	Economic actors *base their decisions* on the stated target
Transparency	Regular disclosure, e.g. **inflation reports**, and a clear statement of which indicators it uses and how. This supports credibility and makes policy moves easier to anticipate	Publishes reports and explains which indicators it uses and how

Relevance, reliability, comparability and consistency are financial-reporting qualities. They are not on the central bank list.

Independence comes in degrees rather than all-or-nothing. Even a fairly independent central bank may have its head appointed by politicians.

- **Operational independence**: the central bank alone sets the **policy rate**.

- **Target independence**: in addition, it defines how inflation is computed, chooses the target inflation level and sets the horizon for reaching it. The ECB has both types. Most other central banks have operational independence alone.

Targeting regimes

KEY CONCEPT

Regime	How it works	Notes
Interest rate targeting	Add money when rates rise above a target band, withdraw money when they fall below it	Used in the past
Inflation targeting	Adjust policy to keep *expected* inflation (about two years ahead) inside a band	Most widely used today, and required by law in some countries. Users include the Bank of England, the ECB and the central banks of Canada, Australia, Brazil and Mexico
Exchange rate targeting	Buy or sell the domestic currency to hold a target exchange rate, often against the US dollar	Common in developing countries

The most common inflation target is **2%**, with a permitted deviation of **±1%** (band 1% to 3%). Because of the deflation risk described under LOS 15.a, the band is set so that its lower edge (target minus deviation) stays at or above zero.

Exchange rate targeting mechanics. If the domestic currency falls *below* the target, the central bank **buys domestic currency with foreign reserves**. This reduces money supply growth and raises interest rates. If the currency rises *above* the target, the bank sells domestic currency, which increases the money supply and lowers rates. As a result, the money supply and interest rates can become more volatile. A country can run out of foreign reserves, so intervention has limits. Monetary policy must follow the peg regardless of domestic conditions, and the pegging country ends up with the anchor country's **inflation rate**.

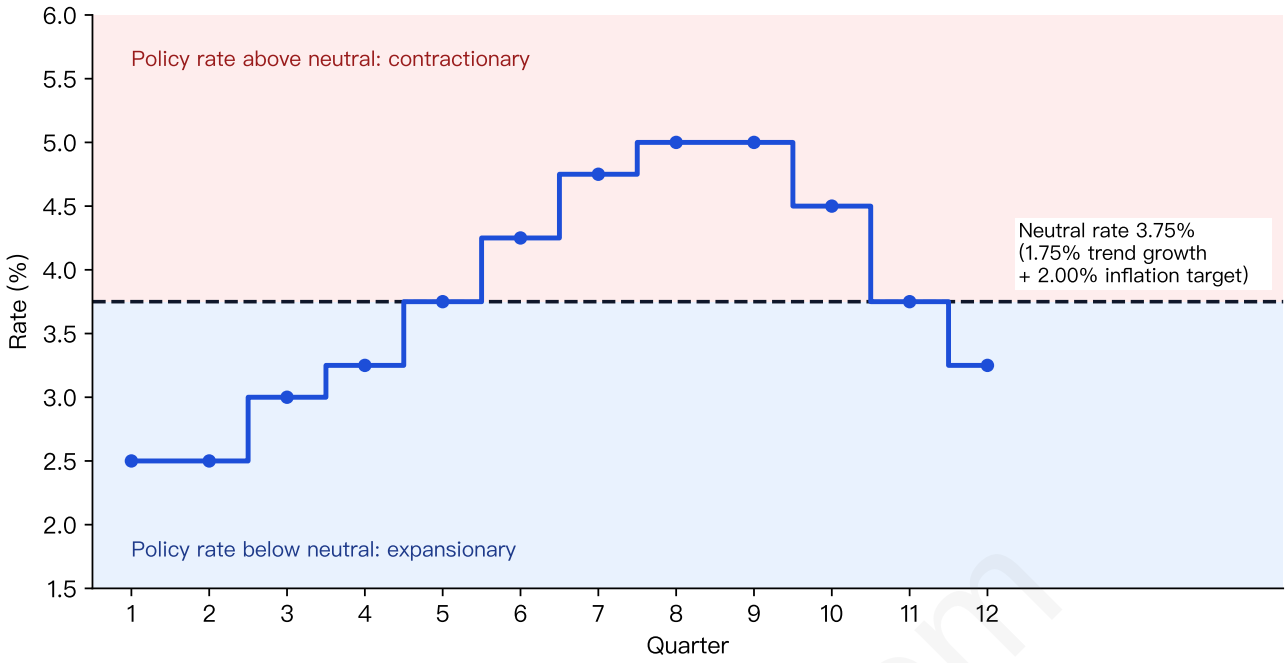
Neutral interest rate and policy stance

Exam convention: the **neutral interest rate** is described as the growth rate of the money supply that neither speeds up nor slows economic growth. Current practice: it is treated as the policy interest rate at which policy neither stimulates nor restrains the economy, which is what the formula and the stance test below use:

KEY CONCEPT

$$\text{neutral interest rate} = \text{real trend rate of economic growth} + \text{inflation target}$$

- Policy rate above neutral: **contractionary**. Policy rate below neutral: **expansionary**. Equal: **neutral**.



Policy rate versus the neutral interest rate

> *Example.* Trend real growth 1.75% and inflation target 2.00% give a neutral rate of $1.75\% + 2.00\% = 3.75\%$. A policy rate of 4.50% (above 3.75%) is contractionary. A policy rate of 3.00% is expansionary.

Limitations of monetary policy

- **Long-term rates may not follow short-term rates.** The central bank controls short-term rates directly, but long-term rates, which drive most household and business decisions, reflect **expected inflation**. Tightening can *lower* long-term yields. If markets believe it will work, expected inflation falls. If they see it as too severe, the higher probability of a recession makes long-term bonds more attractive. Easing that markets see as inflationary can *raise* long-term yields. Investors who sell long-term bonds for this reason are called **bond market vigilantes**. With a credible central bank this effect is small.
- **Liquidity trap.** When the **demand for money becomes very elastic**, people simply hold the extra money as cash, so adding money does not push rates down or stimulate spending. **Deflation** that persists despite expansionary policy points to a liquidity trap.
- **Zero lower bound.** Exam convention: the nominal policy rate cannot be cut below zero, so a central bank whose rate is already at zero has little room left to stimulate. Current practice: some central banks (e.g., the ECB, the Swiss National Bank and the Bank of Japan) have set slightly negative policy rates, but not far below zero. Either way, **deflation** is harder to reverse than inflation. Inflation can be fought by raising rates, which have no upper limit.
- **Banks may not lend** even when excess reserves rise (as after 2008). With short-term rates near zero, central banks turned to **quantitative easing** (**QE**): large-scale purchases of longer-term government bonds, mortgage securities and even securities with credit risk. The aim was to lower long-term rates and create excess reserves that encourage lending. Buying securities with credit risk may also shift risk from the private to the public sector.
- **Developing economies** face extra problems: illiquid government debt markets (making interest rate signals unreliable and open market operations hard), difficulty estimating the neutral rate, rapid financial innovation that changes money demand, and central banks that lack credibility or independence.

Common exam traps

- The benchmark for stance is the neutral interest rate. The inflation target and the trend growth rate are only its components.
- Setting the policy rate alone does not make a central bank target independent.

LOS 15.d — Interaction of monetary and fiscal policy

Monetary	Fiscal	Interest rates	Output	Private sector	Public sector
Tight	Tight	Higher	Lower	Lower	Lower
Easy	Easy	Lower	Higher	Higher	Higher
Tight	Easy	Higher	Higher (exam convention, from the fiscal stimulus)	Lower	Higher
Easy	Tight	Lower	Varies (lower rates tend to raise it)	Higher	Lower

When the two policies pull in *opposite* directions, the output entries follow the exam convention. Exam convention: expansionary fiscal with contractionary monetary policy is taken to raise aggregate demand and output. Contractionary fiscal with expansionary monetary policy has an output effect that varies, although the lower interest rates tend to raise private consumption and output. Current practice: in both mixed cases output rises only if the expansionary policy is the stronger of the two, so the direction depends on their relative strength. The **sector-share** effects (private vs. public share of GDP) and the direction of interest rates do not depend on that assumption.

- **Contractionary fiscal + expansionary monetary**: lower government borrowing and a larger money supply both lower rates, so the **private sector grows** and government spending as a share of GDP falls.
- **Expansionary fiscal + contractionary monetary**: rates rise (heavier government borrowing plus tight money), the **public sector's share of GDP rises** and the private sector is squeezed.
- **Fiscal multipliers** differ: direct government spending has had a larger multiplier than transfers or tax cuts. Among the latter, transfers to the poor have the greatest effect, followed by tax cuts for workers, then broad untargeted transfers. Every type of fiscal stimulus is **more powerful when combined with expansionary monetary policy**, possibly because of higher inflation, falling real interest rates and the extra business investment they bring. Cutting the discount rate or buying securities therefore reinforces expansionary fiscal policy during high unemployment.

Common exam traps

- Selling bonds or raising reserve requirements works *against* an expansionary fiscal stance.

EXAM SHORTCUTS

- To judge the stance, add the real trend growth rate and the inflation target and compare the policy rate with that sum: above it means contractionary, below it expansionary.

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BOTTOM LINE

- An effective central bank has independence, credibility and transparency; operational independence means it sets the policy rate alone, and target independence means it also defines the inflation measure, the target and the horizon for reaching it.
- Inflation targeting, the most widely used regime, keeps expected inflation inside a band, most commonly 2% ± 1%, while an exchange rate target makes the central bank buy its currency with foreign reserves when it falls below target and sell it when it rises above, so the pegging country ends up with the anchor country's inflation rate.
- The neutral interest rate equals the real trend rate of economic growth plus the inflation target, and a policy rate above it is contractionary and below it expansionary; the exam convention describes the neutral rate as the money supply growth rate that neither speeds up nor slows growth, while current practice treats it as the policy rate that neither stimulates nor restrains the economy.
- Monetary policy can fail when long-term rates move against short-term rates because of expected inflation, in a liquidity trap where demand for money is very elastic, and when banks will not lend despite excess reserves, which led central banks to quantitative easing.
- Exam convention: the nominal policy rate cannot be cut below zero, leaving little room to stimulate; current practice: some central banks have set slightly negative rates; either way deflation is harder to reverse than inflation.
- With expansionary fiscal and contractionary monetary policy, interest rates rise and the public sector's share of GDP rises, while contractionary fiscal and expansionary monetary policy lower rates and let the private sector grow.

QUICK CHECK

Question 2

In Taviston, unions and employers routinely write the central bank's announced inflation target into multi-year wage agreements, and lenders price loans on the assumption that inflation will stay close to that target. This behavior indicates that Taviston's central bank has:

A credibility.

B transparency.

C target independence.

Answer: A

A credible central bank is one whose stated targets people believe. They then base their decisions (wages, nominal contracts, loan pricing) on the target inflation rate, which helps make the target self-fulfilling.

Why the other options are wrong

- **B.** Transparency means periodically disclosing the state of the economy (e.g., inflation reports) and the indicators used. It supports credibility, but the behavior described is the definition of credibility itself.
- **C.** Target independence means the central bank determines how inflation is computed, sets the target level and the horizon. It says nothing about whether the public believes the target.

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Key takeaway Decisions based on the announced target = credibility. Regular inflation reports = transparency. Deciding the policy rate or the target = independence.

Practice questions

Question 3

Suppose the Federal Reserve carries out open market sales of Treasury securities. What happens to bank reserves and to the federal funds rate?

A Reserves decrease, banks can lend less, and the federal funds rate rises.

B Reserves decrease, banks can lend less, and the federal funds rate falls.

C Reserves increase, banks can lend more, and the federal funds rate falls.

Question 4

Quellan's central bank has set its policy rate at 4.0% and aims for inflation of 2.4%. Economists expect real GDP to expand at a trend pace of 1.6% a year. How is Quellan's monetary policy *most likely* classified?

A Neutral.

B Expansionary.

C Contractionary.

Question 5

Growth in Tarnovia has stalled. Its government and central bank want both public-sector spending and private-sector consumption and investment to expand. Which policy mix fits this goal, and what is its *most likely* effect on interest rates?

A Expansionary fiscal and contractionary monetary policy; interest rates rise.

B Contractionary fiscal and expansionary monetary policy; interest rates fall.

C Expansionary fiscal and expansionary monetary policy; interest rates usually fall.

Answers: 3. A · 4. A · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/quantitative-easing/. The full bank has 47 questions on this reading.

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ECONOMICS · READING 16

Introduction to Geopolitics

What you'll learn

LOS 16.a Describe geopolitics as the degree of cooperation versus competition among state and nonstate actors, and why countries cooperate.

LOS 16.b Relate geopolitics to globalization versus nationalism and classify countries as autarky, hegemony, bilateralism or multilateralism.

LOS 16.c Describe the functions and objectives of the IMF, the World Bank and the WTO.

LOS 16.d Describe event, exogenous and thematic geopolitical risk and its likelihood, impact and velocity (including black swan risk).

LOS 16.e Describe national security, economic and financial tools of geopolitics and their effects.

LOS 16.f Describe how geopolitical risk affects investments: impact, discrete vs broad effects, business cycle, scenario analysis and signposts.

MODULE 16.1

Geopolitics

This reading describes geopolitics through two lenses, cooperation versus competition and globalization versus nationalism, which combine into four archetypes, and it sets out the roles of the IMF, the World Bank and the WTO. It then classifies geopolitical risks and the tools of geopolitics, and explains how investors judge a risk's likelihood, impact and velocity using scenario analysis and signposts.

LOS 16.a — Geopolitics as cooperation versus competition

Geopolitics is the study of interactions among nations. The players are **state actors** (national governments) and **nonstate actors** (companies, nongovernment organizations and individuals). Geopolitics also looks at how geography shapes these interactions: a country with a long coastline and deep-water ports, for example, tends to produce firms that dominate seaborne shipping.

A useful lens is the degree of **cooperation** among countries. Cooperation means engaging with other countries on diplomatic, military, economic and cultural matters. Economic cooperation includes:

- letting goods, services and capital move freely between countries;
- agreeing on common tariff levels;
- adopting common international rules (e.g., shared financial reporting standards);
- transfers of information and technology.

A country that restricts these flows, for example by banning exports of a particular good, is acting noncooperatively. In practice cooperation varies along a spectrum. A country may cooperate on trade but not on defense, and its stance can shift with elections or a change of leadership, so analysts watch the current decision makers and the length of the political cycle.

Why countries cooperate. A country cooperates when doing so advances its **national interests**, which can be ranked as a hierarchy with survival at the top. The motives fall into three groups:

1. *Security.* Survival comes first, so a country may join its neighbors in a military alliance to protect its citizens from invasion.

2. *Economic interests.* Resource endowments shape these: a country rich in minerals but short of farmland must trade minerals for food, so it wants international trade lanes kept open. Nonstate actors cooperate across borders because they want to put resources to their highest-valued use, and they push for standardized regulations and processes.
3. *Culture.* **Cultural factors** (a shared language, historical migration) and strong, stable **institutions** (laws, organizations, customs) make cooperation easier. Large exporters of films, music and other cultural content usually have strong legal protection for intellectual property. Such exports are one source of **soft power**: influencing other countries without using or threatening force.

Common exam traps

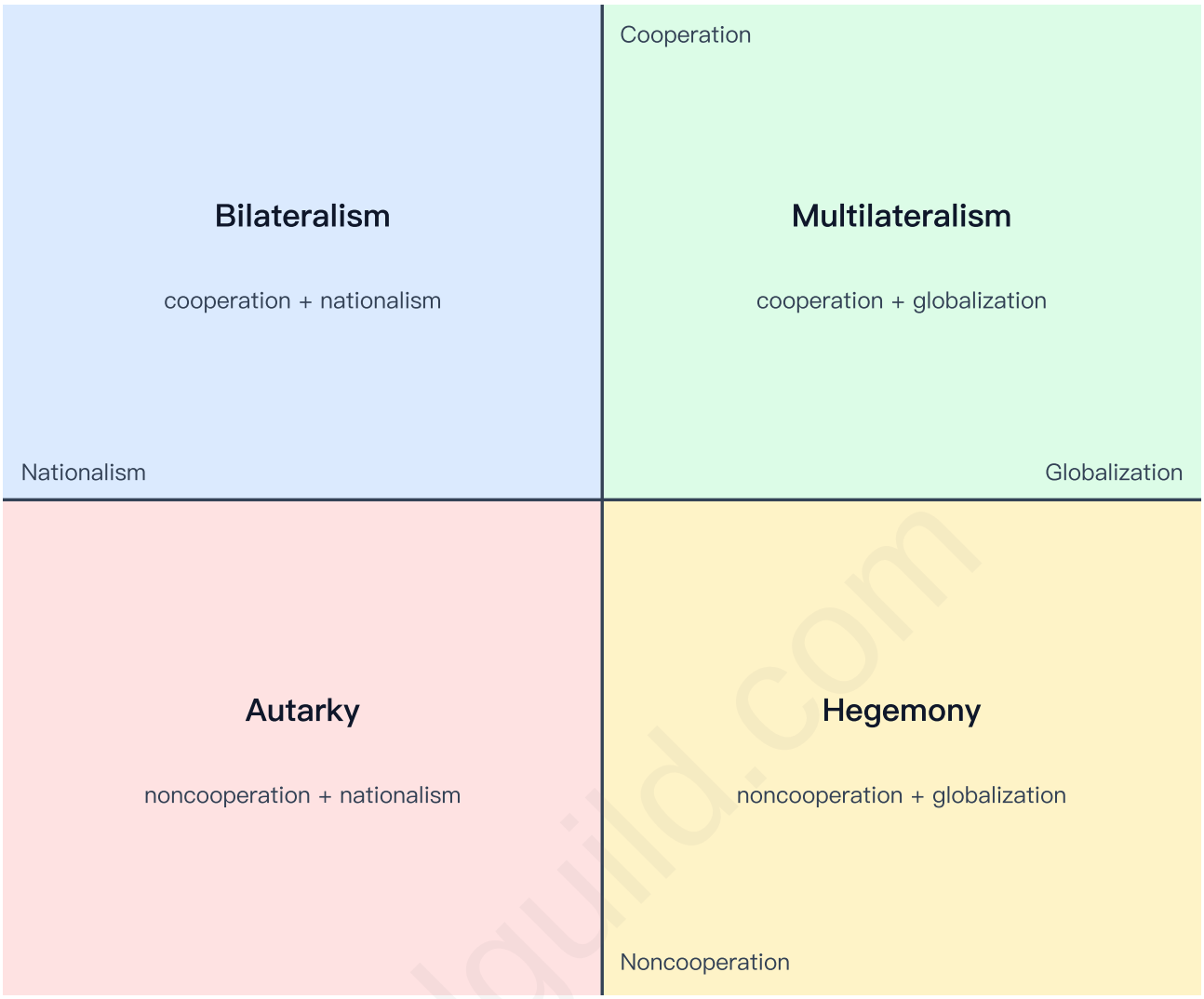
- Treating cooperation as all-or-nothing; a country can be cooperative on some issues and noncooperative on others.
- Assuming cooperation is altruistic; it is driven by national interest.

LOS 16.b — Geopolitics and globalization

Globalization is the long-run trend toward worldwide integration of economic activity and cultures. Economic openness (trade as a share of world output) climbed from roughly a quarter of output in the early 1970s to around 60% before the 2008 financial crisis and has stayed near that level. Its opposite, **nationalism**, here means a country putting its own economic interests first, acting either alone or in competition with other countries. Countries near the globalization end trade more, allow freer movement of capital and currency exchange, and are more open to cultural interaction.

Crossing the cooperation spectrum with the globalization spectrum gives four **archetypes** of geopolitical behavior:

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Four archetypes of geopolitical behavior on the cooperation and globalization axes

KEY CONCEPT

Archetype	Cooperation axis	Globalization axis	Typical behavior
Autarky	Noncooperation	Nationalism	Seeks national self-reliance and self-sufficiency; produces most goods at home, so little or no external trade; often a state-dominated society with government control of industry and media
Hegemony	Noncooperation	Globalization	Open to global trade but large enough to influence other countries without needing to cooperate
Bilateralism	Cooperation	Nationalism	Cooperates through one-to-one relationships with individual countries (possibly many of them) but avoids multicountry arrangements

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Archetype	Cooperation axis	Globalization axis	Typical behavior
Multilateralism	Cooperation	Globalization	Trades extensively and cooperates with many countries at once; **regionalism** is a variant that cooperates multilaterally mainly with neighbors

Nonstate actors may be more globally minded than their governments: firms look abroad for profits, lower costs and new markets; investors look abroad for returns and diversification, through **portfolio investment flows** (buying foreign securities) or **foreign direct investment** (owning production capacity abroad).

Example. Country Q trades heavily with the rest of the world and is so large that it sets terms on its own rather than joining multicountry agreements. It is globalized but noncooperative, so it fits **hegemony**. If instead it signed dozens of separate two-country trade deals but kept away from regional blocs, it would fit **bilateralism**.

Common exam traps

- Bilateralism is *cooperative*, even though it sits on the nationalist side.
- Hegemony is *globalized*, even though it is noncooperative.

LOS 16.c — International organizations that facilitate trade

Organization	Core mission	Key functions
International Monetary Fund (IMF)	International monetary stability	Article I goals: promoting **international monetary cooperation**; facilitating the expansion and **balanced growth of international trade**; promoting **exchange stability**; assisting in the establishment of a **multilateral system of payments**; making resources available (**with adequate safeguards**) to members experiencing **balance of payments difficulties**
World Bank	Fighting poverty	Financial and technical assistance to developing countries: low-interest loans, interest-free credits and grants for education, health, infrastructure, agriculture, public administration and more. Made up of the IBRD (middle-income and creditworthy poorer countries) and the IDA (the poorest countries)
World Trade Organization (WTO)	Global rules of trade	The only international organization dealing with the rules of trade between nations; aims for trade that flows smoothly, predictably and freely; negotiated agreements bind members' trade policies; a **dispute settlement** process resolves trade friction

Common exam traps

- Thinking only the WTO works to expand trade. The IMF's Article I goals include the expansion and balanced growth of international trade.

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- Thinking only one body provides funds. The IMF makes resources available to members with balance of payments difficulties, and the World Bank lends to developing countries for development and poverty reduction.
- Mixing up the roles that belong to one body: fighting poverty is the World Bank's mission (it is not among the IMF's Article I goals), exchange stability is an IMF goal, and settling trade disputes is the WTO's job.

LOS 16.d — Geopolitical risk

Geopolitical risk is the possibility of events that disrupt peaceful international relations. Three types:

KEY CONCEPT

Type	What is known	Example
Event risk	Timing known, outcome unknown	A national election or a scheduled referendum
Exogenous risk	Not anticipated at all	A sudden war, coup or rebellion
Thematic risk	Known factors whose effects play out over a long period	Migration patterns, climate trends, cyber risks

Geopolitical risk changes the **risk premium** investors demand for holding assets in a country or region. A higher risk premium means a higher discount rate, and therefore lower asset values, for the same expected cash flows. Three dimensions describe a risk:

- **Likelihood**: the probability that it occurs.
- **Impact**: how large its effect on investment outcomes would be.
- **Velocity**: how quickly investment values reflect the effect.

The cooperation/globalization framework helps with likelihood: more cooperative and globalized countries face less risk of some events (armed conflict) but more of others (e.g., supply chain disruptions such as those of 2020–2021).

Velocity classes

Velocity	Horizon	Typical effect
High	Short term	Exogenous shocks moving markets quickly; includes **black swan risk** — low-likelihood exogenous events with substantial short-term effects
Medium	Intermediate	Raises costs or disrupts production for specific companies or industries
Low	Long term	Affects companies mainly in the environmental, social, and governance (ESG) realm

Investors with long horizons usually do not need to react to black swan events; short-horizon investors may have to. Long-horizon investors should instead analyze medium- and low-velocity risks.

Common exam traps

- Velocity measures how fast *investment values* respond. How fast the event itself unfolds is a separate matter.
- A black swan is *low* probability with a *short-term* (not long-term) impact.
- Calling a scheduled election exogenous because its result is unknown. A known date makes it event risk.

LOS 16.e — Tools of geopolitics

Tools of geopolitics are the means (mostly used by state actors) to advance national interests. There are three categories, and they are classified in two different ways.

National security tools include armed conflict, espionage, and bilateral or multilateral defense agreements meant to reinforce or prevent armed conflict. They are classified by status: *active* if a country is using the tool now, *threatened* if it is not using it but appears likely to. Armed conflict destroys productive capital and drives migration away from conflict zones.

Economic tools and **financial tools** are classified by whether a country uses them cooperatively:

KEY CONCEPT

Category	Cooperative use	Noncooperative use
Economic tools	Free trade areas, common markets, economic and monetary unions	Domestic content requirements, voluntary export restraints, **nationalization** of companies or industries
Financial tools	Allowing foreign investment and the free exchange of currencies	Restricting foreign investment or currency exchange

Sanctions restrict the financial interests of a specific geopolitical actor. They count as a financial tool, and state actors often combine them with national security tools.

Common exam traps

- Classifying sanctions as a national security tool because they accompany military pressure. They are a *financial* tool.
- Restrictions on currency conversion are a noncooperative *financial* tool; voluntary export restraints are a noncooperative *economic* tool.

LOS 16.f — Geopolitical risk and investments

Analysis costs time and money, so investors should concentrate on risks with potentially high impact, and for those, judge whether the effect is **discrete** (a single company or industry) or **broad** (a country, region or the world). The business cycle matters: a geopolitical risk may have a larger effect on investment values in a recession than in an expansion.

Tools for the analysis:

- **Scenario analysis** (qualitative or quantitative) estimates how a geopolitical risk would affect a portfolio.

- **Signposts** are data or indicators that signal whether an event's likelihood is rising or falling (e.g., market volatility measures), helping investors track a risk over time.

Common exam traps

- Mixing up the two tools. A volatility index watched for a change in the chance of conflict is a signpost; a what-if estimate of the portfolio loss if conflict breaks out is scenario analysis.

EXAM SHORTCUTS

- Place a country on the two axes and the archetype follows: noncooperative and nationalist is autarky, noncooperative and globalized is hegemony, cooperative and nationalist is bilateralism, and cooperative and globalized is multilateralism.
- Classify a geopolitical risk by what is known in advance: a known date with an unknown outcome is event risk, an unanticipated event is exogenous risk, and known factors with long-lasting effects are thematic risk.

BOTTOM LINE

- Geopolitics studies interactions among nations, involving state actors (national governments) and nonstate actors (companies, nongovernment organizations and individuals), and a country's cooperation varies along a spectrum, driven by its national interests in security, economic needs and culture.
- Globalization is the long-run integration of economic activity and cultures across the world, and nationalism here means a country putting its own economic interests first, alone or in competition with others.
- The IMF promotes international monetary cooperation, balanced growth of trade and exchange stability and makes resources available, with adequate safeguards, to members with balance of payments difficulties; the World Bank fights poverty with loans, credits and grants to developing countries; the WTO, the sole international body for the rules of trade between nations, runs a dispute settlement process.
- Geopolitical risk changes the risk premium on a country's or region's assets and is described by its likelihood, its impact and its velocity, meaning how quickly investment values reflect it.
- A black swan is a low-likelihood exogenous event with substantial short-term effects; long-horizon investors usually need not react to one and should instead analyze medium- and low-velocity risks.
- National security tools are classified as active or threatened, while economic and financial tools are classified as cooperative or noncooperative, and sanctions count as a financial tool.
- Investors should focus on geopolitical risks with potentially high impact, judge whether their effects are discrete or broad, estimate them with scenario analysis and track their likelihood with signposts.

QUICK CHECK

Question 1

During the same year, the government of Aldmere (1) signs an agreement with neighboring states to harmonize product standards and lower tariffs among them, and (2) bans all exports of rare-earth minerals mined within its borders. From a geopolitical perspective, which of the two actions counts as

cooperative?

A Neither of the actions.

B Both of the actions.

C Only one of the actions.

Answer: C

In geopolitics, cooperation means interacting and engaging with other countries, for example by allowing goods to move freely across borders, harmonizing tariffs and standardizing rules. The regional agreement is therefore cooperative. Banning exports of a particular good restricts engagement with other countries, so it is noncooperative. Only one of the two actions is cooperative.

Why the other options are wrong

- **A.** The trade and standards agreement is a clear example of economic cooperation, so it is not true that neither action is cooperative.
- **B.** An export ban limits interaction with other countries and is noncooperative, so both actions cannot be cooperative. A country can be cooperative on some issues and noncooperative on others at the same time.

Key takeaway Cooperation is a spectrum and is judged issue by issue: trade agreements and harmonization are cooperative, while export bans and investment restrictions are noncooperative.

Practice questions

Question 2

An analyst compares the goals listed in Article I of the International Monetary Fund's (IMF's) Articles of Agreement with the stated mission of the World Bank and the role of the World Trade Organization (WTO). Which of her conclusions is *most accurate*?

A Of the three, only the World Bank has fighting poverty as its explicit mission.

B Of the three, only the WTO works to expand international trade.

C Of the three, only the IMF lends financial resources to member countries.

Question 3

A portfolio manager is classifying the geopolitical risks that could affect her emerging market holdings. Which of the following is *best* described as an event risk?

A A gradual shift in migration patterns that is expected to play out over several decades.

B A rebellion that breaks out suddenly, with no warning signs identified in advance.

C A referendum on leaving a customs union, scheduled for a date set a year ahead.

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Question 4

Which of the following statements about the effect of geopolitical risk on investments is *most accurate*?

A Scenario analysis is generally not an appropriate technique for estimating how geopolitical risks could affect a portfolio.

B A given geopolitical risk may have a larger effect on investment values in a recession than in an expansion.

C Geopolitical risks affect entire countries or regions, rather than individual companies or industries.

Answers: 2. A · 3. C · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/introduction-to-geopolitics/. The full bank has 24 questions on this reading.

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ECONOMICS · READING 17

Comparative Advantage

What you'll learn

LOS 17.a Describe the benefits (comparative advantage, scale, variety, competition) and costs (import-competing job losses, inequality) of international trade.

LOS 17.b Compare tariffs, quotas, export subsidies, VERs and domestic content rules, the arguments for them, and their welfare effects on consumers, producers and government.

LOS 17.c Explain why countries form trading blocs and rank free trade areas, customs unions, common markets, economic unions and monetary unions by integration.

MODULE 17.1

International Trade

This reading weighs the benefits and costs of international trade, starting from comparative advantage, and compares tariffs, quotas, voluntary export restraints and export subsidies by their effects on prices, consumer and producer surplus and national welfare. It closes with capital restrictions and with the levels of integration in regional trading agreements, from free trade areas to monetary unions.

LOS 17.a — Benefits and costs of international trade

Why countries gain from trade. Traditional trade models focus on **comparative advantage**. A country has a comparative advantage in a good when its *relative* (opportunity) cost of producing that good is lower than other countries'. Comparative advantage comes from differences in technology and resource endowments. When each country specializes in the goods where it has a comparative advantage, exports them, and imports the rest, total output rises.

> *Example.* In one day a worker in Quilland can make 6 tablets or 3 bicycles, and a worker in Dorvik can make 2 tablets or 2 bicycles. To find comparative advantage, compare opportunity costs rather than output:
>
> 1. Cost of one bicycle in tablets given up: Quilland $6/3 = 2$, Dorvik $2/2 = 1$. Dorvik has the lower cost.
> 2. Cost of one tablet in bicycles given up: Quilland $3/6 = 0.5$, Dorvik $2/2 = 1$. Quilland has the lower cost.
> 3. Quilland specializes in tablets and Dorvik in bicycles.

Quilland's workers produce more of both goods, yet Dorvik still has a comparative advantage in bicycles. Gains from trade come from differences in relative cost, so even the less productive country has something to export.

Newer trade models add further sources of gains:

- **Economies of scale**: specialized producers selling to a world market have lower unit costs;
- greater **variety** of goods for consumers;
- more **competition**, which lowers costs, improves quality and limits the pricing power of domestic monopolies;
- a more **efficient allocation of resources**.

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Countries can also trade *the same* kind of good. In markets with differentiated products (monopolistic competition, e.g., cars), a country may export one type and import others, so consumers get more variety at lower cost.

Who gains and who loses.

Group	Effect of freer trade
Domestic consumers of imported goods	Gain: lower prices, more choice
Domestic producers (and workers) in export industries	Gain: larger markets
Domestic producers and workers in **industries competing with imports**	Lose: lower profits, lost jobs, possibly lower wages
Domestic consumers of goods that are now exported	May lose: stronger foreign demand can raise domestic prices

The most cited costs of trade are job losses in import-competing industries and greater income inequality. For example, a high-wage country that imports labor-intensive textiles may see wages and jobs fall in its textile industry.

Net effect. Economic analysis concludes that the overall gains from trade exceed the losses, especially in the long run. In principle the gainers could fully compensate those who lose and still come out ahead, and both the importing and the exporting country share the net gains. The costs are concentrated in specific import-competing industries and are largely short-run: over time, retrained workers move into other industries, and the costs shrink or even reverse.

COMMON EXAM TRAPS

- "Everyone gains from trade" is wrong. Import-competing industries and their workers bear real costs.
- "Only the exporting country gains" is wrong; in the long run both trading partners benefit.
- Expecting openness to raise consumer prices or cut jobs in export industries. Import competition tends to lower consumer prices, export industries expand, and specialization increases.

LOS 17.b — Trade restrictions and their economic implications

Reasons given for trade restrictions

KEY CONCEPT

Supported by economists (at least conceivably valid)	Little or no support in economic theory
Infant industry: temporary protection lets a new industry reach internationally competitive scale and move down the learning curve	**Protecting domestic jobs**: jobs lost to imports are offset by jobs created elsewhere (export and growing industries), so restrictions do not raise the net number of jobs in the long run, and consumers pay more
National security: keep domestic production of goods vital to national defense, even if imports are cheaper	**Protecting domestic industries**: usually the result of political lobbying, at the expense of consumers

Other stated reasons include retaliation against foreign restrictions, raising government revenue from tariffs, countering foreign governments' subsidies, and preventing **dumping**, in which foreign firms sell exports below their cost of production to capture market share. An industry in which a country already has a comparative advantage needs no protection.

Types of trade restrictions

Restriction	What it is
Tariffs	Taxes on imported goods, collected by the government
Quotas	Limits on the amount of imports allowed over some period
Export subsidies	Government payments to firms that export goods
Minimum domestic content	A required percentage of a product's content must come from the domestic country
Voluntary export restraint (VER)	The *exporting* country agrees to limit the quantity of a good it exports, often to avoid tariffs or quotas from its trading partner

Consumer surplus is the difference between what buyers would be willing to pay and the price they actually pay. **Producer surplus** is the difference between the price sellers receive and the lowest price at which they would be willing to sell.

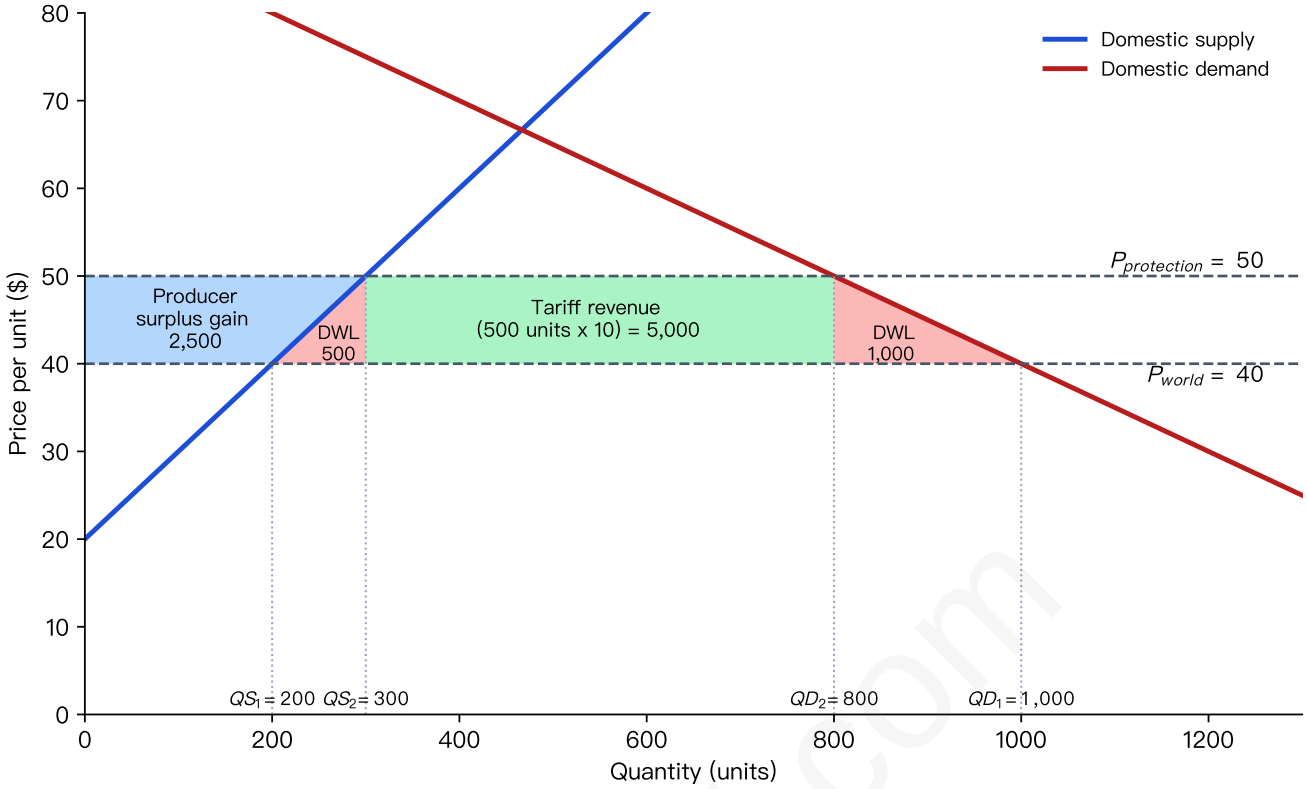
Effects compared. Most effects of the four policies run the same way, even though an export subsidy works in a different market:

KEY CONCEPT

Effect	Tariff	Import quota	VER	Export subsidy
Market affected	Importing country	Importing country	Importing country	Exporting country
Trade volume	Imports fall	Imports fall	Imports fall	Exports rise
Domestic price	Rises	Rises	Rises	Rises
Domestic quantity supplied and producer surplus	Rise	Rise	Rise	Rise
Consumer surplus	Falls	Falls	Falls	Falls
Domestic government	Collects tariff revenue	Collects revenue only if it sells the import licenses	Collects nothing	Pays the subsidy
National welfare	Falls	Falls	Falls	Falls

The national welfare row has one exception: quotas and tariffs imposed by a *large* importing country could raise its welfare under certain assumptions, because its reduced demand can lower the world price.

Tariffs vs quotas. An **equivalent quota** is one that cuts imports by the same amount as a given tariff; both raise the domestic price from the world price P_{world} to $P_{protection}$.



Welfare effects of a $10 tariff in a small importing country (world price $40)

- The loss of consumer surplus is the whole area between P_{world} and $P_{protection}$ to the left of the demand curve.
- Part of it goes to domestic producers (the **producer surplus** gain, left of the supply curve).
- The rectangle (new imports $\times$ price increase) is **tariff revenue** for the government under a tariff.
- The two triangles are the **deadweight loss**: welfare lost to the economy as a whole.

Under a quota, the rectangle goes to whoever holds the import licenses. If the domestic government sells the licenses for their full value, the outcome equals the tariff. If it gives them away, the rectangle becomes **quota rents** earned by the license holders. When the licenses are given free to **foreign exporters**, the rents leave the country and the domestic welfare loss is larger by that amount. A **VER** produces the same welfare loss as an equivalent quota whose licenses are given to foreign exporters free of charge, because the importing country captures none of the quota rents.

Tariff welfare formulas (small country)

For a small importing country (the world price does not change) and straight-line supply and demand between the two prices, let subscript 0 denote free trade at P_{world}, subscript 1 denote the protected price $P_{protection}$, $\Delta P = P_{protection} - P_{world}$, and $M_1 = Q_{D,1} - Q_{S,1}$ denote imports after the restriction. Then:

KEY CONCEPT

$$\text{Loss of consumer surplus} = \frac{Q_{D,0} + Q_{D,1}}{2}\Delta P$$

$$\text{Gain in producer surplus} = \frac{Q_{S,0} + Q_{S,1}}{2}\Delta P$$

$$\text{Tariff revenue} = M_1\Delta P = (Q_{D,1} - Q_{S,1})\Delta P$$

$$\begin{aligned}\text{Deadweight loss} &= \text{consumer surplus loss} - \text{producer surplus gain} - \text{tariff revenue} \\ &= \frac{1}{2}(Q_{S,1} - Q_{S,0})\Delta P + \frac{1}{2}(Q_{D,0} - Q_{D,1})\Delta P\end{aligned}$$

Worked example (small country). The world price of a good is \$40; a tariff of \$10 per unit raises the domestic price to \$50. Domestic supply rises from 200 to 300 units and domestic demand falls from 1,000 to 800 units, so imports fall from 800 to 500.

$$\text{Loss of consumer surplus} = \frac{1{,}000 + 800}{2} \times 10 = \$9{,}000$$

$$\text{Gain in producer surplus} = \frac{200 + 300}{2} \times 10 = \$2{,}500 \qquad \text{Tariff revenue} = 500 \times 10 = \$5{,}000$$

$$\text{Deadweight loss} = 9{,}000 - 2{,}500 - 5{,}000 = \$1{,}500 \left(= \tfrac{1}{2}(100)(10) + \tfrac{1}{2}(200)(10) \right)$$

With an equivalent quota and free licenses granted to foreign exporters, the \$5,000 becomes quota rents for those exporters and the domestic loss rises to \$6,500.

Export subsidies and other effects of trade restrictions

Export subsidies. For a small exporting country, the domestic price rises by the full amount of the subsidy (world price + subsidy), because producers will not sell at home for less than they earn on each exported unit. For a large exporter, the world price falls, so foreign consumers capture part of the benefit and foreign producers are hurt.

Other winners and losers from a tariff. Workers in the protected industry gain along with its producers. Domestic consumers also lose choice, foreign exporters lose sales, and domestic *users* of the protected good (e.g., firms buying protected machinery as an input) face higher costs. Tariff revenue helps explain why trade restrictions remain widespread, but the gains to protected producers and the government are usually smaller than the losses to consumers and other industries.

Capital restrictions limit cross-border flows of financial capital: bans on foreign investment in the country, prohibitions or taxes on the income residents earn on foreign investments, bans on foreign investment in certain industries, and limits on repatriating the earnings of foreign-owned firms. They are generally thought to reduce welfare. Developing countries have sometimes used them in the short run to damp surges of capital inflows in booms and outflows in panics, but these short-run benefits may not offset the longer-run cost of being excluded from global capital markets.

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COMMON EXAM TRAPS

- Assuming every quota earns the government revenue. Revenue needs sold licenses; licenses given free to foreign exporters send the quota rents abroad. In the worked example, free licenses to foreign exporters raise the domestic loss from $1,500 to $6,500.
- Foreign consumers are unaffected by a small (price-taking) country's tariff or quota. A *large* importer's restriction can lower the world price, which benefits foreign consumers rather than harming them.
- A VER is imposed by the *exporting* country, yet it protects producers in the *importing* country.
- Forgetting downstream users. A tariff on imported lumber raises costs for domestic home builders, so they lose even though lumber producers gain.

LOS 17.c — Trading blocs, common markets and economic unions

Motivation. All **regional trading agreements (RTAs)**, also called **trading blocs**, reduce trade barriers among members. Their primary purpose is to raise members' **economic welfare**. Gains come from trading according to comparative advantage and from greater competition among member firms. Some firms and workers lose and may need to retrain. On balance, reducing trade restrictions raises welfare. However, if an agreement *raises* barriers against nonmembers, a member may switch from low-cost imports from a nonmember to higher-priced imports from a member, which reduces the gains and in an extreme case could outweigh them. A free trade area leaves each member's own policy toward nonmembers in place, so it is the form least likely to force such a switch.

Levels of integration (each level adds one feature to the previous one):

KEY CONCEPT

Agreement	Free trade among members	Common trade restrictions with nonmembers	Free movement of labor and capital	Common institutions and economic policy	Single currency
Free trade area	Yes	No	No	No	No
Customs union	Yes	Yes	No	No	No
Common market	Yes	Yes	Yes	No	No
Economic union	Yes	Yes	Yes	Yes	No
Monetary union	Yes	Yes	Yes	Yes	Yes

Examples: NAFTA was a free trade area, the European Union is an economic union, and the eurozone is a monetary union. The single currency of a monetary union removes currency conversion costs and exchange rate risk on trade among members.

Example. Four countries scrap all tariffs on trade among themselves and apply one common tariff schedule to imports from outside, but a worker from one member still needs a permit to take a job in another. Free trade and common external restrictions are present, while free movement of labor is not, so the arrangement is a customs union. Dropping the permits and the barriers to capital flows would turn it into a common market.

COMMON EXAM TRAPS

- A common market already has common external trade restrictions; what an economic union adds is common institutions and economic policy.
- Crediting a customs union with free movement of labor. That feature starts at the common market level.
- Only a monetary union requires a single currency.

EXAM SHORTCUTS

- Find comparative advantage from opportunity costs, not output: the country that gives up less of the other good has the advantage, even if the other country produces more of both goods.
- Each level of integration adds one feature to the level below, so classify an agreement by the highest feature it has: free trade among members, common restrictions on nonmembers, free movement of labor and capital, common institutions and economic policy, then a single currency.

BOTTOM LINE

- A country has a comparative advantage in a good when its opportunity cost of producing it is lower than other countries', and specializing according to comparative advantage raises total output.
- Newer trade models add gains from economies of scale, more variety for consumers, stronger competition and a better allocation of resources.
- Consumers of imported goods and export industries gain from trade while import-competing producers and workers lose, but overall gains exceed losses, especially in the long run, and both trading partners share the net gains.
- Infant industry and national security arguments for trade restrictions have some support among economists, while protecting domestic jobs or industries has little or none.
- Tariffs, import quotas and VERs reduce imports, raise the domestic price, domestic quantity supplied and producer surplus, lower consumer surplus and reduce national welfare, with the possible exception of a tariff or quota imposed by a large importing country.
- A quota whose licenses the government sells for full value has the same outcome as an equivalent tariff, while free licenses to foreign exporters send the quota rents abroad, and a VER causes the same welfare loss as an equivalent quota with free licenses to foreign exporters.
- An export subsidy raises the domestic price by the full subsidy in a small exporting country, while a large exporter's subsidy lowers the world price, benefiting foreign consumers and hurting foreign producers.
- A free trade area removes barriers among members, a customs union adds common restrictions on nonmembers, a common market adds free movement of labor and capital, an economic union adds common institutions and economic policy, and a monetary union adds a single currency.

QUICK CHECK

Question 1

Regarding the gains and losses that international trade creates, which of the following statements is *most accurate*?

A Every group in each of the trading countries is made better off by increased trade.

B The costs of trade fall mainly on firms and workers in domestic industries that compete with imports.

C With respect to domestic employment, the costs of trade outweigh its benefits.

Answer: B

For the economy as a whole, the benefits of trade exceed the costs. The costs are concentrated among firms and workers in domestic industries that compete with imports, who may suffer lower profits or lost jobs.

Why the other options are wrong

- **A.** Not every group gains. Import-competing industries and their workers can lose, even though the gainers could in principle compensate them.
- **C.** Jobs lost in import-competing industries are offset by jobs created in export and growing industries, and over time retrained workers find new work. Economists do not conclude that trade's employment costs exceed its benefits.

Key takeaway Overall gains exceed losses, but the losses are concentrated in import-competing industries.

Practice questions

Question 2

Which of the following statements correctly describes how a tariff differs from a quota?

A A tariff is set by international organizations, whereas a quota is set by an individual country.

B A tariff is imposed by one government, whereas a quota is an international agreement capping the total volume of world trade.

C A tariff is a tax on imported goods, whereas a quota is a limit on the amount of imports allowed over some period.

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Question 3

Tessaly, a small country, is choosing among three measures that would each cut its imports of washing machines by the same quantity: a tariff, an import quota whose licenses the government auctions for their full value, or a voluntary export restraint agreed with the main exporting country. Which measure would cause the greatest loss of national welfare for Tessaly?

A The voluntary export restraint.

B The tariff.

C The import quota with auctioned licenses.

Question 4

Which sequence ranks regional trading agreements correctly, starting with the lowest and ending with the highest level of economic integration?

A Common market, customs union, economic union.

B Free trade area, customs union, common market.

C Customs union, free trade area, monetary union.

Answers: 2. C · 3. A · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/comparative-advantage/. The full bank has 28 questions on this reading.

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ECONOMICS · READING 18

Capital Flows and the FX Market

What you'll learn

LOS 18.a Describe the FX market's functions and participants (sell side, buy side), distinguish nominal from real exchange rates, and calculate percentage changes in a currency's value.

LOS 18.b Describe exchange rate regimes from dollarization to independent floating, and explain how exchange rates affect trade and capital flows (balance of payments identity).

LOS 18.c Describe the common objectives of government restrictions on capital flows.

MODULE 18.1

The Foreign Exchange Market

This reading describes the foreign exchange market and its participants, how to read price/base quotes, spot and forward rates, the difference between nominal and real exchange rates, and how to calculate a currency's percentage change against another. It then covers the IMF's exchange rate regimes, how exchange rates and the balance of payments link trade and capital flows, and the objectives of capital restrictions.

LOS 18.a — The FX market: functions, participants, nominal vs real rates, percentage changes

Why the foreign exchange market exists

Currencies are exchanged for two broad reasons: to pay for goods and services bought across borders (trade flows) and to buy foreign real and financial assets (capital flows). The capital-flow side is much larger than the trade side.

A firm that already has an exposure to a foreign currency (for example, a receivable of CHF 8 million due in 60 days) can take an offsetting position, such as a forward contract to sell those francs. A position that reduces an existing currency risk is **hedging**. A position that creates or increases currency risk (for example, buying a currency forward only because the buyer expects it to rise) is **speculating**. Corporations, investment funds, banks and governments all do both.

Participants: sell side and buy side

KEY CONCEPT

Group	Who	Notes
Sell side	Large multinational banks	The dealers in currencies and the originators of forward FX contracts
Buy side: corporations	Importers, exporters, multinationals	Buy and sell currencies for cross-border business; hedge future receipts and payments with forwards
Buy side: real money accounts	Mutual funds, pension funds, insurance companies and other institutional accounts	Restricted in their use of leverage

Group	Who	Notes
Buy side: leveraged accounts	Hedge funds, proprietary trading firms, other trading firms	Make substantial use of **leverage**, typically through derivatives
Buy side: governments	Governments, **sovereign wealth funds**, government pension funds, **central banks**	Transactional, investment or speculative needs; central banks may intervene to influence the exchange rate
Buy side: retail FX market	Households and relatively small institutions	Tourism, cross-border investment, speculative trading

Exam convention: real money accounts are accounts that do not use derivatives, and leveraged accounts are accounts that use them. Current practice: the dividing line is leverage. Many pension funds and insurers hedge currency exposure with FX forwards and are still real money accounts.

Reading an exchange rate quote

An **exchange rate** is the price of one currency in terms of another. Quotes are written as **price currency / base currency**: 0.9250 CHF/USD means one US dollar (the **base currency**, in the "denominator") costs 0.9250 Swiss francs (the **price currency**). Read "/" as "per".

- From the viewpoint of an investor whose home currency is the price currency, a price/base quote is a **direct quote**; for an investor whose home currency is the base currency it is an **indirect quote**.
- Inverting a quote swaps the roles: 0.9250 CHF/USD is the same as $1/0.9250 = 1.0811$ USD/CHF.
- Treat the base currency as the item being priced and the price currency as the money it is priced in. A rise in the price/base rate means the item has become more expensive: the base currency has appreciated, and the price currency has lost purchasing power over the base country's goods.

Spot and forward rates

- The **spot exchange rate** is for immediate delivery (for most currencies, settlement is two business days after the trade).
- A **forward exchange rate** is agreed today for an exchange on a specified future date (30, 90, 180 days, one year, ...). A forward contract fixes the amounts of both currencies to be exchanged. Example: a company that will receive NZD 4 million in 90 days and sells them forward at 0.5520 EUR/NZD has locked in EUR 2.208 million.

Nominal and real exchange rates

The rate quoted in the market at a point in time is the **nominal exchange rate**. The **real exchange rate** adjusts the nominal rate for changes in the two countries' price levels since a base period, so it tracks the **purchasing power** of one currency over the other country's goods:

KEY CONCEPT

$$\text{real P/B rate} = \text{nominal P/B rate} \times \frac{CPI_{\text{base currency}}}{CPI_{\text{price currency}}}$$

with both CPIs set to 100 in the base period. What moves the real P/B rate:

Change (other things equal)	Real P/B rate	Purchasing power of the price currency over base-country goods
Nominal P/B rate rises	Rises	Falls
Price level in the **base-currency** country rises	Rises	Falls
Price level in the **price-currency** country rises	Falls	Rises

If neither country's price level changes, the CPI ratio stays at 1 and the real rate moves exactly with the nominal rate. The *difference* between nominal and real rates therefore comes only from the ratio of the two price levels (relative inflation).

Example. Base period: both CPIs = 100 and the rate is 1.50 SGD/GBP (price currency SGD, base currency GBP). Two years later the nominal rate is 1.44 SGD/GBP, the UK CPI is 107 and the Singapore CPI is 104.

$$\text{real SGD/GBP} = 1.44 \times \frac{107}{104} = 1.4815$$

The nominal rate fell 4.0%, but faster UK inflation offset part of that, so the real rate fell only about 1.2% (from 1.50 to 1.4815). A Singapore resident gained less purchasing power over UK goods than the nominal move suggests.

For a quick estimate, work in percentage changes: the change in the real P/B rate is approximately the change in the nominal rate, plus inflation in the base-currency country, minus inflation in the price-currency country. In the example this gives $-4.0\% + 7.0\% - 4.0\% = -1.0\%$, close to the exact $1.4815/1.50 - 1 = -1.23\%$.

Percentage change in a currency's value

For a quote P/B, the percentage change in the **base currency** is simply

$$\%\Delta \text{ base} = \frac{\text{new P/B}}{\text{old P/B}} - 1$$

To get the change in the **price currency**, first invert both quotes so that it becomes the base currency:

KEY CONCEPT

$$\%\Delta \text{ price} = \frac{1/\text{new P/B}}{1/\text{old P/B}} - 1 = \frac{\text{old P/B}}{\text{new P/B}} - 1$$

Example. The CHF/USD rate moves from 0.8000 to 0.8600. The USD (base) has appreciated by $0.8600/0.8000 - 1 = 7.50\%$. The CHF (price) has depreciated by $0.8000/0.8600 - 1 = -6.98\%$. The two percentages differ in size.

When a currency buys more of a foreign currency (it has appreciated), foreign goods become cheaper for its residents, and the country's exports become more expensive for foreigners.

Common exam traps

- Reading a rise in USD/EUR as a stronger dollar. The euro is the base currency, so a higher quote means the euro has appreciated.
- Reusing the base currency's percentage change, with the sign flipped, for the price currency. Invert the quotes first; the two percentages differ. In the CHF/USD example, flipping the sign gives −7.50% for the franc instead of −6.98%.
- Putting hedge funds or central banks on the sell side. The sell side is the large dealer banks; hedge funds and central banks are buy-side participants.
- Forgetting that a currency forward is a derivative. An investor whose mandate bars derivatives cannot use one, even to hedge.
- Adjusting a real exchange rate with interest rates, or putting the price country's CPI in the numerator. The base country's CPI goes on top. In the SGD/GBP example, $1.44 \times 104/107 = 1.3996$ instead of 1.4815.
- Reading a pair in the market dealers' order. Dealers usually name the base currency first (EUR/USD 1.25 for 1.25 dollars per euro); the price/base form used here writes the same rate as 1.25 USD/EUR.

EXAM SHORTCUTS

- In a price/base quote a higher number means the base currency (the denominator) has appreciated, so identify the base currency before judging which currency strengthened.
- The percentage change in the real P/B rate is approximately the nominal change plus base-country inflation minus price-country inflation, a quick estimate that needs no CPI ratio.

BOTTOM LINE

- The FX market serves trade flows and the much larger capital flows; a position that reduces an existing currency risk is hedging, and one that creates or increases currency risk is speculating.
- The sell side is the large multinational dealer banks, and the buy side is corporations, real money and leveraged accounts, governments and central banks, and the retail market; the exam convention separates real money from leveraged accounts by derivatives use, while current practice separates them by leverage.
- A P/B quote states how many units of the price currency buy one unit of the base currency, so a higher quote means the base currency has appreciated, and the quote is direct for an investor whose home currency is the price currency.
- A spot rate is for immediate delivery (usually two business days), and a forward rate is agreed today for an exchange of fixed amounts on a specified future date.
- The real P/B rate equals the nominal P/B rate × $CPI_{\text{base}}/CPI_{\text{price}}$, so it rises with the nominal rate and the base-country price level and falls with the price-country price level.
- The base currency changes by new P/B ÷ old P/B − 1 and the price currency by old P/B ÷ new P/B − 1, and the two percentages differ in size.

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QUICK CHECK

Question 1

Buy-side participants in the foreign exchange market include real money accounts and leveraged accounts. Compared with a real money account, an investment account classified as a leveraged account is *most likely* distinguished by its:

A active use of leverage, typically through derivatives.

B role as a dealer quoting two-way prices to other participants.

C restriction to long-term, unleveraged holdings of foreign-currency assets.

Answer: A

Leveraged accounts are investment firms such as hedge funds, proprietary trading firms and other trading firms that make substantial use of leverage, typically through derivatives. Real money accounts, such as mutual funds, pension funds, insurance companies and similar institutional accounts, are restricted in their use of leverage. Both are buy-side participants.

Why the other options are wrong

- **B.** Quoting two-way prices is the role of the dealers on the sell side (large multinational banks). Leveraged accounts are buy-side clients of those dealers.
- **C.** Being restricted to long-term, unleveraged holdings describes a real money account, the opposite of a leveraged account.

Key takeaway Leveraged accounts (hedge funds, trading firms) use leverage extensively, typically via derivatives; real money accounts (mutual funds, pension funds, insurers) are restricted in their use of leverage.

MODULE 18.2

Managing Exchange Rates

LOS 18.b — Exchange rate regimes and effects on trade and capital flows

The IMF's classification of regimes

The IMF lists nine regimes: two for countries that do not issue a currency of their own and seven for countries that do. Moving down the table, the monetary authority generally gains more room for independent monetary policy.

KEY CONCEPT

Regime	Own currency?	How it works	Monetary policy independence
Formal dollarization	No	Uses another country's currency	None; the interest earned on the assets backing the currency goes to the issuing central bank
Monetary union	No	Several countries share one currency (e.g. euro area)	No national policy, but members share in setting the union's policy
Currency board arrangement	Yes	Explicit commitment to exchange domestic currency for a specified foreign currency at a fixed rate; domestic currency issued only when fully backed by foreign reserves	Essentially none (imports the anchor's inflation), some short-term room on interest rates; the board earns interest on its reserves
Conventional fixed peg arrangement	Yes	Pegged to a currency or basket within margins of **±1%**; maintained by **direct intervention** (buying/selling FX) and **indirect intervention** (interest rate policy, FX regulation, persuasion)	More than a currency board, but constrained by the peg
Target zone (pegged exchange rates within horizontal bands)	Yes	Like a peg but with wider fixed bands, e.g. **±2%** or more	More discretion than a conventional peg
Crawling peg	Yes	The peg rate is adjusted periodically, usually for inflation differences (**passive crawling peg**), or along a pre-announced path (**active crawling peg**, which can anchor inflation expectations)	Similar to a fixed peg
Management of exchange rates within crawling bands	Yes	The band around the central rate is **widened over time**; used to move from a fixed peg toward a floating rate when the authority lacks the credibility to float at once	Increases as the bands widen

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Regime	Own currency?	How it works	Monetary policy independence
Managed floating exchange rates	Yes	The authority reacts to indicators (balance of payments, inflation, employment) with no target rate or predetermined path; intervention may be direct or indirect. Trading partners may respond in ways that reduce stability	Considerable
Independently floating	Yes	Market-determined; intervention only to slow the rate of change and damp short-term swings, not to hit a target level	Full

A currency board differs from dollarization because the country still issues its own currency (fully backed), and it differs from a conventional peg because it is an explicit commitment to exchange domestic currency at a fixed rate, with the currency issued only when fully backed, rather than a peg that may move within a ±1% band.

Exchange rates, trade and capital flows

Because an appreciation makes imports cheaper and exports dearer, it also shifts trade volumes. If the USD/GBP rate falls (the dollar appreciates), US imports from the UK tend to rise and US exports to the UK tend to fall. These trade effects build up slowly.

The **balance of payments** requires that capital flows offset any imbalance in trade. A country with a **trade deficit** (imports > exports) must have a **capital account surplus** (foreigners buy more of its assets than its residents buy abroad), and a country with a trade surplus has a **capital account deficit**. The identity:

KEY CONCEPT

$$(X - M) \equiv (S - I) + (T - G)$$

where $X - M$ = exports minus imports, $S - I$ = private saving minus investment in physical capital, and $T - G$ = tax revenue minus government spending.

- A trade deficit means total domestic saving (private plus government) falls short of domestic investment; foreign capital fills the gap. At least one of the two gaps must then be negative: private saving below investment, a government budget deficit, or both.
- A government budget deficit that is not covered by an excess of private saving over investment goes together with a trade deficit.

Example. Private saving 260, investment 300, tax revenue 180, government spending 205 (billions). $(S - I) + (T - G) = -40 + (-25) = -65$: a trade deficit of 65, matched by a capital account surplus of 65.

Timing. Capital flows adjust much faster than spending, saving and asset prices, so they dominate exchange rate moves in the short and intermediate term. Trade flows matter more over the long term.

Reducing a trade deficit

The identity shows what has to change for a trade deficit to shrink on a lasting basis: the right-hand side must rise. That means domestic spending must fall relative to income: a smaller government budget deficit (larger $T - G$) or more private saving relative to domestic investment (larger $S - I$). A bigger capital account surplus does not cure a trade deficit; it is the financing side of the same imbalance.

LOS 18.c — Objectives of capital restrictions

KEY CONCEPT

Governments may limit capital flowing in, out, or both. Commonly cited objectives:

1. **Reduce the volatility of domestic asset prices.** Sudden outflows in a crisis can crash the prices of liquid holdings like equities and bonds, especially in small economies with large foreign holdings.
2. **Maintain fixed exchange rates.** Limiting capital flows makes an exchange rate target easier to defend, freeing monetary and fiscal policy for domestic goals.
3. **Keep domestic interest rates low.** If residents cannot move money abroad for higher yields, the central bank can keep rates low and run a more independent monetary policy.
4. **Protect strategic industries** (e.g. defense, telecommunications) from foreign ownership.

Capital restrictions act on flows of investment capital. Tariffs, quotas, **voluntary export restraints** and **regional trading agreements** are trade-policy tools that act on flows of goods and services.

Common exam traps

- Confusing a crawling peg with crawling bands. In a crawling peg the central rate itself is adjusted; with crawling bands the band around it widens, as a step from a peg toward floating.
- Assuming an independent float means no intervention at all. Intervention can still slow the rate of change; it just has no target level.
- Capital controls aim to keep domestic interest rates low (not high).

EXAM SHORTCUTS

- Because $(X - M) \equiv (S - I) + (T - G)$, a trade deficit implies that private saving falls short of investment, the government runs a budget deficit, or both, and a larger capital account surplus cannot be the cure.

BOTTOM LINE

- The IMF lists two regimes for countries without their own currency (formal dollarization and monetary union) and seven for countries with one, and moving toward floating generally gives more room for independent monetary policy.
- A currency board explicitly commits to swap domestic currency for a named foreign currency at a fixed rate, issuing currency only when fully backed, while a conventional fixed peg allows ±1% margins and a target zone wider bands such as ±2%.

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- A crawling peg adjusts the peg rate itself, crawling bands widen the band over time as a step toward floating, managed floating reacts to indicators without a target rate, and an independent float uses intervention only to slow the rate of change.
- A currency appreciation raises imports and cuts exports, but these trade effects build up slowly; capital flows dominate exchange rate moves in the short and intermediate term, and trade flows matter more in the long term.
- Under $(X - M) \equiv (S - I) + (T - G)$, a trade deficit is matched by a capital account surplus, and shrinking it on a lasting basis requires domestic spending to fall relative to income.
- Capital restrictions aim to reduce the volatility of domestic asset prices, maintain fixed exchange rates, keep domestic interest rates low and protect strategic industries from foreign ownership.

QUICK CHECK

Question 2

Under Pelagia's monetary law, the monetary authority may issue Pelagian dinars only when each new dinar is fully matched by euro reserves, and it must exchange dinars for euros on demand at a fixed rate. The authority earns interest on the euro reserves it holds. The regime Pelagia operates is best classified as:

A a currency board arrangement.

B formal dollarization.

C a conventional fixed peg arrangement.

Answer: A

Under a currency board arrangement the monetary authority commits to convert its own currency into one named foreign currency on demand at a fixed rate, and it may issue domestic currency only when every unit is backed by reserves of that foreign currency. The country keeps its own currency and earns interest on its reserves, but it gives up independent monetary policy and in effect imports the inflation rate of the anchor currency.

Why the other options are wrong

- **B.** Under formal dollarization a country has no currency of its own; it uses another country's currency, and the interest earned on the assets backing that currency goes to the issuing central bank. Pelagia issues its own dinars and earns interest on its reserves.
- **C.** A conventional fixed peg keeps the currency within a narrow band, about 1% either side of the target rate, through intervention and policy, without a rule that every unit of domestic currency be backed by foreign reserves. The monetary authority keeps more policy flexibility than under a currency board.

Key takeaway Currency board: the country has its own currency, fully backed by an anchor currency at a fixed rate; its reserves earn interest, and it has no independent monetary policy.

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Practice questions

Question 3

Tomas Riedl, an economist in Lorvania, tracks the real exchange rate of the Lorvanian crown against its main trading partner's currency, quoted as the number of crowns needed to buy one unit of the partner's currency (domestic/foreign). Holding all other factors constant, which of the following would cause this real exchange rate to fall?

A A rise in the foreign price level.

B A rise in the nominal exchange rate (domestic/foreign).

C A rise in the domestic price level.

Question 4

Which of the following is *least likely* to be cited by a government as an objective of restricting capital flows?

A Maintaining a fixed exchange rate.

B Protecting strategic industries from foreign ownership.

C Keeping domestic interest rates high.

Answers: 3. C · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/capital-flows-and-the-fx-market/. The full bank has 31 questions on this reading.

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ECONOMICS · READING 19

Exchange Rate Calculations

What you'll learn

LOS 19.a Calculate and interpret currency cross rates from two (or more) quotes that share a common currency.
LOS 19.b Explain the no-arbitrage link between spot rates, forward rates and interest rates; calculate forward rates from points, percentages or interest rates; interpret forward premiums and discounts.

MODULE 19.1

Foreign Exchange Rates

This reading calculates currency cross rates from two quotes against a common third currency. It then derives the no-arbitrage forward rate from the spot rate and the two interest rates, converts forward quotes given in points or percentages into outright rates, and interprets forward premiums and discounts.

LOS 19.a — Currency cross-rates

A **cross rate** is the exchange rate between two currencies derived from the rate of each against a third, common currency (usually the USD or EUR). Cross rates are needed when a currency pair has no active market of its own.

Method: make the units cancel. Write every quote as price/base and arrange the quotes so that the common currency cancels, leaving the required pair:

KEY CONCEPT

$$\frac{A}{C} = \frac{A}{B} \times \frac{B}{C}$$

- If a quote has the wrong orientation, invert it first: $B/A = 1/(A/B)$.
- Two quotes with the **same base** (e.g. A/USD and C/USD): divide, $A/C = (A/\text{USD}) \div (C/\text{USD})$.
- Two quotes with the **same price currency** (e.g. USD/A and USD/C): divide the other way, $A/C = (\text{USD}/C) \div (\text{USD}/A)$.
- With three quotes, chain them: $A/D = A/B \times B/C \times C/D$.

Example. Given 1.3620 CAD/USD and 0.7940 EUR/USD, find CAD/EUR:

$$\frac{\text{CAD}}{\text{EUR}} = \frac{\text{CAD}}{\text{USD}} \times \frac{\text{USD}}{\text{EUR}} = 1.3620 \times \frac{1}{0.7940} = 1.7154$$

One euro costs about 1.7154 Canadian dollars. The inverse, EUR/CAD, is $1/1.7154 = 0.5830$.

Sanity check. Ask "how many units of the price currency does one unit of the base currency buy?" If the euro buys more dollars than the Canadian dollar does, the CAD/EUR rate must be above 1.

Common exam traps

- Multiplying where the quotes should be divided, so the units do not cancel. Writing the units next to each number prevents this. In the CAD/EUR example, $1.3620 \times 0.7940 = 1.0814$ instead of 1.7154.
- Forgetting to invert at the end when the answer is asked in the opposite orientation (e.g. the chain gives X/Z but the question asks for Z/X). In the CAD/EUR example, the EUR/CAD rate is 0.5830, not 1.7154.
- "Direct" and "indirect" depend on the investor's home currency; the arithmetic of a cross rate does not change.

LOS 19.b — Spot, forward and interest rates: no-arbitrage, points, percentages, premiums and discounts

The no-arbitrage (interest rate parity) relation

If currencies trade freely and forward contracts are available, the percentage difference between forward and spot rates must approximately equal the difference between the two currencies' **risk-free interest rates**. Otherwise an investor could borrow one currency, convert it at spot, invest at the other currency's rate, and sell the proceeds forward for a riskless profit. The **no-arbitrage condition** rules this out: the round trip cannot earn more than the domestic riskless rate.

The idea behind it: investing at home, and investing abroad with the currency sold forward, are both riskless, so they must end with the same amount of domestic currency. With quotes as domestic/foreign (d/f, price/base):

$$(1 + r_d) = \frac{1}{S_{d/f}} (1 + r_f) F_{d/f}$$

KEY CONCEPT

$$\frac{F_{d/f}}{S_{d/f}} = \frac{1 + r_d}{1 + r_f} \qquad F_{d/f} = S_{d/f} \frac{1 + r_d}{1 + r_f} \qquad S_{d/f} = F_{d/f} \frac{1 + r_f}{1 + r_d}$$

The **price currency's** rate is in the numerator and the **base currency's** rate in the denominator. In percentage terms the same relation reads

$$\frac{F_{d/f}}{S_{d/f}} - 1 = \frac{r_d - r_f}{1 + r_f} \approx r_d - r_f$$

so the forward premium or discount on the base currency is roughly the interest rate differential for the period. Exam convention: the percentage relation is printed as $\%\Delta s = (r_f - r_d)/(1 + r_d)$, which holds only when the spot rate is quoted as foreign currency per unit of domestic currency. Current practice: with domestic/foreign (price/base) quotes, as used throughout this module, the forward premium is $(r_d - r_f)/(1 + r_f)$.

KEY CONCEPT

Interest rates	Forward vs spot (P/B quote)	Base currency	Price currency
$r_{\text{price}} > r_{\text{base}}$	Forward > spot	Forward premium	Forward discount
$r_{\text{price}} < r_{\text{base}}$	Forward < spot	Forward discount	Forward premium

Interest rates	Forward vs spot (P/B quote)	Base currency	Price currency
Equal	Forward = spot	Neither	Neither

The currency with the **higher interest rate** trades at a **forward discount** (it "depreciates" in the forward market by roughly the rate differential).

Periods shorter than a year. Use the interest rate for the period rather than the annual rate. Money market rates are annualized, usually on a 360-day basis, so for a 90-day forward each rate becomes $r \times 90/360$:

$$F = S \times \frac{1 + r_{\text{price}}\,(\text{days}/360)}{1 + r_{\text{base}}\,(\text{days}/360)}$$

Example. Spot is 1.4800 AUD/GBP, the one-year AUD rate is 4.5% and the one-year GBP rate is 5.0%.

$$F = 1.4800 \times \frac{1.045}{1.050} = 1.4730 \text{ AUD/GBP}$$

The GBP (higher rate, base) is at a forward discount of $1.47295/1.4800 - 1 = -0.48\%$, close to the 0.5% rate gap.

Example. Spot is 1.4650 CHF/GBP. The 180-day CHF rate is 1.20% and the 180-day GBP rate is 4.80%, both quoted as annualized rates on a 360-day basis. Find the 180-day forward rate in points.

1. De-annualize each rate: CHF $1.20\% \times 180/360 = 0.60\%$; GBP $4.80\% \times 180/360 = 2.40\%$.
2. Apply parity with the price currency (CHF) in the numerator: $F = 1.4650 \times 1.0060/1.0240 = 1.43925$.
3. Convert to points using this quote's 0.0001 point size (four decimal places): $(1.43925 - 1.4650)/0.0001 \approx -257.5$ points.

The GBP, the higher-yielding base currency, trades at a forward discount of about 1.76% for the half year.

If the quoted forward differs from the no-arbitrage forward, arbitrage is possible: buy the currency that is cheap in the forward market relative to parity and sell the one that is dear. If the forward price of the base currency is **too high**, borrow the price currency, buy the base currency at spot, invest it at the base-currency rate and sell the proceeds forward. The profit is what is left after repaying the loan with interest. Arbitrage trading pushes the spot and forward rates back to parity.

Forward rates are not forecasts. Reading a forward rate as the expected future spot rate would mean the spot rate is expected to move by roughly the interest rate differential, with the higher-yielding currency expected to depreciate. Capital-market reasoning points the other way: higher domestic rates attract capital and strengthen the currency. Historically, interest rate differentials have forecast future spot rates poorly, though they may be unbiased and usually get the direction right. Treat the no-arbitrage forward as the rate that rules out arbitrage at a point in time. It is not the expected future spot rate.

Forward quotes in points or percentages

Forward points are quoted in units of the last decimal place of the spot quote. For a four-decimal quote, one point = 0.0001:

$$F = S + (\text{points} \times \text{point size})$$

The size of a point follows the number of decimals in the quote. For a two-decimal quote such as 148.25 JPY/USD, one point is 0.01, so +35 points means +0.35.

A **percentage** forward quote is applied multiplicatively:

$$F = S \times (1 + \text{percentage})$$

Examples. Spot 0.6480 USD/AUD and +27.5 points gives $0.6480 + 0.00275 = 0.65075$. Spot 0.6480 with -0.35% gives $0.6480 \times (1 - 0.0035) = 0.64573$.

Forward premium or discount

For the **base currency**:

$$\text{forward premium } (+) \text{ or discount } (\text{-}) = \frac{F}{S} - 1$$

- A positive value means the base currency is at a **forward premium** (it buys more of the price currency forward than spot). The price currency is then at a **forward discount**.
- To state the premium or discount of the **price currency**, invert both quotes first so that it becomes the base currency. The two percentages differ slightly in size.
- A premium or discount for a period shorter than a year can be annualized roughly by multiplying by (12 / months).

Common exam traps

- One point is 0.0001 on a four-decimal quote: +14.2 points = +0.00142, not +0.0142 or +0.142.
- Reading negative forward points as a discount on the price currency. Forward below spot puts the *base* currency at a discount and the price currency at a premium.
- Putting the base currency's interest rate in the numerator, which makes the higher-yielding currency appear to trade at a forward premium. In the AUD/GBP example, $1.4800 \times 1.050/1.045 = 1.4871$ instead of 1.4730.
- Using annual rates directly in a 90- or 180-day parity calculation. Scale each rate to the period first (days/360 for rates quoted on a 360-day basis). In the CHF/GBP example, $1.4650 \times 1.0120/1.0480 = 1.41468$ instead of 1.43925.
- When asked for the forward rate in the inverted quote, compute the forward first, then invert.

EXAM SHORTCUTS

- Before computing a cross rate, check its size: if one unit of the base currency buys more of the common currency than one unit of the price currency does, the cross rate must be above 1.
- The currency with the higher interest rate trades at a forward discount, so whether the forward is above or below spot is known before any calculation: with a P/B quote, forward exceeds spot when the price currency's rate is higher.
- The forward premium or discount on the base currency is roughly the interest rate differential for the period, $r_{\text{price}} - r_{\text{base}}$, which checks a computed forward rate.

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BOTTOM LINE

- A cross rate is found by arranging price/base quotes so the common currency cancels, $A/C = A/B \times B/C$, inverting a quote first when it has the wrong orientation.
- Under no-arbitrage, with domestic/foreign (price/base) quotes, $F/S = (1 + r_d)/(1 + r_f)$, with the price currency's rate in the numerator and the base currency's rate in the denominator.
- With price/base quotes the forward premium on the base currency is $(r_d - r_f)/(1 + r_f)$, roughly the rate differential; the exam convention prints the relation as $(r_f - r_d)/(1 + r_d)$, which holds only for quotes of foreign currency per unit of domestic currency.
- For a forward shorter than a year, each annualized money market rate is scaled to the period, usually $r \times \text{days}/360$, before applying parity.
- A forward quoted in points is $F = S + \text{points} \times \text{point size}$, where one point is the last decimal place of the spot quote, and a percentage quote gives $F = S(1 + \text{percentage})$.
- The forward premium (+) or discount (–) on the base currency is $F/S - 1$, and the price currency's premium or discount is found by inverting both quotes first.
- If the quoted forward differs from the no-arbitrage forward, borrowing one currency, converting at spot, investing and selling the proceeds forward earns a riskless profit, and arbitrage trading restores parity.
- The no-arbitrage forward rate is the rate that rules out arbitrage at a point in time, not the expected future spot rate, and interest rate differentials have historically forecast future spot rates poorly.

QUICK CHECK

Question 1

Using the quotes EUR/USD 0.9240 and NZD/EUR 1.7960, an analyst calculates the spot cross rate between the US dollar and the New Zealand dollar. Stated as USD/NZD, it is *closest* to:

A 0.51448.

B 0.60259.

C 1.65950.

Answer: B

Convert EUR/USD into USD/EUR by inverting, then divide by NZD/EUR (equivalently, multiply by EUR/NZD) so the euro cancels and USD/NZD remains.

$$\text{USD/EUR} = \frac{1}{0.9240} = 1.08225$$

$$\frac{\text{USD}}{\text{NZD}} = \frac{\text{USD/EUR}}{\text{NZD/EUR}} = \frac{1.08225}{1.7960} = 0.60259$$

Why the other options are wrong

- **A.** This divides EUR/USD by NZD/EUR without first inverting EUR/USD, so the units do not cancel to USD/NZD.

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- **C.** This multiplies the two quotes: $0.9240 \times 1.7960 = 1.6595$. The units (EUR/USD $\times$ NZD/EUR) give NZD/USD, which is the inverse of the USD/NZD rate asked for ($1/1.6595 = 0.60259$).

Key takeaway Invert first, then chain the quotes so the common currency cancels.

Practice questions

Question 2

The spot exchange rate is 1.3450 CAD/USD. The 180-day money market rate is 4.20% for the Canadian dollar and 5.40% for the US dollar, both quoted as annualized rates on a 360-day basis. The 180-day no-arbitrage forward CAD/USD exchange rate is *closest* to:

A 1.3297.

B 1.3371.

C 1.3529.

Question 3

The central bank of Selvia unexpectedly raises its policy rate, and Selvian money market rates move well above euro-area rates. Two analysts comment on the outlook for the Selvian lev (SLV) against the euro:

- Ruiz: "The higher-yielding currency always trades at a forward discount, so the rate increase tells us the SLV will now weaken against the euro."
- Okoye: "Higher Selvian rates should draw in foreign capital, which tends to push the SLV up against the euro."

Which evaluation of the two comments is *most accurate*?

A Ruiz is right; the forward discount is a reliable forecast of the future spot rate.

B Neither is right; a change in the policy rate leaves the spot rate unaffected.

C Okoye is right; parity sets today's forward rate but does not forecast spot rates.

Question 4

A dealer's SGD/AUD quotes are 0.8870 for spot settlement and 0.8795 for delivery in 12 months. Measured against spot, the forward quote is a:

A premium of 75 points, and the SGD is at a forward discount to the AUD.

B discount of 75 points, and the SGD is at a forward premium to the AUD.

C discount of 7.5 points, and the SGD is at a forward discount to the AUD.

Answers: 2. B · 3. C · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/economics/exchange-rate-calculations/. The full bank has 29 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Corporate Finance

Exam weight 6–9% · 7 readings

How companies are organized and governed and whose interests they serve, how they manage working capital and liquidity, which investments they choose and how they fund them, and how a business model earns its money.

Reading 20	Organizational Forms, Corporate Issuer Features, and Ownership
Reading 21	Shareholder vs Stakeholder
Reading 22	Principal Agent Problem
Reading 23	Cash Conversion Cycle
Reading 24	NPV vs IRR
Reading 25	Modigliani Miller
Reading 26	Business Models

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CORPORATE FINANCE · READING 20

Organizational Forms, Corporate Issuer Features, and Ownership

What you'll learn

LOS 20.a Compare sole proprietorships, general and limited partnerships, and corporations by legal identity, owner-operator separation, liability, taxation and access to capital.

LOS 20.b Describe the key features of corporations: separate legal identity, shareholder voting and limited liability, dividends, and double taxation.

LOS 20.c Compare public (listed) and private companies, and the ways a private company goes public or a public company goes private.

MODULE 20.1

Features of Corporate Issuers

This reading compares sole proprietorships, partnerships and corporations by legal identity, the link between owners and operators, owner liability, taxation and access to capital, and describes the key features of corporate issuers, including the double taxation of dividends. It then contrasts public and private companies and the ways a company goes public or private.

LOS 20.a — Comparing organizational forms

A business's **organizational form** is its legal and organizational set-up. Five features separate one form from another:

1. whether the business is a **separate legal entity** from its owners;
2. whether the owners also run the business, and if not, how owners and operators are related;
3. whether the owners' liability for the business's debts and actions is **limited liability** or **unlimited liability**;
4. how the profits (or losses) are taxed;
5. how easily the business can raise more capital and spread its risk.

The four common forms

- **Sole proprietorship**: one person owns and runs the business. The law does not separate the business from its owner: all profits and losses are hers, profits are taxed as her **personal income**, and she has unlimited liability for the business's obligations. Growth is limited to what one individual can finance.
- **General partnership**: two or more partners pool their capital and share the risk, so the business can operate on a larger scale than a sole proprietorship. The partners run it under a **partnership agreement** (written, verbal, or implied by how the partners act) that sets out duties and profit shares. Every **general partner** has unlimited liability, and each partner's share of profit is taxed as personal income.
- **Limited partnership**: two classes of partner. There must be at least one general partner, who runs the business and has unlimited liability, and at least one **limited partner**, whose liability is capped at the amount invested and whose profit claim is proportional to that investment. Limited partners usually do not take part in management and usually do not appoint or remove the general partners. Exam convention: because general partners run the business, they typically receive a larger portion of the profits than the limited partners. Current practice: the comparison is best read per unit of capital, since general partners usually earn a

promote or carried interest on top of a pro-rata share, while limited partners, who often supply most of the capital, can still receive most of the profits in absolute terms. Profits are taxed as personal income of each partner.

- **Limited liability partnership (LLP)**: permitted in some jurisdictions; no general partner is required, so all partners have a liability shield. Unlike the passive limited partners of an LP, LLP partners may take part in running the firm. Exam convention: in the United States, LLPs are open only to providers of professional services such as law and accounting, with limits on the number of partners and the equity invested. Current practice: the rules are set state by state; some states restrict LLPs to designated professions, while others allow any partnership to register as an LLP.
- **Corporation** (a **limited company**): a legal entity separate from its owners and managers. All **shareholders** have limited liability: the most an owner can lose is the amount invested in the shares. Owners and managers are separated: shareholders elect a **board of directors**, which hires senior managers to run the company in shareholders' interests. Corporations have the widest access to capital, both **debt** (borrowed capital) and **equity** (ownership capital), which is why most large firms are corporations. Because owners do not have to run the business, a corporation can raise equity from many investors who want a share of the profits but no role in management.

KEY CONCEPT

Feature	Sole proprietorship	General partnership	Limited partnership	Corporation
Separate legal entity	No	No*	No*	Yes
Owners also operate?	Yes	Yes (all partners)	General partners only	No; board hires managers
Owner liability	Unlimited	Unlimited	GPs unlimited, LPs limited	Limited
Taxation of profits	Personal income	Personal income	Personal income	Corporate tax, then possibly tax on dividends
Access to capital	Limited (one owner)	Limited (partners)	Limited (partners)	Greatest (debt and equity)

*Exam convention: a partnership is an extension of its partners, and only the corporation is a separate legal entity. Current practice: many US states, under the Revised Uniform Partnership Act, treat a partnership as a separate entity, but its general partners remain personally liable.

A **public corporation** (public limited company) has shares that are sold to the public. Most public limited companies are listed, so their shares trade on a stock exchange. A **private limited company** has a limited number of shareholders, and the transfer of its shares is restricted.

Common exam traps

- A limited partnership needs a minimum of one general partner and one limited partner. Two of either class is not required.

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- Assuming a general partner's liability is capped because the business is a limited partnership. Only the limited partners have limited liability.
- Treating separate legal identity and limited liability as one feature. Where a partnership is a separate entity, its general partners are still personally liable; limited liability for every owner is what sets the corporation apart.
- Applying double taxation to a partnership. Partnership profit is taxed once, as the partners' personal income.

LOS 20.b — Key features of corporate issuers

- *Legal identity.* A corporation is formed by filing **articles of incorporation** with a regulator. As a legal person it can hire employees, sign contracts, borrow and lend.
- *Shares and voting.* Issuing shares lets a corporation raise large amounts of capital. Shareholders have **voting rights** to elect the board; listed shares are easy to transfer.
- *Dividends.* The board *may* distribute part of earnings as **dividends**; the corporation is not required to do so.
- **Double taxation.** Where a country taxes corporate earnings *and* taxes dividends as personal income, distributed profit is taxed twice. The burden is smaller when a company pays out less and reinvests more.

KEY CONCEPT

$$\text{effective tax rate on distributed profit} = t_c + t_d(1 - t_c)$$

More generally, with payout ratio p: total tax per unit of pretax profit $= t_c + t_d \times p \times (1 - t_c)$.

Example. Pretax profit is \$1,000, the corporate rate is 30% and dividends are taxed at 10%.

- Full payout: corporate tax \$300; dividends \$700; dividend tax \$70; total \$370, an effective rate of 37.0%.
- Half of after-tax profit paid out: corporate tax \$300; dividends \$350; dividend tax \$35; total \$335, an effective rate of 33.5%.

Common exam traps

- The dividend tax applies to the after-tax profit that is actually paid out. Applying it to pretax profit overstates the tax. In the full-payout example, a dividend tax on the \$1,000 pretax profit gives a total of \$400 (40.0%) instead of \$370 (37.0%).

LOS 20.c — Public versus private companies

Most public limited companies are **listed companies**: their shares are listed on a stock exchange, a rules-based open market with price and volume transparency. Shareholders may be individuals, other corporations, nonprofits or governments. Shares that are actively traded (those not held by insiders, strategic investors or sponsors) form the **free float**, usually quoted as a percentage of total shares outstanding:

$$\text{free float (\%)} = \frac{\text{shares outstanding} - \text{shares held by insiders, strategic investors and sponsors}}{\text{shares outstanding}}$$

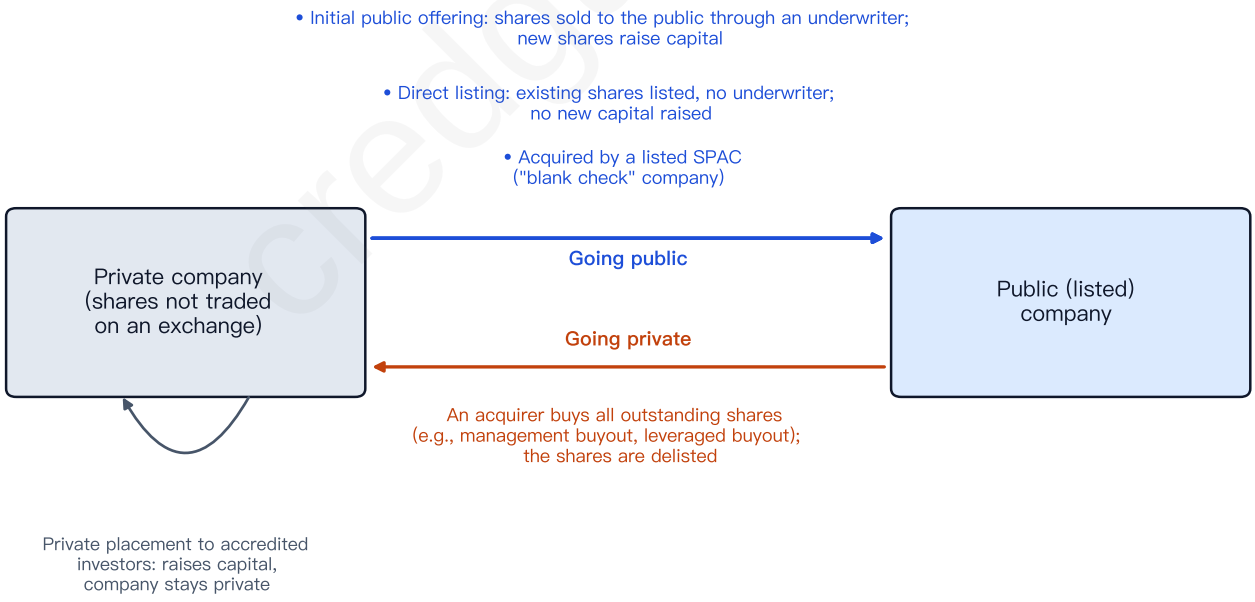
Public companies must meet compliance and reporting requirements, such as filing quarterly or annual financial reports with a regulator and disclosing material changes in the business or its ownership. **Private companies** face fewer regulatory requirements, disclose less, and can take a longer-term view; but their shares do not trade on an exchange, so value is hard to observe and exit is difficult (investors usually wait for an IPO or a sale of the company).

KEY CONCEPT

	Public (listed) company	Private company
Share trading	On an exchange; price observable	No exchange; transfer difficult
Periodic reporting to regulators	Required	Generally not required
Raising equity	Public offerings	**Private placements** to **accredited investors** (corporate and institutional investors, high-net-worth individuals)
Investment horizon	Pressure from public investors	Can take a longer-term view

Moving between private and public

The figure summarizes the routes described below.



Routes between private and public ownership

Ways to go public:

- **Initial public offering (IPO)**: the company's shares are first sold to the public, usually through an **underwriter** (investment bank). The company must meet the listing requirements of the chosen exchange. Once the shares are listed, investors buy and sell them among themselves without dealing with the company. An IPO typically includes new shares that raise equity capital; existing shareholders may also sell some of their shares in the offering.
- **Direct listing**: an exchange lists the company's existing shares. It is faster than an IPO and needs no underwriter. Exam convention: a direct listing raises no new capital for the company. Current practice: that describes the traditional (secondary) direct listing; some exchanges (e.g., NYSE) also permit a primary direct listing, in which the company sells new shares in the opening auction.
- Acquisition by a **special purpose acquisition company (SPAC)**, a "blank check" company that raises money in its own IPO, holds it in trust, and must use it to acquire a (not-yet-identified) private company within a set time. Because the SPAC is already listed, the acquired business becomes part of a listed company when the deal closes.

Going private. An acquirer buys all outstanding shares, for example in a **management buyout** or a **leveraged buyout**, and the company is delisted, often to restructure an underperforming firm with a lighter regulatory burden.

Common exam traps

- Assuming that going public always raises money for the company. Only newly issued shares do: a direct listing of existing shares raises none, and existing shares sold in an IPO pay the selling shareholders.
- A private placement raises capital but does not make the company public; buyouts take companies private.
- Computing free float from all shares outstanding. Holdings of insiders, strategic investors and sponsors are not actively traded and must be left out.

EXAM SHORTCUTS

- To decide whether going public raises money for the company, ask whether new shares are issued: new shares in an IPO raise capital, while existing shares sold in an IPO pay the selling shareholders, and under the exam convention a direct listing raises no new capital (current practice also allows a primary direct listing that sells new shares).

BOTTOM LINE

- Organizational forms differ in whether the law treats the business as distinct from its owners, whether owners also operate it, whether owner liability is limited, how profits are taxed and how easily it can raise capital.
- In a sole proprietorship and a general partnership the owners run the business, have unlimited liability and pay personal income tax on its profits.
- A limited partnership needs at least one general partner, who runs the business with unlimited liability, and at least one limited partner, whose liability is capped at the amount invested and who usually takes no part in management.
- A corporation exists as a legal person distinct from its owners and managers, all shareholders have limited liability, the shareholders elect a board that hires managers, and it has the widest access to debt and equity capital.

- Where both corporate earnings and dividends are taxed, the effective rate on distributed profit is $t_c + t_d(1 - t_c)$, and paying out less of the profit reduces the double-taxation burden.
- Most public companies are listed, so their shares trade on an exchange, and they must meet compliance and reporting requirements, while private companies disclose less, raise equity through private placements to accredited investors and offer investors no easy exit.
- Free float is the actively traded part of the shares outstanding, which excludes holdings of insiders, sponsors and strategic investors.
- A private company goes public through an IPO, a direct listing or acquisition by a SPAC, and a public company goes private when an acquirer, as in a management or leveraged buyout, buys all its shares and delists it.

QUICK CHECK

Question 1

Two siblings plan to buy and operate a vineyard. They will manage it themselves and accept unlimited liability for its debts. Four cousins will contribute capital but will take no part in running the business and want their liability capped at the amount they invest. The *most appropriate* organizational form is a:

A limited partnership.

B general partnership.

C limited liability partnership.

Answer: A

A limited partnership combines general partners, who manage the business and have unlimited liability (the siblings), with limited partners, whose liability is limited to their investment (the cousins). That is the mix described.

Why the other options are wrong

- **B.** In a general partnership every partner is a general partner with unlimited liability, so the cousins could not cap their liability.
- **C.** A limited liability partnership has no general partner: every partner has a liability shield. It does not fit owners who choose to accept unlimited liability, and some US states restrict LLPs to designated professions such as law and accounting.

Key takeaway Mixed roles (managers with unlimited liability + passive investors with limited liability) point to a limited partnership.

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Practice questions

Question 2

An analyst drafts a list of features of Kittredge Marine, a corporation. Which item on her list is *incorrect*?

A Kittredge's shareholders do not have to take part in managing the company.

B Kittredge has no legal identity apart from that of its shareholders.

C Kittredge's profits may be taxed at the corporate level and again when paid out as dividends.

Question 3

Brenmoor Robotics is privately held. Its board wants to bring in new equity capital and, at the same time, allow its shares to be bought and sold by the public on a stock exchange. The board is *most likely* to pursue:

A a direct listing of its existing shares.

B a management buyout.

C an initial public offering.

Answers: 2. B · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/corporate-finance/organizational-forms-corporate-issuer-features-and-ownership/. The full bank has 18 questions on this reading.

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CORPORATE FINANCE · READING 21

Shareholder vs Stakeholder

What you'll learn

LOS 21.a Compare the claims, risk and upside of lenders (debtholders) and shareholders, and why their interests can conflict.

LOS 21.b Describe a company's stakeholder groups (shareholders, lenders, board, managers, employees, suppliers, customers, governments) and compare their interests under shareholder and stakeholder theory.

LOS 21.c Describe the environmental, social and governance (ESG) factors investors consider and how ESG risks affect equity and debt investors.

MODULE 21.1

Stakeholders and ESG Factors

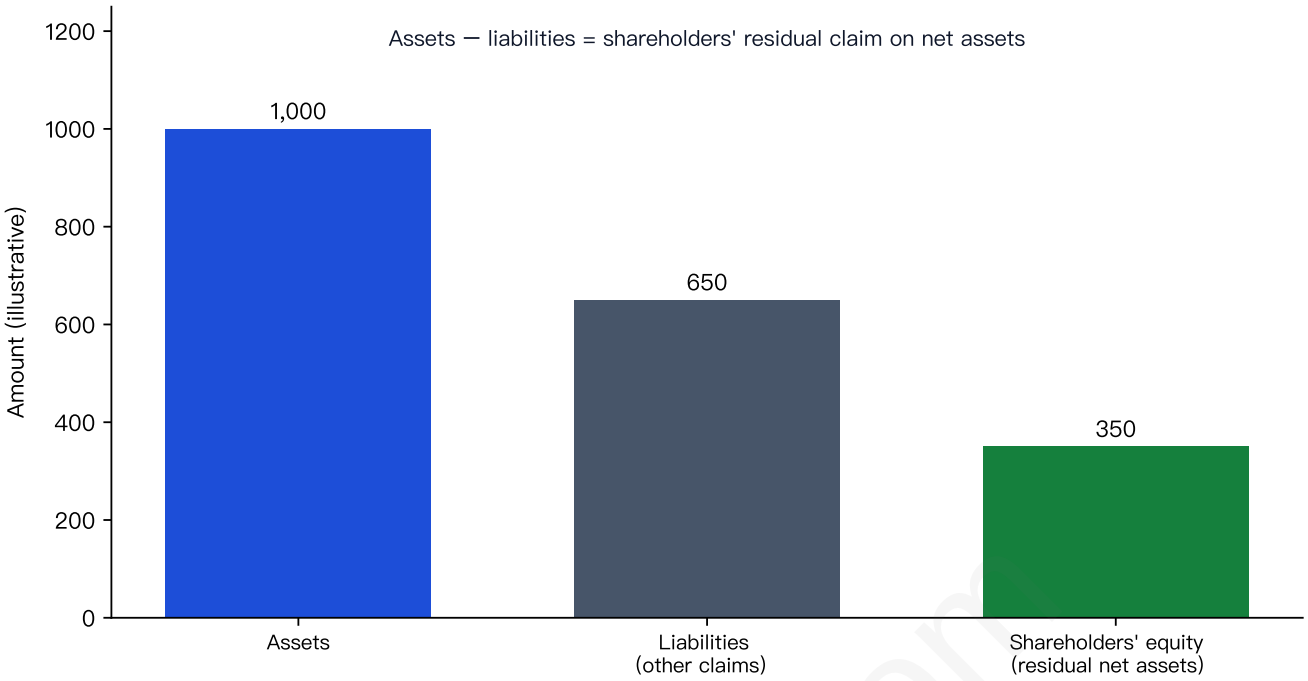
This reading contrasts what lenders and shareholders are owed and what each of them wants, including how leverage changes return on equity. It then describes a company's other stakeholder groups and board structures, and the environmental, social and governance factors that investors evaluate.

LOS 21.a — Lenders versus shareholders

Lenders (**debtholders**) have a legal, **contractual claim** to the interest and principal the company has promised to pay. **Shareholders** (equity holders, owners) have a **residual claim**: whatever is left of the company's net assets after all other claims are met. Lenders therefore rank ahead of owners, debt is less risky than equity, and debt is the cheaper form of capital. In most tax systems interest is also deducted in computing the company's taxable income, whereas dividends are paid out of after-tax profit.

The figure illustrates the balance-sheet claim: shareholders' residual is the company's net assets after liabilities, not an automatic distribution of annual net income.

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Shareholders' residual claim on net assets (illustrative values)

Both groups can lose their whole investment if the firm fails, but no more than they put in. The difference lies in the upside:

KEY CONCEPT

	Debtholders	Equity holders
Claim	Contractual: promised interest + principal	Residual: net assets after all other claims
Priority	Higher	Lower
Best case	Receive exactly what was promised	Theoretically unlimited gains
Worst case	Lose the investment	Lose the investment
Attitude to risk-increasing growth	May oppose: more default risk, no extra return	May favor: they keep the upside

Firm value = value of debt + value of equity. While the firm is worth more than its debt, extra value accrues to equity and the debt value stays roughly constant; if the firm becomes worth less than its debt, equity is worth zero and debt value falls with firm value.

Example: leverage and ROE. A firm needs \$800 of assets; revenue is \$600 and cash operating expenses are \$480 (no taxes), so operating income is \$120 (a 15% return on assets).

- All-equity: ROE $= 120/800 = 15\%$.
- Half debt at 6%: interest $= 0.06 \times 400 = \$24$; net income $= \$96$; ROE $= 96/400 = 24\%$.

- If revenue falls 20% to $480 while operating expenses stay at $480, operating income is zero: ROE is 0% unlevered but $-24/400 = -6\%$ levered, and lenders are still owed their $24.

Leverage raises ROE as long as the firm earns more on its assets than it pays on its debt; when it earns less, leverage lowers ROE and magnifies any loss. Lenders get only their promised return either way, so they have no upside and only downside. Shareholders therefore may favor adding debt to fund growth (it also avoids diluting their ownership), while existing lenders resist; lenders protect themselves with **covenants**, such as a maximum leverage ratio or a minimum interest coverage ratio.

Common exam traps

- Only interest and principal are contractual; common and preferred dividends are paid at the board's discretion.

LOS 21.b — Stakeholder groups and their interests

Under **shareholder theory**, corporate governance focuses on shareholders' interest in maximizing the market value of common equity, so the main conflict is the one between shareholders (the owners) and the managers they employ. Under **stakeholder theory**, the focus is broader: governance must balance the competing interests of shareholders and the other groups affected by the company's actions: creditors, employees, suppliers, customers, communities and governments.

KEY CONCEPT

Stakeholder	Main interests
Shareholders	Residual claim; vote to elect the board; want profitability and growth in share value. Residual claim with theoretically unlimited upside, and they can diversify their portfolios cheaply, so they may favor riskier growth.
Lenders — public **bondholders** and **private debtholders** (banks)	Timely interest and principal; solvency. Private lenders may see nonpublic information (less **information asymmetry**), which makes them a key funding source for small and medium-sized firms; some also hold equity and may be more willing to renegotiate loan terms. Bondholders rely on public information and have little or no influence over operations. Both are protected by covenants.
Board of directors	Protect shareholders' interests; hire, fire and pay senior managers; set strategy; monitor performance.
Senior managers	Salary, bonus, perks, continued employment. Bonuses tied to firm performance give them a stake in its financial success. Their employment income is tied to the firm, so they may choose less business risk than shareholders would.
Employees	Pay, advancement, training, working conditions, firm sustainability; may own shares through employee stock plans; in some industries they bargain through unions.

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Stakeholder	Main interests
Suppliers	Ongoing, profitable trade relationship; as short-term creditors, the firm's solvency and stability.
Customers	Quality product at a fair price; after-sale support; increasingly, the firm's social and environmental record.
Governments / regulators	Tax revenue, job creation, economic growth, social welfare; compliance with laws.

Board structure. Boards include **inside directors** (executives, founders) and **independent directors** with no material relationship to the company, who better protect shareholders. A **one-tier board** puts both on one board; exchanges typically require a majority of independent directors and may also require a range of backgrounds and skills. In a **two-tier board** (continental Europe), a **supervisory board** oversees a **management board** of insiders. In a **staggered board**, only a fraction of directors is elected each year, which makes a rapid overhaul by shareholders harder; companies defend it as giving continuity and a longer-term view of strategy. Minority shareholders generally prefer a majority-independent board elected in full.

Common exam traps

- Stakeholder theory does not put other groups in place of shareholders; it balances all groups, shareholders included.
- Lenders are less keen on a riskier strategy than suppliers. Both are creditors, but suppliers also gain from a customer's growth through more trade, while lenders can receive no more than the promised payments.
- When the firm's financial position weakens, shareholders absorb losses first, and the groups whose claims depend on its solvency are directly exposed: lenders, suppliers as short-term creditors, and managers and employees whose pay and jobs are at stake. Customers tend to be less affected than these groups. They mainly want a good product at a fair price, although they can lose after-sale support.

LOS 21.c — ESG factors considered by investors

Investors who take a stakeholder perspective evaluate **environmental, social, and governance (ESG)** factors because (1) governments increasingly push climate and social policy through regulation, (2) ESG issues can hit results through lost customer goodwill, fines and legal judgments, (3) poor governance lets managers exploit shareholders, and (4) many younger investors want ESG considered. **Negative externalities**, costs a firm imposes on others without bearing them, increasingly have to be recognized, explicitly or implicitly.

KEY CONCEPT

Factor	Examples
Environmental	Climate change, air and water pollution, deforestation, energy efficiency, waste management, water scarcity
Social	Customer privacy and data security, customer satisfaction, employee engagement, diversity and inclusion, labor relations, community relations

Factor	Examples
Governance	Board and audit committee composition, executive compensation, bribery and corruption, political contributions, lobbying

Climate-related risks come in two forms: **physical risk**, which is damage to assets or operations from more frequent severe weather, and **transition risk**, which arises when regulation or consumer choices force a move from high-carbon to low-carbon activity. Assets made unviable by such changes are **stranded assets**; an example is a plant closed because retrofitting it to new emissions rules is uneconomic. Resource-intensive industries affect the environment directly, while other industries may affect it indirectly. Weak safety policies or governance raise the risk of events such as oil spills or groundwater contamination, whose penalties, cleanup, litigation and reputational costs can be large.

Social factors shape how the company is seen to treat employees, customers and communities. Lowering social risk can cut costs through higher productivity, lower staff turnover, more loyal customers and fewer lawsuits. Governance systems need checks that keep managers acting ethically, lawfully and in shareholders' interests.

Who bears ESG risk. Analysts identify and measure the ESG risks a company faces and how they could affect the cash flows it generates in the future. Adverse ESG outcomes fall most heavily on equity investors. Debt investors are hurt mainly if losses are large enough to cause default. Because some ESG risks may take years to appear (a plant that complies today may become obsolete later), holders of longer-maturity debt may be more exposed than short-term lenders.

Common exam traps

- A carbon tax or a shift in customer demand toward low-carbon products is transition risk even though climate change is the underlying cause. Physical risk is damage done by the weather itself.

EXAM SHORTCUTS

- Compare the return on assets with the cost of debt before computing ROE: if the firm earns more on its assets than it pays on its debt, leverage raises ROE, and if it earns less, leverage lowers ROE.
- Classify a climate risk by its channel: damage done by the weather itself is physical risk, while a carbon tax, a new rule or a shift in demand toward low-carbon products is transition risk.

BOTTOM LINE

- Lenders have a contractual claim to promised interest and principal and rank ahead of shareholders, whose claim is residual, so debt is less risky than equity and the cheaper form of capital.
- Lenders and shareholders can each lose their whole investment but no more, yet lenders can receive at most the promised payments while equity has theoretically unlimited upside, so shareholders may favor risk-increasing growth that lenders resist with covenants.
- Shareholder theory focuses governance on maximizing the market value of common equity and on the conflict between shareholders and managers, while stakeholder theory balances the interests of shareholders and all other affected groups.
- Private lenders such as banks may see nonpublic information, which makes them a key funding source for small and medium-sized firms, while bondholders rely on public information and have little influence over operations.

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- Independent directors better protect shareholders than inside directors; a two-tier board has a supervisory board overseeing a management board, and a staggered board makes a rapid overhaul by shareholders harder.
- Investors evaluate ESG factors because regulation increasingly targets climate and social issues, ESG problems can cause lost goodwill, fines and judgments, poor governance lets managers exploit shareholders, and many younger investors want ESG considered.
- Climate change brings physical risk from severe weather and transition risk from regulation or consumer choices, and assets made unviable by these changes are stranded assets.
- Adverse ESG outcomes fall most heavily on equity investors, debt investors are hurt mainly if losses cause default, and holders of longer-maturity debt may be more exposed than short-term lenders.

QUICK CHECK

Question 1

Orrin Freight has bank loans, preferred stock and common stock outstanding. Which of the following payments are contractual obligations of Orrin?

A Interest and principal on its bank loans.

B Interest on its loans and dividends on its common stock.

C Interest and principal on its loans, and dividends on its preferred stock.

Answer: A

Lenders have a legal, contractual claim to the interest and principal payments the company has promised. Owners have only a residual claim: the company may pay dividends, but it is not required to.

Why the other options are wrong

- **B.** Common stock dividends are paid at the board's discretion. They are not a contractual obligation.
- **C.** Preferred stock dividends are also not a contractual obligation of the company. Only interest and principal owed to lenders are.

Key takeaway Contractual = interest + principal to lenders. Dividends (common or preferred) = discretionary distributions to owners.

Practice questions

Question 2

Which of the following *best* describes the primary focus of the stakeholder theory of corporate governance?

A Serving the interests of groups other than shareholders in place of the interests of shareholders.

B Maximizing the market value of the company's common equity.

C Resolving the conflicting interests of the company's managers and the various groups affected by what the company does.

Question 3

Ardent Power generates most of its electricity from coal. New national emissions limits and a steady shift in customer demand toward low-carbon energy threaten to make several of its plants uneconomic before the end of their useful lives. The climate-related risk Ardent faces is *best* described as:

A physical risk.

B transition risk.

C governance risk.

Answers: 2. C · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/corporate-finance/shareholder-vs-stakeholder/. The full bank has 16 questions on this reading.

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CORPORATE FINANCE · READING 22

Principal Agent Problem

What you'll learn

LOS 22.a Describe the principal-agent relationship and the conflicts between shareholders and managers/directors, between groups of shareholders, and between creditors and shareholders.

LOS 22.b Describe corporate governance and the mechanisms (shareholder, creditor, board committee, employee, customer/supplier, government) used to manage stakeholder relationships.

LOS 22.c Describe the risks of poor corporate governance and stakeholder management and the benefits of effective governance.

MODULE 22.1

Corporate Governance

This reading describes principal-agent relationships and the conflicts that arise between shareholders and managers or directors, between groups of shareholders, and between creditors and shareholders. It then sets out corporate governance and the mechanisms each stakeholder group uses, including the board and its committees, and the risks of poor governance and the benefits of effective governance.

LOS 22.a — Principal-agent relationships and conflicts

When one party (the principal) hires another (the agent) to act on its behalf, they are in a **principal-agent relationship**. A **principal-agent conflict** can arise because the agent's own interests may differ from the principal's. Example: a sales agent paid per contract signed has an incentive to sign up poor-quality customers the firm would rather refuse; the firm responds with acceptance standards and by dropping agents who act against it. **Agency costs** are the costs of such conflicts. *Direct* agency costs include the cost of monitoring the agent; *indirect* agency costs include business lost because of the conflict.

Shareholders vs. managers and directors

In a corporation, shareholders are the principals, and managers and directors are their agents. Conflicts also arise when inside directors side with management against shareholders, or when directors favor one group of shareholders over another. Typical conflicts:

KEY CONCEPT

Conflict	What happens
Insufficient effort	Managers give the firm too little time or attention (for example, because of outside commitments); projects and risks are poorly evaluated and total costs rise
Risk appetite	Managers' income depends on one firm, and managers paid mainly in cash share little in the gains, so they may take too little risk; managers paid mainly in **options** (no downside) may take too much
Empire building	Pay tied to company size encourages unnecessary or poor acquisitions

Conflict	What happens
Entrenchment	Managers take inadequate risk, copy competitors, or undertake projects that would not otherwise pay off but rely on their personal expertise; directors go along with management rather than challenge it
Self-dealing	Managers use company resources for personal benefit

Information asymmetry, the fact that managers know more about the firm than shareholders do, makes monitoring harder. It is more severe for large, multi-business or multinational firms, firms with complex products, and firms with low institutional ownership or low free float.

Between groups of shareholders

Controlling shareholders hold most of the votes and may act against minority shareholders. A controlling owner with most of his wealth in the firm may want it to diversify into unrelated businesses; already-diversified minority owners would rather it did not waste resources doing so. A **dual-class structure** gives one share class (often founders') more votes than its economic claim. CFA Institute opposes dual-class voting structures: they let one group of shareholders pursue its own interests at the expense of the others.

Example. A company has 100 million Class A shares with one vote each, held by the public, and 25 million Class B shares with ten votes each, held by its founders. The founders own $25/125 = 20\%$ of the shares but cast $250/(100 + 250) = 71\%$ of the votes, so they decide board elections.

Creditors vs. shareholders

Creditors' upside is capped while shareholders' is not, so shareholders may want the company to take more business risk than creditors would like. Actions that shift value from creditors to shareholders include issuing new debt that raises existing lenders' default risk, or paying larger dividends that shrink the asset base. The risk is greatest for long-term debtholders.

Common exam traps

- Directors are agents of the shareholders even though they hire and oversee the managers.
- Entrenchment shows up as too little risk-taking (copying competitors, otherwise unprofitable projects that depend on the manager's own knowledge). Excessive risk-taking points to option-heavy pay instead.

LOS 22.b — Corporate governance and stakeholder mechanisms

Corporate governance is the set of internal controls and procedures that governs how a company is run: a framework defining the rights, roles and responsibilities of groups within the organization, aimed at managing and minimizing conflicts of interest among stakeholders. Stakeholder management means understanding stakeholders' interests and communicating with them effectively. Public companies report through annual reports, proxy statements and public notices (performance, related-party transactions, executive pay, governance structure); transparency reduces information asymmetry. Private companies have lighter reporting requirements and usually give information to their investors directly.

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Shareholder mechanisms

- The annual general meeting (AGM) is held after fiscal year-end: audited statements presented, questions answered, votes cast. Corporate law sets when the meeting is held and how shareholders are notified; any shareholder may normally attend, speak and vote. **Proxy voting** is the main shareholder mechanism: a shareholder who does not attend assigns her vote to another person (often a director, a manager or her adviser), either with instructions on each issue or at that person's discretion.
- **Ordinary resolutions**, such as appointing the auditor or electing directors, need a simple majority of votes cast.
- **Extraordinary general meetings** are called for major decisions: changes to the bylaws, mergers and takeovers, special board elections requested by shareholders, and liquidation.
- Activist shareholders, increasingly hedge funds with large stakes, push for change through lawsuits, board seats, shareholder resolutions, a **proxy contest**, or a tender offer for a controlling stake. Shareholders can replace both senior managers and directors when they think performance would improve. The threat of a **hostile takeover** (one management does not support) disciplines management but may prompt takeover defenses such as a **staggered board** or a **poison pill** (rights that let existing shareholders other than the bidder buy extra shares at a low price, diluting the bidder).

Creditor mechanisms

The **bond indenture** sets out bondholders' rights and the issuer's obligations, including **covenants** (required or prohibited actions); bonds may be backed by collateral; a trustee monitors compliance. Creditor committees protect bondholders when the issuer is in distress; some countries require one in bankruptcy, and bondholders may form an ad hoc committee that does not represent every holder but shares their interests.

Board of directors and its committees

The board, which shareholders elect to represent them, is responsible for:

- selecting senior managers, setting their pay and evaluating their performance, and planning management and CEO succession;
- setting the company's strategic direction;
- approving changes in capital structure, significant acquisitions and large investment outlays;
- reviewing company performance and taking corrective action;
- setting up and supervising risk management and internal controls;
- making sure that financial reporting and internal audit are of high quality.

It delegates work to committees but keeps overall responsibility. How many committees a board needs, and how large they are, depends on the scale and complexity of the business; regulation often requires an audit committee.

KEY CONCEPT

Committee	Main responsibilities
Audit committee	Financial reporting and accounting policies; internal controls and internal audit; recommending the external auditor and its fee; remedies from audit findings

Committee	Main responsibilities
Nominating/governance committee	Governance code and board elections; nomination policy and recruiting board candidates; code of ethics and conflict-of-interest policies; monitoring laws and ensuring compliance
Compensation committee / remuneration committee	Recommends pay of directors and senior managers (including the CEO); benefit plans; evaluating senior managers. Should consist of independent directors only (required in many countries), because managers should not evaluate or set their own pay
Risk committee (financial services)	Risk policy and tolerance; enterprise-wide risk management
Investment committee (insurance)	Investment and capital management policies

Audit, compensation and governance committees are often made up only of nonexecutive or independent directors.

Other mechanisms

- *Employees*: labor laws, employment contracts, unions, employee board representatives (some countries), employee stock ownership plans (ESOPs).
- *Customers and suppliers*: contracts; increasingly, social media pressure.
- *Governments*: regulation and regulatory agencies; corporate governance codes (often "comply or explain"); exchange listing requirements.

Common exam traps

- Recommending the external auditor is the audit committee's job. The nominating committee does not do it.
- Routine director elections are ordinary resolutions at the AGM, but a special board election proposed by shareholders is an extraordinary general meeting matter.

LOS 22.c — Risks of poor governance, benefits of good governance

KEY CONCEPT

Poor governance and stakeholder management	Effective governance and stakeholder management
Weak control systems (audits, board oversight); some stakeholders gain at others' expense; accounting fraud or poor recordkeeping	Strong controls and compliance avoid many legal and regulatory risks
Less effective decision making: unmonitored managers choose sub-optimal risk; incentive pay that serves managers	Incentives aligned with shareholders improve operational efficiency

Poor governance and stakeholder management	Effective governance and stakeholder management
Related-party transactions benefiting insiders	Formal conflict-of-interest and related-party policies improve results
Legal risk and **reputational risk** from non-compliance and lawsuits	
Default risk and bankruptcy from mismanaging creditors' rights	Lower default risk and a **lower cost of debt**
Weaker financial performance and firm value	Better financial performance and higher company value

Common exam traps

- Related-party transactions are a risk of poor governance. Good governance limits them through formal policies; making such transactions more efficient is not one of its benefits.
- Counting control by the most interested stakeholder group as a benefit. Weak governance lets some stakeholders gain at the expense of others; effective governance brings strong control and monitoring through audits and board oversight.

EXAM SHORTCUTS

- Match the symptom to the conflict: too little risk-taking, copying competitors or projects that depend on the manager's own knowledge point to entrenchment, excessive risk-taking points to option-heavy pay, and unnecessary acquisitions point to pay tied to company size.

BOTTOM LINE

- A principal-agent conflict arises when an agent's interests differ from the principal's, and its agency costs are direct, such as monitoring, or indirect, such as business lost.
- Shareholders are the principals and managers and directors their agents; typical conflicts are insufficient effort, the wrong risk appetite, empire building, entrenchment and self-dealing, and information asymmetry makes them harder to monitor.
- Controlling shareholders may act against minority shareholders, and a dual-class structure gives one class more votes than its economic claim, which CFA Institute opposes.
- Because creditors' upside is capped, shareholders may want more business risk, and new debt or larger dividends can shift value from creditors to shareholders, a risk greatest for long-term debtholders.
- Corporate governance is the set of internal controls and procedures defining the rights, roles and responsibilities of groups within a company to manage conflicts among stakeholders, and proxy voting is the main shareholder mechanism.
- Ordinary resolutions such as electing directors or appointing the auditor need a simple majority of votes cast, while bylaw changes, mergers, special board elections and liquidation go to extraordinary general meetings.

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- The audit committee recommends the external auditor and oversees reporting and internal controls, and the compensation committee should consist of independent directors only, since managers should not set their own pay.
- Poor governance brings weak controls, related-party transactions, legal, reputational and default risk, while effective governance improves efficiency, lowers default risk and the cost of debt, and raises performance and company value.

QUICK CHECK

Question 1

Between which two parties associated with a company does a principal-agent relationship *most likely* exist?

A Its customers and its suppliers.

B Its directors and its regulators.

C Its shareholders and its managers.

Answer: C

In a principal-agent relationship one party hires another to act on its behalf. Shareholders (the principals), acting through the board of directors, hire managers (the agents) to run the company in the shareholders' best interests.

Why the other options are wrong

- **A.** Customers and suppliers deal with the company under contracts; neither hires the other to act on its behalf.
- **B.** Regulators oversee companies to enforce laws. Directors are not agents hired by regulators; they are agents of the shareholders.

Key takeaway Shareholders = principals; managers and directors = agents. Their differing interests create agency costs.

Practice questions

Question 2

Ridgeway Partners, a hedge fund, has built a 7% stake in Almont Paper and has started a proxy contest to replace three directors. Ridgeway's main aim is *most likely* to:

A keep the current board and management in place.

B reduce Almont's debt to protect its bondholders.

C increase the value of Almont's shares.

Question 3

The board of Castleford Water is drafting the charter of its new audit committee. Which of the following duties should be assigned to a different board committee instead?

A Overseeing the financial reporting process and the application of accounting policies.

B Reviewing the effectiveness of internal controls and of the internal audit function.

C Overseeing the company's corporate governance code, including board elections.

Answers: 2. C · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/corporate-finance/principal-agent-problem/. The full bank has 16 questions on this reading.

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CORPORATE FINANCE · READING 23

Cash Conversion Cycle

What you'll learn

LOS 23.a Explain the cash conversion cycle (DOH + DSO - DPO), what lengthens or shortens it, the cost of trade credit, and compare CCCs across issuers.

LOS 23.b Explain liquidity, primary and secondary liquidity sources, drags and pulls on liquidity, the cost of liquidity, and the current, quick and cash ratios.

LOS 23.c Describe working capital objectives and compare conservative, moderate and aggressive approaches to financing working capital, and factors affecting short-term funding.

MODULE 23.1

Liquidity Measures and Management

This reading explains the cash conversion cycle and its components, the cost of trade credit and measures of working capital, and how to compare issuers on them. It then covers primary and secondary sources of liquidity, drags and pulls on liquidity, liquidity ratios, and conservative, moderate and aggressive approaches to managing working capital.

LOS 23.a — The cash conversion cycle

The **cash conversion cycle (CCC)**, also called the **net operating cycle**, measures how long a company's cash is tied up between paying for inventory and other resources and collecting cash from the resulting sales:

KEY CONCEPT

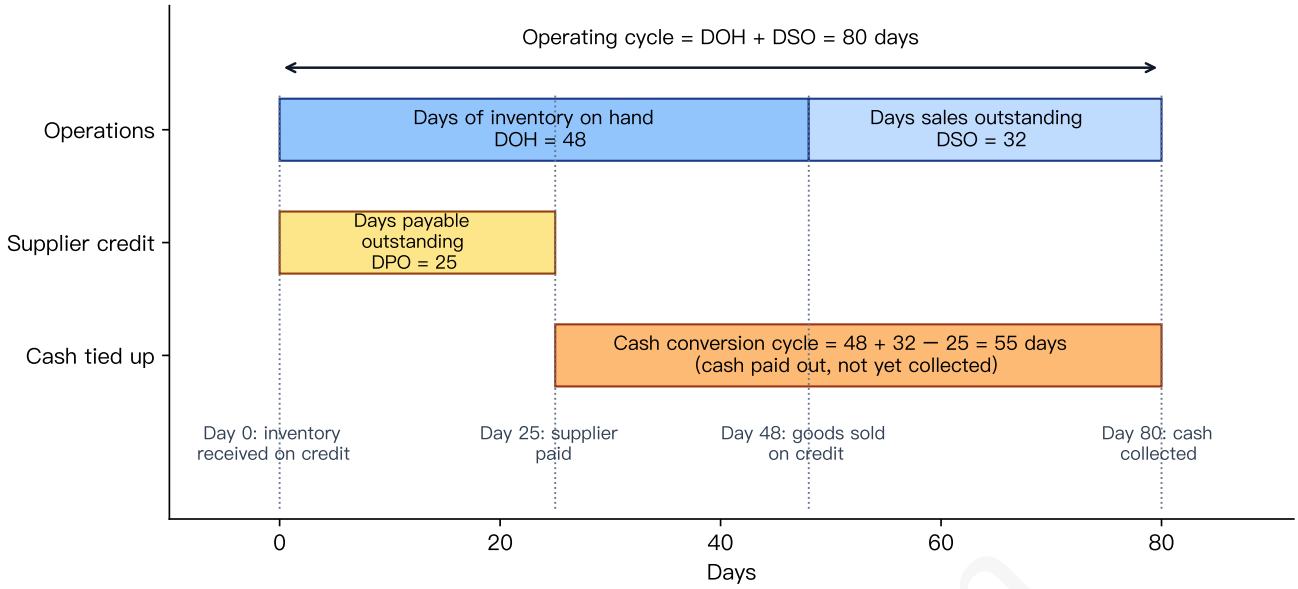
$$\text{CCC} = \text{DOH} + \text{DSO} - \text{DPO}$$

- **Days of inventory on hand (DOH)**: how long inventory is held before it is sold;
- **Days sales outstanding (DSO)**: how long customers take to pay (average days of receivables);
- **Days payable outstanding (DPO)**: how long the company takes to pay its suppliers (average days of payables).

The **operating cycle** is the time from receiving materials to collecting cash from the sale, so it equals $\text{DOH} + \text{DSO}$. Suppliers finance the first DPO days of it, so the CCC is the part of the operating cycle that the company must finance itself.

Example. $\text{DOH} = 48$, $\text{DSO} = 32$ and $\text{DPO} = 25$ give an operating cycle of 80 days and a CCC of 55 days, as the timeline shows.

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Operating cycle and cash conversion cycle on a timeline (DOH 48, DSO 32, DPO 25 days)

From turnover ratios to days. Activity data are often given as turnover ratios, so convert them to days first (365-day year):

1. $\text{DOH} = 365/\text{inventory turnover}$, where $\text{inventory turnover} = \text{cost of goods sold}/\text{average inventory}$;
2. $\text{DSO} = 365/\text{receivables turnover}$, where $\text{receivables turnover} = \text{revenue}/\text{average receivables}$;
3. $\text{DPO} = 365/\text{payables turnover}$, where $\text{payables turnover} =$ $\text{cost of goods sold (many analysts use purchases)}/\text{average trade payables}$;
4. $\text{CCC} = \text{DOH} + \text{DSO} - \text{DPO}$.

> *Example.* Inventory turnover of 6.0, receivables turnover of 9.125 and payables turnover of 7.3 give DOH $= 365/6.0 = 60.8$, DSO $= 365/9.125 = 40.0$ and DPO $= 365/7.3 = 50.0$ days. The operating cycle is 100.8 days and the CCC is $60.8 + 40.0 - 50.0 = 50.8$ days. A higher turnover means fewer days.

A lower CCC is generally better: less capital is tied up in working capital and cash is generated faster. A CCC that is high relative to peers may signal excessive investment in working capital and possible cash flow strain.

Change	Effect on CCC
DOH ↑ (slower inventory turnover)	↑
DSO ↑ (slower collection, looser customer credit)	↑
DPO ↓ (suppliers tighten terms, demand faster payment)	↑
DOH ↓, DSO ↓ or DPO ↑	↓

Because the CCC is a linear combination of its components, scaling every component by the same factor scales the CCC by that factor. A percentage change in one component is not the percentage change in the CCC. In the timeline example, if DPO rises from 25 to 35 days (the firm negotiates longer supplier terms), the CCC falls to 45 days: DPO rose 40%, but the CCC fell about 18%.

A company can shorten its CCC by working on any of the three components, but each lever has a cost:

Lever	How the CCC is shortened	Cost or risk
DOH ↓	Hold less raw material and finished goods	Production bottlenecks if supplies are disrupted; unable to meet spikes in demand
DSO ↓	Tighten the credit given to customers	Lost sales
DPO ↑	Take longer to pay suppliers	Early-payment discounts are given up (see the cost of trade credit below)

Trade credit and its cost

Accounts payable are an **implicit source of credit** from suppliers (bank loans are an explicit source). Terms **a/b net c** mean a discount of a (a percentage) if paid within b days, otherwise the full amount is due by day c. Skipping the discount is borrowing $(1 - a)$ per \$1 of invoice for $(c - b)$ days at a cost of a:

KEY CONCEPT

$$\text{EAR of supplier financing} = \left(1 + \frac{a}{1 - a}\right)^{365/(c-b)} - 1$$

where a is the discount as a decimal, b the days until the discount expires and c the days until full payment is due.

Example. Terms 1/10 net 40, bank rate 7%. $\text{EAR} = (1 + 0.01/0.99)^{365/30} - 1 \approx 13.0\% > 7\%$, so borrow from the bank and take the discount.

Comparing CCCs

CCCs differ by industry. Pharmaceutical firms, for example, carry large inventories (long CCC); airlines collect cash before providing the service and hold little inventory (short CCC). Compare a firm with its own industry or with its own history. Scale working capital by sales to compare firms of different size, and compare only firms in similar lines of business, because the ratio varies by industry. A company with a short CCC ties up less working capital for each unit of sales, so the two measures usually point the same way. Analysts often prefer net working capital, built from operating items only, because it ties closely to the CCC:

$$\text{total working capital} = \text{current assets} - \text{current liabilities}$$

$$\text{net working capital} = (\text{current assets} - \text{cash and marketable securities}) - (\text{current liabilities} - \text{short-term and current debt})$$

Example. Current assets of 645 are cash and marketable securities 55, receivables 260, inventory 310 and prepaid expenses 20. Current liabilities of 485 are accounts payable 290, accrued expenses 45, short-term notes 120 and the current portion of long-term debt 30. Total working capital $= 645 - 485 = 160$. Net working capital $= (645 - 55) - (485 - 150) = 590 - 335 = 255$.

Common exam traps

- Adding DPO to the operating cycle. Supplier credit shortens the CCC, so DPO is subtracted. In the timeline example, $48 + 32 + 25 = 105$ days instead of 55.
- A CCC can be negative. With prepaid sales and little inventory, the firm collects from customers before it pays suppliers, so supplier credit funds its operations.
- For net working capital, remove only cash and marketable securities from current assets and only short-term and current debt from current liabilities. Receivables, prepaid items, accounts payable and accrued expenses stay in.

LOS 23.b — Liquidity

Liquidity of an asset is its nearness to cash; of a liability, its nearness to settlement. Inventory is less liquid than receivables (it must be sold, then collected).

Current assets ranked by nearness to cash

Current asset	What must happen before it becomes cash	Liquidity
Cash	Nothing	Highest
Marketable securities	Sell them in the market	Very high
Accounts receivable	Collect from customers	Lower
Inventory	Process (if needed) and sell, then collect the receivable	Lowest

For an issuer, liquidity is the availability of cash and liquid assets to meet short-term obligations. Long-term solvency depends on generating enough cash from the business to service liabilities, and analysts judge liquidity management mainly from the statement of cash flows. Companies normally rely on primary sources and turn to secondary sources only when needed.

Primary liquidity sources	Secondary liquidity sources
Cash and marketable securities on hand	Suspending dividends
Bank borrowings (e.g., credit lines)	Delaying or reducing capital investment
Cash generated by the business	Selling assets; issuing equity
	Renegotiating/restructuring debt; filing for bankruptcy protection

Secondary sources are costlier and send a negative signal to the market. Cash buffers cost money (idle capital) but reduce the risk of having to rely on secondary sources when the CCC deviates from normal (seasonality, demand shocks).

- A **drag on liquidity** delays cash *inflows*: slow-moving or obsolete inventory (DOH ↑), slow or uncollectible receivables (DSO ↑).
- A **pull on liquidity** accelerates cash *outflows*: suppliers cut credit lines or demand faster payment (DPO ↓).

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The **cost of liquidity** is the discount to fair value accepted when assets must be sold quickly. *Example.* $50 of cash (0% cost), $150 of receivables and inventory sold at a 10% discount, and $200 of equipment sold at a 25% discount give proceeds of $50 + 135 + 150 = \$335$ against a fair value of $400; cost $= \$65 = 16.25\%$.

Liquidity ratios

$$\text{current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{quick ratio} = \frac{\text{cash and marketable securities} + \text{receivables}}{\text{current liabilities}}$$

$$\text{cash ratio} = \frac{\text{cash and marketable securities}}{\text{current liabilities}}$$

A current ratio above 1 means current assets are enough to cover current liabilities. The **quick ratio** excludes **inventories**, the least liquid current asset, so it is more conservative than the **current ratio**; the **cash ratio** also excludes receivables and is the most stringent. *Example.* With cash 30, receivables 50, inventory 90 and current liabilities 100, the current ratio is 1.70, the quick ratio 0.80 and the cash ratio 0.30.

Common exam traps

- Classing delayed capital expenditure or suspended dividends as primary sources. Firms turn to them only under pressure, so they are secondary sources.
- Suppliers cutting credit lines is a pull on liquidity, not a drag, even though it lengthens the CCC just as slow collections do.
- Leaving receivables out of the quick ratio. Only the cash ratio excludes them. In the ratio example, $30/100 = 0.30$ instead of 0.80.

LOS 23.c — Managing working capital and liquidity

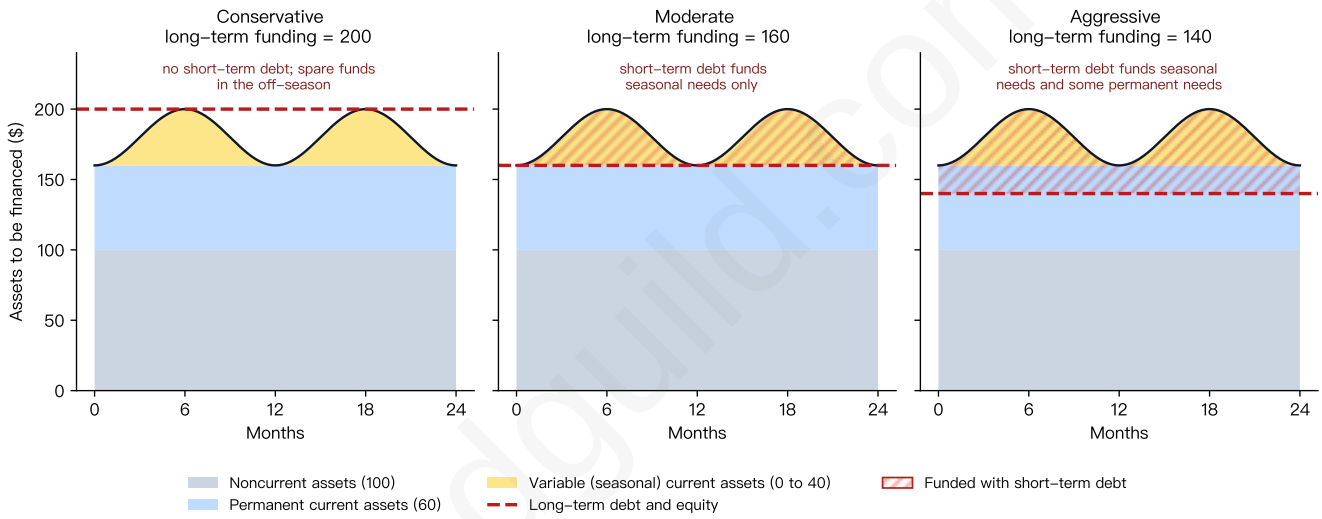
Working capital management tries to maximize profit while keeping enough liquidity to run the business and meet obligations. Holding more short-term assets is safer but earns less; financing with short-term debt is cheaper than long-term debt or equity but carries **rollover risk**. Compare firms by scaling short- and long-term assets by sales.

KEY CONCEPT

	Conservative approach	Moderate approach	Aggressive approach
Short-term assets held	High (relative to long-term assets)	Middle	Low
Financing of working capital	Long-term debt and equity	Permanent current assets: long-term; variable (seasonal) current assets: short-term	Short-term debt

	Conservative approach	Moderate approach	Aggressive approach
Benefits	Permanent capital, little rollover need; flexibility in market disruptions; high probability of meeting short-term obligations	Balance	Lower financing costs, higher returns
Costs / risks	Higher costs, lower profitability; long-term lenders may impose covenants (e.g., minimum interest coverage)	Balance	Risk of failing to meet obligations; vulnerable to market disruptions

Permanent current assets are the level of current assets the business needs all year; **variable (seasonal) current assets** are the extra amount needed at peak times. For the same assets, the approaches differ in how far long-term debt and equity reach up this stack of needs; the rest is funded short term:



Funding the same seasonal working capital needs under the conservative, moderate and aggressive approaches

Factors in choosing short-term funding: company size (small firms have fewer options), creditworthiness, the legal system (lender protections), regulation (e.g., banks and utilities), and the underlying assets available as collateral. Firms should keep several funding sources, know their costs, and arrange them before they are needed.

Common exam traps

- Treating long-term funding of recurring needs such as inventory, wages and rent as the moderate approach. Funding all working capital long term is conservative; the moderate approach funds only permanent needs long term and seasonal needs short term.

EXAM SHORTCUTS

- Compare the EAR of skipping the discount with the bank rate: if the EAR is higher, borrowing from the bank to take the discount is cheaper.

- Scaling every CCC component by the same factor scales the CCC by that factor, but a percentage change in one component is not the CCC's percentage change, so recompute the CCC in days.

BOTTOM LINE

- The cash conversion cycle is $\text{CCC} = \text{DOH} + \text{DSO} - \text{DPO}$; the operating cycle is DOH + DSO, and the CCC is the part of it the company must finance itself.
- With a 365-day year, DOH, DSO and DPO equal 365 divided by inventory turnover, receivables turnover and payables turnover.
- A lower CCC is generally better; it rises with DOH or DSO and falls with DPO, and each lever has a cost: production bottlenecks, lost sales or forgone early-payment discounts.
- Forgoing a discount on terms a/b net c costs $(1 + a/(1 - a))^{365/(c-b)} - 1$ per year.
- CCCs vary by industry, so compare a firm with its industry or its own history; net working capital removes cash and marketable securities and short-term and current debt, which ties it closely to the CCC.
- Primary liquidity sources are cash and marketable securities, bank borrowings and cash generated by the business, while secondary sources such as suspending dividends, delaying capital investment or selling assets are costlier and send a negative signal.
- A drag on liquidity delays inflows (higher DOH or DSO), a pull accelerates outflows (lower DPO), and the quick ratio excludes inventories while the cash ratio also excludes receivables.
- A conservative approach holds more short-term assets financed long term, an aggressive approach holds fewer financed with short-term debt, and a moderate approach funds permanent current assets long term and seasonal ones short term.

QUICK CHECK

Question 1

The CFO of Brightwater Distributors wants to shorten the company's cash conversion cycle (CCC). Which of the following changes would achieve this goal?

A Days sales outstanding increases.

B Days payable outstanding decreases.

C Days of inventory on hand decreases.

Answer: C

The CCC equals DOH plus DSO minus DPO and measures how long cash is tied up in inventory and receivables before it returns from sales. Selling inventory faster lowers DOH and therefore shortens the CCC.

$$\text{CCC} = \text{DOH} + \text{DSO} - \text{DPO}$$

A lower DOH shortens the CCC; a higher DSO or a lower DPO lengthens it.

Why the other options are wrong

- **A.** Slower collection of receivables raises DSO and lengthens the CCC.

- **B.** Paying suppliers sooner lowers DPO; because DPO is subtracted, the CCC gets longer.

Key takeaway CCC falls when DOH or DSO falls, or when DPO rises.

Practice questions

Question 2

Kellan Foods buys ingredients from a supplier that offers terms of 3/15, net 75. Kellan can also borrow from its bank at an annual rate of 9%. Which source of financing is cheaper for Kellan?

A Borrowing from the bank.

B Forgoing the discount and using the supplier's credit.

C The two sources cost the same.

Question 3

Talbridge Electronics' main component suppliers have cut the trade credit line they extend to it and now require payment within 15 days instead of 45 days. For Talbridge, this development is *best* described as a:

A drag on liquidity.

B pull on liquidity.

C secondary source of liquidity.

Question 4

Three companies describe how they handle their working capital. Whose practice fits a conservative approach *best*?

A A distributor pays its warehouse rent and staff wages with rolling 90-day bank loans.

B A manufacturer keeps far more long-term assets than short-term assets relative to its sales.

C A retailer finances its inventory with a ten-year term loan.

Answers: 2. A · 3. B · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/corporate-finance/cash-conversion-cycle/. The full bank has 22 questions on this reading.

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CORPORATE FINANCE · READING 24

NPV vs IRR

What you'll learn

LOS 24.a Describe the four types of capital investments (going concern, regulatory/compliance, expansion, other) and the analysis each requires.

LOS 24.b Describe the capital allocation process; calculate and interpret NPV, IRR and ROIC and contrast their use in capital allocation.

LOS 24.c Describe the principles of capital allocation (after-tax incremental cash flows, sunk and opportunity costs, externalities, timing) and common pitfalls.

LOS 24.d Describe the types of real options relevant to capital investments: timing, abandonment, expansion, flexibility and fundamental options.

MODULE 24.1

Capital Investments and Project Measures

This reading classifies capital investments, sets out the capital allocation process, and calculates and compares NPV, IRR and ROIC. It then covers the principles for estimating a project's cash flows, common capital allocation pitfalls, and the real options that can add value to a project.

LOS 24.a — Types of capital investments

A **capital investment** is spending on long-lived assets whose cash flows arrive over more than one year. Capital investments fall into four groups, which differ mainly in how much analysis they need. The first two keep the existing business running; the last two aim to grow it and need the most analysis.

KEY CONCEPT

Type	Purpose	Typical depth of analysis
Going concern projects	Keep the existing business running, or cut costs in it (e.g., replace a worn or obsolete but still usable machine with a cheaper-to-run model)	Low: maintenance spending needs little analysis; efficiency (replacement) decisions need a cost comparison
Regulatory/compliance projects	Required by a government agency or an insurer, often for safety or environmental reasons	Moderate: the firm must do it, so the analysis compares alternative ways of carrying it out; the project itself brings in little to no revenue
Expansion projects	Grow the business: new markets or new products in the current market	High: needs forecasts of demand, revenues and expenses
Other projects	Investments outside existing lines of business (start-up-like ventures, buying a firm in a new industry)	High: great uncertainty, risk of overpaying in acquisitions

For pure maintenance spending the only questions are whether to keep the existing operations going and, if so, whether to keep the current processes.

Two further points:

- Firms often use **match funding**: financing a project with sources whose life matches the project's life, which reduces financing risk.
- Analysts often use annual **depreciation expense** as a rough estimate of the capital spending a firm needs just to maintain its business (going concern spending).

Common exam traps

- Little or no revenue is not a reason to reject a regulatory/compliance project.
- Replacing a machine that still works, to lower operating costs, is a going concern project, not an expansion project, even though it involves buying new equipment.

LOS 24.b — The capital allocation process, NPV, IRR and ROIC

The process

Capital allocation is the process of identifying and evaluating capital projects, meaning projects whose cash flows are received over a period longer than one year. The same framework can be applied to any corporate decision that affects future earnings. It matters because the assets are costly and long-lived, because the same thinking applies to other decisions (working capital, mergers and acquisitions), and because good capital allocation serves management's primary goal of maximizing shareholder value.

The four administrative steps are:

1. **Idea generation**: the most important step; ideas come from senior management, divisions, employees or outsiders.
2. **Analyzing project proposals**: forecast each project's cash flows to judge its profitability.
3. **Creating the firm-wide capital budget**: rank and schedule profitable projects given cash flow timing, available resources and the firm's strategy; a project that looks attractive alone may not fit the strategic plan.
4. **Monitoring decisions and conducting a post-audit**: compare actual with forecast results; the **post-audit** exposes systematic forecasting errors and pushes managers to improve operations.

Arranging the financing for projects is not one of these four steps.

Net present value (NPV)

The **net present value (NPV)** adds up the present values of all the expected incremental after-tax cash flows of a project, discounted at the project's required rate of return (the firm's cost of capital adjusted for project risk):

KEY CONCEPT

$$NPV = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \cdots + \frac{CF_n}{(1+r)^n} = \sum_{t=0}^{n} \frac{CF_t}{(1+r)^t}$$

where CF_0 is usually a negative initial outlay. **Decision rule** for independent projects: accept if $NPV > 0$ (expected to increase shareholder wealth), reject if $NPV < 0$. A zero-NPV project is expected to leave shareholder wealth unchanged. In theory, accepting a project raises the firm's value by its NPV, so the share price rises by the NPV per share.

Example. A company with 40 million shares trading at $25 announces a project that costs $90 million; the present value of its after-tax inflows is $96 million. If investors had not expected the project and accept these estimates, NPV $= 96 - 90 = \$6$ million, or $6/40 = \$0.15$ per share, so the price rises to $25.15.

Internal rate of return (IRR)

The **internal rate of return (IRR)** is the discount rate at which the inflows and the outflows have equal present values, i.e., the rate at which NPV = 0:

$$0 = \sum_{t=0}^{n} \frac{CF_t}{(1+IRR)^t}$$

Decision rule: accept if IRR > required rate of return (the **hurdle rate**), reject if IRR < hurdle rate. The required rate is usually the firm's cost of capital, raised or lowered when the project is riskier or safer than the firm's average project. Because NPV falls as the discount rate rises, the two rules agree for a conventional independent project:

Discount rate used vs. IRR	Sign of NPV
Discount rate < IRR	NPV > 0
Discount rate = IRR	NPV = 0
Discount rate > IRR	NPV < 0

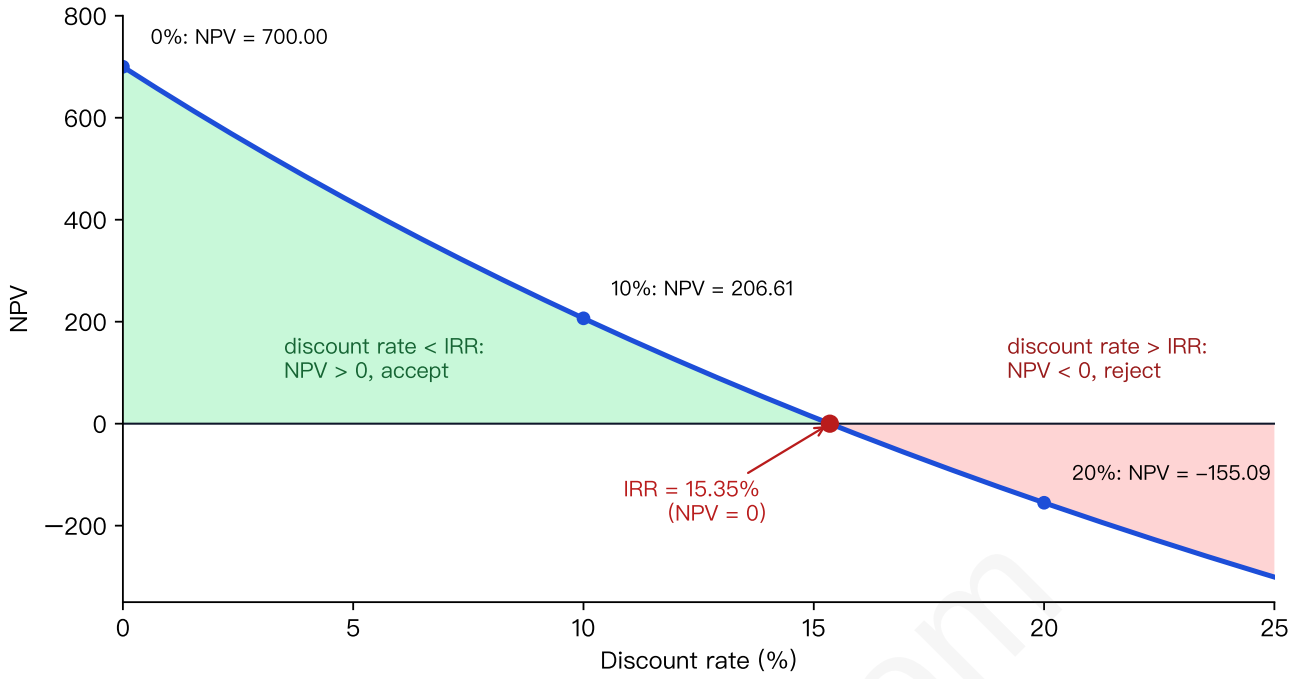
Example. A project costs 2,000 and returns 700, 900 and 1,100 at the ends of years 1–3; the required return is 10%.

$$NPV = -2{,}000 + \frac{700}{1.10} + \frac{900}{1.10^2} + \frac{1{,}100}{1.10^3} = -2{,}000 + 636.36 + 743.80 + 826.45 = 206.61$$

Calculator (TI BA II Plus): [CF] [2ND][CLR WORK]; CF0 = −2,000; C01 = 700, F01 = 1; C02 = 900, F02 = 1; C03 = 1,100, F03 = 1; [NPV] I = 10, [↓][CPT] = 206.61; [IRR][CPT] = 15.35%. Both rules say accept (NPV > 0 and 15.35% > 10%).

The figure plots this project's NPV at discount rates from 0% to 25%; it shows the table above as a curve.

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NPV of the example project (outlay 2,000; inflows 700, 900, 1,100) at different discount rates

Calculator tips for uneven cash flows

- Use the frequency registers (F01, F02, ...) for repeated equal cash flows.
- A year with no cash flow must be entered as a cash flow of 0; skipping it shifts every later cash flow one year too early.
- If a sale (or clean-up cost) happens in the same year as an operating cash flow, net them into one cash flow for that year; entering the sale as an extra period puts it one year too late.

Conventional vs. unconventional cash flows

A **conventional cash flow pattern** changes sign once (outflow(s) followed by inflows). An **unconventional cash flow pattern** changes sign more than once, e.g., an outflow at the end for decommissioning. Cash flows spaced at uneven time intervals are also treated as unconventional. Spreadsheets handle both cases best.

NPV vs. IRR

KEY CONCEPT

	NPV	IRR
Main advantage	Direct measure of the expected increase in firm value	A percentage return; shows the margin of safety (how far the return can fall before NPV turns negative)
Reinvestment assumption	Interim cash flows reinvested at the required rate of return (more realistic)	Interim cash flows reinvested at the IRR

	NPV	IRR
Unconventional cash flows	No problem	Possible **multiple IRRs** that are hard to interpret

Return on invested capital (ROIC)

Return on invested capital (ROIC) measures, for the firm as a whole, the after-tax operating return on all capital (debt and equity):

$$ROIC = \frac{\text{NOPAT}}{\text{average book value of invested capital}} = \underbrace{\frac{\text{NOPAT}}{\text{Sales}}}_{\text{operating margin after tax}} \times \underbrace{\frac{\text{Sales}}{\text{average invested capital}}}_{\text{capital turnover}}$$

Net operating profit after tax (NOPAT) = net income + after-tax interest expense $= NI + \text{interest} \times (1 - t)$. Invested capital is the average book value of the firm's **total capital**, the debt and equity that finance it. Exam convention: invested capital is long-term debt plus equity, and working capital is left out of it, meaning non-interest-bearing current liabilities such as accounts payable are not counted as capital. Current practice: debt and equity finance receivables and inventory as well as fixed assets, so operating working capital is not subtracted from capital; invested capital is usually total interest-bearing debt, including short-term borrowings, plus equity, often net of excess cash. The ratio of sales to invested capital is also called **asset turnover**, so a firm can raise ROIC by improving its after-tax operating margin or by turning over its capital faster. If ROIC exceeds the investors' blended required return, the firm is adding value over time.

Example. Net income 18m, interest expense 5m, tax rate 20%, sales 275m. Long-term debt plus equity is 190m at the start of the year and 210m at the end. NOPAT $= 18 + 5(0.80) = 22$m; average invested capital $= (190 + 210)/2 = 200$m; ROIC $= 22/200 = 11\% = (22/275) \times (275/200) = 8\% \times 1.375$.

ROIC is attractive to outside investors because it uses published accounting data and applies to the whole firm, which is what they can buy. However, it is not comparable across different accounting treatments, it is backward-looking and volatile, and good projects can hide bad ones in the company-wide figure. NPV and IRR are project-specific.

Common exam traps

- NPV is the PV of inflows minus the initial outlay. The PV of inflows alone is a common wrong answer. In the three-year example, the PV of the inflows alone is 2,206.61 instead of an NPV of 206.61.
- Do not add unadjusted (undiscounted) cash flows. In the three-year example, $-2{,}000 + 700 + 900 + 1{,}100 = 700$ instead of 206.61.
- For NOPAT, add back interest after tax; adding the full pretax interest overstates NOPAT. In the ROIC example, $18 + 5 = 23$m gives an ROIC of 11.5% instead of 11%.

EXAM SHORTCUTS

- For a conventional independent project, NPV is positive exactly when the discount rate is below the IRR, so once the IRR is compared with the required return the sign of the NPV is known.

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BOTTOM LINE

- Capital investments are going concern, regulatory/compliance, expansion and other projects, and the last two, which aim to grow the business, need the most analysis.
- The capital allocation process is idea generation (the most important step), analysis of proposals, the firm-wide capital budget, and monitoring with a post-audit that exposes systematic forecasting errors.
- NPV is the sum of a project's incremental after-tax cash flows discounted at its required rate of return, $\sum CF_t/(1+r)^t$, and an independent project is accepted when NPV > 0.
- The IRR is the discount rate at which NPV = 0, an independent project is accepted when the IRR exceeds the hurdle rate, and for a conventional independent project the NPV and IRR rules agree.
- NPV measures the expected increase in firm value and assumes reinvestment at the required return, while the IRR assumes reinvestment at the IRR and can give multiple values for unconventional cash flows.
- ROIC = NOPAT ÷ average invested capital = after-tax operating margin × capital turnover, with NOPAT = net income + interest × $(1-t)$; under the exam convention invested capital is long-term debt plus equity excluding working capital, while current practice usually counts all interest-bearing debt plus equity.

QUICK CHECK

Question 1

Tamsin Foods is reviewing three capital projects. Which one would typically need the *most* detailed analysis?

A Replacing a packaging machine that still works with a newer model that costs less to run.

B Launching a new product line in a region where the company does not yet sell.

C Installing fire-suppression equipment that the company's insurer now requires.

Answer: B

An expansion project that brings a new product into a new market requires forecasts of demand, revenues and expenses that are all highly uncertain. It therefore needs the most detailed analysis of the three.

Why the other options are wrong

- **A.** Replacing a usable machine to lower costs is a going concern project. It needs a cost comparison, but its cash flows are much easier to estimate than those of a new product in a new market.
- **C.** Insurer-mandated safety equipment is a regulatory/compliance project. The firm must do it, so the analysis centers on comparing alternative ways of complying.

Key takeaway Depth of analysis rises with uncertainty. Projects that maintain the business need little analysis, regulatory/compliance projects call for comparing alternative ways to comply, and expansion and other projects need the most detailed analysis.

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MODULE 24.2

Capital Allocation Principles and Real Options

LOS 24.c — Principles of capital allocation and common pitfalls

Principles

1. Decisions are based on cash flows rather than accounting income, because accounting income is built on accruals and ignores when cash is received or paid. The cash flows are measured after tax, since the firm's value rests on the cash it keeps rather than the cash it pays in taxes. Tax savings from non-cash deductions such as depreciation and amortization are part of the cash flows.
2. Only incremental cash flows count: the cash flows that change because the project is accepted.
   - **Sunk costs** are *excluded*. These are amounts already spent that cannot be recovered whatever the decision.
   - **Opportunity costs** are *included*. An opportunity cost is the value of the best alternative use of a resource the project will use, such as rent forgone on a building the firm owns.
   - Effects on the rest of the firm are *included*: **cannibalization** is a negative effect in which the new product takes sales from an existing product, as when a "light" version of an existing snack is launched; a **positive externality** is an increase in sales of other product lines (e.g., a printer maker adding a scanner that lifts sales of its cables).
3. The timing of cash flows is important: money received sooner is worth more (time value of money).
4. Financing costs are not deducted from the cash flows; they are reflected in the project's **required rate of return**, the rate used to discount the cash flows. Subtracting interest as well would count the cost of financing twice.

KEY CONCEPT

Item	Include in project cash flows?
Sunk cost	No
Opportunity cost	Yes
Cannibalization (negative externality)	Yes (as a reduction)
Positive externality	Yes
Tax effects, including depreciation tax savings	Yes
Interest on financing	No; captured in the discount rate

Common pitfalls

Cognitive errors (calculation errors):

- **Poor forecasting**, for example misallocating overhead or ignoring how competitors will respond.
- **Not considering the cost of internal funds**: retained earnings are not free; their cost is the cost of equity (they could have been paid out as dividends). Firms reluctant to pay dividends risk spending retained earnings on poor projects.

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- **Incorrectly accounting for inflation**: real cash flows need a real discount rate; nominal with nominal.

Behavioral biases (errors of judgment):

- **Pet projects** of senior management get optimistic forecasts and less scrutiny.
- **Inertia in setting the entire capital budget**: anchoring each year's budget to last year's rather than to the opportunities available; a warning sign is a static or rising budget alongside falling returns. A value-focused firm returns surplus cash to shareholders when positive-NPV projects are scarce and argues for a bigger budget when they are plentiful.
- **Basing investment decisions on EPS or ROE**: managers paid on short-term EPS or ROE may pass up positive-NPV projects that depress those ratios early on.
- **Failure to generate alternative investment ideas**: stopping at the first "good" idea instead of looking for a better one.

Common exam traps

- A consulting or research fee paid *before* the decision is a sunk cost even if it was paid "for" the project.
- Cannibalization is counted even though the lost sales belong to another product line. The test is whether the firm's total cash flows change.
- Depreciation is not itself a cash flow, but the tax it saves is. Leave out the depreciation charge and include the tax saving.

LOS 24.d — Real options

Real options are future actions a firm may take because it invested in a project today. Like financial options, they give the right but not the obligation to act. Because the firm will simply not exercise an option whose exercise would destroy value, a real option can never have a negative value; at worst it is worth zero.

KEY CONCEPT

Real option	What it lets management do	Example
Timing options	Delay an investment to wait for better information	Postpone building a plant until a pilot's results are known
Abandonment options	Exit a project when the PV of the cash flows from exiting exceeds the PV of continuing	Close a loss-making branch and sell its equipment
Expansion options (also called **growth options**)	Make further investments later if they will create value	Buy land next to a new plant so capacity can be doubled later
Flexibility options: **price-setting options** and **production-flexibility options**	Change operating decisions: raise prices when demand is high; use overtime, switch input materials or produce a different variety of product	A bakery line that can switch between two flours depending on cost

Real option	What it lets management do	Example
Fundamental options	The project itself is an option because its payoff depends on the price of an underlying asset	A gold mine that is opened when the gold price is high and closed when it is low

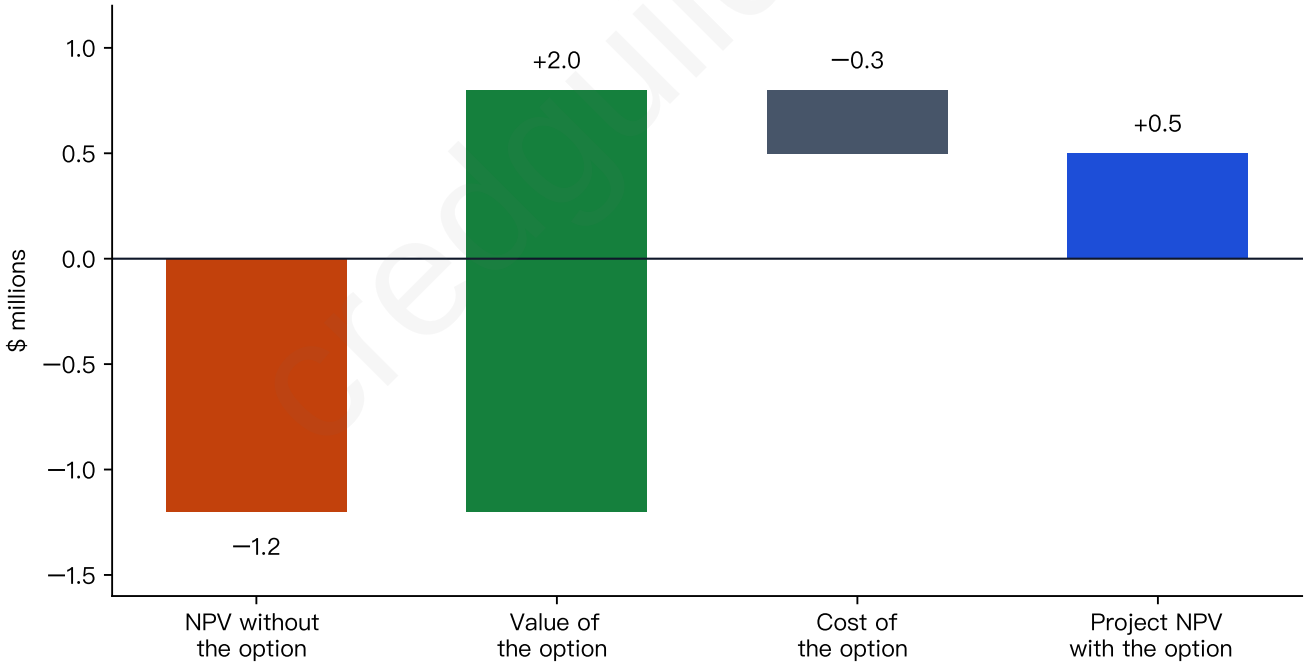
Valuing a project with real options. One approach is:

KEY CONCEPT

$$\text{Project value} = NPV_{\text{without options}} + \text{value of options} - \text{cost of options}$$

using option pricing models or decision trees. Another is to treat the NPV without options as the project's minimum value. If that minimum is already positive, the project can be accepted without valuing its options, because an option can only add value.

Example. A project's NPV on its own is −1.2m. Paying 0.3m now for an option to expand later adds an estimated 2.0m of option value. Adjusted NPV $= -1.2 + 2.0 - 0.3 = +0.5$m, so the project with the option is worth doing.



Building up project NPV with a real option (numbers from the example)

Common exam traps

- Buying rights now that let the firm invest in *future* projects (sequels, extra capacity, a second phase) = expansion option.

- Paying overtime or switching inputs is a flexibility option, not an expansion option: it changes how the existing project runs rather than adding a new investment.
- A resource project undertaken because the commodity price is high = fundamental option.

EXAM SHORTCUTS

- Test each cost by asking whether the firm's total cash flows change if the project is accepted: amounts already spent are sunk and excluded, while forgone rent, cannibalized sales and extra sales of other products are included.
- If a project's NPV without options is already positive, it can be accepted without valuing its real options, because an option can only add value.

BOTTOM LINE

- Capital allocation uses after-tax incremental cash flows rather than accounting income, including the tax savings from depreciation and amortization.
- Sunk costs are excluded, while opportunity costs, cannibalization and positive externalities are included in project cash flows.
- Financing costs are reflected in the required rate of return and are not also deducted from the project's cash flows.
- Cognitive errors include poor forecasting, ignoring the cost of internal funds (the cost of equity) and mixing real and nominal terms, while behavioral biases include pet projects, inertia in the capital budget, decisions based on EPS or ROE and failure to generate alternatives.
- Real options give the right but not the obligation to act and so never have negative value; they include timing, abandonment, expansion, flexibility (price-setting and production-flexibility) and fundamental options.
- A project with real options is worth its NPV without options plus the value of the options minus their cost.

QUICK CHECK

Question 2

For the past eight years, Farrow Plastics' capital budget has been set at roughly the prior year's amount plus 3%, even though the returns on its recent projects have been falling. This practice is best described as which capital allocation pitfall?

A Failure to generate alternative investment ideas.

B Pet projects of senior management.

C Inertia in setting the entire capital budget.

Answer: C

Anchoring each year's capital budget to the prior year's, rather than to the positive-NPV opportunities actually available, is the behavioral bias of inertia. A budget that stays the same or keeps rising while returns fall is the usual warning sign. A firm focused on shareholder value would return excess funds

when good projects are scarce.

Why the other options are wrong

- **A.** This bias is stopping at the first acceptable idea without searching for better ones. It concerns which projects are proposed rather than how the size of the budget is set.
- **B.** Pet projects are individual projects backed by influential executives that get optimistic forecasts and little scrutiny. Nothing in the scenario points to a favored project.

Key takeaway A static or steadily rising capital budget combined with declining returns signals inertia (anchoring to last year's budget).

Practice questions

Question 3

Harwick Instruments has estimated that a proposed sensor line has an internal rate of return of 11.5% and a net present value of +$1.9 million. The company's 9.0% cost of capital is appropriate for the risk of the line. Harwick should *most likely*:

A reject the line, based on both its NPV and its IRR.

B accept the line, but only on the basis of its NPV.

C accept the line, based on both its NPV and its IRR.

Question 4

Fenmoor Packaging earns an operating margin after tax of 8.0%, and its sales are 1.5 times its invested capital. The blended rate of return that its debt and equity investors require is 10.0%. Which statement about Fenmoor is *most accurate*?

A Its ROIC is 5.3%, so it is losing value over time.

B Its ROIC is 8.0%, so it is losing value over time.

C Its ROIC is 12.0%, so it is adding value over time.

Question 5

Kinloss Cold Storage is evaluating a refrigerated warehouse that would cost $8.20 million today. Discounted at Kinloss's required rate of return, the warehouse's expected after-tax cash flows have a present value of $7.65 million. For an additional $0.45 million paid now, Kinloss can buy the adjoining parcel of land, which would allow it to double the warehouse's capacity later if demand justifies it. Using a decision tree, Kinloss's analysts value this expansion option at $1.30 million. The NPV of the warehouse project including the expansion option is *closest* to:

A +$0.30 million.

B −$0.55 million.

C −$1.00 million.

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Answers: 3. C · 4. C · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/corporate-finance/npv-vs-irr/. The full bank has 35 questions on this reading.

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CORPORATE FINANCE · READING 25

Modigliani Miller

What you'll learn

LOS 25.a Calculate and interpret a company's weighted-average cost of capital using after-tax cost of debt and target (market) weights.

LOS 25.b Explain internal, external and life-cycle factors that affect capital structure, debt capacity and the WACC.

LOS 25.c Explain the Modigliani–Miller propositions on capital structure, without and with taxes, and the role of costs of financial distress.

LOS 25.d Describe optimal and target capital structures: static trade-off theory, deviations from target, asymmetric information, agency costs and pecking order theory.

MODULE 25.1

Weighted-Average Cost of Capital

This reading calculates the weighted-average cost of capital and explains the internal and external factors, including the life cycle stage, that shape a company's capital structure. It then sets out the Modigliani–Miller propositions with and without taxes, the costs of financial distress, the static trade-off and pecking order theories, and how a target capital structure is estimated.

LOS 25.a — Calculating and interpreting the WACC

A company's **weighted-average cost of capital (WACC)** is the blended required return of all its capital providers. With debt and common equity only:

KEY CONCEPT

$$WACC = w_d\, r_d(1 - t) + w_e\, r_e$$

where w_d and w_e are the weights of debt and equity ($w_d + w_e = 1$), r_d is the **pretax cost of debt**, t is the marginal tax rate, and r_e is the **cost of equity**. If the firm has preferred stock, add $w_p r_p$. Preferred dividends are not tax-deductible, so this term has no tax adjustment.

Key points:

- Debt costs less than equity because debtholders are paid first from the firm's cash flows and have a senior claim on its assets.
- Interest is tax-deductible in most jurisdictions, so the pretax cost of debt is converted to an **after-tax cost of debt**, $r_d(1 - t)$. The cost of equity gets no tax adjustment.
- The weights can be **target weights** or **market value weights**. Market values reflect conditions today, so they are the right basis for measuring today's opportunity cost of capital. LOS 25.d explains how an analyst estimates a target capital structure that the company does not disclose.
- The WACC is the discount rate for computing the NPV of an expansion that mirrors the existing business and is financed in the firm's normal proportions.

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Example. A firm's debt has a market value of $32 million and its equity $48 million, so $w_d = 32/80 = 0.40$ and $w_e = 48/80 = 0.60$. Its pretax cost of debt is 7%, its cost of equity is 12%, and its tax rate is 21%.

$$WACC = 0.40(0.07)(1 - 0.21) + 0.60(0.12) = 0.02212 + 0.07200 = 9.41\%$$

Common exam traps

- Forgetting the tax adjustment (using r_d instead of $r_d(1 - t)$) overstates the WACC. In the example, $0.40(0.07) + 0.60(0.12) = 10.00\%$ instead of 9.41%.
- Multiplying by t instead of $(1 - t)$ understates it badly. In the example, $0.40(0.07)(0.21) + 0.60(0.12) = 7.79\%$ instead of 9.41%.
- Swapping the weights, or weighting by the debt-to-equity ratio instead of by each source's share of total capital, $D/(D + E)$ and $E/(D + E)$. In the example, swapping them gives $0.60(0.07)(0.79) + 0.40(0.12) = 8.12\%$ instead of 9.41%.

LOS 25.b — Factors affecting capital structure and the WACC

Firms usually aim for the capital structure that minimizes the WACC, and they try to match the maturity of their financing to the life of their assets. The factors that shape capital structure are essentially those that determine a company's **capacity to service debt**.

- **Internal factors**: business and industry characteristics, **life cycle stage**, existing leverage, and the corporate tax rate.
- **External factors**: conditions in the market and the business cycle, regulation, and industry norms.

The corporate tax rate matters because interest is tax-deductible: the higher the rate, the larger the tax saving from each unit of debt.

Features of a company that let it carry more debt:

- Revenue that is growing or stable, and cash flow that is growing and predictable.
- Low **business risk** (operational risk and demand risk).
- Plenty of liquid, **tangible assets** that can be pledged as collateral. Creditors prefer tangible assets that can be sold without much loss of value, or that are fungible, over intangible assets. A firm that owns its productive assets outright has more collateral than one that uses assets owned by others, as in a franchise model.
- Low cost and ready availability of debt.
- Low existing leverage and low earnings volatility. Analysts gauge debt capacity with leverage ratios (debt-to-equity, debt-to-operating profit) and coverage ratios, such as interest coverage $= \text{EBIT}/\text{interest expense}$.

Other things equal:

KEY CONCEPT

Supports more debt	Supports less debt
Noncyclical industry	Cyclical industry
Low **operating leverage** (low fixed costs as a share of total costs)	High fixed operating costs

Supports more debt	Supports less debt
Subscription-based revenue	Pay-per-use revenue
Liquid tangible assets owned outright	Mostly intangible assets, or assets owned by others

The life cycle stage shows these factors at work:

KEY CONCEPT

Stage	Cash flows and risk	Typical financing
Start-up	Low or negative earnings and cash flow, high business risk, few assets	Almost all equity; possibly convertible debt (if the share price is rising fast) or leasing
Growth	Rising revenue and cash flow, business risk falling	Some debt, used conservatively, often secured by fixed assets or receivables
Mature	Slower growth, stable and significant cash flow, low business risk	Significant debt, including unsecured debt at relatively low cost

A start-up's debt would be very risky, so lenders would demand high interest rates, and its small base of receivables and fixed assets offers little collateral. **Convertible debt** suits a start-up with a fast-rising share price because it costs less than straight debt and postpones the dilution of existing shareholders.

Top-down factors are macroeconomic conditions (inflation, real GDP growth, monetary policy and exchange rates) that move benchmark rates and credit spreads. In downturns, lenders demand wider spreads, especially from companies in cyclical industries. Some industries benefit from particular conditions; for example, spreads of oil producers tend to narrow when oil prices rise.

Common exam traps

- A start-up is financed almost exclusively with equity, even if it is growing fast.
- High fixed operating costs point to less debt, not more: operating leverage and financial leverage both make earnings more volatile.

BOTTOM LINE

- With debt and common equity only, $WACC = w_d r_d(1 - t) + w_e r_e$, where each weight is that source's share of total capital; preferred stock adds $w_p r_p$ with no tax adjustment.
- Debt costs less than equity because debtholders have a prior claim, and because interest is tax-deductible in most jurisdictions the cost of debt enters the WACC after tax, while the cost of equity does not.
- WACC weights can be target or market value weights, and market values are the right basis for today's opportunity cost of capital.
- The WACC is the discount rate for a project that mirrors the existing business and is financed in the firm's normal proportions.

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- Capital structure reflects the capacity to service debt, shaped by internal factors (business characteristics, life cycle stage, existing leverage, tax rate) and external factors (market and business cycle conditions, regulation, industry norms).
- Stable or growing revenue and cash flow, low business risk, low operating leverage and liquid tangible assets owned outright support more debt, so start-ups are financed almost entirely with equity, growth firms use some debt and mature firms significant debt.

QUICK CHECK

Question 1

Harlan Steelworks has no preferred stock. An equity analyst wants a WACC figure that measures Harlan's current opportunity cost of capital, and she has collected these data:

Harlan Steelworks: capital and cost data

Item	Value
Debt, market value	$175 million
Debt, book value	$200 million
Common equity, market value	$525 million
Common equity, book value	$900 million
Yield to maturity on Harlan's bonds (pretax cost of debt)	5.6%
Cost of common equity	13.2%
Marginal tax rate	25%

Interest expense is tax-deductible. Harlan's WACC is *closest* to:

A 10.95%.

B 11.30%.

C 11.56%.

Answer: A

Market-value weights reflect current market conditions, so they are the right weights for measuring the current opportunity cost of capital. Total capital at market value is $175 + 525 = \$700$ million, so debt is 25% and equity 75%. Because interest is tax-deductible, the cost of debt enters the WACC after tax.

Market-value weights: $w_d = 175/700 = 0.25$; $w_e = 525/700 = 0.75$.

After-tax cost of debt: $5.6\% \times (1 - 0.25) = 4.2\%$.

$$WACC = w_d\, r_d(1 - t) + w_e\, r_e = 0.25(4.2\%) + 0.75(13.2\%) = 1.05\% + 9.90\% = 10.95\%$$

Why the other options are wrong

- **B.** 11.30% uses the correct market-value weights but the 5.6% pretax cost of debt: $0.25(5.6\%) + 0.75(13.2\%) = 1.40\% + 9.90\%$. It ignores the tax deductibility of interest.
- **C.** 11.56% weights debt and equity by book values, $200/1{,}100 = 18.18\%$ and $900/1{,}100 = 81.82\%$: $0.1818(4.2\%) + 0.8182(13.2\%) = 0.76\% + 10.80\%$. Book values do not reflect current market conditions; here the book value of equity is far above its market value, so book weights overstate the weight of equity, the more expensive source.

Key takeaway For the current cost of capital, weight each source by its market value (or by management's target weights) and use the after-tax cost of debt. Book values can differ sharply from market values, especially for equity.

MODULE 25.2

Capital Structure Theories

LOS 25.c — The Modigliani–Miller propositions

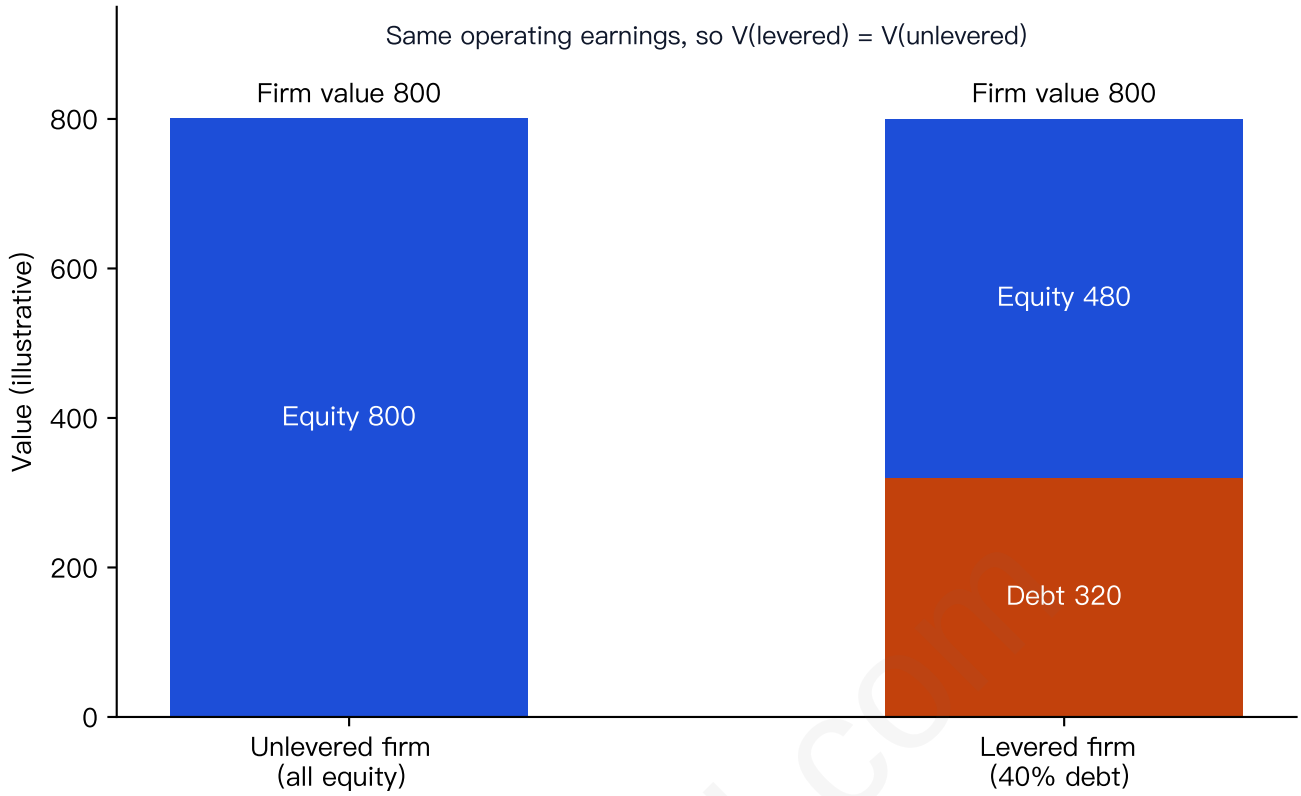
The assumptions

The capital structure propositions of **Modigliani and Miller (MM)** rest on these assumptions: capital markets are perfectly competitive, with no taxes, transactions costs or bankruptcy costs; all investors hold homogeneous expectations; borrowing and lending are available to investors at the risk-free rate; agency costs are absent; and financing choices have no effect on investment decisions, so the firm's operating income does not depend on its mix of debt and equity.

MM Proposition I (no taxes): capital structure irrelevance

With these assumptions in place, changing the capital structure does not change firm value: $V_L = V_U$, where V_L is the value of the levered firm and V_U the value of an otherwise identical unlevered (all-equity) firm. The size of the "pie" (the value of the firm's operating earnings, EBIT) stays the same however it is divided between debtholders and shareholders. An investor who owned all of the debt and equity of a levered firm would receive the same EBIT as the owner of an identical all-equity firm.

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MM Proposition I without taxes: the same firm value under two financing mixes (illustrative numbers)

MM Proposition II (no taxes): cost of equity and leverage

The cost of equity rises linearly with the debt-to-equity ratio. Debtholders have a prior claim, so the residual cash flows left to shareholders become riskier as debt increases:

KEY CONCEPT

$$r_e = r_0 + \frac{D}{E}(r_0 - r_d)$$

where r_e is the cost of equity, r_0 is the cost of equity of the firm if it had no debt (all equity), r_d is the cost of debt and D/E is the debt-to-equity ratio. Promised payments to debtholders are more certain than the residual cash flows to shareholders, so $r_d < r_e$. The formula follows from Proposition I: if the WACC equals r_0 whatever the mix, then $\frac{D}{V}r_d + \frac{E}{V}r_e = r_0$, and solving for r_e gives the line above. r_0 reflects the business risk of the firm's operations, which the financing mix does not change. As D/E rises, the cost of equity rises while the cost of debt stays the same. The benefit of using more low-cost debt is exactly offset by the higher cost of equity, so the WACC is unchanged. This is consistent with Proposition I.

MM with taxes

When interest is tax-deductible, debt creates a **debt tax shield** worth $t \times D$:

$$V_L = V_U + tD$$

With taxes, the government's slice of the pie shrinks as debt rises. Firm value is therefore maximized, and the WACC minimized, at 100% debt. The cost of equity still rises with leverage, but more slowly:

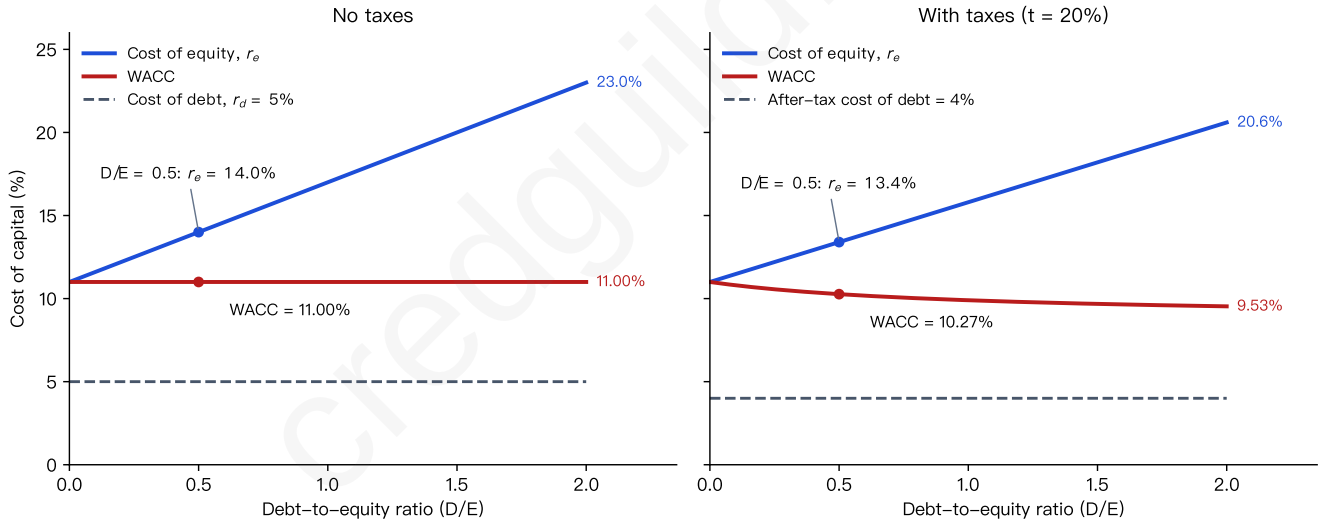
$$r_e = r_0 + \frac{D}{E}(r_0 - r_d)(1 - t)$$

Example. Let $r_0 = 11\%$, $r_d = 5\%$, $D/E = 0.5$ and $t = 20\%$. Without taxes, $r_e = 11\% + 0.5(6\%) = 14.0\%$. With taxes, $r_e = 11\% + 0.5(6\%)(0.8) = 13.4\%$. Separately, an unlevered firm worth 500 that adds 200 of debt at a 25% tax rate is worth $500 + 0.25(200) = 550$ under MM with taxes.

KEY CONCEPT

	MM without taxes	MM with taxes
Firm value	Unaffected by capital structure	Rises with debt; maximum at 100% debt
WACC	Constant	Falls as leverage rises; minimum at 100% debt
Cost of equity	Rises linearly with D/E	Rises with D/E, but less steeply (factor $1 - t$)

The figure draws both cases with the example inputs ($r_0 = 11\%$, $r_d = 5\%$, $t = 20\%$).



MM Proposition II without and with taxes (r0 = 11%, rd = 5%, t = 20%)

Costs of financial distress

In practice firms do not use 100% debt or anything close to it. MM themselves suggested that differences in investors' tax rates on dividends and on interest income could help explain observed capital structures. Current theory points instead to a cost the model leaves out, the **costs of financial distress**: the extra costs a firm faces when falling earnings make its fixed financing costs hard to pay. These costs rise with leverage. Expected costs of financial distress have two components:

1. The costs incurred if distress occurs. **Direct costs** are the legal and administrative fees of bankruptcy. **Indirect costs** include forgone investment opportunities and the lost trust of customers, suppliers, creditors and employees. Conflicts between managers (acting for shareholders) and debtholders add the **agency**

costs of debt.

2. The **probability of financial distress**, which rises with operating and financial leverage and with weak management and corporate governance.

Other things equal, higher expected costs of financial distress discourage heavy use of debt.

LOS 25.d — Optimal and target capital structures

Static trade-off theory

The **static trade-off theory** balances the value of the debt tax shield against the expected costs of financial distress:

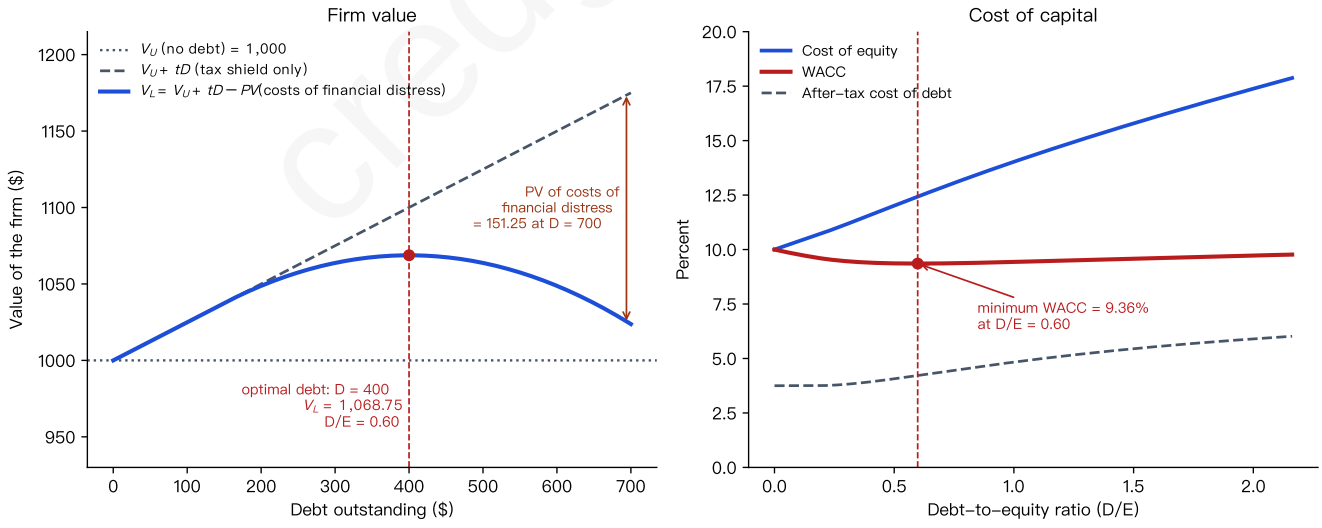
KEY CONCEPT

$$V_L = V_U + tD - PV(\text{costs of financial distress})$$

As debt increases, the tax shield keeps increasing, but beyond some point the extra expected distress costs outweigh the extra tax benefit. The **optimal capital structure** is the debt level at which firm value is maximized and the WACC is minimized. Because V_U is constant, this is also where $V_L - V_U$ is largest.

- Firm value first rises and then falls as leverage increases; the WACC first falls and then rises (a U shape).
- The cost of equity rises throughout as leverage increases.
- The optimum differs across firms. It depends on business risk, the tax rate, corporate governance, industry and other factors.

The figure shows firm value and the costs of capital with illustrative numbers: an unlevered firm worth 1,000, a 25% tax rate and distress costs that start once debt passes 150.



Static trade-off theory: firm value and cost of capital as debt increases (illustrative numbers)

Target capital structure

The **target capital structure** is the mix a firm aims for on average over time, reflecting management's view of the optimal structure. WACC weights should reflect the target, measured at market values. An analyst can use management's stated target when one is given, but most firms do not publish a target, so an external analyst estimates it from (1) the current market-value capital structure, (2) the current structure adjusted for a visible trend, or (3) industry-average weights.

Book values are also common in practice. Analysts often estimate target weights from the book values of debt and equity, and managers often think in book values for three reasons: short-term market swings do not change the appropriate amount of debt, management cares about how capital is deployed to projects, and rating agencies use book values.

Actual capital structures fluctuate around the target because of:

- changes in market values, especially of equity;
- management exploiting opportunities, such as issuing equity after a temporary share-price rise;
- external capital being raised in minimum-size lots.

Deviations caused by these three reasons are a normal part of how an actual capital structure moves around its target.

Asymmetric information, signaling and pecking order theory

Costs of asymmetric information arise because managers know more about the firm's prospects than investors do. They are higher for firms with complex products or opaque financial statements, and they raise required returns on both debt and equity. The larger the share of equity in the capital structure, the higher the cost of asymmetric information. Investors read financing choices as signals. Taking on debt, a commitment to fixed payments, signals confidence; issuing equity is often read as a sign that management believes the shares are overvalued.

Pecking order theory says managers prefer the financing least likely to send a negative signal, ranked by visibility to investors:

1. Internally generated funds (retained earnings), the most preferred source;
2. Debt;
3. New external equity, the least preferred source.

Capital structure is then a by-product of individual financing decisions rather than a deliberate target.

Agency costs of equity

Agency costs of equity stem from conflicts between managers and shareholders. Managers without an ownership stake do not bear the cost of excessive pay or of taking too much or too little risk, so shareholders spend money to limit the conflict. The **net agency costs of equity** that result have three components:

Component	Meaning	Examples
Monitoring costs	Supervising management; lower with strong corporate governance	Reporting to shareholders, board of directors' pay
Bonding costs	Assuring shareholders that managers act in their interest	Insurance premiums guaranteeing performance; implicit costs of noncompete agreements

Component	Meaning	Examples
Residual losses	Losses that remain despite monitoring and bonding	Imperfect guarantees

Under the **free cash flow hypothesis**, more debt reduces agency costs because interest payments leave managers less free cash flow to spend for their own benefit.

Common exam traps

- MM with taxes and no distress costs leads to 100% debt. The static trade-off theory leads to an optimum below 100% debt.
- Bonding and monitoring costs belong to the net agency costs of equity. They are not part of pecking order theory.
- Under pecking order theory debt ranks ahead of new equity even though equity carries no default risk. The ranking follows how visible the source is and how negative a signal it sends, not its cost.

EXAM SHORTCUTS

- Match the theory to its conclusion: MM without taxes makes capital structure irrelevant, MM with taxes points to 100% debt, the static trade-off theory gives an optimum below 100% debt, and pecking order theory has no target, with internal funds used first.

BOTTOM LINE

- The MM propositions assume perfectly competitive markets with no taxes, transactions or bankruptcy costs, homogeneous expectations, borrowing and lending at the risk-free rate, no agency costs, and operating income unaffected by financing.
- Without taxes, MM Proposition I gives $V_L = V_U$, and Proposition II gives $r_e = r_0 + (D/E)(r_0 - r_d)$, so the cost of equity rises with leverage while the WACC is unchanged.
- With taxes, $V_L = V_U + tD$, firm value is maximized and the WACC minimized at 100% debt, and the cost of equity rises more slowly, $r_e = r_0 + (D/E)(r_0 - r_d)(1 - t)$.
- Expected costs of financial distress combine the direct and indirect costs of distress with its probability, which rises with operating and financial leverage and weak management or governance, and they discourage heavy use of debt.
- Under the static trade-off theory $V_L = V_U + tD - PV(\text{costs of financial distress})$, so the optimal capital structure, where firm value is highest and the WACC lowest, lies below 100% debt; analysts without a stated target estimate it from the current market-value structure, its trend or industry averages.
- Pecking order theory ranks internal funds first, then debt, then new external equity, by how visible and negative a signal each sends, and under the free cash flow hypothesis more debt reduces the agency costs of equity.

QUICK CHECK

Question 2

Sorrel Systems would have a cost of equity of 10.5% if it had no debt. The company borrows at 6.0%, has a debt-to-equity ratio of 0.6, and faces a 25% corporate tax rate. Under Modigliani and Miller's Proposition II with taxes, Sorrel's cost of equity is *closest* to:

A 13.20%.

B 12.53%.

C 11.77%.

Answer: B

MM Proposition II says the cost of equity rises linearly with the debt-to-equity ratio. With taxes, the leverage premium is scaled down by $(1 - t)$ because the debt tax shield absorbs part of the extra risk borne by shareholders.

$$r_e = r_0 + \frac{D}{E}(r_0 - r_d)(1 - t) = 10.5\% + 0.6(10.5\% - 6.0\%)(1 - 0.25)$$

$$= 10.5\% + 0.6(4.5\%)(0.75) = 10.5\% + 2.025\% = 12.525\% \approx 12.53\%$$

Why the other options are wrong

- **A.** 13.20% is the no-tax version of Proposition II; it omits the $(1 - t)$ factor.
- **C.** 11.77% uses the debt-to-total-capital ratio ($0.6/1.6 = 0.375$) instead of the debt-to-equity ratio of 0.6.

Key takeaway MM II with taxes: $r_e = r_0 + (D/E)(r_0 - r_d)(1 - t)$. The leverage term uses D/E; using D/V is a common error.

Practice questions

Question 3

Treloar Bakeries' revenue growth has slowed to a few percent a year, its operating cash flow is large and stable, and its business risk is low. Given its life cycle stage, Treloar is *most likely* to be financed with:

A equity almost exclusively, possibly with some convertible debt.

B a significant amount of debt, including unsecured debt at relatively low cost.

C limited debt, used conservatively and secured by receivables or fixed assets.

Question 4

Halvard Retail generates far more cash than it can invest profitably. One of its directors argues that the company should take on more debt, because the required interest and principal payments would leave management with less spare cash to spend on lavish perks and low-return acquisitions. The director's argument is *most* consistent with:

A pecking order theory.

B the use of debt to signal management's confidence.

C the free cash flow hypothesis.

Answers: 3. B · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/corporate-finance/modigliani-miller/. The full bank has 28 questions on this reading.

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CORPORATE FINANCE · READING 26

Business Models

What you'll learn

LOS 26.a Describe the key features of a business model: customers, product, channels, pricing strategy, key assets and suppliers, value proposition and value chain.

LOS 26.b Describe types of business models, including private label manufacturing, licensing, value-added resellers, network effects and crowdsourcing.

MODULE 26.1

Business Model Features and Types

This reading sets out the key features of a business model (its customers, product, key assets and suppliers, channels and pricing) together with the value proposition and value chain. It then describes conventional and other types of business models, including private label manufacturing, licensing, value-added resellers, network effects and crowdsourcing.

LOS 26.a — Key features of business models

A **business model** explains how a firm provides a product or service, finds customers, delivers to them and makes a profit. A useful checklist asks *who*, *what*, *how*, *where* and *how much*:

KEY CONCEPT

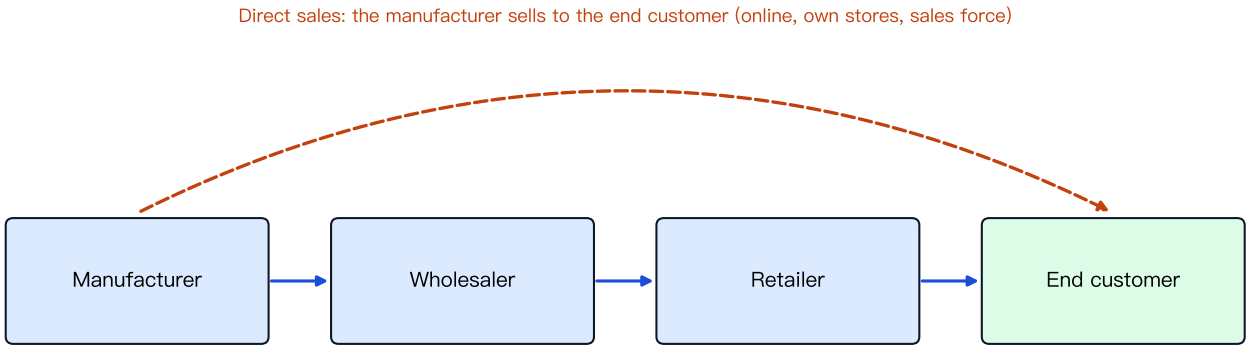
Question	What the model should cover
Who: customers	The target customers (**segmenting** the market into customer groups), how they are acquired, the cost of acquiring them, and how satisfaction is monitored and kept
What: product or service	The need it meets and what differentiates it from competitors (low price, premium quality, innovative features)
How: key assets and suppliers	The assets that matter (patents, software, skilled staff) and the critical suppliers
Where: channels	Where and how the firm sells and delivers, which is its **channel strategy**
How much: pricing	The **pricing strategy** and why buyers will pay that price given the competition

The model describes how the firm works and earns money. Detailed forecasts of revenues and expenses belong in a financial plan rather than in the business model.

Channel strategy

The **channel strategy** covers how the firm sells (online, physical stores, direct mail, trade shows, sales representatives), whether it sells directly to buyers (**direct sales**) or through intermediaries (wholesalers, retailers, agents, franchisees), and how it delivers the product. A strategy that combines digital and physical channels, such as ordering online and collecting in store, is an **omnichannel strategy**.

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A traditional channel through intermediaries compared with direct sales

Firms selling to other businesses are **B2B** (business to business) firms; firms selling to consumers are **B2C** (business to consumer) firms.

Pricing strategies

Commodity producers with undifferentiated products are **price takers**, so premium pricing does not fit their business model. Firms with few competitors or highly differentiated products have **pricing power**. Added services or features that customers value can support a premium price.

Price discrimination means charging different prices to different customers or groups of customers. The common forms are:

- **Tiered pricing**: the price depends on the volume purchased, with lower unit prices for larger orders.
- **Dynamic pricing**: the price varies with when the purchase is made, for example the hour or the weekday (peak and off-peak prices, or cheap airfares on flights at very early or very late hours).
- **Value-based pricing**: the price is set by the value customers perceive in the product, such as a premium for a new drug that protects slightly better than existing ones.
- **Auction pricing**: buyers bid.

Pricing models for multiple products:

KEY CONCEPT

Model	How it works	Example
Bundling	Complementary products sold together at one price	A hotel room priced with breakfast and parking included
Razors-and-blades	Equipment sold at a low price and margin; profit made on consumables used with it	A coffee machine sold cheaply, with profits from the capsules
Add-on pricing	Core product sold at a normal margin; high-margin options added after the purchase decision	A sofa with pricey fabric protection and an extended warranty offered at checkout

Other pricing models:

Model	How it works
Penetration pricing	Low margins (even losses) for a period to build market share and scale
Freemium pricing	Basic product free; extra functionality sold or unlocked for a fee
Hidden revenue	Product is free to the user; revenue comes from advertisers or from selling user data

Two models offer alternatives to outright purchase. Under the **subscription model**, customers pay regularly for access, as with software sold as a monthly subscription. Under **licensing and franchising**, a firm lets others produce or sell its product. A biotech firm may license a new drug to an established drug maker that already has a sales force. A **franchisee** sells in a set area and pays the franchisor a share of sales in return for product and marketing support.

Value proposition and value chain

A firm's **value proposition** is how customers will value the characteristics of its product or service, given competing products and their prices. The **value chain** is how the firm *executes* that value proposition: its assets and organization, and how they add value and exploit its competitive advantage through product quality and functionality, included service, the sales process and pricing relative to competitors. Michael Porter's value chain analysis names five activities a firm should execute well. Exam convention: inbound logistics, operations, outbound logistics, marketing, and sales and service. Current practice: Porter's own list groups the last two as marketing and sales, and service; the first three activities are the same in both.

> *Example.* A meal-kit company's value proposition is "restaurant-quality dinners at home in 20 minutes for less than takeout." Its value chain consists of the sourcing contracts, the packing centers, the refrigerated delivery network and the recipe team that deliver on that promise.

Common exam traps

- A discount is not automatically tiered pricing. A discount for buying at off-peak times is dynamic pricing; only a discount tied to the quantity bought is tiered pricing.
- A free product is not automatically freemium pricing. If no paid version exists and all revenue comes from advertisers or user data, the model is hidden revenue.
- Timing separates add-on pricing from bundling: add-ons are offered after the customer has decided to buy, while a bundle is priced as one package from the start.
- "Direct sales" describes the channel strategy, and "B2B" or "B2C" describes the customers. Neither is a pricing strategy.

LOS 26.b — Types of business models

Conventional business models are largely industry-specific: manufacturers, distributors and retailers, natural resource producers, banks and brokers, software firms and service providers. Other common models:

KEY CONCEPT

Business model	Description
Private label manufacturers (contract manufacturers)	Make products that other companies sell under their own brand names, such as a store brand
Licensing agreements	A company lets another company use its brand on that company's products for a fee
Value-added resellers	Resell complex equipment together with installation, service, support or customization

Business model innovation often comes from new entrants that use new technology to challenge incumbents. Examples are **software as a service (SaaS)**, ultra-low-cost airlines and discount brokers.

Network effects are the increase in a network's value as its user base grows, as with messaging apps, online marketplaces and social media. Network effects favor an initial penetration pricing strategy to build the user base quickly, and they exploit both size and scope. A network can be two-sided or multi-sided, as with a platform that matches travelers with hosts.

Crowdsourcing models benefit from users' contributions, such as articles for an online encyclopedia, traffic reports for a navigation app, or code for open-source software.

Common exam traps

- A manufacturer that makes goods for a retailer's own brand is a private label (contract) manufacturer. In a licensing agreement, the brand is what is rented out.
- A value-added reseller adds services such as installation and support to equipment it resells. It does not manufacture the equipment.
- A platform that needs enough users on each side (drivers and riders, hosts and guests) relies on network effects. Crowdsourcing is different: there users contribute content or improvements.

EXAM SHORTCUTS

- Classify a price difference by what triggers it: the quantity bought means tiered pricing, the time of purchase means dynamic pricing, and the value customers perceive means value-based pricing.
- For a free product, a paid upgrade means freemium pricing and revenue from advertisers or user data means hidden revenue; for extras, options offered after the purchase decision mean add-on pricing, while a package priced from the start is bundling.

BOTTOM LINE

- A business model covers who the customers are, what the product is, how the firm operates (key assets and suppliers), where it sells (channels) and how much it charges, while detailed revenue and expense forecasts belong in a financial plan.
- The channel strategy covers how the firm sells, whether directly or through intermediaries, and how it delivers; an omnichannel strategy combines digital and physical channels, and B2B or B2C describes the customers.

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- Commodity producers are price takers, firms with few competitors or highly differentiated products have pricing power, and price discrimination takes the forms of tiered, dynamic, value-based and auction pricing.
- Pricing models include bundling, razors-and-blades and add-on pricing for multiple products, penetration, freemium and hidden-revenue models, and subscription, licensing and franchising as alternatives to outright purchase.
- The value proposition is how customers value the product given competing products and prices, and the value chain is how the firm executes it; the exam convention lists Porter's activities as inbound logistics, operations, outbound logistics, marketing, and sales and service, while Porter's own list groups the last two as marketing and sales, and service.
- Besides industry-specific conventional models, firms can be private label (contract) manufacturers, license their brand to others for a fee, or act as value-added resellers that add installation, service or customization to complex equipment.
- Network effects raise a network's value as its user base grows and favor initial penetration pricing, while crowdsourcing models benefit from content or improvements contributed by users.

QUICK CHECK

Question 1

An analyst is writing up the business model of Seabrook Outfitters. Which of the following items is *least likely* to belong in the description of the business model?

A Detailed forecasts of revenue and expenses for the next five years.

B The customer groups the company targets and how it reaches them.

C The key assets and critical suppliers the company relies on.

Answer: A

A business model explains how a firm provides its product, finds and serves customers, and makes a profit: who the customers are, what is offered, how and where it is delivered, and how it is priced. Detailed forecasts of revenue and expenses belong in a financial plan, not in the business model.

Why the other options are wrong

- **B.** Target customers and how they are acquired and kept are core parts of a business model.
- **C.** The assets and suppliers that matter to the firm are part of how the business model works.

Key takeaway Business model = who, what, how, where and how much; detailed financial forecasts belong to the financial plan.

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Practice questions

Question 2

A laptop maker has sold a premium keyboard, a high-resolution display and three years of on-site support as separate options. For its next model it decides to stop selling them separately and to include all three with the laptop for one combined base price. Which pricing model *best* reflects this decision?

A Value-based pricing.

B Bundling.

C Dynamic pricing.

Question 3

Fairfield Mills produces breakfast cereals that are sold only under the store brands of several large supermarket chains. Fairfield's name does not appear on any package. Which business model *best* describes Fairfield Mills?

A Value-added reseller.

B Licensing agreement.

C Private label manufacturer.

Answers: 2. B · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/corporate-finance/business-models/. The full bank has 15 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Financial Statement Analysis

Exam weight 11–14% · 12 readings

Reading the income statement, balance sheet and cash flow statement; the accounting for inventories, long-lived assets, leases, pensions and income taxes; judging reporting quality; ratio analysis; and a first financial statement model.

Reading 27	Introduction to Financial Statement Analysis
Reading 28	Analyzing Income Statements
Reading 29	Analyzing Balance Sheets
Reading 30	Analyzing Statements of Cash Flows I
Reading 31	Free Cash Flow
Reading 32	LIFO vs FIFO
Reading 33	Straight Line Depreciation
Reading 34	Finance Lease vs Operating Lease
Reading 35	Deferred Tax Liability
Reading 36	Financial Reporting Quality
Reading 37	Dupont Analysis
Reading 38	Pro Forma Financial Statements

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FINANCIAL STATEMENT ANALYSIS · READING 27

Introduction to Financial Statement Analysis

What you'll learn

LOS 27.a Describe the six steps of the financial statement analysis framework.

LOS 27.b Distinguish the role of financial reporting from the role of financial statement analysis.

LOS 27.c Explain why regulatory filings, financial statement notes and supplementary information, management's commentary and audit reports matter to analysts.

LOS 27.d Describe what differing reporting systems (IFRS vs US GAAP) mean for analysis and why analysts must track changes in reporting standards.

LOS 27.e Describe the information sources analysts use beyond annual and interim financial reports.

MODULE 27.1

Financial Statement Roles

This reading sets out the six steps of the financial statement analysis framework and separates the role of financial reporting from the role of financial statement analysis. It then covers the information an analyst works with: regulatory filings, footnotes, segment data, management commentary and the audit report, the differences between IFRS and US GAAP as standards evolve, and sources outside the company's filings.

LOS 27.a — The financial statement analysis framework

The framework is a generic, repeatable process that works for any analytical purpose, whether valuing equity, judging a bond issue or assessing a merger. It has six steps, taken in this order:

KEY CONCEPT

#	Step	What happens in it
1	**State the objective and context**	Define the questions to be answered, the format of the output, and the time and resources available
2	**Gather data**	Obtain the financial statements plus industry and economic data; talk to management, suppliers and customers; visit sites
3	**Process the data**	Adjust the statements where needed, compute ratios and run statistical work, build exhibits such as graphs and common-size statements
4	**Analyze and interpret the data**	Use the processed data to answer the step-1 questions and decide which conclusions or recommendations the evidence supports
5	**Report the conclusions or recommendations**	Write up and communicate the findings to the intended audience, in line with the CFA Institute Code and Standards
6	**Update the analysis**	Repeat the process periodically and revise the conclusions when facts change

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Each step works on the output of the one before it. The questions written down in step 1 decide which data are collected in step 2, and those data decide which adjustments, ratios and exhibits can be prepared in step 3. Skipping step 1 therefore leaves the later steps without a target.

Example. A credit analyst at a regional bank is asked whether to raise a borrower's credit line. She first writes down the question and the deadline (step 1). She downloads three years of filings and sector data (step 2), restates leases and computes coverage ratios (step 3), and concludes that the borrower can service more debt (step 4). She sends a memo to the credit committee (step 5) and schedules a review after the next annual report (step 6).

Common exam traps

- Obtaining statements or interviewing suppliers belongs to gathering data. It is not processing.
- Adjusting statements and calculating ratios is processing the data. Using the ratios to reach a conclusion is analyzing and interpreting the data.
- Ratios and common-size statements are outputs of processing the data and inputs to analyzing and interpreting it. Revised reports and recommendations come from updating the analysis.
- Deciding whether a security suits a particular client is not a step of this framework.

LOS 27.b — Financial reporting and financial statement analysis

- **Financial reporting** is the way a company prepares and publishes financial statements that show its performance and position to investors, creditors and anyone else with an interest in it. Its role is to give a wide range of users useful information about the company's performance and financial position.
- **Financial statement analysis** is the use of that information, together with other relevant data, to make economic decisions about the company: whether to buy or recommend its securities, extend trade or bank credit, or assign a credit rating. Analysts evaluate past performance and the current position to judge how well the firm is likely to generate profits and cash flow in the future, and they identify the risk factors along the way.

Financial statement analysis as described here is done by external users. Management also analyzes financial information for day-to-day decisions, but it can draw on internal data kept in other formats that outsiders never see.

	Financial reporting	Financial statement analysis
Who	The company (management)	Investors, lenders, rating agencies, analysts
Activity	Prepare and present statements	Evaluate statements plus other data
Output	Information useful to many users	An economic decision or recommendation

Common exam traps

- "Provides users with useful information about performance and financial position" describes reporting.
- "Uses the statements to decide whether to lend or invest" describes analysis.
- The statements may give indirect evidence about the skill of individual executives, but judging managers is not a stated role of the analysis.

- Giving reasonable assurance that statements are free of material error is the auditor's job, not the analyst's.

LOS 27.c — Filings, notes, management commentary and audit reports

Standard setters and regulators

Standard-setting bodies are private professional bodies that write the reporting rules. The **FASB** issues **US GAAP**; the **IASB** issues **IFRS**, and older IASB standards are called IAS. Other national standard setters also exist. **Regulatory authorities** are government agencies with legal power to enforce the rules, such as the **Securities and Exchange Commission (SEC)** in the United States and the UK's Financial Conduct Authority. Most national regulators belong to the **International Organization of Securities Commissions (IOSCO)**, whose members between them regulate over 95% of global financial markets. IOSCO itself is not a regulator. Its members cooperate across borders to make national regulation and enforcement more uniform. Its Objectives and Principles of Securities Regulation rest on three objectives: protecting investors; ensuring that markets are fair, efficient and transparent; and reducing systemic risk. IOSCO expects issuers to give full, accurate and timely disclosure of financial results, risks and other decision-relevant information, prepared under high-quality, internationally accepted accounting standards.

The **Sarbanes-Oxley Act**, enforced by the SEC, limits the non-audit services a company's external auditor may provide. The purpose of that limit is to avoid conflicts of interest and protect auditor independence. The act also requires executives to certify that the statements are fairly presented and to include a statement on the effectiveness of internal control over financial reporting.

Exam convention: the external auditor must also confirm the effectiveness of the company's internal controls. Current practice: the auditor attestation in Section 404(b) applies to accelerated and large accelerated filers; non-accelerated filers and emerging growth companies (EGCs) are exempt.

Securities regulation in the EU is set by each member state separately. Exam convention: all EU countries are required to report using IFRS. Current practice: the EU IAS Regulation requires IFRS for the consolidated statements of listed companies. Exam convention: the body that advises the European Commission on securities regulation is called the European Securities Commission. Current practice: its official name is the European Securities Committee. The **European Securities and Markets Authority (ESMA)** coordinates regulation within the EU.

Key SEC filings:

Filing	Content
Form S-1	Registration statement filed before new securities are sold to the public: the securities offered, audited statements, risk assessment, underwriters, and the estimated amount and use of the proceeds
Form 10-K	Required annual filing: the business, its risks and management, audited statements and disclosures, and legal matters. The annual report to shareholders does not replace it. Foreign issuers file Form 40-F (Canadian) or Form 20-F (others)
Form 10-Q	Quarterly filing of US firms with interim statements, which need not be audited, and disclosures such as significant legal proceedings or accounting policy changes; non-US issuers typically file Form 6-K semiannually

Filing	Content
Form DEF-14A	The **proxy statement** filed when shareholders must vote
Form 8-K	Material events: major asset acquisitions or disposals, changes in management or corporate governance, and matters involving the company's accountants, its financial statements or the markets where its securities trade
Form 144	Exam convention: a notice to the SEC that the company will sell securities to certain qualified buyers without registering them. Current practice: Form 144 is the notice an affiliate files before a proposed sale of restricted or control securities under Rule 144
Forms 3, 4 and 5	Beneficial ownership of the company's securities by officers and directors; used to track insider purchases and sales

Financial statement notes

Financial statement notes (footnotes) give the detail behind the summary numbers so that users can judge the amount, timing and uncertainty of estimates. They cover the basis of presentation (fiscal period, IFRS or US GAAP, consolidated entities) and the **accounting methods, assumptions and estimates** chosen by management. They also give details on contingencies and commitments, legal actions, acquisitions and disposals, employee benefit plans, related-party transactions, significant customers, segments and post-balance-sheet events. Footnotes are audited together with the primary statements.

Segment reporting

Both IFRS (IFRS 8) and US GAAP require segment disclosures. For each segment, a smaller set of items than the company-wide statements is reported: a measure of profit or loss; revenue, split into sales to outside customers and sales to other segments; measures of assets and liabilities; interest revenue and interest expense; income tax expense; depreciation and amortization; other noncash expenses; spending on PP&E and intangibles; and the share of results of equity-accounted investments. A **business segment (operating segment)** is a portion of the company whose risk and return characteristics are distinguishable from those of the company's other lines of business.

Exam convention: a business segment is one whose revenues, assets or income exceed 10% of the company's totals. Current practice (IFRS 8 and ASC 280): an operating segment is a part of the company that earns revenues and incurs expenses and whose results management reviews separately. It must be reported separately, as a **reportable segment**, when it meets any one of three size tests. It accounts for 10% or more of:

1. the combined revenue (external and intersegment) of all operating segments;
2. the combined assets of all operating segments; or
3. the larger of (i) the combined profit of all segments that report a profit and (ii) the combined loss of all segments that report a loss, with the segment's own profit or loss taken in absolute amount.

Reportable segments must together cover at least 75% of the company's external revenue; if they do not, more segments are reported. **Geographic segments** are identified on the same size criterion when an area's business environment differs from that of the rest of the company. Segment profit margins, asset turnover and return on assets give a clearer picture of the whole firm, and growth rates of segment revenues and profits help forecast future sales and profits.

Management commentary

Management commentary is also called the management report, the operating and financial review, or **management's discussion and analysis (MD&A)**. IFRS guidance recommends, but does not require, that it cover the nature of the business, management's objectives, past performance, performance measures, and the company's key relationships, resources and risks. Parts of it may be unaudited.

For US public companies, the SEC requires a discussion of trends and significant events and uncertainties affecting liquidity, capital resources and results of operations. Exam convention: management must also discuss any material effects of inflation and changing prices, contractual and off-balance-sheet obligations (purchase commitments, for example), accounting policies that call for significant judgment, and forward-looking expenditures and divestitures. Current practice: since the SEC's MD&A amendments took effect in 2021, inflation, off-balance-sheet arrangements and the contractual obligations table are no longer separate items; material effects are covered in the known-trends, liquidity and capital resources discussion. SEC rules also require a description of unusual or infrequent events or transactions that materially affected reported income from continuing operations. Information that belongs in other filings is not an MD&A requirement; the compensation of individual directors, for example, appears in the proxy statement.

Audits and the auditor's report

An **audit** is an independent review that aims to give an opinion on whether the statements are fair and reliable. Management prepares the statements and is responsible for them. The auditor is an independent accounting firm engaged by the board of directors. It reviews the company's accounting systems and internal controls, verifies its assets and liabilities, and tries to determine that the statements contain no material errors. The standard opinion states that:

1. the statements are management's responsibility, and the auditor performed an independent review;
2. generally accepted auditing standards were followed, giving **reasonable assurance** (not a guarantee) that there are no material errors; and
3. the statements conform to accepted accounting principles, and the principles and estimates chosen are reasonable.

The report must also explain any accounting method that has not been applied consistently between periods.

KEY CONCEPT

Opinion	When
Unqualified opinion (unmodified, clean)	Free of material omissions and errors
Qualified opinion	Specific exceptions to accounting principles, explained in the report; also a **scope limitation** (the auditor cannot obtain enough evidence on some items) whose possible effect is material but confined to those items
Adverse opinion	Statements are not presented fairly or are materially nonconforming
Disclaimer of opinion	Auditor cannot express an opinion at all, as when a scope limitation is pervasive and affects the statements as a whole

Any opinion other than unqualified is a **modified opinion**. Exam convention: an **explanatory paragraph** is added when a material loss is probable but cannot be reasonably estimated; the uncertainty may concern the **going concern** assumption (that the company will keep operating for the foreseeable future), the valuation or realization of assets, or litigation. Current practice: auditing standards still require a separate paragraph or section when there is substantial doubt about the company's ability to continue as a going concern, but for other uncertainties, such as pending litigation, an emphasis-of-matter paragraph is at the auditor's discretion. Such a paragraph can signal serious problems and deserves close attention from the analyst.

Reports also include **key audit matters** (international) or **critical audit matters** (US). These are the accounting areas of most significance to users: areas that need significant management judgment and estimates, the accounting for significant transactions, and areas the auditor found especially challenging or subjective, where the risk of misstatement is highest.

Internal controls are the processes that make the statements accurate, and they are management's responsibility. Weak controls open the door to low-quality or fraudulent reporting. Exam convention: for US publicly traded firms, the auditor must express an opinion on internal control, either as a fourth element of the standard opinion or separately. Current practice: this applies to accelerated and large accelerated filers; non-accelerated filers and emerging growth companies are exempt.

Common exam traps

- Management's view of results and outlook appears in management commentary. The footnotes cover methods, estimates and supporting detail.
- Footnotes are audited; management commentary and 10-Q interim statements may not be.
- IFRS recommends the content of management commentary. Securities regulators, not accounting standards, can require topics.
- A statement that the accounts follow GAAP is part of the standard report. It is not an explanatory paragraph.
- Being just below 10% on several segment tests does not make a segment reportable. One test at 10% or more does.

LOS 27.d — Alternative reporting systems and evolving standards

The IASB and FASB work toward convergence, but significant differences remain, so comparisons of firms in different jurisdictions need care:

KEY CONCEPT

Item	US GAAP	IFRS
Developed by	FASB	IASB
Approach	Rules-based	Principles-based
Inventory cost methods	FIFO, LIFO, weighted average	LIFO prohibited
Product development costs	Expensed	May be capitalized
Interest paid (cash flow)	CFO	CFO or CFF

Item	US GAAP	IFRS
Reversal of inventory write-downs	Prohibited	Allowed

Exam convention: under IFRS, interest paid may be classified in CFO or CFF (the IAS 7 choice). Current practice: IFRS 18, effective for annual periods beginning on or after 1 January 2027, removes that choice for companies whose main business is not financing; they report interest paid in CFF.

Standards keep changing, and new products and transactions may not fit existing standards. The analyst can use the financial reporting framework as a guide to their likely effect on the statements. The analyst should also follow the IASB and FASB websites, professional journals and CFA Institute position papers, and monitor company disclosures of significant accounting policies and estimates.

Common exam traps

- New transactions that existing standards do not yet cover are not unusual. The analyst uses the reporting framework to judge their likely effect rather than ignoring them until a standard is issued.

LOS 27.e — Other information sources

Besides annual reports, quarterly and semiannual reports and other regulatory filings, analysts use:

Category	Examples
Issuer sources	**Earnings calls** and earnings guidance, ad hoc presentations, **press releases**, contact with management and investor relations; unlikely to have been audited
Public third-party sources	Free industry reports and trade journals, government agency economic and industry statistics, media, social media
Proprietary third-party sources	Analyst reports, data platforms, specialist agency and consultancy reports
Proprietary primary research	Studies the analyst commissions, hands-on product experience, in-house technical specialists

Many companies publish **earnings guidance** ahead of their results. Once results are out, management often holds a call with analysts, takes their questions and gives detail beyond what the filings contain. Outside the company, trade publications, statistical data services and government statistics help the analyst judge the wider economy and the industry and benchmark the firm against its rivals.

Proxy statements are sent to shareholders when a vote is needed. They are a good source of information on board elections and qualifications, compensation, management qualifications and stock option issuance. Proxy statements are also filed with the SEC (Form DEF-14A), like the 10-K, 10-Q and 8-K. Press releases are issuer communications distributed by the company itself.

Common exam traps

- Press releases and earnings calls come from management and are unlikely to have been audited. Treat them as less objective than government statistics or audited statements.

- Analyst research reports are proprietary third-party sources. Trade journals and government statistics are public third-party sources.

BOTTOM LINE

- The financial statement analysis framework has six steps in a fixed order: first the objective and context are stated and data are gathered, then the data are processed, analyzed and interpreted, and finally the conclusions or recommendations are reported and the analysis is updated.
- Adjusting the statements and computing ratios and common-size statements is processing the data; using those outputs to reach a conclusion is analyzing and interpreting the data.
- Financial reporting gives a wide range of users useful information on a company's performance and financial position, while financial statement analysis uses that information, with other relevant data, to make economic decisions about the company.
- Financial statement footnotes give the accounting methods, assumptions and estimates management chose, plus detail such as contingencies, legal actions and related-party transactions, and they are audited together with the primary statements.
- Form 10-K is the required annual SEC filing with audited statements, Form 10-Q is the quarterly filing whose interim statements need not be audited, Form 8-K reports material events, and Form DEF-14A is the proxy statement.
- An audit gives reasonable assurance, not a guarantee, against material errors in the statements, and the opinion is unqualified (clean), qualified (specific exceptions, or a scope limitation whose possible effect is material but confined to some items), adverse (not presented fairly) or a disclaimer (no opinion can be expressed).
- US GAAP, issued by the FASB, is rules-based and allows LIFO; IFRS, issued by the IASB, is principles-based, prohibits LIFO, allows inventory write-downs to be reversed and allows product development costs to be capitalized.
- Proxy statements are the source for board elections, compensation, management qualifications and stock option issuance, while press releases and earnings calls come from the issuer and are unlikely to have been audited.

QUICK CHECK

Question 1

An analyst converts a competitor's inventory figures to the same cost basis as her subject company, then calculates liquidity and profitability ratios and prepares common-size statements. In the financial statement analysis framework, this work is *best* described as:

A processing the data.

B analyzing and interpreting the data.

C gathering the data.

Answer: A

Adjusting the reported statements where appropriate, calculating ratios, and preparing exhibits such as graphs and common-size statements are the activities of step 3, process the data. No conclusion has yet been drawn from the ratios, so the analyst has not reached the analysis-and-interpretation step.

The six steps:

1. State the objective and context: questions to answer, output format, time and resources.
2. Gather data: statements, industry and economic data, discussions with management, suppliers and customers, site visits.
3. Process the data: adjustments, ratios, statistical work, exhibits such as graphs and common-size statements.
4. Analyze and interpret the data: answer the step-1 questions and decide what the evidence supports.
5. Report the conclusions or recommendations: communicate to the audience in line with the Code and Standards.
6. Update the analysis: repeat periodically and revise conclusions as needed.

The analyst's work (adjustment, ratios, common-size statements) fits step 3.

Why the other options are wrong

- **B.** Analyzing and interpreting (step 4) means using the processed figures to answer the questions posed in step 1 and deciding what conclusions or recommendations they support. The analyst is still preparing the figures.
- **C.** Gathering data (step 2) is acquiring the statements and other industry and economic information and asking questions of management, suppliers and customers. It comes before any adjustment or ratio work.

Key takeaway Adjusting, calculating and building exhibits is processing the data. Using the results to answer the questions is analyzing and interpreting the data.

Practice questions

Question 2

An analyst is reading the notes that accompany a retailer's audited annual financial statements. Which of the following statements about those notes is *least* accurate? The notes:

A disclose contingencies, such as losses that could arise from pending claims.

B are covered by the independent auditor's examination.

C present management's own assessment of the year's operating performance and financial results.

Question 3

Ilse Brandt covers Corrigan Freight, a US public company. She wants management's own account of known trends and of the significant events and uncertainties that could affect Corrigan's liquidity, capital resources and results of operations. She is *most likely* to find this in:

A management's discussion and analysis (MD&A).

B the proxy statement.

C the notes to the financial statements.

Question 4

Priya Nandakumar is researching Corvell Biotech and wants information she can treat as objective. Which of the following sources should she be *least likely* to rely on for that purpose?

A Industry output statistics published by a government agency.

B A proxy statement that Corvell filed with the SEC.

C A press release issued by Corvell's investor relations team.

Answers: 2. C · 3. A · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/introduction-to-financial-statement-analysis/. The full bank has 29 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 28

Analyzing Income Statements

What you'll learn

LOS 28.a Explain the general principles of revenue recognition, the five-step model, specific recognition issues, and their implications for financial analysis.

LOS 28.b Explain the matching principle and expense recognition choices (inventory, depreciation, capitalization vs. expensing, estimates) and their effects on financial statements and ratios.

LOS 28.c Explain how discontinued operations, unusual or infrequent items, accounting changes and error corrections are reported and how analysts should treat them.

LOS 28.d Calculate and interpret basic and diluted EPS for simple and complex capital structures, including the treatment of antidilutive securities.

LOS 28.e Evaluate a company's performance using common-size income statements and income-statement ratios such as gross and net profit margin.

MODULE 28.1

Revenue Recognition

This reading covers how revenue and expenses are recognized, from the five-step revenue model to the choice between capitalizing and expensing, and how nonrecurring items and accounting changes are reported. The calculations to master are revenue on long-term contracts, basic and diluted EPS, and common-size income statements with their margins.

LOS 28.a — Revenue recognition: principles, applications and analysis

The accrual idea

Revenue belongs in the period in which it is **earned**, that is, when the company has done what it promised. This need not be the period in which cash arrives.

- **Credit sale:** revenue is recognized at the sale, and an asset, accounts receivable, is recorded.
- **Cash received in advance** (subscriptions, deposits, gift cards): no revenue is recognized yet. The company records cash and a liability, **unearned (deferred) revenue**. As the goods or services are delivered, the liability is reduced and revenue is recognized.
- Revenue is reported net of expected returns, discounts and allowances. Exam convention: estimated warranty provisions count as one of the allowances subtracted from revenue when the sale is made. Current practice: a standard (assurance-type) warranty is recognized at the sale as an expense with a matching provision, and a warranty sold as a separate service is a performance obligation whose share of the price is deferred.

The converged standard (IFRS 15 and ASC 606)

The IASB and FASB jointly developed this converged revenue standard. It is **principles-based**. Instead of many industry-specific rules, it rests on one core principle: recognize revenue when (or as) control of a promised good or service passes to the customer, at the amount the firm expects to be entitled to. The principle is applied through a five-step model:

KEY CONCEPT

Step	What the firm does	Key definition
1	Identify the **contract** with the customer	An agreement between two or more parties that creates enforceable rights and obligations; collection must be *probable* (IFRS and US GAAP define "probable" differently, so identical deals can be treated differently)
2	Identify the **performance obligations**	A promise to transfer a *distinct* good or service: one the customer can benefit from on its own (or with readily available resources) and that is separately identifiable from the other promises
3	Determine the **transaction price**	The amount the firm expects to receive. It is usually fixed but may include **variable consideration** (bonuses such as one for early delivery, rebates, penalties), included only to the extent that a significant later reversal is highly unlikely
4	**Allocate** the price to the obligations	Spread the transaction price across the performance obligations identified in Step 2
5	**Recognize** revenue when (or as) each obligation is satisfied	An obligation is satisfied when control transfers

Indicators that control has transferred are that the customer has physical possession, has accepted the item, bears the risks and rewards of ownership and holds legal title, and that the seller has a present right to payment.

Exam convention: revenue is recognized only when a later reversal is highly unlikely, so a firm may have to record a **refund liability**, with an asset for its right to the returned goods, when revenue from a sale cannot be estimated reliably. Current practice: under IFRS 15 and ASC 606, every sale with a right of return gives revenue only for the amount not expected to be refunded, a refund liability for expected refunds and an asset for the goods expected back.

Revenue over time (long-term contracts)

KEY CONCEPT

An obligation is satisfied **over time** if any one of these conditions holds:

1. The customer receives and consumes the benefit as the firm performs, as with a cleaning or maintenance contract.
2. The firm's work creates or enhances an asset that the customer controls as it is built.
3. The asset has no alternative use to the seller, and the seller can enforce payment for work done to date.

Progress can be measured by **inputs** (cost incurred as a share of total expected cost) or by **outputs** (milestones, units delivered). Costs of securing a long-term contract, such as sales commissions, are capitalized and then expensed over the life of the contract.

A building is usually one performance obligation. Site preparation, foundations, wiring and the roof are not distinct promises, because each is an input to a single combined output that the customer contracted for.

Example. Fenwick Builders agrees to construct a cold-storage facility for $30 million, plus a $3 million bonus if the facility is finished within 18 months. The facility uses a new refrigeration design and the finish date depends on a supplier's delivery schedule, so Fenwick cannot conclude that a reversal of revenue from the bonus is highly unlikely. The bonus is left out of the transaction price, which stays at $30 million. The expected total cost is $25 million. Costs incurred are $8 million in Year 1 and $7 million in Year 2.

$$\text{Cumulative revenue}_t = \frac{\text{Cumulative cost to date}}{\text{Total expected cost}} \times \text{Transaction price}$$

- Year 1: $8/25 = 32\%$, so revenue is $0.32 \times 30 = \$9.6$ million.
- Year 2: $(8 + 7)/25 = 60\%$, so cumulative revenue is $\$18.0$ million and Year 2 revenue is $18.0 - 9.6 = \$8.4$ million.

The formula always uses the transaction price as currently estimated. Had the bonus become includable by the end of Year 2, cumulative revenue would be $0.60 \times 33 = \$19.8$ million and Year 2 revenue $19.8 - 9.6 = \$10.2$ million. Year 1 revenue is not restated, so the bonus share of the Year 1 work is recognized in Year 2.

The amounts billed or collected do not affect these figures. The result is the same as under the older percentage-of-completion method, although the converged standard no longer uses that name.

Specific applications

Situation	Treatment
Principal or agent	A *principal* controls the good before transfer and reports **gross** revenue, with the cost as an expense. An *agent* only arranges the sale, is not responsible for providing the good or service and bears no inventory or credit risk; it reports its **net** fee or commission.
Franchise fees	An upfront fee covering several years is deferred and recognized over the franchise term; royalties are recognized as they become payable. Revenue is disaggregated into categories with similar characteristics (company stores, royalties, supplies).
Software license or service	An "as-is" license, with updates and support under a separate contract, gives license revenue at the start; the support revenue is recognized as the service is provided, usually over the contract. A license under which the supplier keeps updating the software, the customer is exposed to the effects of those updates, and the updates are not a separate transfer of goods or services gives revenue over the contract. Cloud access without taking possession of the software is a service recognized over time.
Bill-and-hold	The customer pays before shipment. Such a payment is normally deferred revenue. Revenue before shipment is allowed only if the seller has completed its obligations, the customer has control, the customer requested the arrangement, and the goods are identified as the customer's, are ready for delivery and cannot be redirected.

Example. An online travel platform sells an €800 cruise cabin and keeps a €60 commission. As an agent it reports revenue of €60, a gross margin of 100%. If it were the principal, it would report €800 of revenue and €740 of cost, a gross margin of 7.5%. Gross profit is €60 either way, but revenue and margins differ sharply, which creates a comparability problem.

Disclosures and analysis

Firms must disclose contracts by category, contract assets and liabilities (and changes in them), remaining performance obligations with the price allocated to them, and the **judgments** used on timing and amount. Analysts should watch the choices that move revenue between periods: the measure of progress, estimates of variable consideration, contract modifications, and principal-versus-agent classification.

COMMON EXAM TRAPS

- Treating cash received as revenue. Advance payments create a liability.
- Confusing the contract (step 1) with a performance obligation (a distinct promise within the contract, step 2).
- Calling the converged standard "rules-based." It is principles-based.
- In over-time questions, reporting cumulative revenue for the year instead of subtracting revenue already recognized. For Fenwick, this reports Year 2 revenue of $18.0 million instead of $8.4 million.
- Reporting an agent's revenue at the gross sale price. Agents report net commission revenue. For the cruise cabin, this reports €800 of revenue instead of €60.

EXAM SHORTCUTS

- In an over-time contract measured by inputs, amounts billed or collected can be ignored: cumulative revenue depends only on the cumulative cost to date, the total expected cost and the current transaction price.

BOTTOM LINE

- Under IFRS 15 and ASC 606, revenue is recognized when (or as) control of a promised good or service passes to the customer, at the amount the firm expects to be entitled to.
- The five steps are: identify the contract with the customer; identify its performance obligations; determine the transaction price; allocate that price to the obligations; and recognize revenue when (or as) each obligation is satisfied.
- Cash received before goods or services are delivered is recorded as a liability, unearned (deferred) revenue, and becomes revenue as the delivery takes place.
- Variable consideration such as a bonus enters the transaction price only to the extent that a significant later reversal is highly unlikely.
- With an input measure of progress, cumulative revenue equals cumulative cost to date divided by total expected cost, times the transaction price, and the period's revenue is that figure less revenue already recognized.
- A principal controls the good before transfer and reports gross revenue with the cost as an expense; an agent only arranges the sale and reports its net fee or commission.

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QUICK CHECK

Question 1

On November 15, Halden Yacht Services receives $24,000 in cash from a customer for a winter storage and maintenance package that Halden will provide from January through March. Halden has not yet performed any of the service. At November 30, the effect of this transaction on Halden's balance sheet is *best* described as:

A cash and retained earnings each increase by $24,000.

B cash and unearned revenue each increase by $24,000.

C accounts receivable and revenue each increase by $24,000.

Answer: B

Revenue is recognized when it is earned, that is, as the promised service is provided. Cash received before any service is delivered creates a liability, unearned (deferred) revenue, because Halden still owes the customer the service. The asset cash rises by $24,000 and the liability rises by the same amount; no revenue, and therefore no retained earnings, is recognized yet.

Why the other options are wrong

- **A.** This treats the prepayment as revenue that raises net income and retained earnings. No service has been performed, so no revenue has been earned by November 30.
- **C.** The customer has already paid, so there is no receivable, and revenue is not recognized before the service is provided.

Key takeaway Cash received in advance is a liability, unearned revenue. It becomes revenue only as the goods or services are delivered.

MODULE 28.2

Expense Recognition

LOS 28.b — Expense recognition, capitalization and expensing

How expenses are recognized under accrual accounting

An **expense** is a decrease in economic benefits (assets used up or liabilities incurred) that lowers equity other than through distributions to owners. Under the accrual basis, an expense is recognized when the benefit is **consumed**, whether or not cash has been paid. Three routes achieve this:

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KEY CONCEPT

Route	Idea	Examples
Matching	Costs that generate revenue are expensed in the same period as that revenue	Cost of goods sold is expensed when the goods are sold; estimated warranty cost and bad debt expense are recorded in the period of the sale
Expensing as incurred (**period costs**)	Costs that cannot be tied to specific revenue are expensed in the period they relate to	Head-office salaries, rent for the year (paid or not), interest accrued on a loan (paid or not)
Capitalization	Costs that bring benefits over several periods are recorded as an asset and expensed gradually	PP&E (depreciation), finite-life intangibles (amortization), natural resources (depletion)

Production or purchase costs stay on the balance sheet as inventory until the goods are sold. If nothing is sold in a period, none of those costs reaches the income statement.

A policy that recognizes expenses later is **aggressive** because it raises current income. A policy that recognizes them sooner is **conservative**, so expensing an outlay is more conservative than capitalizing it.

Recording a credit sale

When goods costing 70 are sold on credit for 100, receivables rise by 100 and inventory falls by 70, so total assets rise by 30. Retained earnings (equity) rise by the profit of 30, and liabilities are unaffected (ignoring taxes).

Inventory cost-flow methods

Specific identification assigns each unit its own actual cost. It suits goods that can be tracked one by one, such as vehicles recorded by serial number. When individual units cannot be tracked this way, the firm assumes a cost flow:

Method	COGS contains	Physical flow it resembles	With rising prices
FIFO	Oldest costs	Units sold in the order produced (perishables, dated goods)	Lowest COGS, highest gross margin and net income; ending inventory near current cost
LIFO (US GAAP only; not permitted under IFRS)	Newest costs	Units taken from the top of a heap (road salt, sand)	Highest COGS, lowest margins and income
Weighted average cost	Average cost	Units drawn from a mixed stock	In between

Revenue is unaffected by the method, so differences in COGS pass directly into margins. Analysts comparing firms must adjust for different methods.

Depreciation methods

The depreciable base is cost minus salvage value. Straight-line spreads it evenly over the useful life.

$$\text{Straight-line} = \frac{\text{Cost} - \text{Salvage}}{\text{Useful life}}$$

> *Example.* A press costs \$60 million, has a salvage value of \$12 million and a 6-year life. Straight-line depreciation is $(60 - 12)/6 = \$8.0$ million a year.

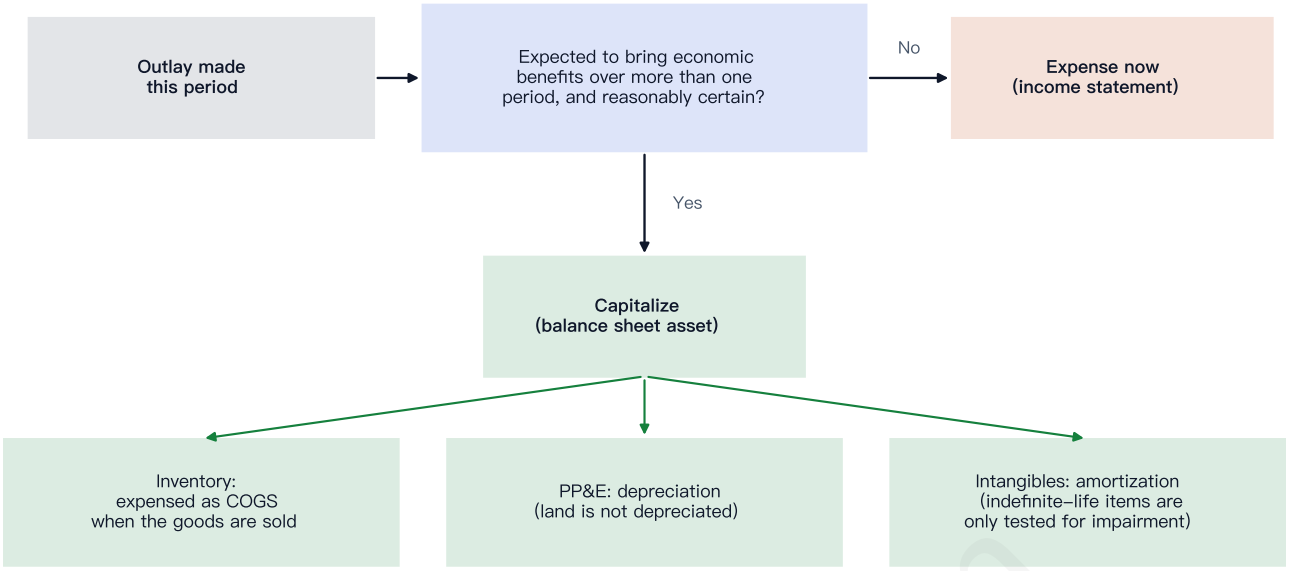
Intangible assets

- **Finite-life intangibles** (patents, licenses, customer lists) are amortized over their useful lives.
- **Indefinite-life intangibles** (acquired goodwill, some trademarks or licenses renewable at little cost) are not amortized. They are tested for impairment at least annually under both IFRS and US GAAP.
- Most internally generated intangibles are expensed. **Research** costs (aimed at new scientific or technical knowledge) are expensed. Under IFRS, **development** costs (turning research findings into a plan or design for a new product or process) may be capitalized. Among other criteria, the firm must show that it is able to finish the asset and plans to use or sell it. Under US GAAP, R&D is generally expensed; the main exception is software developed for sale, which is capitalized after technological feasibility is established.
- When one firm capitalizes development costs and a peer expenses them, to make the two comparable, an analyst expenses the period's development spending and removes the amortization of earlier capitalized amounts from the income statement. On the balance sheet the capitalized asset is removed, which lowers assets and equity. In the cash flow statement the outlay moves from CFI to CFO, so CFO falls.

Capitalizing and expensing

An outlay expected to bring economic benefits over more than one period is capitalized. If the future benefit is unlikely or highly uncertain, it is expensed. The chart shows the decision and the route by which each kind of capitalized cost later reaches the income statement (see the figure below). A capitalized asset's cost includes the purchase price plus the costs needed to get the asset ready for use (freight, installation, non-recoverable taxes). Except for land and indefinite-life intangibles, that cost is then spread over the asset's life as depreciation, depletion or amortization. Later outlays that extend the asset's life or add capacity are also capitalized. Routine repairs, maintenance and staff training are expensed.

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Capitalize or expense: the decision and how capitalized costs are expensed later

Example. A bakery buys an oven for $48,000. It also pays $1,500 for delivery, $900 for installation, $400 for test bakes before the oven goes into use and $1,200 to train its staff. The oven is recorded at $48{,}000 + 1{,}500 + 900 + 400 = \$50{,}800$. The $1,200 of training is expensed, because it does not get the oven ready for use.

KEY CONCEPT

Effect of capitalizing (compared with expensing)	Year of outlay	Later years
Net income, net margin	Higher	Lower (depreciation continues)
Total assets, equity	Higher (the extra asset is matched by higher retained earnings; liabilities are unchanged)	Higher, gap shrinks to zero
ROA, ROE	Higher	Lower
Asset turnover	Lower	Lower, gap shrinks
Earnings volatility	Lower (smoother)	Not applicable
Cash flow from operations	Higher (outflow sits in CFI)	Same if tax ignores the choice; higher if depreciation is deducted for tax
Cash flow from investing	Lower	Same
Debt-to-assets, debt-to-equity	Lower	Lower

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Total cash flow is the same under both choices, provided the tax treatment does not depend on the accounting choice; only the classification between CFO and CFI differs. If the tax deduction follows the accounting choice, expensing brings the whole tax saving into the year of the outlay. Total cash flow is then higher in that year under expensing, CFO is higher in later years under capitalization (depreciation keeps reducing tax), and cumulative cash is the same by the end of the asset's life.

Example. A new firm buys a machine for $20,000 with a 5-year life and no salvage value. Operating profit before the machine is $25,000 a year, the tax rate is 25%, and depreciation is tax deductible.

- Capitalize: depreciation is $4,000, so net income is $(25{,}000 - 4{,}000)(1 - 0.25) = \$15{,}750$ every year. CFO is $15{,}750 + 4{,}000 = \$19{,}750$ every year, and CFI is −$20,000 in Year 1.
- Expense: Year 1 net income is $(25{,}000 - 20{,}000)(1 - 0.25) = \$3{,}750$, which is also Year 1 CFO. Years 2–5 net income and CFO are $25{,}000 \times 0.75 = \$18{,}750$.
- Year 1 total cash flow is −$250 when capitalizing and $3,750 when expensing. Capitalization then gives $1,000 more CFO in each of Years 2–5, and five-year cash totals are $78,750 either way.
- Cumulative net income over the five years is also $78,750 under both choices $(5 \times 15{,}750 = 3{,}750 + 4 \times 18{,}750)$. The choice moves income between years; it does not change the total.

The table describes a single outlay. A firm that capitalizes similar costs every year keeps reporting higher income than an expensing peer for as long as the amount capitalized each period exceeds that period's depreciation or amortization of the capitalized costs.

Capitalized interest

Interest incurred while a firm builds an asset for its own use must be capitalized as part of the asset's cost under both IFRS and US GAAP. In limited cases this also applies to an asset built for sale. Capitalized interest is not shown as interest expense. It later reaches income through depreciation (asset held for use) or COGS (asset held for sale), and the cash paid appears in investing cash flow. As with any capitalized cost, net income is higher while the asset is being built and lower in later years. In those later years EBIT is also lower, because the interest now sits inside depreciation, an operating expense. Exam convention: interest paid that is not capitalized is an operating outflow under US GAAP and an operating or financing outflow under IFRS. Current practice: under IFRS 18, effective for annual periods beginning on or after 1 January 2027, a company whose main business is not financing reports interest paid in CFF.

For interest coverage, analysts add the interest capitalized in the period to interest expense. They also add the depreciation of previously capitalized interest back to EBIT, because it is economically an interest cost rather than an operating cost.

KEY CONCEPT

$$\text{Adjusted coverage} = \frac{\text{EBIT} + \text{Depreciation of capitalized interest}}{\text{Interest expense} + \text{Interest capitalized}}$$

Example. A utility reports EBIT of $90 million and interest expense of $30 million. It capitalized $15 million of interest this year, and depreciation includes $6 million of interest capitalized in earlier years. Reported coverage is $90/30 = 3.0$. Adjusted coverage is $(90 + 6)/(30 + 15) = 2.13$. Had the $15 million been

expensed, CFO would be $15 million lower and CFI $15 million higher (less negative), ignoring taxes and with interest paid classified in CFO, as under US GAAP.

Analyzing estimates

Bad-debt, warranty, useful-life and salvage estimates give management room to shift income. Analysts should ask why an estimate changed and compare it with peers. The footnotes and MD&A disclose accounting policies and significant estimates.

Example. A heater maker has Year 1 sales of $1,000,000 and expects warranty repairs to cost 4% of sales. It records warranty expense of $40,000 in Year 1, the year of the sales, together with a warranty provision (a liability) of the same amount. Repairs of $35,000 carried out in Year 2 are charged against the provision, so they are not Year 2 expense. Had management assumed 2%, Year 1 pretax income would have been $20,000 higher. A warranty rate well below that of peers may reflect better products or more aggressive expense recognition, and the analyst should find out which.

COMMON EXAM TRAPS

- Believing costs are expensed when paid. Accrued interest and rent are expenses of the period even if unpaid.
- Expensing inventory costs when incurred. They are expensed when the goods are sold.
- Reversing the FIFO and LIFO results. With rising prices, FIFO gives the highest margins and LIFO the lowest.
- Amortizing goodwill. Indefinite-life intangibles are only tested for impairment.
- Thinking capitalized interest is an IFRS-only or US GAAP-only rule. Both require it.
- Forgetting to deduct salvage value when computing straight-line depreciation. For the press, this gives $10.0 million a year instead of $8.0 million.

BOTTOM LINE

- Under accrual accounting, an expense is recognized when the benefit is consumed, whether or not cash has been paid: by matching it with revenue, as a period cost, or by capitalizing it and expensing it gradually.
- With rising prices, FIFO gives the lowest COGS and the highest gross margin and net income, LIFO (US GAAP only) gives the highest COGS and the lowest margins, and weighted average cost falls in between.
- Straight-line depreciation equals cost minus salvage value, divided by the useful life.
- Indefinite-life intangibles such as acquired goodwill are not amortized but are tested for impairment at least annually; research costs are expensed, development costs may be capitalized under IFRS, and US GAAP generally expenses R&D except software developed for sale once technological feasibility is established.
- Compared with expensing, capitalizing an outlay gives higher net income, ROA, ROE and CFO but lower CFI in the year of the outlay, higher assets and equity, lower debt ratios, and lower net income, ROA and ROE in later years.

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- Interest incurred while a firm builds an asset for its own use is capitalized under both IFRS and US GAAP, and adjusted interest coverage is (EBIT + depreciation of capitalized interest) / (interest expense + interest capitalized).

QUICK CHECK

Question 2

Three U.S. hardware retailers are identical except for their inventory cost-flow assumption: one uses FIFO, one uses LIFO and one uses weighted average cost. Over the past several years, both the unit costs they pay suppliers and the number of units they sell have climbed steadily. An analyst who ranks the three on reported gross profit margin, without adjusting for inventory accounting, would *most likely* find the lowest margin at the retailer using:

A LIFO.

B Weighted average cost.

C FIFO.

Answer: A

With rising purchase prices, LIFO charges the most recent, and therefore highest, costs to cost of goods sold. Revenue is the same whatever cost-flow method is used, so the higher COGS under LIFO produces the lowest gross profit (and net profit) margins.

Why the other options are wrong

- **B.** Weighted average cost blends older, cheaper units with newer, dearer ones, so its COGS and margins fall between those under FIFO and LIFO.
- **C.** FIFO charges the oldest, cheapest costs to COGS when prices are rising, so FIFO firms report the lowest COGS and the highest margins.

Key takeaway With rising prices, FIFO gives the lowest COGS and the highest margins, LIFO the highest COGS and the lowest margins, and average cost falls in between. Sales are unaffected by the choice.

MODULE 28.3

Nonrecurring Items

LOS 28.c — Nonrecurring items and accounting changes

Analysts forecast sustainable earnings, so they need to know which income statement items are likely to repeat. The standards give certain items special placement or treatment.

Unusual or infrequent items

Unusual or infrequent items are material items that are unusual in nature or infrequent in occurrence. Examples are:

- gains or losses on selling assets or parts of the business outside ordinary operations;

- impairments, write-downs and write-offs;
- restructuring charges.

They are reported within income from continuing operations, before tax, often as a separate line. Analysts should judge whether they are really one-off: a company with "unusual" charges almost every year is in effect reporting a recurring cost.

Discontinued operations

A **discontinued operation** is a component that management has decided to dispose of (and that is classified as held for sale) or has already disposed of during the period. Exam convention: the business must be physically and operationally separable from the rest of the firm in its assets, operations, and investing and financing activities. Current practice: IFRS 5 also requires a separate major line of business or geographical area (or a subsidiary bought only for resale), and US GAAP requires a strategic shift with a major effect on operations, so selling a minor product line or individual assets is not a discontinued operation.

- The **measurement date** is the date a formal disposal plan is adopted. The **phaseout period** runs from then to the actual disposal.
- The operation's income or loss, and any gain or loss on disposal, is reported after income from continuing operations, net of tax. Prior-period income statements presented are restated to show it separately.
- Once the component is held for sale, its assets are measured at the lower of carrying amount and fair value less costs to sell. An expected loss on disposal is therefore recognized immediately as a write-down. An expected gain is recognized only when the sale is completed.
- Exam convention: on the measurement date the firm accrues the estimated operating losses of the phaseout period along with any estimated loss on the sale. Current practice: IAS 37 prohibits provisions for future operating losses other than for onerous contracts, and US GAAP reports the results of a discontinued operation in the periods in which they occur. Under both frameworks, phaseout-period operating losses are therefore reported in discontinued operations as they are incurred.
- Analysts usually exclude discontinued operations when forecasting earnings, although the disposal itself can signal something about future cash flows.

Accounting changes

KEY CONCEPT

Type	Example	Treatment
Change in accounting policy (principle)	Switching inventory from FIFO to weighted average cost; adopting a new standard issued by the standard setter	Retrospective: unless this is impractical, prior periods presented are restated as if the new policy had always applied, so the periods stay comparable. A new standard may allow *modified retrospective* adoption, with no restatement and the cumulative effect adjusted in opening balances.
Change in accounting estimate	Revised useful life, salvage value, bad-debt rate or warranty rate	Prospective: current and future periods only, with no restatement. Usually no cash flow effect.

Type	Example	Treatment
Correction of a prior-period error	Moving from an incorrect method to an acceptable one; fixing omitted or misstated amounts	**Prior-period adjustment**, applied retrospectively: prior statements presented are restated, and the nature of the error and its effect on net income are disclosed. Usually no cash flow effect. Errors may indicate weak internal controls.

Example. Aldercrest Mills bought a machine for $900,000 with no salvage value and a 9-year life. After 3 years, when the carrying value is $600,000, it concludes that the machine will last 5 more years (8 years in total). This is a change in estimate.

$$\text{New annual depreciation} = \frac{\text{Carrying value}}{\text{Remaining life}} = \frac{600{,}000}{5} = \$120{,}000$$

The depreciation of $100,000 per year for Years 1–3 is not restated; the new amount applies from Year 4 onward.

Changes in scope and exchange rates

Acquisitions (changes in scope, which alter the size of the combined entity) and currency movements (which change the reporting-currency value of foreign sales, purchases and subsidiaries' results) can distort year-on-year comparisons, even though the standards do not require their effects to be isolated. Analysts should adjust for them where possible.

Where items appear

KEY CONCEPT

Item	Location	Tax presentation
Unusual or infrequent items	In income from continuing operations	Pre-tax
Discontinued operations	Below income from continuing operations	Net of tax
Policy change or error correction	Restated prior periods (and opening retained earnings)	Not a current-period income line
Estimate change	Current and future period expenses	Ordinary lines

Example. Brackwell Foods earns operating income of $52 million before a $12 million restructuring charge. Interest expense is $6 million and the tax rate is 25%. A frozen-meals division that is held for sale lost $4 million after tax.

Line	Amount (millions)
Operating income before the restructuring charge	52.0
Restructuring charge (unusual or infrequent, pretax)	(12.0)

Line	Amount (millions)
Operating income	40.0
Interest expense	(6.0)
Income before tax	34.0
Income tax at 25%	(8.5)
Income from continuing operations	25.5
Loss from discontinued operations, net of tax	(4.0)
Net income	21.5

The forecast therefore starts from income from continuing operations of $25.5 million. If the restructuring is truly a one-off, recurring after-tax income is $25.5 + 12 \times (1 - 0.25) = \$34.5$ million; if Brackwell restructures most years, the $25.5 million figure stands.

COMMON EXAM TRAPS

- Changes in useful life or salvage value are changes in estimates, applied prospectively. They are not policy changes.
- Error corrections are neither unusual items nor policy changes. They are prior-period adjustments with restatement.
- Unusual or infrequent items are pre-tax and inside continuing operations. Only discontinued operations are shown net of tax below that line.

BOTTOM LINE

- Unusual or infrequent items are shown pre-tax within income from continuing operations, and an analyst judges whether they really are one-off.
- Discontinued operations are reported net of tax below income from continuing operations, prior-period income statements presented are restated to show them separately, and analysts usually exclude them when forecasting earnings.
- Assets of a component held for sale are measured at the lower of carrying amount and fair value less costs to sell, so an expected loss on disposal is recognized at once and an expected gain only when the sale is completed.
- A change in accounting policy is applied retrospectively, restating the prior periods presented unless that is impractical, while a change in accounting estimate is applied prospectively with no restatement.
- A correction of a prior-period error is a prior-period adjustment: prior statements presented are restated, and the nature of the error and its effect on net income are disclosed.
- After a change in an asset's estimated life, new annual depreciation equals carrying value divided by remaining life, and earlier depreciation is not restated.

QUICK CHECK

Question 3

During the current year, Brantley Industrial recorded a $14 million restructuring charge to close two of its warehouses. Its board also approved a formal plan to sell Brantley's marine-engine business, a separate major line of business with its own plants, management and financing that can be clearly separated from the rest of the company. The business meets the criteria to be classified as held for sale, and the sale is expected to close next year at a gain. Which statement about Brantley's income statement for the current year is *most accurate*?

A The marine-engine business's results are reported net of tax after income from continuing operations, and the prior-year income statement presented is restated to show them separately.

B The restructuring charge is reported net of tax below income from continuing operations.

C The expected gain on the sale of the marine-engine business is recognized in the current year, when the disposal plan is approved.

Answer: A

The marine-engine business is a separate major line of business, physically and operationally distinct, and it is held for sale under a formal disposal plan. It therefore qualifies as a discontinued operation. Its income or loss is reported separately, net of tax, after income from continuing operations, and the prior-period income statements presented are restated so that the discontinued business is shown separately in every year.

Placement of the two items:

Item	Where it is reported	Tax presentation
Restructuring charge (an unusual or infrequent item)	Within income from continuing operations	Pre-tax
Marine-engine business (discontinued operation)	After income from continuing operations	Net of tax

The measurement date is the date the formal plan was approved; the period until the sale closes is the phaseout period. Had a loss on the sale been expected, the held-for-sale assets would be written down at once to fair value less costs to sell. Operating results of the phaseout period are recognized as they occur, and future operating losses are not accrued in advance. An expected gain is not recognized until the sale is completed.

Why the other options are wrong

- **B.** A restructuring charge is an unusual or infrequent item, so it is reported before tax, within income from continuing operations. Only discontinued operations are shown net of tax below that line.
- **C.** An expected gain on disposal cannot be recognized until the sale is actually completed. Only an expected loss on disposal is recognized early, as a write-down of the held-for-sale assets to fair value less costs to sell. Operating losses of the phaseout period are not accrued in advance either; they are recognized as incurred.

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Key takeaway Unusual or infrequent items: pre-tax, inside continuing operations. Discontinued operations: net of tax, below continuing operations, with prior periods restated. Analysts usually exclude discontinued operations when forecasting earnings.

MODULE 28.4

Earnings Per Share

LOS 28.d — Basic and diluted earnings per share

Earnings per share (EPS) shows how much of the period's profit belongs to each common share. Only companies with publicly traded shares must report it, and it is always expressed per common share. There is no EPS figure for preferred shares.

Simple and complex capital structures

Capital structure	What it contains	EPS the firm must report
Simple capital structure	Common stock, *nonconvertible* bonds and *nonconvertible* preferred stock; nothing that could become new common shares	Basic EPS only
Complex capital structure	At least one **potentially dilutive security**: convertible bonds, convertible preferred stock, stock options or warrants	Both basic and diluted EPS

Straight (nonconvertible) debt and nonconvertible preferred stock cannot create new common shares, so they do not make a capital structure complex. Other features, such as a call feature, a cumulative dividend or a sinking fund, do not change this.

Exam convention: a company with a simple capital structure reports basic EPS only. Current practice: that is the US GAAP rule (ASC 260). IFRS (IAS 33) requires every company within its scope to present both basic and diluted EPS; with no dilutive potential shares the two figures are equal.

Basic EPS

KEY CONCEPT

$$\text{Basic EPS} = \frac{\text{Net income} - \text{Preferred dividends}}{\text{Weighted average number of common shares outstanding}}$$

- The numerator is **income available to common shareholders**. Subtract the current year's preferred dividends.
- Common dividends are not subtracted. They are a distribution of income that already belongs to common shareholders.
- Convertible securities do not enter basic EPS at all: there is no add-back in the numerator and no extra shares in the denominator.

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Weighted average shares outstanding

Each block of shares is weighted by the fraction of the year it was outstanding. Strictly, the weight is days outstanding divided by days in the year; exam questions normally use months.

- New shares (sold for cash, issued in an acquisition, or treasury shares reissued) count from the issue date.
- Repurchased (treasury) shares drop out from the repurchase date.
- **Stock dividends** and **stock splits**, including a **reverse stock split**, are applied retroactively. Every share outstanding before the event is restated as if the event had happened at the start of the year, and prior-year figures are restated too, so EPS does not appear to fall because of a largely cosmetic change in the share count. Shares issued or repurchased after the event are not adjusted for it.
- A split or stock dividend brings in no new resources and leaves each holder's percentage of the company unchanged. Its date within the year therefore does not change how the shares outstanding before it are restated: a 2-for-1 split in late March and one in early November both double the opening shares for the full year.
- A 3-for-2 split and a 50% stock dividend have the same effect: each old share becomes 1.5 new shares. A 1-for-4 reverse split multiplies pre-event shares by 0.25.
- Issuing preferred shares has no effect on the common share count.

Example. Wrenfield Tools starts the year with 60,000 common shares. It sells 12,000 new shares on 1 March, executes a 3-for-2 stock split on 1 June and repurchases 9,000 shares on 1 October. Net income is $450,000, and dividends on its nonconvertible preferred stock are $39,000. Common dividends of $50,000 were also paid.

Block	Split factor	Months	Weighted shares
60,000 opening shares	×1.5	12/12	90,000
12,000 issued 1 March	×1.5	10/12	15,000
9,000 repurchased 1 October	none (after split)	3/12	−2,250
Weighted average			**102,750**

$$\text{Basic EPS} = \frac{450{,}000 - 39{,}000}{102{,}750} = \frac{411{,}000}{102{,}750} = \$4.00$$

The $50,000 of common dividends plays no part in the calculation. When comparing EPS across years, check whether a change comes from earnings or from the share count: repurchases can raise EPS while net income and margins stay flat.

Diluted EPS and the if-converted method

Diluted EPS shows the worst case for existing shareholders: what EPS would be if every dilutive security had been converted or exercised at the start of the year, or on its issue date if it was issued during the year.

KEY CONCEPT

$$\text{Diluted EPS} = \frac{\text{NI} - \text{Pref. div.} + \text{Conv. pref. div.} + \text{Conv. debt interest} \times (1 - t)}{\text{Wtd. avg. shares} + \text{Shares from conv. pref.} + \text{Shares from conv. debt} + \text{Shares from options/warr}}$$

Convertibles are handled with the **if-converted method**, which assumes conversion even if the terms did not allow it during the period.

Convertible preferred stock. If the shares were converted, the preferred dividends would not be paid. Add them back to the numerator (they were subtracted to get income available to common) and add the conversion shares to the denominator. There is no tax adjustment, because preferred dividends are not tax-deductible.

Convertible bonds. If the bonds were converted, the interest would not be paid, but the tax deduction on that interest would also be lost. Add back the after-tax interest, $\text{Interest} \times (1 - t)$, and add the conversion shares. If the bonds were issued part-way through the year, weight both the interest add-back and the shares by the fraction of the year the bonds were outstanding.

Options and warrants. These are handled with the **treasury stock method**. There is no numerator adjustment, because exercise saves no expense. Assume exercise, and assume that the cash received is used to repurchase shares at the **average market price** for the period:

KEY CONCEPT

$$\text{Incremental shares} = N - \frac{N \times X}{\bar{P}} = \frac{\bar{P} - X}{\bar{P}} \times N$$

where N is the number of shares issuable, X is the **exercise price** and $\bar{P}$ is the average market price. Options are dilutive only when $X < \bar{P}$, that is, when they are in the money on average. The year-end price plays no part.

Testing for dilution

A security is **dilutive** if including it lowers EPS, and **antidilutive** if including it would raise EPS or reduce a loss per share. Antidilutive securities are excluded from diluted EPS, so diluted EPS can never exceed basic EPS.

A quick test for a convertible is to compute its per-share effect and compare it with basic EPS:

$$\text{Conv. preferred: } \frac{\text{Preferred dividends}}{\text{Shares on conversion}} \qquad \text{Conv. bonds: } \frac{\text{Interest} \times (1 - t)}{\text{Shares on conversion}}$$

If the ratio is below basic EPS, the security is dilutive; if it is above, the security is antidilutive. Each potentially dilutive security is tested on its own.

When a company reports a net loss available to common shareholders, adding shares would make the loss per share smaller. All potential shares are then antidilutive, and diluted EPS equals basic EPS.

Example. Ridgeback Instruments has net income of $3,000,000 and 900,000 weighted average common shares. It also has:

- 6% convertible preferred stock paying $300,000 of dividends, convertible into 250,000 common shares;
- 100,000 options exercisable at $40, with an average share price of $50 (year-end price $58);
- $2,000,000 of 8% convertible bonds, convertible into 30,000 shares, with a tax rate of 25%.

Basic EPS $= (3{,}000{,}000 - 300{,}000)/900{,}000 = \$3.00$.

Security	Numerator add-back	Extra shares	Per-share effect	Result
Conv. preferred	$300,000	250,000	$1.20	Dilutive (below $3.00)
Options	$0	$100{,}000 \times (50 - 40)/50 = 20{,}000$	$0	Dilutive (X below average price)
Conv. bonds	$160{,}000 \times 0.75 = 120{,}000$	30,000	$4.00	Antidilutive, so excluded

$$\text{Diluted EPS} = \frac{2{,}700{,}000 + 300{,}000}{900{,}000 + 250{,}000 + 20{,}000} = \frac{3{,}000{,}000}{1{,}170{,}000} = \$2.56$$

Including the bonds would raise EPS to $3{,}120{,}000/1{,}200{,}000 = \$2.60$, which confirms that they are antidilutive. The $58 year-end price plays no role.

COMMON EXAM TRAPS

- Subtracting common dividends from net income. Only preferred dividends are subtracted. For Wrenfield, subtracting the $50,000 gives $3.51 instead of $4.00.
- Forgetting to restate pre-event shares for a split or stock dividend, or restating shares issued after it. For Wrenfield, leaving the opening and 1 March shares unrestated gives 67,750 weighted shares and EPS of $6.07 instead of 102,750 and $4.00.
- Adding back pre-tax instead of after-tax interest for convertible bonds, or applying a tax adjustment to preferred dividends.
- Using a full year of interest and shares for convertibles issued during the year.
- Using the year-end price instead of the average price in the treasury stock method, or including out-of-the-money options. For Ridgeback, the $58 year-end price gives about 31,034 incremental shares and diluted EPS of $2.54 instead of 20,000 and $2.56.
- Including convertible shares in the basic EPS denominator.
- Including an antidilutive security. Diluted EPS must be less than or equal to basic EPS.

EXAM SHORTCUTS

- A convertible is dilutive when its per-share effect (preferred dividends, or interest × (1 – t), divided by the shares on conversion) is below basic EPS, so each convertible can be tested without computing diluted EPS with and without it.
- Options or warrants whose exercise price is not below the average market price are not dilutive, so they can be dropped before applying the treasury stock method.

- When a company reports a net loss available to common shareholders, every potential share is antidilutive and diluted EPS equals basic EPS, so no dilution test is needed.

BOTTOM LINE

- Basic EPS equals net income minus preferred dividends, divided by the weighted average number of common shares outstanding; common dividends are not subtracted.
- Stock dividends and stock splits are applied retroactively to all shares outstanding before the event, as if it had happened at the start of the year, and shares issued or repurchased after the event are not adjusted.
- Exam convention: a company with a simple capital structure reports basic EPS only. Current practice: that is the US GAAP rule, while IFRS requires every company within its scope to present both basic and diluted EPS.
- Under the if-converted method, diluted EPS adds back convertible preferred dividends and convertible debt interest × (1 – t) to the numerator and adds the conversion shares to the denominator.
- Under the treasury stock method, options and warrants add $(\bar{P} - X)/\bar{P} \times N$ shares, where $\bar{P}$ is the average market price, and they are dilutive only when $X < \bar{P}$.
- A security that would raise EPS or reduce a loss per share is antidilutive and is excluded, so diluted EPS can never exceed basic EPS.

QUICK CHECK

Question 4

Ravensworth Holdings entered 2025 with 3,000,000 common shares. Its capital transactions during the year, listed from latest to earliest, were: a sale of 240,000 new common shares on 31 October; an issue of €15 million of 6% convertible bonds on 31 May, convertible into common shares at a conversion price of €10; and a 20% stock dividend distributed on 28 February. The number of shares Ravensworth should use for its 2025 *basic* EPS is:

A 3,540,000.

B 3,640,000.

C 5,140,000.

Answer: B

Basic EPS ignores potential shares from convertible securities, so the convertible bonds are not counted. The stock dividend is applied retroactively to the opening shares, and the new shares sold on 31 October are weighted for the remaining two months.

Opening shares restated: $3{,}000{,}000 \times 1.20 = 3{,}600{,}000$ (all year).

New shares: $240{,}000 \times \frac{2}{12} = 40{,}000$.

$$\text{WASO} = 3{,}600{,}000 + 40{,}000 = 3{,}640{,}000$$

(Monthly form: $[3{,}000{,}000(12) + 600{,}000(12) + 240{,}000(2)]/12 = 43{,}680{,}000/12 = 3{,}640{,}000$.) The €15 million of convertible bonds would convert into 1,500,000 shares, which matters only for diluted EPS.

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Why the other options are wrong

- **A.** 3,540,000 treats the 600,000 dividend shares as outstanding only from 28 February (3,000,000 × 2/12 + 3,600,000 × 10/12 + 40,000). A stock dividend is applied retroactively to the start of the year.
- **C.** 5,140,000 adds the 1,500,000 shares issuable on conversion of the convertible bonds (€15,000,000/€10). Potential shares from convertibles are excluded from basic EPS.

Key takeaway Basic EPS denominator excludes convertibles; stock dividends are retroactive; new issues are time-weighted.

MODULE 28.5

Ratios and Common-Size Income Statements

LOS 28.e — Common-size income statements and margin analysis

Vertical common-size income statement

In a **vertical common-size income statement**, every line is divided by **revenue** (sales) for the same period, so revenue $= 100\%$. This removes the effect of size and lets an analyst compare:

- the same firm across years (**time-series** or trend analysis), and
- different firms in the same period (**cross-sectional** analysis), however different their scale.

KEY CONCEPT

$$\text{Common-size line item} = \frac{\text{Line item}}{\text{Revenue}}$$

A common-size figure shows the cost per unit of revenue. It does not show the absolute amount. If cost of sales goes from 55% to 60% of revenue, it has risen relative to sales; whether it rose in currency terms depends on what happened to revenue.

Tax is the exception. Tax expense is more informative relative to pretax income:

$$\text{Effective tax rate} = \frac{\text{Income tax expense}}{\text{Pretax income}}$$

The rate can be computed directly from common-size percentages because revenue cancels out. For example, tax of 6% of sales divided by pretax income of 24% of sales gives 25%.

Margin ratios

Any subtotal divided by revenue is a margin:

$$\text{Gross profit margin} = \frac{\text{Revenue} - \text{COGS}}{\text{Revenue}} \qquad \text{Operating profit margin} = \frac{\text{Operating profit (EBIT)}}{\text{Revenue}}$$

$$\text{Pretax margin} = \frac{\text{Earnings before tax}}{\text{Revenue}} \qquad \text{Net profit margin} = \frac{\text{Net income}}{\text{Revenue}}$$

The differences between successive margins isolate each cost layer, all as a percentage of revenue:

KEY CONCEPT

Gap	What it measures
100% − gross margin	Cost of goods sold
Gross margin − operating margin	Operating expenses (SG&A, R&D, depreciation)
Operating margin − pretax margin	Net **nonoperating** expense (mainly interest, net of investment income)
Pretax margin − net margin	Income tax expense

Gross margin improves when a firm raises prices or cuts unit production costs. Product differentiation (brand, technology, patents) supports higher prices. Administrative costs do not affect gross margin. Net margin reflects every cost, including financing and tax. Both should be compared over time and against peers.

The table below shows what moves gross margin when there are no fixed production costs. A product sells for $50 and costs $30 per unit to make, a 40% gross margin.

Change	New gross margin
Unit volume +15%	Still $(50 - 30)/50 = 40\%$, because revenue and COGS grow together
Unit cost −6% (to $28.20)	$(50 - 28.20)/50 = 43.6\%$
Selling price +5% (to $52.50)	$(52.50 - 30)/52.50 \approx 42.9\%$
Administrative costs −10%	Unchanged; admin is below gross profit (it raises operating and net margin instead)

Common-size statements are also a starting point for strategy analysis. A firm that spends a larger share of revenue on research or marketing may be building the product differentiation that later shows up as a higher gross margin.

Example. Ashgrove Outfitters reports these margins as a percentage of revenue:

	Year 1	Year 2	Year 3
Gross margin	42%	42%	42%
Operating margin	16%	18%	20%
Pretax margin	14%	16%	18%
Net margin	10.5%	12%	13.5%

- COGS is a steady 58% of revenue.
- Operating expenses fell from 26% to 22% of revenue, so the whole improvement comes from cost control below gross profit.

- Nonoperating expense is a stable 2%, so financing costs are unchanged relative to sales.
- The effective tax rate is $3.5/14 = 4/16 = 4.5/18 = 25\%$ every year.

Example. Firm P has revenue of $50 million, gross profit of $15 million and operating profit of $4 million. Firm Q has revenue of $8 million, gross profit of $5.2 million and operating profit of $2.4 million. P earns more in absolute terms, but Q's gross margin (65% against 30%) and operating margin (30% against 8%) are far higher. Q is relatively more profitable, perhaps because its product is differentiated.

COMMON EXAM TRAPS

- A rising common-size percentage does not prove that the absolute amount rose; revenue may have fallen.
- A higher tax-to-sales ratio does not mean a higher tax rate. Divide tax by pretax income.
- An improvement in net margin can come from any layer. Check which gap actually moved.
- A constant interest-to-sales ratio suggests stable reliance on debt relative to activity. It does not by itself measure leverage, which needs the balance sheet.
- Vertical common-size income statements use revenue as the base. Assets and gross profit are not used as the base.

EXAM SHORTCUTS

- The effective tax rate can be read straight from common-size figures: tax as a percentage of sales divided by pretax income as a percentage of sales, because revenue cancels.

BOTTOM LINE

- A vertical common-size income statement divides every line by revenue for the same period, which removes the effect of size for time-series and cross-sectional comparison.
- The effective tax rate is income tax expense divided by pretax income, not tax as a percentage of revenue.
- Gross, operating, pretax and net profit margin are gross profit, operating profit (EBIT), earnings before tax and net income, each divided by revenue.
- As a percentage of revenue, gross margin minus operating margin is operating expenses, operating margin minus pretax margin is net nonoperating expense, and pretax margin minus net margin is income tax expense.
- Gross margin improves when a firm raises prices or cuts unit production costs, product differentiation supports higher prices, and administrative costs do not affect gross margin.

QUICK CHECK

Question 5

An analyst reviews the following margins for Halvorsen Marine:

Halvorsen Marine — selected margins from common-size income statements

Margin (% of revenue)	Year 1	Year 2	Year 3
Gross profit margin	30%	32%	34%
Operating profit margin	21%	23%	25%
Pretax margin	19%	19%	19%
Net profit margin	14%	14%	14%

Halvorsen's net profit margin has stayed flat even though its gross profit margin has widened. Based only on these data, the *most likely* explanation is an increase, relative to revenue, in:

A selling, general and administrative expenses.

B income tax expense.

C nonoperating expenses such as interest.

Answer: C

The gap between operating margin and pretax margin measures net nonoperating expense, chiefly interest, as a share of revenue. That gap widens from 2% to 6% of revenue and absorbs the whole 4-point gain in operating margin, so pretax and net margins stay flat.

Each cost layer as % of revenue:

Layer	Formula	Yr 1	Yr 2	Yr 3
COGS	100% − gross margin	70%	68%	66%
Operating expenses	gross − operating margin	9%	9%	9%
Nonoperating expense	operating − pretax margin	2%	4%	6%
Income tax	pretax − net margin	5%	5%	5%

Effective tax rate $= 5/19 \approx 26.3\%$ in every year.

Only the nonoperating layer grows (+4 points), offsetting the +4-point improvement in gross (and operating) margin.

Why the other options are wrong

- **A.** Operating expenses below gross profit (the gross-minus-operating-margin gap) are a constant 9% of revenue in all three years, so the operating margin rises one-for-one with the gross margin.
- **B.** The pretax-minus-net-margin gap is a constant 5% of revenue, and the effective tax rate is unchanged at about 26.3% of pretax income. Taxes do not explain the flat net margin.

Key takeaway Subtract adjacent margins to isolate each cost layer. Gross minus operating margin gives operating expenses; operating minus pretax margin gives nonoperating items; pretax minus net margin gives taxes.

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Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/analyzing-income-statements/. The full bank has 152 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 29

Analyzing Balance Sheets

What you'll learn

LOS 29.a Explain how intangible assets are recognized, measured and disclosed under IFRS and US GAAP, including R&D costs.

LOS 29.b Explain how goodwill arises, is measured and tested for impairment, and how analysts adjust for it.

LOS 29.c Explain the classification and measurement of financial instruments (marketable securities) under US GAAP and IFRS.

LOS 29.d Explain the reporting of non-current liabilities, including amortized cost, fair value exceptions and deferred tax liabilities.

LOS 29.e Calculate and interpret common-size balance sheets and balance-sheet-based liquidity and solvency ratios.

MODULE 29.1

Intangible Assets and Marketable Securities

This reading covers intangible assets, goodwill, financial instruments and non-current liabilities, including deferred tax liabilities, and ends with common-size balance sheets and balance sheet ratios. A candidate must be able to compute goodwill, state how each class of financial asset is measured and where its gains and losses are reported, and calculate the liquidity and solvency ratios.

LOS 29.a — Intangible assets

Intangible assets are nonmonetary assets that lack physical substance. Securities are financial assets, so they are not intangible assets.

Type	Features	Examples
Identifiable intangible assets	Can be acquired separately, or arise from legal rights or privileges	Patents, trademarks, copyrights, licenses, franchises
Unidentifiable intangible assets	Cannot be acquired separately; may have an unlimited life	Goodwill

Purchased intangibles: measurement after acquisition

KEY CONCEPT

	IFRS	US GAAP
Cost model (cost − accumulated amortization − impairment)	Allowed	Required
Revaluation model (carry at fair value)	Allowed only if an **active market** exists for the asset	Not allowed

Internally created intangibles. Under US GAAP, research and development costs are **expensed as incurred** (apart from certain legal costs). IFRS splits the project into two stages:

- **Research stage** (seeking new scientific or technical knowledge): expense the costs.
- **Development stage** (applying research to design or plan products): capitalize the costs once the project is technically feasible, the firm has the **intention and resources to complete** it and to use or sell the product, and a market exists. Capitalized development cost includes materials, direct labor and production overhead.

Both IFRS and US GAAP require these costs to be expensed when incurred: start-up costs, training costs, administrative overhead, advertising and promotional spending, relocation and reorganization costs, and termination costs.

Amortization and impairment. **Finite-lived intangibles** are amortized over the period they are expected to be used, and impairment testing follows the same rules as for PP&E. The amortization method and useful life are reassessed at least once a year. **Indefinite-lived intangibles** are **not amortized** but are **tested for impairment at least annually**.

> *Example.* Oriel Diagnostics (IFRS) spends €450,000 on research for a new blood test until it proves feasible. After that, with resources, intent and buyers in place, it spends €300,000 on materials and direct labor, €90,000 on production overhead, €40,000 training technicians and €25,000 on administrative overhead.

- Capitalized (development): €300,000 + €90,000 = €390,000
- Expensed: research €450,000 + training €40,000 + admin €25,000 = €515,000
- Under US GAAP the whole €905,000 would be expensed.

A purchased patent costing $360,000 with a six-year useful life is carried at $360{,}000 - 360{,}000/6 = \$300{,}000$ after one year under the cost model (straight-line, no residual value), even if its market value has risen.

Some analysts remove intangibles when analyzing balance sheets, but each one should first be judged on its value to the firm.

LOS 29.b — Goodwill

KEY CONCEPT

$$\text{Goodwill} = \text{purchase price} - \text{fair value of identifiable net assets acquired}$$

Buyers pay above fair value for items that are not on the target's balance sheet (reputation, customer loyalty, in-process R&D kept off the books) and for expected **synergies**, such as closing duplicate facilities.

- **Goodwill** is recognized only in an acquisition. **Internally generated goodwill** is expensed as incurred and never appears on the balance sheet.
- If the price is below the fair value of identifiable net assets (a **bargain purchase**), the difference is recognized immediately as a gain in the income statement, and no goodwill arises.
- Goodwill has an indefinite life. It is not amortized but is tested for impairment at least annually. An **impairment loss** reduces goodwill and net income but has no cash flow effect. Its carrying amount is the original goodwill less any impairment losses. An impairment suggests that the acquired business is now worth less than the price paid, because the company has lowered its estimate of the future excess returns that business is expected to generate. A loss is recognized only when the impairment test shows the goodwill is impaired.

Measuring goodwill at the acquisition date:

1. Start from the target's assets and remove any goodwill already on its balance sheet. That goodwill came from the target's own past acquisitions and cannot be separated, so it is not an identifiable asset.
2. Restate the remaining identifiable assets, and the liabilities, to fair value.
3. Identifiable net assets = fair value of identifiable assets − fair value of liabilities.
4. Subtract identifiable net assets from the purchase price, as in the goodwill formula above.

Example. Hale Partners pays $9.2 million for a target whose balance sheet shows total assets of $10.4 million, including $0.6 million of goodwill, and liabilities of $3.5 million. The target's plant is worth $1.2 million more than its carrying amount; every other item is already at fair value. Identifiable assets $= 10.4 - 0.6 + 1.2 = \$11.0$ million, identifiable net assets $= 11.0 - 3.5 = \$7.5$ million, and goodwill $= 9.2 - 7.5 = \$1.7$ million. Had Hale paid $7.1 million, it would instead report a $0.4 million gain.

	Accounting goodwill	Economic goodwill
Source	Past acquisitions	Expected future performance of the firm
On the balance sheet?	Yes (purchased only)	No

Is goodwill an asset? One view treats it as the present value of the above-normal returns the target is expected to generate, and therefore a genuine asset. The other view notes that it cannot be sold separately and may simply reflect overpaying for the target. Goodwill impairments help an analyst judge how successful past acquisitions have been.

Manipulation risk. Because goodwill is not amortized, management can raise future net income by assigning more of the price to goodwill and less to depreciable or amortizable identifiable assets, especially when fair values are subjective. Depreciation and amortization fall and reported income rises, but the risk of a later impairment is higher.

Analyst adjustments. For comparability, analysts remove goodwill from the balance sheet and remove goodwill impairment losses from the income statement before computing ratios. An acquisition is assessed by comparing what was paid with the earning power obtained.

LOS 29.c — Financial instruments

Financial instruments are contracts that create a financial asset for one party and a financial liability or equity instrument for another. On the asset side they range from investment securities (bonds and stocks) to derivatives, receivables and loans. Three measurement bases are used: historical cost, amortized cost and **fair value** (**mark-to-market**).

US GAAP classifications

KEY CONCEPT

Measurement basis	Items	Unrealized gains and losses
Historical cost	Unquoted equity investments whose fair value cannot be reliably measured; loans and notes receivable	Not recognized

Measurement basis	Items	Unrealized gains and losses
Amortized cost	Held-to-maturity securities	Not recognized, because the security is not remeasured to market
Fair value	Trading securities and derivatives	Income statement
Fair value	Available-for-sale securities	Other comprehensive income

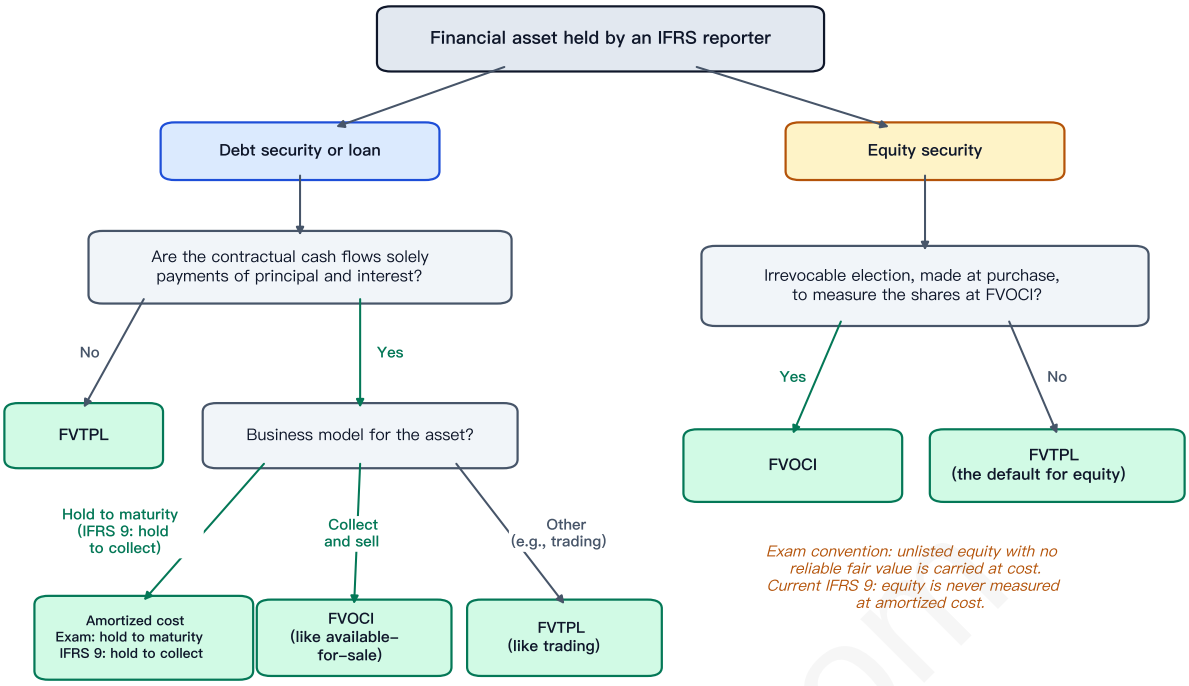
- **Amortized cost** = issue price − principal repayments + amortized discount (or − amortized premium) − impairment losses. Later market price changes are ignored.
- **Held-to-maturity securities** are debt securities the firm intends to hold until they mature; they are carried at amortized cost.
- **Trading securities** (held-for-trading) are debt securities bought to sell in the near term. They are reported at fair value, and unrealized gains and losses go to the income statement. Exam convention: all listed equity holdings without significant influence are treated this way, and US GAAP does not allow equity to be classified as available-for-sale. Current practice: ASC 321 measures these equity securities at fair value through net income without using the trading label, so the balance sheet value and the income statement effect are the same. Derivatives are treated like trading securities.
- **Available-for-sale securities** are debt securities that are neither held to maturity nor traded. They are reported at fair value, and unrealized gains and losses go to other comprehensive income (part of equity).
- Under US GAAP, for every classification, **interest and dividend income** and **realized gains and losses** go to the income statement. The categories differ only in balance sheet value and in where **unrealized gains and losses** (holding period gains and losses) are reported.

IFRS classifications and their US GAAP counterparts:

IFRS category	US GAAP counterpart	Used for
Amortized cost	Held-to-maturity	Exam convention: debt whose cash flows are solely principal and interest and that the firm intends to hold to maturity, plus loans and notes receivable. Current practice: IFRS 9 calls this business model "hold to collect" (holding the asset to collect its contractual cash flows) and applies the same solely-principal-and-interest (SPPI) test.
Fair value through other comprehensive income (FVOCI)	Available-for-sale	Debt held both to collect interest and to sell; equity securities if the firm makes an **irrevocable election at purchase**
Fair value through profit or loss (FVTPL)	Trading	Everything that fits neither category above (the default for all equity investments, listed or unlisted, and for derivatives); plus any financial asset the firm irrevocably designates at initial recognition under the **fair value option**

The IFRS categories can be read as a decision sequence (see the figure below).

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Amortized–cost business model: exam convention 'hold to maturity'; current IFRS 9 'hold to collect'. Fair value option conditions are described in the text.

IFRS classification of financial assets as a decision sequence

Exam convention: interest, dividends and realized gains and losses reach the income statement under every IFRS category too, as under US GAAP. Current practice: IFRS 9 treats the two kinds of FVOCI asset differently. For a debt security at FVOCI, interest goes to profit or loss, and the cumulative gain or loss held in OCI is reclassified (recycled) to profit or loss when the security is sold. For an equity investment designated at FVOCI, dividends go to profit or loss, but fair value gains and losses stay in OCI and are never recycled, even on sale; the cumulative amount may be transferred within equity, for example to retained earnings.

Exam convention: unlisted equity securities whose fair value cannot be determined reliably are carried at historical cost under both frameworks, alongside loans and notes receivable. Current practice: IFRS 9 measures every equity investment at fair value (FVTPL, or FVOCI by election), with no cost-measurement exemption. In limited circumstances, such as too little recent information, cost may be an appropriate estimate of fair value, but the measurement basis remains fair value.

Fair value option. Exam convention: under IFRS a firm may irrevocably choose, at purchase, to carry any financial asset at FVTPL, and US GAAP offers no such choice. Current practice: the option exists under both frameworks, with conditions. A firm may, at initial recognition, irrevocably elect to carry a financial asset at fair value and report its unrealized gains and losses in profit or loss.

- IFRS: designation at FVTPL is allowed for a financial asset that would otherwise be at amortized cost or FVOCI only if it eliminates or significantly reduces an **accounting mismatch** (a measurement or recognition inconsistency with a related liability or asset).
- US GAAP: the fair value option (FASB Statement 159, now ASC 825) may be elected instrument by instrument for eligible financial assets, including debt that would otherwise be held-to-maturity.
- Absent an election, debt held to maturity is at amortized cost under both frameworks (current IFRS 9 calls this business model "hold to collect").

Example. Linwood Corp buys $500,000 of 5% bonds at par; at year-end their fair value is $488,000.

- Held-to-maturity / amortized cost: balance sheet $500,000; interest income of $25,000 in net income.
- Trading / FVTPL: balance sheet $488,000; interest of $25,000 and the unrealized loss of $12,000 both in net income (net +$13,000).
- Available-for-sale / FVOCI: balance sheet $488,000; interest of $25,000 in net income; the $12,000 loss in OCI.

LOS 29.d — Non-current liabilities

Long-term financial liabilities (bank loans, notes payable, bonds payable, some derivatives) are usually reported at amortized cost: issue price − principal repayments + amortized discount (or − amortized premium). Premiums and discounts are amortized through interest expense, so the carrying amount converges on face value at maturity. A loan issued at face value simply stays at that amount, less repayments.

Liabilities reported at fair value include **held-for-trading liabilities** (e.g., a short position in a stock), **derivative liabilities**, and non-derivative liabilities whose exposures are hedged with derivatives.

Deferred tax liabilities (DTL) are income taxes payable in future periods because of temporary timing differences between financial and tax reporting. A DTL arises when income tax expense in the income statement exceeds taxes payable. This happens when:

- expenses are deducted for tax before they appear in the income statement (e.g., accelerated depreciation for tax, straight-line for reporting), or
- revenues or gains appear in the income statement in an earlier period than they are taxed (e.g., credit sales booked at sale but taxed on cash collection; subsidiary earnings recognized before dividends are paid).

DTLs reverse when the tax is eventually paid. **Permanent differences** (e.g., revenue that is never taxable) do not create deferred taxes.

COMMON EXAM TRAPS

- Training costs are expensed even when incurred during the development stage.
- An annual impairment test is not an annual write-down. A loss is recognized only when the test shows that the carrying amount is too high.
- Treating an intangible as indefinite-lived is not the conservative choice. No amortization is charged, so reported income is higher unless an impairment is later found.
- A consultant's valuation of internally built goodwill is never recorded.
- Computing goodwill from the target's book values, or counting goodwill already on the target's books as an identifiable asset. For Hale, book values give goodwill of $2.9 million, and treating the target's $0.6 million of goodwill as identifiable gives $1.1 million, instead of $1.7 million.
- "Unlisted equity is always at cost" and "listed equity is always at FVTPL" are both too absolute under IFRS.
- Moving held-to-maturity debt to FVTPL when the facts state that no fair value option was elected at initial recognition.
- An available-for-sale bond whose price falls still raises net income through interest income; the loss bypasses net income.
- Interest income is in net income for held-to-maturity securities too.

- Bonds issued at a premium or discount are carried at amortized cost. Face value and fair value are both wrong answers.
- When tax treatment matches book treatment, no deferred tax arises.

EXAM SHORTCUTS

- Under US GAAP, interest and dividend income and realized gains and losses go to the income statement for every classification of security, so only the balance sheet value and the location of unrealized gains and losses depend on the classification.

BOTTOM LINE

- After acquisition, IFRS allows purchased intangibles to be carried under the cost model or, only if an active market exists, the revaluation model, while US GAAP requires the cost model.
- US GAAP expenses R&D costs when incurred (apart from certain legal costs); IFRS expenses research costs and capitalizes development costs once the project is technically feasible, the firm has the intention and resources to complete it and to use or sell the product, and a market exists.
- Finite-lived intangibles are amortized over the period they are expected to be used, while indefinite-lived intangibles, including goodwill, are not amortized but are tested for impairment at least annually.
- Goodwill equals the purchase price minus the fair value of the identifiable net assets acquired; it arises only in an acquisition, and a price below that fair value is a bargain purchase recognized immediately as a gain.
- US GAAP carries held-to-maturity debt at amortized cost, trading securities and derivatives at fair value through the income statement, and available-for-sale debt at fair value through other comprehensive income; the IFRS counterparts are amortized cost, FVTPL and FVOCI respectively.
- A deferred tax liability arises when income tax expense exceeds taxes payable because of a temporary difference, and permanent differences create no deferred taxes.

QUICK CHECK

Question 1

Norvale Biotech, which reports under IFRS, is developing a rapid diagnostic kit. Early in the year it spent €640,000 on laboratory research, and that work showed the kit to be technically feasible. Norvale intends to finish the kit, has the resources to do so, and has signed a supply agreement with a hospital network. For the next phase it expects the following costs:

- staff training: €72,000
- materials and direct labor: €415,000
- production overhead: €185,000
- general administrative overhead: €38,000

The total amount Norvale should expense under IFRS is *closest* to:

A €640,000.

B €712,000.

C €750,000.

Answer: C

Under IFRS, costs of the research phase are expensed, while development-phase costs are capitalized once the project is technically feasible, a market exists, and the firm has the intention and resources to complete and sell the product. All of these conditions are met here. Training costs and administrative overhead are expensed whenever they are incurred, even during development; materials, direct labor and production overhead of the development phase are capitalized.

Classify each cost:

Cost	Phase	Treatment
Laboratory research €640,000	Research	Expense
Staff training €72,000	Any	Expense
Administrative overhead €38,000	Any	Expense
Materials and direct labor €415,000	Development	Capitalize
Production overhead €185,000	Development	Capitalize

$$\text{Expensed} = 640{,}000 + 72{,}000 + 38{,}000 = €750{,}000$$

Capitalized development cost $= 415{,}000 + 185{,}000 = €600{,}000$. (Under US GAAP all €1,350,000 would be expensed.)

Why the other options are wrong

- **A.** €640,000 expenses only the research-phase costs and wrongly capitalizes the training and administrative overhead, which must be expensed as incurred.
- **B.** €712,000 correctly expenses research and training but forgets that general administrative overhead is also expensed as incurred.

Key takeaway Research costs are expensed. Once the development criteria are met, materials, direct labor and production overhead are capitalized. Training, administrative overhead, advertising, start-up, relocation and termination costs are always expensed.

MODULE 29.2

Common-Size Balance Sheets

LOS 29.e — Common-size balance sheets and balance sheet ratios

Vertical common-size balance sheet

A **vertical common-size balance sheet** divides every line item by total assets, so total assets equal 100%. It removes the effect of size, which allows **time-series analysis** (the same firm over time) and **cross-sectional analysis** (different firms at one date). A common-size income statement uses revenue (sales) as the base instead.

KEY CONCEPT

$$\text{Common-size item} = \frac{\text{balance sheet item}}{\text{total assets}}$$

What to read from it:

- **Working capital** = current assets − current liabilities, as a % of assets. A high value means more liquidity, but also more money tied up in low-return assets.
- Composition of current assets: cash, receivables and inventory. Unusually high inventory may signal obsolescence.
- Reliance on fixed assets (PP&E share) and on acquisitions (goodwill share).
- Capital structure: the shares of liabilities and equity. Equity as a % of assets is the reciprocal of the **financial leverage ratio** (total assets / total equity).
- Equity detail: **common stock** (par value) plus **additional paid-in capital** equals the amount received from issuing common shares. Equity also includes retained earnings and accumulated other comprehensive income; preferred stock is shown separately.

Example.

Item	Nolan Co.	%	Pryce Co.	%
Current assets	2,600	32.5%	1,170	18.0%
Non-current assets	5,400	67.5%	5,330	82.0%
Total assets	8,000	100%	6,500	100%
Current liabilities	1,200	15.0%	455	7.0%
Long-term debt	2,800	35.0%	1,495	23.0%
Equity	4,000	50.0%	4,550	70.0%

Nolan's working capital is $32.5\% - 15.0\% = 17.5\%$ of assets vs. $18.0\% - 7.0\% = 11.0\%$ for Pryce; Pryce is more asset-heavy (82% non-current). Nolan is more leveraged: financial leverage $= 1/0.50 = 2.0$ vs. $1/0.70 \approx 1.43$.

Liquidity ratios (short-term obligations)

Each common-size percentage is a **balance sheet ratio**, one balance sheet item divided by another (here, total assets). Balance sheet ratios are used, over time and against peers, to judge liquidity and solvency. **Liquidity ratios** measure the ability to meet short-term obligations as they fall due.

KEY CONCEPT

$$\text{current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{quick ratio} = \frac{\text{cash} + \text{marketable securities} + \text{receivables}}{\text{current liabilities}}$$

$$\text{cash ratio} = \frac{\text{cash} + \text{marketable securities}}{\text{current liabilities}}$$

The **quick ratio** (**acid-test ratio**) excludes inventory; the **cash ratio** also excludes receivables. Read them together: when Firm A's current ratio exceeds Firm B's while A's quick ratio is below B's, the difference must come from A holding relatively more inventory. Current liabilities include accounts payable and short-term debt.

Solvency ratios (long-term obligations)

Solvency ratios measure the ability to meet long-term obligations. The four below should be read together.

KEY CONCEPT

$$\text{long-term debt-to-equity} = \frac{\text{long-term debt}}{\text{total equity}} \qquad \text{total debt-to-equity} = \frac{\text{total debt}}{\text{total equity}}$$

$$\text{debt ratio} = \frac{\text{total debt}}{\text{total assets}} \qquad \text{financial leverage ratio} = \frac{\text{total assets}}{\text{total equity}}$$

In these ratios, debt means interest-bearing obligations (short- and long-term borrowings). Accounts payable and accruals are excluded. The financial leverage ratio captures all liabilities, whether or not they bear interest. A higher long-term debt-to-equity but lower total debt-to-equity than a peer means the peer relies more on short-term debt. When only percentages of assets are known, $\text{total liabilities/equity} = (1 - e)/e$, where e is equity as a fraction of assets.

Limitations

- Accounting standards and estimates differ across peers.
- Ratios may not be comparable across industries.
- Interpretation requires judgment.
- The balance sheet is a snapshot at a single point in time.

COMMON EXAM TRAPS

- The denominator of a common-size balance sheet is total assets. Equity and sales are wrong bases.
- Quick ratio numerator excludes inventory; the denominator is all current liabilities, including both payables and short-term debt.
- Using the prior year's balances when the question asks for the latest year.
- Inverting a ratio, for example computing current liabilities / current assets instead of current assets / current liabilities. For Nolan, the inverted current ratio is 0.46 instead of 2.17.
- Treating common stock plus additional paid-in capital as total equity.

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EXAM SHORTCUTS

- If Firm A's current ratio is above Firm B's while A's quick ratio is below B's, A holds relatively more inventory, with no further data needed.
- From a common-size balance sheet alone, with e equal to equity as a fraction of total assets, the financial leverage ratio is $1/e$ and total liabilities to equity is $(1 - e)/e$.

BOTTOM LINE

- A vertical common-size balance sheet divides every line item by total assets, while a common-size income statement uses revenue as the base.
- The current ratio is current assets divided by current liabilities; the quick ratio uses cash, marketable securities and receivables in the numerator, and the cash ratio uses only cash and marketable securities.
- In the debt-to-equity and debt ratios, debt means interest-bearing short- and long-term borrowings, excluding accounts payable and accruals, while the financial leverage ratio, total assets divided by total equity, captures all liabilities.
- Equity as a percentage of total assets is the reciprocal of the financial leverage ratio.
- Balance sheet ratios are limited because accounting standards and estimates differ across peers, ratios may not be comparable across industries, interpretation requires judgment, and the balance sheet is a snapshot at a single point in time.

QUICK CHECK

Question 2

Part of the most recent common-size balance sheet of Harrowgate Mills is shown below.

Harrowgate Mills — excerpt from common-size balance sheet (most recent year)

Item	% of total assets
Preferred stock	10%
Common stock	2%
Additional paid-in capital	23%

Which interpretation of these data is *most appropriate*?

A Preferred stock accounts for 10% of shareholders' equity.

B Shareholders' equity equals 35% of total assets.

C The amount received from issuing common stock equals 25% of total assets.

Answer: C

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A common-size balance sheet states each item as a percentage of total assets. Capital raised by issuing common shares is recorded partly as common stock (at par value) and partly as additional paid-in capital (the excess over par), so together they show that common share proceeds are $2\% + 23\% = 25\%$ of total assets.

Common-share proceeds $= \text{common stock} + \text{additional paid-in capital} = 2\% + 23\% = 25\%$ of total assets.

Why the other options are wrong

- **A.** The 10% is a percentage of total assets. As a share of shareholders' equity, preferred stock would be larger than 10%.
- **B.** Adding the three items (35%) leaves out other parts of equity, such as retained earnings and accumulated OCI, so total equity is very likely larger than 35% of assets.

Key takeaway Common stock (par) + additional paid-in capital = contributed capital from common shares. Every common-size balance sheet percentage is relative to total assets.

Practice questions

Question 3

Which statement *best* describes how available-for-sale securities are accounted for under US GAAP after purchase?

A They are carried at amortized cost, and changes in market value are ignored.

B They are carried at fair value, and unrealized gains and losses are recognized in other comprehensive income.

C They are carried at fair value, and unrealized gains and losses are recognized in the income statement.

Question 4

In Veloria, companies record revenue in their financial statements when a sale is made. Montclair Tiles sells mainly on 60-day credit terms. Which Velorian tax rule would *most likely* cause Montclair to report a deferred tax liability?

A Revenue is taxed when the sale is made.

B Revenue is taxed when cash is collected from the customer.

C Revenue is exempt from tax.

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Question 5

The liabilities and equity section of Galloway Instruments' balance sheet is shown below.

Galloway Instruments — liabilities and equity

Item	Amount
Accounts payable	400
Accrued expenses	150
Short-term notes payable	300
Long-term debt	1,500
Total equity	2,650
Total liabilities and equity	5,000

Treating only interest-bearing obligations as debt, as is usual for solvency ratios, Galloway's debt ratio is *closest* to:

A 36%.

B 47%.

C 68%.

Answers: 3. B · 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/analyzing-balance-sheets/. The full bank has 35 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 30

Analyzing Statements of Cash Flows I

What you'll learn

LOS 30.a Describe how the cash flow statement links to the income statement and balance sheet, and how timing differences between accruals and cash appear in balance sheet accounts.

LOS 30.b Describe the steps in preparing direct and indirect cash flow statements, including computing operating, investing and financing cash flows from income statement and balance sheet data.

LOS 30.c Demonstrate how an indirect-method statement of cash flows is converted to the direct method.

LOS 30.d Contrast statements of cash flows prepared under IFRS and US GAAP (classification of interest, dividends, taxes and bank overdrafts; CFO presentation).

MODULE 30.1

Cash Flow Introduction and Direct Method CFO

This reading explains how the statement of cash flows connects with the income statement and balance sheet, and how operating, investing and financing cash flows are prepared. A candidate must be able to compute CFO with both the direct and the indirect method, derive investing and financing flows from balance sheet changes, convert an indirect statement to the direct format, and classify items under IFRS and US GAAP.

LOS 30.a — How the cash flow statement links to the income statement and balance sheet

Why a separate cash flow statement?

The income statement is prepared on the **accrual basis**, so net income differs from the cash a firm generates. The **statement of cash flows** reports the actual **cash receipts** and **cash payments** of the period and sorts them into three activities:

KEY CONCEPT

Activity	What it captures	Balance sheet accounts it relates to
Cash flow from operations (CFO)	Cash effects of the firm's day-to-day revenue and expense transactions	Mostly **working capital** accounts (current operating assets and current operating liabilities)
Cash flow from investing activities (CFI)	Buying and selling long-term assets and certain investments	**Noncurrent assets** (PP&E, intangibles, long-term investments)
Cash flow from financing activities (CFF)	Transactions with creditors and shareholders that change the capital structure	**Noncurrent liabilities** and **equity**

The terms **cash flow from operations**, **cash flow from operating activities** and **operating cash flow** mean the same thing.

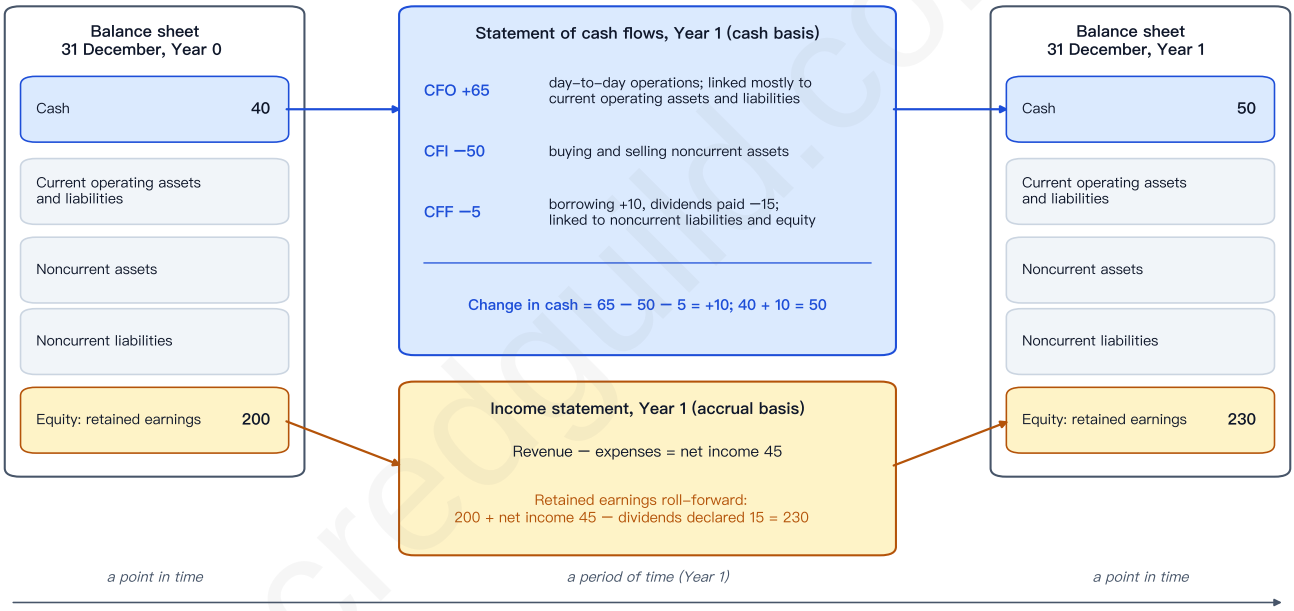
The classification depends on what a transaction means for the company reporting it. When a machinery maker sells a press to a printing firm, the maker reports an operating inflow, because selling presses is its business. The printing firm reports an investing outflow, because it is buying a long-term asset.

The statement reconciles the opening and closing cash balances. Each section is a signed net amount, so a net outflow enters as a negative number:

KEY CONCEPT

$$\text{CFO} + \text{CFI} + \text{CFF} = \Delta\text{Cash}; \quad \text{Beginning cash} + \Delta\text{Cash} = \text{Ending cash}$$

The income statement is also a **flow** statement (sometimes called a dynamic statement): like the statement of cash flows, it covers the period between two balance sheet dates, and net income reaches the closing balance sheet through retained earnings (see the figure below). Because of accrual accounting, net income is not the cash the firm generated, and many investing and financing cash flows, such as buying equipment or borrowing, do not touch the income statement when they occur.



The income statement and the statement of cash flows link two balance sheets

Analysts use the statement of cash flows to judge **liquidity**, **solvency** and **financial flexibility**, meaning whether operations can fund the business, repay maturing debt, absorb shocks and take up opportunities without new financing. It also shows the **quality of earnings**. Earnings are of higher quality when CFO is close to, or above, net income; earnings that persistently exceed CFO are not backed by cash.

Timing differences show up on the balance sheet

When revenue or expense recognition and the cash flow happen in different periods, a balance sheet account absorbs the difference:

Situation	Balance sheet effect
Sales exceed **cash collected from customers** (sales on credit)	**Accounts receivable** increase

Situation	Balance sheet effect
Cash collected exceeds sales	Accounts receivable decrease
Purchases exceed **cash paid to suppliers**	**Accounts payable** increase
Cash paid to suppliers exceeds purchases	Accounts payable decrease
Customers pay before revenue is recognized	**Unearned revenue** (a liability) increases
Premiums paid before the expense is recognized	**Prepaid expenses** (an asset) increase

Every such account can be written as a roll-forward, e.g. for receivables:

$$\text{Beginning AR} + \text{Sales} - \text{Cash collected} = \text{Ending AR}$$

so that cash collected = sales − increase in AR (or + decrease in AR). Knowing any three items gives the fourth.

Revenue recognition rules decide when a sale reaches the income statement, not when the customer pays. Cash received in advance, for instance for a multi-year service contract, is reported in the statement of cash flows when it arrives, while the revenue is recognized gradually as the contract runs. Tracing each account this way also helps an analyst detect aggressive accounting, since reported revenue or expenses that do not fit the related cash and balance sheet changes stand out.

> *Example.* Sales are $540,000; AR rise from $62,000 to $77,000; unearned revenue rises from $9,000 to $13,000. Cash collected $= 540{,}000 - 15{,}000 + 4{,}000 = \$529{,}000$.

LOS 30.b — Preparing CFO with the direct method

Sources and uses of cash

CFO can be presented with the **direct method** or the **indirect method**. The presentation differs but the CFO total is the same, and CFI and CFF are presented the same way whichever method is chosen. Sources of cash are shown as positive numbers (inflows) and uses of cash as negative numbers (outflows).

KEY CONCEPT

Change in account	Effect on cash
Increase in an operating **asset**	**Use of cash** (subtract)
Decrease in an operating asset	**Source of cash** (add)
Increase in an operating **liability**	Source of cash (add)
Decrease in an operating liability	Use of cash (subtract)

The direct method

The **direct method** lists the operating cash receipts and payments themselves. Typical lines are **cash collected from customers** (usually the largest component), **cash paid to suppliers** (cash inputs), cash paid for operating expenses such as wages, **cash paid for interest** and **cash paid for taxes**. It gives the analyst more detail than the indirect method, which helps with forecasting.

Steps: begin with the first line of the income statement; for each revenue or expense line (treat expenses as negative numbers), find the related operating asset or liability, apply the source/use rules to its change, and move to the next line. **Noncash items** such as **depreciation** and **gains and losses** on asset disposals are ignored because they are not operating cash flows. Sum the adjusted lines to get CFO. Net income never appears in a direct-method CFO section.

Key formulas. Each formula starts from the income statement amount and corrects it for the part that was not received or paid in cash this period, using the change in the related balance sheet account.

$$\text{Cash collected} = \text{Net sales} - \Delta\text{AR} + \Delta\text{Unearned revenue}$$

$$\text{Purchases} = \text{COGS} + \text{Ending inventory} - \text{Beginning inventory} = \text{COGS} + \Delta\text{Inventory}$$

$$\text{Cash paid to suppliers} = \text{Purchases} - \Delta\text{AP} = \text{COGS} + \Delta\text{Inventory} - \Delta\text{AP}$$

$$\text{Cash paid for wages} = \text{Wage expense} - \Delta\text{Wages payable}$$

$$\text{Cash paid for interest} = \text{Interest expense} - \Delta\text{Interest payable}$$

$$\text{Cash paid for insurance} = \text{Insurance expense} + \Delta\text{Prepaid insurance}$$

$$\text{Cash taxes paid} = \text{Tax expense} - \Delta\text{Taxes payable} - \Delta\text{Deferred tax liability} + \Delta\text{Deferred tax asset}$$

If purchases are given directly, no further inventory adjustment is made, because the inventory change is already built into purchases. The same identities can be run backwards: expense = cash paid − increase in the related prepaid asset, or = cash paid + increase in the related payable.

Example. A retailer reports sales of $820,000, COGS of $510,000, wage expense of $140,000, depreciation of $35,000 and tax expense of $30,000. AR fell by $12,000, inventory rose by $18,000, AP rose by $9,000, wages payable fell by $4,000, taxes payable were unchanged.

Line	Computation	Cash flow
Cash collected	$820{,}000 + 12{,}000$	832,000
Cash paid to suppliers	$-510{,}000 - 18{,}000 + 9{,}000$	(519,000)
Cash paid for wages	$-140{,}000 - 4{,}000$	(144,000)
Depreciation	noncash, ignored	0
Cash taxes paid	$-30{,}000$	(30,000)
CFO		**139,000**

COMMON EXAM TRAPS

- Linking operating activities to noncurrent assets (investing) or to long-term debt and equity (financing).

- Subtracting the ending balance of AR from sales instead of the change in AR. In the LOS 30.a example, this gives cash collected of $467,000 instead of $529,000.
- Forgetting that an increase in unearned revenue means cash arrived before the sale was recorded, so the increase is added. In the LOS 30.a example, subtracting the $4,000 increase gives $521,000 instead of $529,000.
- Including depreciation, a **noncash charge**, as a cash expense. For the retailer, this gives CFO of $104,000 instead of $139,000.
- Adjusting purchases a second time for the inventory change.
- Getting the sign of a liability change wrong. A decrease in wages or interest payable means more cash was paid than expensed. For the retailer, adding the $4,000 decrease in wages payable gives cash paid for wages of $136,000 instead of $144,000, and CFO of $147,000 instead of $139,000.
- Dropping interest paid and taxes paid from direct-method CFO. Under US GAAP both are operating outflows.

EXAM SHORTCUTS

- A balance sheet roll-forward such as beginning AR + sales − cash collected = ending AR links four items, so any three of them give the fourth.

BOTTOM LINE

- CFO relates mostly to working capital accounts, CFI to noncurrent assets, and CFF to noncurrent liabilities and equity.
- CFO + CFI + CFF equals the change in cash, and beginning cash plus that change equals ending cash.
- Earnings are of higher quality when CFO is close to, or above, net income; earnings that persistently exceed CFO are not backed by cash.
- An increase in an operating asset is a use of cash and an increase in an operating liability is a source of cash; decreases work the other way.
- Cash collected from customers equals net sales − ΔAR + Δunearned revenue, and cash paid to suppliers equals COGS + Δinventory − ΔAP.
- The direct method lists operating cash receipts and payments and ignores noncash items, for example depreciation or a gain on an asset sale, so net income never appears in a direct-method CFO section.

QUICK CHECK

Question 1

Alvern Truck Works builds delivery trucks for sale. It sells 25 new trucks for cash to Ardwell Couriers, which will use them in its delivery business for about eight years. Both companies report under US GAAP. How will the cash that changes hands be classified in each company's statement of cash flows? Answer in the form: Alvern; Ardwell.

A Operating inflow; investing outflow.

B Operating inflow; operating outflow.

C Investing inflow; investing outflow.

Answer: A

Cash flows are classified by what the transaction means for the company that reports it. Selling trucks is Alvern's ordinary business: the trucks are its inventory, the sale produces revenue and cost of goods sold, and the cash is collected from a customer, so it is an operating inflow. For Ardwell the trucks are long-term assets bought for use over several years, so the payment is an investing outflow. One transaction can therefore fall in different sections of the two companies' statements.

- Alvern: trucks held for sale are inventory. Cash received from customers is part of cash flow from operations (CFO).
- Ardwell: trucks bought for use are property, plant, and equipment, a noncurrent asset. Purchases of long-term assets belong in cash flow from investing activities (CFI). Ardwell will later depreciate the trucks, which reduces net income but is a noncash expense.

Why the other options are wrong

- **B.** This treats Ardwell's purchase as an operating payment. Buying a long-term asset for use over several years is an investing activity; only Alvern, for which the trucks are inventory, reports an operating cash flow.
- **C.** This treats Alvern's sale as the disposal of a long-term asset. Trucks that Alvern builds in order to sell them are inventory, not PP&E, so the proceeds are cash collected from customers, an operating inflow.

Key takeaway Classify a cash flow by its role for the reporting company. The sale of inventory is operating for the seller, while the same asset bought for long-term use is an investing outflow for the buyer.

MODULE 30.2

Indirect Method CFO

LOS 30.b — Preparing CFO with the indirect method

Starting point: net income

The **indirect method** starts with **net income** and reconciles it to CFO. Net income already contains noncash items and nonoperating gains and losses, so these are unwound, and then the changes in operating working capital are applied:

KEY CONCEPT

$$\text{CFO} = \text{NI} + \text{NCC} - \text{WC}_{\text{INV}}$$

where NCC are **noncash charges** and WC_{INV} is the **investment in working capital** (the net increase in operating assets less operating liabilities).

Step 1–2: noncash charges, gains and losses

KEY CONCEPT

Add back to net income	Subtract from net income
Depreciation, depletion and **amortization**	**Gains on asset disposals**
Losses on asset disposals	Gains on early retirement of debt
Asset impairments and write-downs	Reversals of impairments and write-downs
Losses on early retirement of debt	Amortization of premiums on bonds issued (amortized cost method)
Amortization of discounts on bonds issued (amortized cost method)	Decreases in deferred tax liabilities, increases in deferred tax assets
Increases in **deferred tax liabilities**, decreases in deferred tax assets	
Losses of equity-accounted associates	

Noncash components of revenue are subtracted in the same way. Although these items are called noncash "charges", they can raise or lower net income.

An inventory write-down reduces both net income and the inventory balance, so it must be counted only once; the two consistent ways to handle it are set out under LOS 30.c.

A gain on the sale of equipment is removed because the whole cash proceeds from the sale belong in CFI. The gain is the part of those proceeds above **carrying value**, and it also sits in net income. Leaving it in would count it twice and misclassify investing cash as operating cash. A loss reduced net income without any operating cash outflow, so it is added back.

Step 3: working capital changes

KEY CONCEPT

Add	Subtract
Decreases in current operating assets (AR, inventory, prepaid expenses)	Increases in current operating assets
Increases in current operating liabilities (AP, wages payable, interest payable, taxes payable, unearned revenue)	Decreases in current operating liabilities

Only operating working capital counts. Cash and short-term investments are ignored, except trading securities, whose flows are CFO. Short-term interest-bearing debt and **dividends payable** are financing items and are also ignored.

The definition of **working capital** depends on the context. For ratio analysis it is total current assets minus total current liabilities. For cash flow work, and in corporate finance, it means operating assets minus operating liabilities, often called **noncash working capital**. The working capital changes can be applied line by line or as

one net figure: the increase in noncash working capital is subtracted.

Some items never enter CFO under the indirect method because they are not in net income or are not operating: issuing or repaying bonds, issuing or buying back stock, **dividends paid** (CFF under US GAAP), and purchases of PP&E or land (CFI). They matter for CFO only through any gain or loss they create.

Example

Net income $64,000; depreciation $15,000; gain on sale of a delivery van $3,000; AR up $7,000; inventory down $4,000; prepaid rent up $1,000; AP up $6,000; wages payable down $2,000; dividends paid $10,000.

$$\text{CFO} = 64{,}000 + 15{,}000 - 3{,}000 - 7{,}000 + 4{,}000 - 1{,}000 + 6{,}000 - 2{,}000 = \$76{,}000$$

The $10,000 dividend is a financing outflow and is not deducted. When there are no disposals, the increase in **accumulated depreciation** equals the period's depreciation, which is the add-back seen from the balance sheet.

Direct vs. indirect presentation

	Direct method	Indirect method
Starts with	Cash receipts from customers	Net income
Depreciation and gains/losses	Not considered (noncash)	Adjusted (added back / subtracted)
Working capital changes	Used to convert each revenue/expense line	Used to adjust net income
Main advantage	Shows actual operating receipts and payments	Shows why net income and CFO differ (earnings quality)

The CFO totals agree because both methods start from the same income statement and use the same working capital changes. The direct method converts each revenue and expense line to cash and leaves out noncash items. The indirect method takes net income, in which those lines are already netted, and unwinds the noncash items and gains instead.

> *Example.* The retailer in the direct-method example of Module 30.1 has net income of $820{,}000 - 510{,}000 - 140{,}000 - 35{,}000 - 30{,}000 = \$105{,}000$. Indirect method: $105{,}000 + 35{,}000 + 12{,}000 - 18{,}000 + 9{,}000 - 4{,}000 = \$139{,}000$, the same CFO that the direct method gives.

Most firms present CFO with the indirect method. What each framework requires of the choice is compared under LOS 30.d.

COMMON EXAM TRAPS

- Adding a gain (or subtracting a loss) instead of the reverse. In the example, adding the $3,000 gain gives CFO of $82,000 instead of $76,000.
- Subtracting depreciation, or adding it under the direct method. In the example, subtracting the $15,000 of depreciation gives $46,000 instead of $76,000.
- Deducting dividends paid or bond repayments in CFO. In the example, deducting the $10,000 dividend gives $66,000 instead of $76,000.

- Using the full sale proceeds of an asset in CFO instead of removing only the gain.
- Computing a gain from an out-of-date carrying value. The carrying value on the sale date includes the depreciation charged in the year of sale.

EXAM SHORTCUTS

- The direct and indirect methods give the same CFO total, so a CFO figure found with one method need not be recomputed with the other.
- Dividends paid, debt and stock issues or repayments, and purchases of PP&E or land can be skipped when computing indirect CFO; they matter only through a gain or loss they create.

BOTTOM LINE

- Under the indirect method, CFO equals net income plus noncash charges minus the investment in working capital.
- Depreciation, amortization, impairments and losses on asset disposals are added back to net income, while gains on disposals are subtracted, because the whole sale proceeds belong in CFI.
- Decreases in current operating assets and increases in current operating liabilities are added to net income; increases in operating assets and decreases in operating liabilities are subtracted.
- Only operating working capital enters CFO: short-term interest-bearing debt and dividends payable are financing items, and cash is ignored.
- The direct method shows the actual operating receipts and payments, while the indirect method shows why net income and CFO differ.

QUICK CHECK

Question 2

An analyst is rebuilding the operating section of Castellan Paper's statement of cash flows, which is prepared using the indirect method. Which of the following items recorded by Castellan during the year requires an addition to net income in arriving at cash flow from operating activities?

A A loss on the early retirement of its bonds.

B A gain on the sale of a delivery truck.

C A decrease in its deferred tax liability.

Answer: A

The indirect method starts with net income and removes items that affected income without a matching operating cash flow. A loss on retiring debt early reduced net income, but the cash paid to retire the bonds is a financing outflow, so the loss is added back. Otherwise operating cash flow would be reduced by an amount that belongs in financing activities.

Why the other options are wrong

- **B.** A gain on the sale of a truck raised net income, but the full sale proceeds are an investing inflow. The gain is subtracted so that it is not counted in operating cash flow as well.

- **C.** A decrease in a deferred tax liability means income tax expense was lower than the tax payable for the period, so net income overstates the cash left after taxes. The decrease is subtracted from net income.

Key takeaway Indirect method: add back losses and subtract gains that arise from investing or financing transactions, such as asset sales and debt retirement. Add increases and subtract decreases in deferred tax liabilities.

MODULE 30.3

Investing and Financing Cash Flows and IFRS/US GAAP Differences

LOS 30.b — Cash flow from investing and financing activities

Investing activities (CFI)

Cash flow from investing activities covers buying and selling long-term assets and investments: purchases and sales of PP&E and intangibles, purchases and sales of investments in bonds and shares (trading securities excepted), and loans the firm makes to others and later collects. Only the cash paid or received counts. For an asset sale the cash is the **disposal proceeds**:

KEY CONCEPT

$$\text{Proceeds} = \text{Carrying value of asset sold} + \text{Gain (or} - \text{Loss)}$$

When the proceeds must be backed out of the balance sheet:

$$\text{Gross cost of disposals} = \text{Beginning gross PP\&E} + \text{Purchases} - \text{Ending gross PP\&E}$$

$$\text{Accumulated depreciation removed} = \text{Beginning AD} + \text{Depreciation expense} - \text{Ending AD}$$

Carrying value sold = gross cost − accumulated depreciation removed; then add the gain or subtract the loss. The three steps can be combined:

$$\text{Carrying value disposed} = \text{Beginning net PP\&E} - \text{Depreciation} + \text{Purchases} - \text{Ending net PP\&E}$$

Land is not depreciated, so its carrying value is its cost. With no land purchases, the carrying value of land sold is the fall in the land balance.

WORKED EXAMPLE: PROCEEDS FROM AN EQUIPMENT SALE

A firm's gross PP&E is $480,000 at the start of the year and $515,000 at the end, and it bought $95,000 of equipment for cash during the year. Accumulated depreciation is $160,000 at the start and $178,000 at the end, and depreciation expense for the year is $52,000. The income statement shows a $7,000 gain on the sale of equipment. Find the carrying value of the equipment sold, the cash proceeds and the firm's CFI, assuming no other investing transactions.

Step 1. Gross cost of disposals $= 480{,}000 + 95{,}000 - 515{,}000 = \$60{,}000$.

Step 2. Accumulated depreciation removed $= 160{,}000 + 52{,}000 - 178{,}000 = \$34{,}000$.

Step 3. Carrying value sold $= 60{,}000 - 34{,}000 = \$26{,}000$. The one-line check gives the same figure: beginning net PP&E of $320,000 less depreciation of $52,000, plus purchases of $95,000, less ending net PP&E of $337,000 $= \$26{,}000$.

Step 4. Proceeds $= 26{,}000 + 7{,}000 = \$33{,}000$, an investing inflow; the $7,000 gain is subtracted from net income in indirect CFO.

Result. CFI $= -95{,}000 + 33{,}000 = -\$62{,}000$.

Financing activities (CFF)

Cash flow from financing activities covers transactions with the providers of capital:

$$\text{CFF} = (\text{New borrowing} - \text{Principal repaid}) + (\text{Stock issued} - \text{Stock repurchased}) - \text{Cash dividends paid}$$

Dividends paid often have to be derived:

$$\text{Dividends declared} = \text{Beginning retained earnings} + \text{Net income} - \text{Ending retained earnings}$$

$$\text{Dividends paid} = \text{Dividends declared} - \Delta\text{Dividends payable}$$

Example. Net income is $640, retained earnings rise from $1,500 to $1,880, and dividends payable fall from $70 to $40. Dividends declared $= 1{,}500 + 640 - 1{,}880 = \$260$. Dividends paid $= 260 - (40 - 70) = \$290$, a financing outflow. More cash went out than was declared because part of the payable left over from last year was also settled.

The Level I convention assumes bonds are issued at par and shares are repurchased at their issue price, so the changes in bonds payable and in **contributed capital** (common stock at par plus additional paid-in capital) give the principal and equity flows. For a bond issued at a discount or premium, the amortization is noncash: interest expense = coupon + discount amortized − premium amortized, and the carrying value of the bond changes without any financing cash flow. When buybacks occur at a price different from the issue price, the difference goes through retained earnings. The total of CFO, CFI and CFF must equal the change in the cash balance, which serves as a check figure.

Noncash investing and financing activities

Transactions with no cash, such as buying an asset in exchange for bonds or stock or converting bonds into preferred or common stock, do not appear in CFI or CFF. They are disclosed in a footnote or supplementary schedule. If an asset is bought partly for cash, only the cash portion is an investing outflow.

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US GAAP classification summary

KEY CONCEPT

	Operating	Investing	Financing
Inflows	Cash from customers; **interest received**; **dividends received**; sales of trading securities	Sale of PP&E and intangibles; sale of debt/equity investments; collection of loans made	Debt issued; stock issued
Outflows	Suppliers and employees; other expenses; **interest paid**; **taxes paid**; purchases of trading securities	Purchase of PP&E and intangibles; purchase of investments; loans made	Principal repaid; stock repurchased; **dividends paid**

Example. A firm buys machinery for $42,000, sells a warehouse (carrying value $30,000) for $36,000, issues bonds of $25,000, repays a bank loan of $8,000 and pays dividends of $11,000. CFI $= -42{,}000 + 36{,}000 = -\$6{,}000$; CFF $= 25{,}000 - 8{,}000 - 11{,}000 = \$6{,}000$. The $6,000 gain is removed from net income in CFO.

LOS 30.c — Converting an indirect statement to the direct method

Only the CFO section differs between the two formats, so the conversion uses the income statement plus the working capital changes from the indirect statement:

1. Total the revenues and gains, and separately total the expenses and losses.
2. Remove all noncash items (depreciation, amortization, gains and losses, **inventory writedowns**, deferred tax changes) and break the rest into individual lines.
3. Convert each line from accrual to cash using the related working capital change.

Direct-method line	Adjustment
Cash collected from customers	Start with **net sales**; subtract an increase (add a decrease) in AR; add an increase (subtract a decrease) in unearned revenue
Cash paid to suppliers (as a positive amount)	Start with COGS; add an increase (subtract a decrease) in inventory; subtract an increase (add a decrease) in AP; subtract noncash items inside COGS such as an **inventory writedown** or depreciation included in COGS. When the writedown is removed from COGS, the inventory change must also exclude the writedown (see below)

An inventory writedown (lower of cost or market / net realizable value) raises COGS but involves no cash, so it is taken out of COGS when computing cash paid to suppliers. A decrease in inventory means some goods sold came out of existing stock and were not bought this period, so it also reduces the cash paid.

The writedown also lowers the ending inventory balance, so it must not be counted twice. There are two consistent routes:

1. Subtract the writedown from COGS and use the inventory change before the writedown (purchases less cost of units sold), or
2. Keep COGS including the writedown and use the raw balance sheet inventory change (after the writedown), with no separate writedown adjustment.

Example. Beginning inventory 200, cash purchases 90, cost of units sold 70, writedown 20, ending inventory 200, AP unchanged. Reported COGS is 90 and the balance sheet inventory change is 0. Route 1: $90 - 20 + (+20) = 90$. Route 2: $90 + 0 = 90$. Cash paid = 90 either way; mixing the routes $(90 - 20 + 0 = 70)$ is wrong.

LOS 30.d — IFRS versus US GAAP

KEY CONCEPT

Item	US GAAP	IFRS
Interest received	CFO	CFO or CFI
Interest paid	CFO	CFO or CFF
Dividends received	CFO	CFO or CFI
Dividends paid	CFF	CFO or CFF
Bank overdraft	Treated as debt (a financing item)	Included in cash and cash equivalents when repayable on demand and part of cash management
Taxes paid	All CFO	May be split among CFO, CFI and CFF according to the transaction that caused the tax to become payable (CFO unless tied to an investing or financing transaction)
Presentation of CFO	Direct encouraged; indirect allowed; direct users must disclose the indirect reconciliation	Direct encouraged; indirect allowed

Under these IAS 7 choices, interest and dividends received are never financing and interest and dividends paid are never investing, and every US GAAP classification of the four items is also allowed under IFRS. Exam convention: the IAS 7 choices in the table above. Current practice: IFRS 18, effective for annual periods beginning on or after 1 January 2027, amends IAS 7 and removes most of this choice for companies whose main business is not investing or financing (interest and dividends received go to CFI; interest and dividends paid go to CFF).

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Example. Investment property is sold for $3.2 million and tax of $0.25 million is paid on the gain. Under US GAAP the statement shows a CFI inflow of $3.2 million and a CFO outflow of $0.25 million. Under IFRS the firm may classify the tax in investing activities, so the sale adds a net $2.95 million to CFI (total taxes paid are still disclosed).

COMMON EXAM TRAPS

- Putting interest paid in CFF for a US GAAP firm.
- Treating dividends paid as CFO for a US GAAP firm (only IFRS allows it).
- Including noncash exchanges (bonds for land, bonds converted to stock) in CFI or CFF.
- Using dividends declared as the CFF outflow without adjusting for the change in dividends payable. In the dividend example, this gives an outflow of $260 instead of $290.
- Using the gain, or the carrying value, instead of the sale proceeds in CFI. In the CFI example, the gain gives CFI of −$36,000 and the carrying value gives −$12,000, instead of −$6,000.

EXAM SHORTCUTS

- The carrying value of PP&E disposed of can be found in one line as beginning net PP&E − depreciation + purchases − ending net PP&E, instead of working through gross cost and accumulated depreciation separately.

BOTTOM LINE

- Proceeds from an asset sale equal the carrying value of the asset sold plus the gain, or minus the loss, and only the proceeds are reported in CFI.
- CFF equals new borrowing minus principal repaid, plus stock issued minus stock repurchased, minus cash dividends paid, where dividends paid equal dividends declared (opening retained earnings plus net income less closing retained earnings) minus the change in dividends payable.
- Noncash investing and financing transactions, such as buying an asset with bonds or converting bonds into stock, are not in CFI or CFF but are disclosed in a footnote or supplementary schedule.
- Under US GAAP, interest received, dividends received, interest paid and taxes paid belong in CFO, and dividends paid belong in CFF.
- Exam convention: IFRS allows interest and dividends received in CFO or CFI and interest and dividends paid in CFO or CFF. Current practice: IFRS 18, effective for annual periods beginning on or after 1 January 2027, removes most of this choice for companies whose main business is not investing or financing.
- An inventory writedown is noncash and must be counted only once when cash paid to suppliers is computed: either subtract it from COGS and use the inventory change before the writedown, or keep it in COGS and use the balance sheet inventory change.

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QUICK CHECK

Question 3

On December 31, Tarn Retail, which reports under IFRS, borrowed €45,000 on a 90-day bank note and used the cash to buy merchandise inventory. On the same day it issued €200,000 of seven-year bonds and used the proceeds to buy delivery vans. No interest was paid during the year. The combined effect of these transactions on Tarn's statement of cash flows for the year is that:

A cash flow from financing activities increases by €200,000.

B cash flow from operating activities decreases by €45,000.

C cash flow from operating activities is unchanged.

Answer: B

Classification follows the nature of each cash flow, not the maturity of the borrowing. Buying inventory is part of day-to-day operations, so the €45,000 paid is an operating outflow. Borrowing, whether on a short-term note or long-term bonds, is a financing inflow, and buying vans is an investing outflow.

CFO: inventory purchase −€45,000.

CFI: vans −€200,000.

CFF: note +€45,000 and bonds +€200,000, a total of +€245,000.

The net change in cash is zero.

Why the other options are wrong

- **A.** Financing cash flow rises by €245,000, because the 90-day note is also borrowing and belongs in financing activities together with the bonds.
- **C.** The cash used to buy inventory is an operating outflow, so operating cash flow falls by €45,000 even though the purchase was funded by a bank note.

Key takeaway Borrowing of any maturity is financing; paying for inventory is operating; buying long-term assets is investing. Match each leg of a transaction to its own section.

Practice questions

Question 4

Dmitri Volkov is deciding whether to present the operating section of a projected statement of cash flows using the direct or the indirect format. Which of the following statements is *most accurate*?

A Under the direct method, depreciation is added to cash collected from customers because it is a noncash expense.

B Under the indirect method, depreciation is added back to net income because it is a noncash expense.

C Under the indirect method, changes in accounts receivable need no adjustment because net income already reflects them.

Question 5

Under US GAAP and under IFRS, how may **interest paid** be classified in the statement of cash flows?

A US GAAP: CFO or CFF; IFRS: CFO.

B US GAAP: CFO; IFRS: CFO or CFF.

C US GAAP: CFF; IFRS: CFO or CFF.

Answers: 4. B · 5. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/analyzing-statements-of-cash-flows-i/. The full bank has 59 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 31

Free Cash Flow

What you'll learn

LOS 31.a Analyze and interpret reported and common-size cash flow statements, including sources and uses of cash and the sustainability of operating cash flow.

LOS 31.b Calculate and interpret free cash flow to the firm, free cash flow to equity, and cash flow performance and coverage ratios.

MODULE 31.1

Analyzing Statements of Cash Flows II

This reading shows how an analyst evaluates a statement of cash flows: the sources and uses of cash in each section, what CFO says about earnings quality, and the two formats of common-size cash flow statement. It then defines free cash flow to the firm and to equity and the cash flow performance and coverage ratios, all of which a candidate must be able to calculate.

LOS 31.a — Reading reported and common-size cash flow statements

What the statement of cash flows tells an analyst

The cash flow statement gives an analyst information for assessing a firm's **liquidity**, **solvency** and **financial flexibility**. It answers practical questions:

- Do ordinary operations produce enough cash to keep the business running?
- Can maturing debt be repaid from internally generated cash, or will new financing be needed?
- Could the firm absorb an unexpected obligation or seize a new opportunity?

The analysis starts by listing the **sources and uses of cash** in each of the three sections (operating, investing and financing) and asking whether the pattern fits the firm's stage in its **life cycle**.

Stage	Typical pattern
Early growth	Operating cash flow often negative (cash tied up in receivables and inventory); gap funded by issuing debt or equity, which cannot continue indefinitely
Mature	Operating cash flow positive and larger than capital spending; surplus returned to lenders and shareholders

Operating activities: sources of CFO

Sustainable operating cash flow comes from the firm's **earnings-generating activities**. A firm can also boost CFO temporarily by shrinking **noncash working capital**:

- collecting receivables faster or running down inventory (these cannot fall below zero), or
- paying suppliers more slowly.

Such cash is classified as operating cash flow. It is not a sustainable source of cash, because suppliers will eventually refuse to extend more credit if payment keeps slipping.

Earnings quality. A stable relationship between CFO and net income signals good **earnings quality**. For a mature firm whose working capital is not growing, CFO normally exceeds net income, because depreciation and amortization reduce net income without using cash ($\text{CFO} = \text{NI} + \text{NCC} - \text{WC}_{\text{INV}}$). If net income persistently runs well ahead of CFO, the firm may be making aggressive or improper choices, such as recognizing revenue too early or deferring expenses. The variability of both net income and CFO is also an indicator of risk.

Investing and financing activities

- Rising **capital expenditures** usually indicate growth; cutting capex or selling long-lived assets frees cash now but may mean heavier outflows later when old assets must be replaced. Investing flows also include acquisitions and purchases/sales of securities.
- The financing section (**cash flow from financing activities**) shows whether cash is being raised by issuing debt or equity and whether cash is going to repay debt, repurchase shares or pay dividends. Borrowing to pay dividends or buy back stock deserves scrutiny.

Common-size cash flow statements

Two formats are used:

Format	Denominator	Main use
Revenue-based	Every line item as a % of **net revenue** for the period	Spotting trends and **forecasting**: once revenue is forecast, revenue-linked cash flows can be projected
Inflow/outflow-based	Each **cash inflow** as a % of **total cash inflows**; each **cash outflow** as a % of **total cash outflows**	Seeing which sources dominate and where cash goes

Neither format uses total assets, the net change in cash, or CFO as the denominator. CFO is itself a net figure that mixes inflows and outflows, so it cannot serve as a base for individual payments.

> *Example.* Cumbria Home Goods reports revenue of $4,000, cash paid to suppliers of $2,200 and total cash outflows of $3,600. Payments to suppliers are $2{,}200/4{,}000 = 55\%$ of revenue in the revenue-based format and $2{,}200/3{,}600 \approx 61.1\%$ of total outflows in the inflow/outflow format.

If a revenue-based statement shows CFO falling as a % of revenue while the inventory line (a use of cash) grows, the firm is tying up cash in stock. This is consistent with growth, but the analyst should ask whether the build-up was intended.

LOS 31.b — Free cash flow and cash flow ratios

Free cash flow to the firm (FCFF)

Free cash flow is the cash left for discretionary use once the firm has paid for its capital expenditures. It is widely used in valuation, and the two most common measures are FCFF and FCFE.

Free cash flow to the firm (FCFF) is the cash available to all capital providers (debtholders and shareholders) after the firm pays its operating costs and taxes and makes its investments in fixed capital and working capital.

From net income:

$$FCFF = NI + NCC + [Int \times (1 - t)] - FC_{INV} - WC_{INV}$$

From operating cash flow (because $CFO = NI + NCC - WC_{INV}$):

KEY CONCEPT

$$FCFF = CFO + [Int \times (1 - t)] - FC_{INV}$$

where NCC = **noncash charges** (depreciation and amortization), Int = cash interest paid, t = tax rate, FC_{INV} = **fixed capital investment** = cash paid for new fixed assets less cash proceeds from fixed assets sold, and WC_{INV} = **working capital investment** (an increase in receivables or inventory, or a decrease in payables, is a use of cash).

- After-tax interest is added back because interest is paid to debtholders, who are among the capital providers FCFF serves. If the tax rate is not given, it can be inferred as income tax expense ÷ income before tax.
- Fixed capital investment differs from CFI, which may also include purchases of securities or loans made.
- Under IFRS, if interest paid is classified in financing activities, CFO already excludes it, so no interest is added back. If dividends paid were classified in operating activities, they are added back to CFO with no tax adjustment, because dividends are not tax deductible.

Free cash flow to equity (FCFE)

Free cash flow to equity (FCFE) is the cash available to common shareholders after all obligations, including those to lenders:

KEY CONCEPT

$$FCFE = CFO - FC_{INV} + \text{net borrowing}$$

$$FCFE = FCFF - [Int \times (1 - t)] + \text{net borrowing}$$

Net borrowing = debt issued − debt repaid (subtract it if repayments exceed new borrowing). If no borrowing information is given, net borrowing is zero.

The chart shows how the three measures connect: start from CFO, give back the after-tax interest and pay for fixed capital to reach FCFF, then pay the lenders and add net borrowing to reach FCFE (see the figure below).

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Shortcut: FCFE = CFO − Fixed capital investment + Net borrowing

* Add back interest only if it was included in CFO; if classified as financing, do not add it again.

From operating cash flow to FCFF and FCFE

Example. A firm reports CFO of $920, cash interest paid of $80 (tax rate 25%), purchases of equipment of $310, proceeds from equipment sold of $40 and net new borrowing of $50.

- $FC_{INV} = 310 - 40 = 270$
- $FCFF = 920 + 80(1 - 0.25) - 270 = 920 + 60 - 270 = 710$
- $FCFE = 920 - 270 + 50 = 700$; check: $710 - 60 + 50 = 700$

Cash flow ratios

Cash flow ratios are compared over time for one firm or across firms, and fall into two groups.

Performance ratios (how well operations generate cash):

Ratio	Formula
Cash flow-to-revenue ratio	CFO / net revenue
Cash return-on-assets ratio	CFO / average total assets
Cash return-on-equity ratio	CFO / average total equity
Cash-to-income ratio	CFO / operating income
Cash flow per share	(CFO − preferred dividends) / weighted average number of common shares

Coverage ratios (ability to meet obligations and fund spending from CFO):

KEY CONCEPT

Ratio	Formula	Measures
Debt coverage ratio	CFO / total debt	Financial risk and leverage
Interest coverage ratio	(CFO + interest paid + taxes paid) / interest paid	Ability to meet interest obligations

Ratio	Formula	Measures
Reinvestment ratio	CFO / cash paid for long-term assets	Ability to acquire long-term assets from operating cash
Debt payment ratio	CFO / cash long-term debt repayment	Ability to repay long-term debt from operating cash
Dividend payment ratio	CFO / dividends paid	Ability to pay dividends from operating cash
Investing and financing ratio	CFO / cash outflows from investing and financing activities	Ability to purchase assets, satisfy debts and pay dividends

If interest paid is in financing activities (IFRS), no interest add-back is needed in the interest coverage ratio. If common dividends were placed in operating activities, they are added back to CFO for cash flow per share.

Example. CFO of $1,500 and $600 of cash spent on new long-term assets give a reinvestment ratio of $1{,}500/600 = 2.5$: operating cash covers capital spending 2.5 times.

COMMON EXAM TRAPS

- Recording the cash from paying suppliers more slowly as a financing inflow, or treating it as a lasting source of cash.
- The reinvestment ratio divides CFO by cash paid for long-term assets. Net CFI (which nets in asset sales and, for an IFRS firm, may include interest and dividends received) and total investing-plus-financing outflows (the investing and financing ratio) are wrong denominators.
- In FCFF from CFO, after-tax interest is added, never subtracted. In the example, subtracting it gives FCFF of 590 instead of 710.
- FC_{INV} is net of proceeds from asset sales; forgetting the proceeds understates free cash flow. In the example, ignoring the proceeds of 40 gives FCFF of 670 instead of 710.
- From net income, working capital investment must be deducted: a rise in receivables and a fall in payables both reduce free cash flow.
- The cash-to-income ratio is a performance ratio; the debt payment ratio is a coverage ratio.

EXAM SHORTCUTS

- When FCFF is already known, FCFE follows directly as FCFF − Int × (1 − t) + net borrowing, without starting again from CFO.

BOTTOM LINE

- The statement of cash flows helps an analyst assess a firm's liquidity, solvency and financial flexibility.
- Cash generated by collecting receivables faster, running down inventory or paying suppliers more slowly is classified as operating cash flow, but it is not a sustainable source of cash.
- For a mature firm whose working capital is not growing, CFO normally exceeds net income; net income that persistently runs well ahead of CFO may point to aggressive or improper accounting choices.

- Common-size cash flow statements come in two formats: every line divided by net revenue, or inflows divided by total inflows and outflows divided by total outflows.
- FCFF equals CFO + Int × (1 − t) − FCInv, or NI + NCC + Int × (1 − t) − FCInv − WCInv, where FCInv is cash paid for fixed assets less cash proceeds from fixed assets sold.
- FCFE equals CFO − FCInv + net borrowing, where net borrowing is debt issued minus debt repaid.
- When interest paid is classified in financing activities under IFRS, CFO already excludes it, so no after-tax interest is added back in FCFF and no interest is added back in the interest coverage ratio.
- Performance ratios, such as cash flow-to-revenue and cash-to-income, measure how well operations generate cash, while coverage ratios, such as debt coverage and the reinvestment ratio (CFO / cash paid for long-term assets), measure the ability to meet obligations and fund spending from CFO.

QUICK CHECK

Question 1

An analyst reviews three years of results for Pinecrest Media:

Pinecrest Media — net income and cash flow from operations ($ millions)

Year	Net income	Cash flow from operations	Accounts receivable (year-end)
Year 1	42	45	60
Year 2	51	38	84
Year 3	63	31	117

Which conclusion is *most appropriate* based on these data?

A Earnings quality appears high, because net income has grown steadily in every year.

B The widening gap is expected and harmless, because growing firms always report operating cash flow above net income.

C Earnings quality may be deteriorating; the firm could be recognizing revenue too early or delaying expense recognition.

Answer: C

A stable relationship between net income and operating cash flow indicates good earnings quality. Here net income rises from 42 to 63 while CFO falls from 45 to 31, and receivables nearly double; earnings that increasingly exceed operating cash flow can signal aggressive or even improper accounting, such as recognizing revenue too early or delaying the recognition of expenses. The build-up in receivables points to revenue being booked well ahead of cash collection.

Net income minus CFO: Year 1 $= 42 - 45 = -3$; Year 2 $= 51 - 38 = 13$; Year 3 $= 63 - 31 = 32$.

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Receivables grow from 60 to 117 (+95%) while net income grows 50%, so revenue is being booked faster than it is collected.

Why the other options are wrong

- **A.** Steady growth in reported income says little about quality when operating cash flow is moving the opposite way; the diverging trend is the warning sign.
- **B.** Growing firms often have operating cash flow below net income because working capital expands, so the direction of the statement is wrong. A persistent, widening gap also calls for investigation.

Key takeaway Compare net income with CFO over several years: a persistent, widening excess of earnings over CFO is a red flag for earnings quality.

Practice questions

Question 2

Amara Osei is valuing Kestrel Logistics, which reports under US GAAP (interest paid is included in operating activities). She collects the following data:

Kestrel Logistics — selected data for the year

Item	Amount ($)
Net cash provided by operating activities	4,600
Fixed capital investment (net)	950
Cash interest paid	300
Income before tax	5,000
Income tax expense	1,250
Net income	3,750

Kestrel's free cash flow to the firm (FCFF) is *closest* to:

A $3,875.

B $3,425.

C $3,025.

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Question 3

Vasquez Dental Supply reports under US GAAP, so interest paid and income taxes paid are both included in its cash flow from operating activities. For the latest year, CFO was $7,800, cash interest paid was $600, and cash income taxes paid were $1,500. Vasquez's (cash flow) interest coverage ratio is *closest* to:

A 16.5.

B 14.0.

C 13.0.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/free-cash-flow/. The full bank has 14 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 32

LIFO vs FIFO

What you'll learn

LOS 32.a Describe measurement of inventory at the lower of cost and net realizable value (and lower of cost or market under US GAAP) and the effects of write-downs and reversals on financial statements and ratios.

LOS 32.b Calculate and explain how inflation and deflation of inventory costs affect the financial statements and ratios of companies using FIFO, LIFO or weighted average cost, including LIFO liquidations.

LOS 32.c Describe inventory presentation and disclosures and the issues analysts consider when examining inventory disclosures, inventory mix and inventory ratios.

MODULE 32.1

Inventory Measurement

This reading covers how inventory is measured on the balance sheet, how FIFO, LIFO and weighted average cost change the financial statements when prices rise or fall, and what inventory disclosures and ratios tell an analyst. The calculations to master are the inventory write-down, the LIFO-to-FIFO restatement and inventory turnover.

LOS 32.a — Lower of cost or net realizable value (and lower of cost or market)

Review: which costs go into inventory?

Before measuring inventory, separate capitalized costs from period expenses. Inventory cost includes the purchase price net of trade discounts and rebates, import duties, transport and handling needed to reach the present location and condition, direct labor, and fixed and variable production overhead. Normal waste is included in production cost. Expense abnormal waste; storage unless it is necessary during production; administrative overhead; and selling costs. Capitalizing a cost that should be expensed overstates inventory and current profit. When the goods are sold, their capitalized cost becomes cost of goods sold.

Which measurement rule applies?

Framework / cost method	Balance-sheet measure	Later recovery in value
IFRS (FIFO, weighted average cost, specific identification; LIFO is not permitted)	**Lower of cost or net realizable value**	**Write-up allowed**, but only up to the amount previously written down (carrying value never exceeds original cost)
US GAAP, any method other than LIFO or the retail method (e.g., FIFO, weighted average cost)	**Lower of cost or NRV**	**No write-up**
US GAAP, **LIFO** or the **retail method**	**Lower of cost or market**	**No write-up**

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KEY CONCEPT

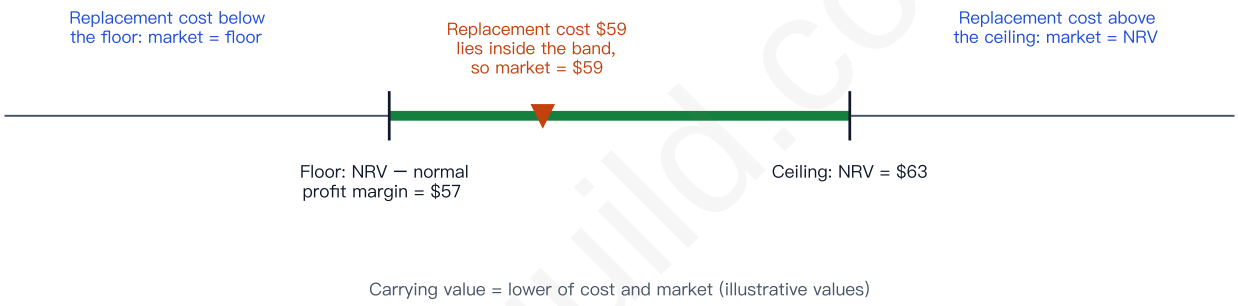
Net realizable value (NRV) = estimated selling price − estimated **selling costs** (disposal costs) − estimated **completion costs**.

Lower of cost or market (US GAAP, LIFO and retail firms)

Market is normally **replacement cost**, constrained to a band:

KEY CONCEPT

- Ceiling: NRV. If replacement cost is above NRV, market = NRV.
- Floor: NRV − **normal profit margin**. If replacement cost is below the floor, market = NRV − normal profit margin.
- In between, market = replacement cost.



Lower of cost or market: where replacement cost falls decides market (illustrative values)

Then carrying value = lower of (original cost, market). The width of the band equals the normal profit margin, so the lowest possible "market" figure is selling price − selling (and completion) costs − normal profit margin.

Example

Per unit: original cost $64, expected selling price $70, selling costs $5, completion costs $2, replacement cost $59, normal profit margin $6.

- NRV $= 70 - 5 - 2 = 63$. Lower of cost or NRV: $\min(64, 63) = 63$, a write-down of $1 per unit.
- Market band: floor $= 63 - 6 = 57$, ceiling $= 63$. Replacement cost of 59 lies inside, so market = 59. Lower of cost or market: $\min(64, 59) = 59$, a write-down of $5 per unit.

Next year NRV recovers to $67. An IFRS firm writes the unit back up to $64 (a $1 gain, capped by the original write-down); a US GAAP firm keeps it at its new cost basis and simply earns a higher margin when the unit is sold.

How the write-down is recorded

- The loss is recognized in the income statement, usually by increasing **cost of goods sold** (or as a separate line item if large). The **inventory equation** shows why: ending inventory = beginning inventory + purchases − COGS. Ending inventory falls with beginning inventory and purchases unchanged, so COGS must rise.

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- Firms often use a **valuation allowance**, a **contra-asset account** similar to accumulated depreciation, so that original cost and carrying value are both visible.
- Under IFRS a write-up (reversal) is reported in profit or loss, as its own line or as a reduction of COGS.
- Under US GAAP, the written-down amount becomes the new cost basis.
- During inflation, LIFO firms are less likely to need write-downs, because their inventory carries older, lower costs.
- Producers and dealers of commodities (agricultural and forest products, mineral ores, precious metals) are an exception under both IFRS and US GAAP: they may report inventory at NRV even when it is above historical cost, with unrealized gains and losses in the income statement. The quoted price in an active market is used where one exists; otherwise recent market transactions are used.

Effects of a write-down on ratios (period of the write-down, loss charged to COGS)

Item / ratio	Effect	Why
Current assets, total assets, equity	Decrease	Inventory falls; the loss reduces retained earnings
Current ratio	Decrease	Inventory is in current assets
Quick ratio	No effect	Inventory is excluded from the numerator (cash + marketable securities + receivables) and current liabilities are unchanged
Inventory turnover	Increase	Higher COGS over a lower inventory; days of inventory on hand and the cash conversion cycle fall
Total asset turnover	Increase	Revenue unchanged, total assets lower
Debt-to-assets, debt-to-equity	Increase	Debt unchanged, assets and equity lower
Gross, operating and net profit margins	Decrease	COGS higher
ROA, ROE	Decrease	The percentage drop in net income is typically larger than the drop in assets or equity

In later periods the lower carrying amount flows through COGS, so COGS is lower and income higher. Combined with a smaller asset and equity base, ROA and ROE rise in those periods. Write-downs therefore distort comparisons of ratios across periods.

COMMON EXAM TRAPS

- NRV deducts both selling (disposal) costs and completion costs from the selling price. In the example, deducting only the selling costs gives an NRV of $65, above the $64 cost, so the $1 write-down is missed.
- Lower of cost or market applies under US GAAP only to LIFO and retail-method firms; a US GAAP FIFO firm uses lower of cost or NRV.
- Replacement cost is not automatically "market": check the ceiling (NRV) and the floor (NRV – normal profit margin).
- IFRS reversals are limited to the original write-down, so carrying value can never exceed cost. US GAAP allows no reversal at all.

- Expecting an inventory write-down to lower the quick ratio.

EXAM SHORTCUTS

- Under lower of cost or market, market is the middle value of replacement cost, NRV and NRV minus the normal profit margin.
- An inventory write-down leaves the quick ratio unchanged, because the quick ratio excludes inventory, so an answer in which the quick ratio moves can be ruled out.

BOTTOM LINE

- IFRS firms, and US GAAP firms not using LIFO or the retail method, measure inventory at the lower of cost or NRV; US GAAP LIFO and retail-method firms use the lower of cost or market.
- NRV is the estimated selling price minus selling costs and completion costs.
- Market is replacement cost, but not above NRV and not below NRV minus the normal profit margin.
- IFRS allows a write-up only up to the amount previously written down; US GAAP allows no reversal.
- A write-down raises COGS, lowers the current ratio, margins, ROA and ROE, raises inventory turnover and leaves the quick ratio unchanged.

QUICK CHECK

Question 1

Brenvale Creamery makes aged cheese. Its costs for the year are shown below.

Brenvale Creamery: costs incurred during the year ($)

Cost	Amount
Milk and other direct materials	410,000
Direct labor	185,000
Fixed production overhead	260,000
Storage of cheese wheels during the aging stage of production	38,000
Normal spoilage during production	6,200
Spoilage caused by a refrigeration breakdown (abnormal)	29,000
Storage of packaged cheese awaiting delivery to customers	17,500

The cost that Brenvale should include in inventory is *closest* to:

A $928,200.

B $916,700.

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C $899,200.

Answer: C

Inventory cost includes the costs of purchase and conversion needed to bring the goods to their present location and condition: materials, direct labor, production overhead, normal spoilage and storage that is a necessary part of production, such as aging. Abnormal waste and storage of finished goods waiting for delivery are expensed as incurred.

$$410{,}000 + 185{,}000 + 260{,}000 + 38{,}000 + 6{,}200 = \$899{,}200$$

Expensed: abnormal spoilage $29,000 and storage of packaged cheese $17,500.

Why the other options are wrong

- **A.** $928,200 also capitalizes the $29,000 of abnormal spoilage from the refrigeration breakdown, $899{,}200 + 29{,}000$. Abnormal waste is expensed when it occurs.
- **B.** $916,700 also capitalizes the $17,500 of storage after production, $899{,}200 + 17{,}500$. Storage is part of inventory cost only when it is required in the production process.

Key takeaway Put purchase and conversion costs, normal waste and production-related storage into inventory; expense abnormal waste, post-production storage, administrative overhead and selling costs.

MODULE 32.2

Inflation Impact on FIFO and LIFO

LOS 32.b — How inflation and deflation affect FIFO, LIFO and weighted average cost

Review: the three cost flow assumptions

Method	Costs assigned to COGS	Costs left in ending inventory	Permitted
FIFO (first-in, first-out)	Oldest costs	Most recent costs	IFRS and US GAAP
LIFO (last-in, first-out)	Most recent costs	Oldest costs	US GAAP only (prohibited under IFRS)
Weighted average cost	Average cost of goods available for sale	Average cost	IFRS and US GAAP

The methods differ in which costs are assigned to units sold and to units kept. The physical flow of goods does not have to match.

Example (periodic system)

Beginning inventory 100 units at $10; purchases of 200 units at $12, then 200 units at $14. Sales: 350 units. Goods available: 500 units costing $1{,}000 + 2{,}400 + 2{,}800 = 6{,}200$.

Method	COGS	Ending inventory (150 units)
FIFO	$1{,}000 + 2{,}400 + 50 \times 14 = 4{,}100$	$150 \times 14 = 2{,}100$
LIFO	$2{,}800 + 150 \times 12 = 4{,}600$	$50 \times 12 + 1{,}000 = 1{,}600$
Weighted average	$350 \times 12.40 = 4{,}340$	$150 \times 12.40 = 1{,}860$

Average cost $= 6{,}200/500 = 12.40$. In every case COGS + ending inventory = 6,200. With rising costs, weighted average lies between FIFO and LIFO. The difference between FIFO and LIFO inventory ($2{,}100 - 1{,}600 = 500$) is the **LIFO reserve**.

A US GAAP firm that uses LIFO discloses its LIFO reserve, which lets an analyst restate its figures on a FIFO basis before comparing it with FIFO or IFRS peers:

KEY CONCEPT

1. FIFO inventory = LIFO inventory + LIFO reserve, at each balance sheet date.
2. FIFO COGS = LIFO COGS − (ending LIFO reserve − beginning LIFO reserve).
3. Recompute the ratios with the FIFO figures.

Example. A LIFO retailer reports COGS of $5,300, beginning inventory of $900 and ending inventory of $1,100, and its LIFO reserve rose from $300 to $420. FIFO COGS $= 5{,}300 - (420 - 300) = \$5{,}180$, and FIFO inventories are $900 + 300 = \$1{,}200$ and $1{,}100 + 420 = \$1{,}520$. Inventory turnover falls from $5{,}300/1{,}000 = 5.30$ on the reported LIFO basis to $5{,}180/1{,}360 \approx 3.81$ on a FIFO basis.

Rising prices, stable or increasing quantities

Item	FIFO vs. LIFO
Cost of goods sold	FIFO **lower**, LIFO **higher**
Ending inventory	FIFO **higher**, LIFO **lower**
Gross profit, operating profit, pretax and net income	FIFO **higher**
Income taxes	FIFO **higher**
Cash flow (after tax)	LIFO **higher** (lower taxes paid)
Working capital, current ratio	FIFO **higher**
Stockholders' equity (retained earnings), total assets	FIFO **higher**
Debt-to-equity, debt ratio	FIFO **lower**
Inventory turnover (COGS / average inventory)	LIFO **higher** (higher numerator, lower denominator); FIFO has higher days of inventory on hand

Weighted average cost figures fall between FIFO and LIFO. When prices are falling, every effect reverses: LIFO then has lower COGS, higher gross profit and net income, higher inventory, higher working capital, higher taxes and lower cash flow than FIFO; FIFO gives the lowest income.

Stable prices. If purchase costs have not changed (and beginning inventory was bought at the same cost, or is zero), FIFO, LIFO and weighted average cost give identical COGS, ending inventory and gross profit.

Minimizing reported pretax income (and taxes) in inflation. A US GAAP firm uses LIFO. Under IFRS, where LIFO is prohibited, weighted average cost gives lower income than FIFO.

Which figure is "most useful"?

- Balance sheet: FIFO inventory best approximates current cost (economic value), because it contains the most recent purchase costs.
- Income statement: LIFO COGS best approximates current cost. It matches recent costs against current revenue, which gives a better measure of current income and future profitability.
- This holds whether prices are rising or falling.

LIFO liquidation

A **LIFO liquidation** occurs when a LIFO firm's inventory quantity declines, so units are drawn from older, lower-cost **LIFO layers** into COGS. In an inflationary environment this:

- lowers COGS and raises gross and operating profit margins and income taxes;
- produces profits that are not sustainable, because the firm cannot keep selling old layers without replenishing them;
- may be deliberate (management drawing down inventory to boost earnings) or involuntary (strikes, supplier shortages, lower expected orders).

Analysts look for a decrease in the LIFO reserve in the footnotes as a signal of a possible liquidation and adjust margins if its effect is material.

Example. Layers of 100 units at \$8 and 100 units at \$11; this year the firm buys 400 units at \$15 and sells 450 units. LIFO COGS $= 400 \times 15 + 50 \times 11 = 6{,}550$; had it bought 450 units, COGS would have been $450 \times 15 = 6{,}750$. The liquidation adds $50 \times (15 - 11) = 200$ to gross profit.

WORKED EXAMPLE: RESTATING A LIFO FIRM ON A FIFO BASIS

A US GAAP retailer that uses LIFO reports revenue of \$26,000, COGS of \$18,600, beginning inventory of \$3,100 and ending inventory of \$3,500. Its LIFO reserve rose from \$900 to \$1,250 during the year.

Step 1. FIFO inventories: $3{,}100 + 900 = \$4{,}000$ at the start of the year and $3{,}500 + 1{,}250 = \$4{,}750$ at the end.

Step 2. FIFO COGS $= 18{,}600 - (1{,}250 - 900) = \$18{,}250$.

Step 3. Gross margin: $(26{,}000 - 18{,}600)/26{,}000 = 28.5\%$ on LIFO and $(26{,}000 - 18{,}250)/26{,}000 = 29.8\%$ on FIFO.

Step 4. Inventory turnover: $18{,}600/3{,}300 = 5.64$ on LIFO and $18{,}250/4{,}375 = 4.17$ on FIFO, so days of inventory on hand are 64.8 and 87.5.

Result. On a FIFO basis the retailer shows a higher gross margin and slower inventory turnover. Both differences match the rising-price pattern: LIFO COGS is higher and LIFO inventory is lower than under FIFO.

COMMON EXAM TRAPS

- Higher cash flow under LIFO in inflation comes from lower taxes. Pretax cash costs are the same under every method.
- Every inflation result reverses when prices are falling.
- A LIFO firm can report a higher margin than a FIFO firm even with rising prices if it has a LIFO liquidation.
- LIFO is not available to IFRS reporters.
- **Specific identification**, which assigns each unit its own actual cost, can give values above or below the other methods, depending on which units are sold.

EXAM SHORTCUTS

- With rising prices, start from LIFO's higher COGS: lower income, lower taxes and higher after-tax cash flow follow, and every comparison reverses when prices fall.
- With prices moving in one direction, weighted average cost figures always lie between FIFO and LIFO.

BOTTOM LINE

- FIFO puts the oldest costs in COGS and the most recent in ending inventory, LIFO does the reverse, and IFRS prohibits LIFO.
- With rising prices and stable or increasing quantities, LIFO gives higher COGS, lower income and taxes and higher after-tax cash flow, while FIFO gives higher inventory, working capital and equity.
- FIFO inventory = LIFO inventory + LIFO reserve, and FIFO COGS = LIFO COGS − (ending − beginning LIFO reserve).
- FIFO inventory best approximates current cost on the balance sheet; LIFO COGS best approximates current cost on the income statement.
- A LIFO liquidation in inflation lowers COGS and raises margins and taxes, but the extra profit is not sustainable; a decrease in the LIFO reserve signals it.

QUICK CHECK

Question 2

Tolworth Hardware Supply reports under US GAAP and uses the LIFO method. For the year just ended it reported cost of goods sold of $4,815,000. Its inventory footnote shows a LIFO reserve of $612,000 at the beginning of the year and $538,000 at the end. Purchase costs fell during the year, and inventory quantities were unchanged. An analyst restates Tolworth's results to a FIFO basis to compare it with a peer that reports under IFRS. Tolworth's cost of goods sold on a FIFO basis is *closest* to:

A $4,889,000.

B $4,741,000.

C $4,277,000.

Answer: A

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FIFO cost of goods sold equals LIFO cost of goods sold minus the change in the LIFO reserve over the year. The reserve fell by $74,000, so the change is negative and FIFO COGS is higher than LIFO COGS. That fits falling purchase costs: FIFO charges the older, higher costs to COGS, while LIFO charges the newest, lower costs. Inventory quantities did not change, so no old LIFO layers were drawn down: the fall in the reserve reflects lower purchase costs, not a LIFO liquidation. The restatement formula would be the same in either case.

Change in the LIFO reserve $= 538{,}000 - 612{,}000 = -74{,}000$.

$$\text{FIFO COGS} = \text{LIFO COGS} - (\text{ending LIFO reserve} - \text{beginning LIFO reserve})$$

$$\text{FIFO COGS} = 4{,}815{,}000 - (-74{,}000) = 4{,}889{,}000$$

Check with the inventory equation, COGS = beginning inventory + purchases − ending inventory. Purchases are the same under both methods. FIFO inventory exceeds LIFO inventory by $612,000 at the start of the year and by $538,000 at the end, so FIFO COGS exceeds LIFO COGS by $612{,}000 - 538{,}000 = 74{,}000$.

Why the other options are wrong

- **B.** $4,741,000 subtracts $74,000 from LIFO COGS, as if the reserve had risen. The reserve fell, so the change is negative and subtracting it raises COGS.
- **C.** $4,277,000 subtracts the whole ending LIFO reserve of $538,000. The ending reserve is the cumulative difference between FIFO and LIFO inventory and is the balance sheet adjustment. Only the change in the reserve during the year passes through COGS.

Key takeaway $\text{FIFO COGS} = \text{LIFO COGS} - \Delta\text{LIFO reserve}$ and $\text{FIFO inventory} = \text{LIFO inventory} + \text{LIFO reserve}$. A falling reserve makes FIFO COGS higher than LIFO COGS.

MODULE 32.3

Presentation and Disclosure

LOS 32.c — Inventory presentation, disclosures and what analysts look for

Required disclosures (broadly the same under IFRS and US GAAP)

Inventory disclosures, mostly in the footnotes, help the analyst judge how well inventory is managed and adjust the statements so the firm can be compared with its peers.

- The **cost flow method** used (FIFO, LIFO, weighted average cost, etc.), so the analyst does not need to infer it.
- Total **carrying value** of inventory and carrying value by classification (**raw materials**, **work in progress**, **finished goods**) where appropriate.
- Carrying value of inventories reported at **fair value less selling costs**.
- The amount of inventory cost expensed in the period (COGS).
- The amount of **inventory write-downs** (valuation allowances) during the period.
- **Reversals of write-downs** during the period, with a discussion of the circumstances that led to the reversal. Only IFRS firms report this item, since reversals are prohibited under US GAAP.

- Carrying value of **inventories pledged as collateral** for liabilities. This is a disclosure only: the inventory stays in current assets and is not offset against the debt.

Reading the inventory mix

Merchandisers (retailers, wholesalers) report a single inventory line. Manufacturers report raw materials, work in progress and finished goods, and the mix can signal future demand:

Pattern	Likely signal
Raw materials and/or work in progress rising	Management expects higher demand, so higher future revenue and earnings
Finished goods rising while raw materials and work in progress fall	Weakening demand; possible future write-downs
Finished goods growing faster than sales	Declining demand, excess or **obsolete inventory**; lower future earnings when written down; extra storage, insurance and tax costs and idle cash

Analysts corroborate these signals with **management's discussion and analysis**, industry data and trade publications, other parts of the annual report, communication with management, and peer companies' accounts.

Inventory ratios

KEY CONCEPT

$$\text{Inventory turnover} = \frac{\text{COGS}}{\text{average inventory}}$$

$$\text{Days of inventory on hand (DOH)} = \frac{365}{\text{inventory turnover}}$$

Example. COGS of 4,800 and average inventory of 600 give turnover of 8.0 and $DOH = 365/8 \approx 45.6$ days.

Observation	Possible interpretation
Turnover too low / DOH high or rising	Slow-moving or **obsolete** inventory; future write-downs
Turnover high with sales growth at or above the industry	Efficient inventory management
Turnover high with sales growth below the industry	Too little inventory and lost sales; also possible if write-downs have shrunk inventory

Other checks: the gross profit margin (rising input costs squeeze margins), the valuation allowance as a share of finished goods cost (a rising share points to obsolescence), and liquidity. The **current ratio** includes inventory while the **quick ratio** excludes it, so a sharp inventory drop can lower the current ratio while the quick ratio improves. Inventory is one of the least liquid current assets, since turning it into cash takes roughly days of inventory on hand plus days of sales outstanding. For that reason some analysts prefer the quick ratio.

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COMMON EXAM TRAPS

- Looking for write-down reversals in a US GAAP firm's notes.
- Netting pledged inventory against the loan it secures, or moving it out of current assets.
- Reading a build-up of raw materials and work in progress as a sign of weak demand.

BOTTOM LINE

- Inventory disclosures include the cost flow method, carrying value by class, the cost expensed, write-downs, reversals (IFRS only) and inventory pledged as collateral.
- Rising raw materials and work in progress suggest management expects higher demand; finished goods growing faster than sales suggest weak demand or obsolete inventory.
- Inventory turnover = COGS ÷ average inventory, and days of inventory on hand equal 365 divided by that turnover.
- Low turnover points to slow-moving or obsolete inventory; high turnover with sales growth below the industry points to too little inventory.

QUICK CHECK

Question 3

An analyst reviews the following data for Merrow Cycle Parts, whose industry peers have maintained steady inventory levels:

Merrow Cycle Parts — inventory data ($ thousands)

Item	Year 1	Year 2
Cost of goods sold	7,300	7,665
Average inventory	1,000	1,680
Finished goods (year-end)	610	975
Sales growth	4%	3%

Merrow's days of inventory on hand in Year 2, and the *most likely* interpretation, are:

A About 50 days; inventory is being managed more efficiently.

B About 80 days; inventory is being managed more efficiently.

C About 80 days; inventory may be slow-moving or obsolete.

Answer: C

Days of inventory on hand equals 365 divided by inventory turnover (COGS / average inventory). DOH has risen from 50 to 80 days while sales growth has slowed and finished goods have grown far faster than sales. These are signs of slow-moving or obsolete inventory that could require write-downs and depress future earnings.

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Year 1: turnover $= 7{,}300/1{,}000 = 7.30$; DOH $= 365/7.30 = 50$ days.

Year 2: turnover $= 7{,}665/1{,}680 = 4.5625$; DOH $= 365/4.5625 = 80$ days.

Finished goods grow $975/610 - 1 \approx 59.8\%$ versus sales growth of 3%.

Why the other options are wrong

- **A.** About 50 days is the Year 1 figure; in Year 2 turnover fell to about 4.56, so DOH rose to about 80 days. A rise in DOH is not a sign of greater efficiency.
- **B.** The DOH figure is right, but a higher DOH (lower turnover) means inventory is sitting longer. Together with finished goods outpacing sales, this points to weaker inventory management.

Key takeaway Low turnover or rising DOH, together with finished goods outpacing sales, points to obsolescence and future write-downs.

Practice questions

Question 4

Three companies each report that the net realizable value of their inventory is now above its original cost. Which company is *most likely* permitted to carry that inventory on its balance sheet at an amount above historical cost?

A Ridgeback Electronics, a US GAAP retailer of smartphones.

B Coldwater Timber, an IFRS producer of forest products that trade in an active market.

C Marigold Apparel, an IFRS reporter that wrote its clothing down to net realizable value last year.

Question 5

An analyst suspects that Cordell Hardware, a LIFO user facing rising purchase costs, drew down old inventory layers this year to lift its reported profit. The *most* reliable evidence of such a LIFO liquidation is a:

A decrease in the LIFO reserve reported in the inventory footnote.

B rise in the gross profit margin.

C fall in inventory that is out of line with the change in sales.

Answers: 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/lifo-vs-fifo/. The full bank has 44 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 33

Straight Line Depreciation

What you'll learn

LOS 33.a Compare the accounting for intangible assets that are purchased, developed internally, or acquired in a business combination.

LOS 33.b Explain and evaluate how impairment, revaluation and derecognition of PP&E and intangible assets affect the financial statements and ratios.

LOS 33.c Analyze and interpret disclosures about PP&E and intangible assets, including estimates of average age and useful lives.

MODULE 33.1

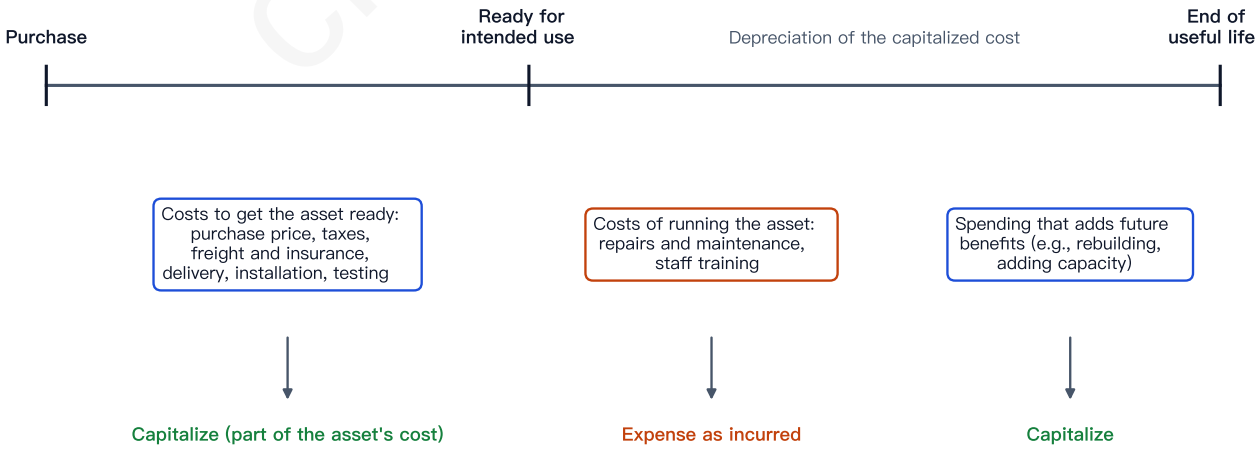
Intangible Long-Lived Assets

This reading covers how intangible assets are recognized according to how they were obtained, how PP&E and intangibles are impaired, revalued and derecognized, and what the disclosures about them show. A candidate must be able to measure an impairment loss under IFRS and US GAAP, compute the gain or loss when an asset is derecognized, and estimate average age, total useful life and remaining useful life from the disclosures.

LOS 33.a — Intangible assets: purchased, internally developed, and acquired in a business combination

Review of assumed knowledge: capitalizing a long-lived asset

This reading assumes that the capitalize-versus-expense decision is already familiar. A cost is **capitalized** (recorded as part of an asset) when it brings future benefits, and **expensed** when its benefit is used up in the current period. For a long-lived asset, the costs of getting it ready for its intended use are capitalized; the costs of running it are expensed as incurred.



Capitalize or expense: costs along the life of a long-lived asset

Long-lived assets are then measured after acquisition under the **cost model** (cost less accumulated depreciation and impairment), which US GAAP requires. IFRS lets a company choose the cost model or the **revaluation model** for each class of asset.

What an intangible asset is

Intangible assets are long-term assets without physical substance, such as patents, copyrights, brand names, **trademarks**, licenses, **franchise rights** and customer lists. Two classifications drive the accounting:

KEY CONCEPT

Classification	Meaning	Accounting consequence
Finite-lived intangible asset	Benefits end at a predictable date (legal, contractual or economic limit)	Cost is **amortized** over the useful life
Indefinite-lived intangible asset	No foreseeable limit to the period of benefit (e.g., **goodwill**, a trademark or license renewable at minimal cost)	**Not amortized**; **tested for impairment at least annually**
Identifiable intangible asset	Separable (or arises from contractual/legal rights), controlled by the firm, expected to give probable future benefits, cost reliably measurable	Can be recognized separately
Unidentifiable intangible asset	Cannot be bought or sold separately from the business	Main example: **goodwill**

A purchased patent or a franchise with a fixed contractual term is finite-lived and is amortized over its remaining useful life until it is derecognized. A patent whose protection lapses later this year is still an asset and is amortized for the months that remain. Only a right that has already expired and brings no further benefit fails to qualify as an asset.

How the asset was obtained decides whether it is on the balance sheet

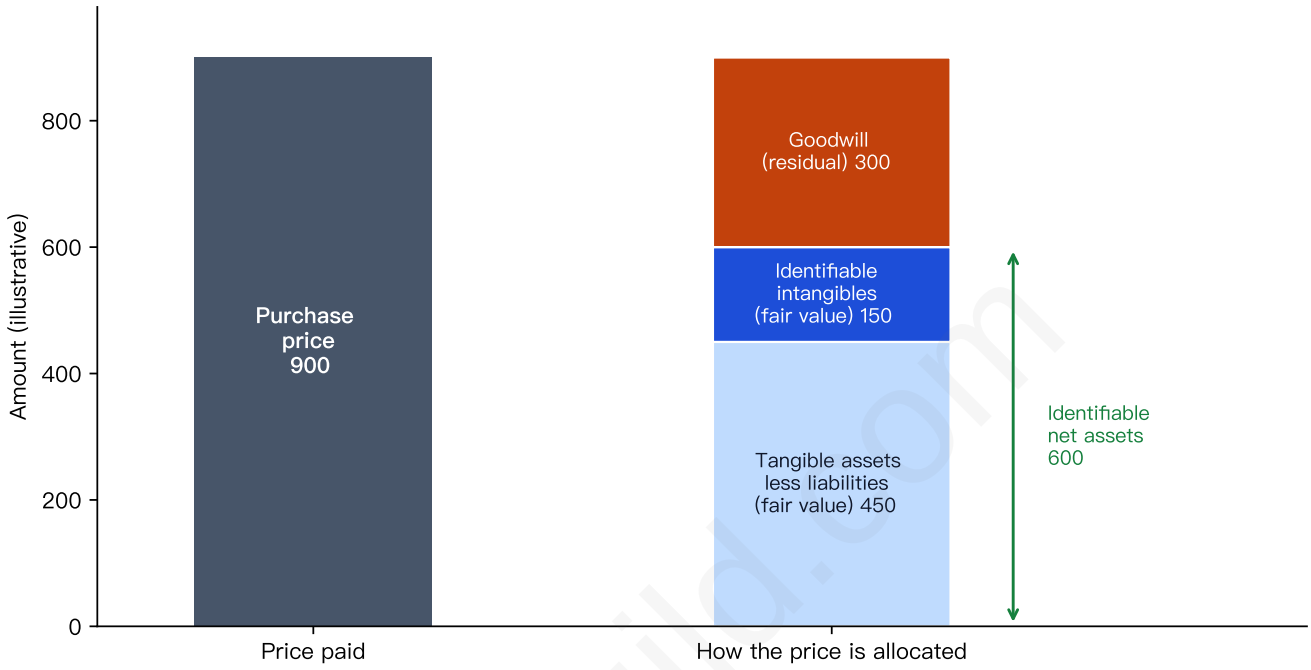
Source of the intangible	Treatment
Purchased from another party	Recorded at cost (normally fair value at purchase); a group purchase is allocated by relative fair values
Internally developed	Costs generally **expensed as incurred**, so internally created brands, trademarks and "goodwill" do not appear on the balance sheet
Acquired in a business combination	**Acquisition method**: purchase price allocated to the fair values of the target's identifiable assets and liabilities, including identifiable intangibles the target built internally and had expensed; the remainder is **goodwill**

Internally built intangibles rarely pass the recognition tests. The future benefits of building a brand or a customer base are hard to show as probable, and the cost is hard to measure reliably because it is mixed in with ordinary operating spending. A purchase or a business combination fixes a measurable amount for each identifiable item, so the same kind of asset can then be capitalized.

Goodwill

$$\text{Goodwill} = \text{Purchase price} - \text{Fair value of identifiable net assets acquired}$$

Recognizing more identifiable intangibles at acquisition leaves less of the price allocated to goodwill, because goodwill is the residual.



Goodwill is the residual of the purchase price (illustrative values)

When intangibles are bought as a group, the analyst usually cares more about what kind of asset was acquired than about the amount allocated to it. Newly acquired franchise rights, for instance, say something about future operating performance whatever value they are booked at.

Research, development and software

KEY CONCEPT

Cost	IFRS	US GAAP
Research costs (discovering new knowledge)	Expensed	Expensed
Development costs (turning research into a product or process plan)	May be **capitalized** once criteria are met (e.g., the firm can complete the asset and intends to use or sell it)	Generally expensed
Software developed for sale	Development-cost rules	Expensed until **technological feasibility** is established, **capitalized** afterward

Cost	IFRS	US GAAP
Software developed for internal use	Development-cost rules	Expensed until it is **probable** the project will be completed and the software used as intended, capitalized afterward

Technological feasibility is a harder hurdle than "probable completion", so software for sale tends to have more of its costs expensed.

Capitalizing versus expensing — effects on the statements

When a cost is **capitalized**, it becomes an asset and flows to the income statement later as **amortization** (or depreciation). Compared with **expensing** the same cost:

KEY CONCEPT

Item	Year the cost is incurred	Later years of the asset's life
Net income	Higher	Lower (amortization expense)
Total assets and equity	Higher	Higher (until fully amortized)
ROA and ROE	Usually higher	Lower (lower numerator, higher denominator)
Cash flow from operations (CFO)	Higher; the outlay is an investing outflow	Unchanged for a single outlay; amortization is noncash (added back)
Cash flow from investing (CFI)	Lower	No effect
Debt-to-assets, debt-to-equity	Lower	Lower

Total cash flow is identical; only the classification differs (ignoring any tax differences). A firm that capitalizes new outlays every year reports higher CFO every year, because each year's outlay is an investing outflow.

Example. Kestrel Apps spends $300,000 in Year 1 on development that qualifies for capitalization and amortizes it straight-line over three years. Capitalizing, Year 1 expense is $300{,}000/3 = \$100{,}000$ instead of $300,000, so Year 1 pre-tax income is $200,000 higher. In Years 2 and 3 the capitalizer reports $100,000 of amortization while the expenser reports nothing, so its income is lower, and because its assets are also higher, its ROA in those years is lower.

A company that develops its intangibles internally shows lower assets than an otherwise identical company that buys them, and analysts should adjust for this when comparing the two.

COMMON EXAM TRAPS

- Internally developed trademarks, brands and goodwill are expensed; the same items bought or acquired in a business combination are capitalized.

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- Indefinite-lived intangibles (including goodwill and cheaply renewable trademarks) are not amortized; they are tested for impairment instead.
- US GAAP generally expenses R&D, with software as the exception; for software to be sold the trigger is technological feasibility.
- "Capitalizing raises net income" is true only in the first year; afterward net income and ROA are lower.
- Capitalized outlays are investing cash outflows, so CFO is higher under capitalization in the period the outlay is paid. Later amortization is noncash and does not raise CFO again.

BOTTOM LINE

- Finite-lived intangibles are amortized over their useful life, while indefinite-lived intangibles, such as goodwill, are not amortized and are tested for impairment at least annually.
- Purchased intangibles are recorded at cost, and internally developed intangibles are generally expensed as incurred, so internally created brands, trademarks and goodwill are not on the balance sheet.
- In a business combination the price is allocated to the fair values of the target's identifiable assets and liabilities, including identifiable intangibles the target built and expensed, and the remainder is goodwill.
- Research costs are expensed under both IFRS and US GAAP; development costs may be capitalized under IFRS once the criteria are met and are generally expensed under US GAAP.
- Under US GAAP, software developed for sale is capitalized once technological feasibility is established, and software for internal use once it is probable the project will be completed and the software used as intended.
- Compared with expensing, capitalizing a cost gives higher net income, assets, equity and CFO and lower CFI in the year it is incurred, then lower net income and ROA in later years, with total cash flow identical apart from any tax differences.

QUICK CHECK

Question 1

Holloway Brands owns three trademarks, each with a finite useful life. One was bought from a competitor, one was recognized when Holloway acquired Pinecrest Foods, and one was created by Holloway's own marketing department. Which trademark is *least likely* to be carried on Holloway's balance sheet at amortized cost?

A The trademark bought from a competitor.

B The trademark recognized in the acquisition of Pinecrest Foods.

C The trademark created by Holloway's own marketing department.

Answer: C

The costs of creating an intangible asset such as a trademark internally are generally expensed as incurred, so no asset is recognized and there is nothing to amortize. A trademark bought from another party is recorded at cost, and one obtained in a business combination is recorded at its fair value under the acquisition method; both are then amortized because they have finite lives.

Why the other options are wrong

- **A.** A purchased intangible asset is recognized at cost (its fair value when bought). Because this trademark has a finite life, that cost is amortized, so it does appear at amortized cost.
- **B.** Under the acquisition method, identifiable intangibles of the target are recognized at fair value on the acquisition date, even if the target built them itself. A finite-lived trademark recognized this way is then amortized.

Key takeaway Intangibles that are purchased or acquired in a business combination are on the balance sheet; those created internally are expensed, apart from the R&D and software exceptions.

MODULE 33.2

Impairment and Derecognition

LOS 33.b — Impairment and derecognition of PP&E and intangible assets

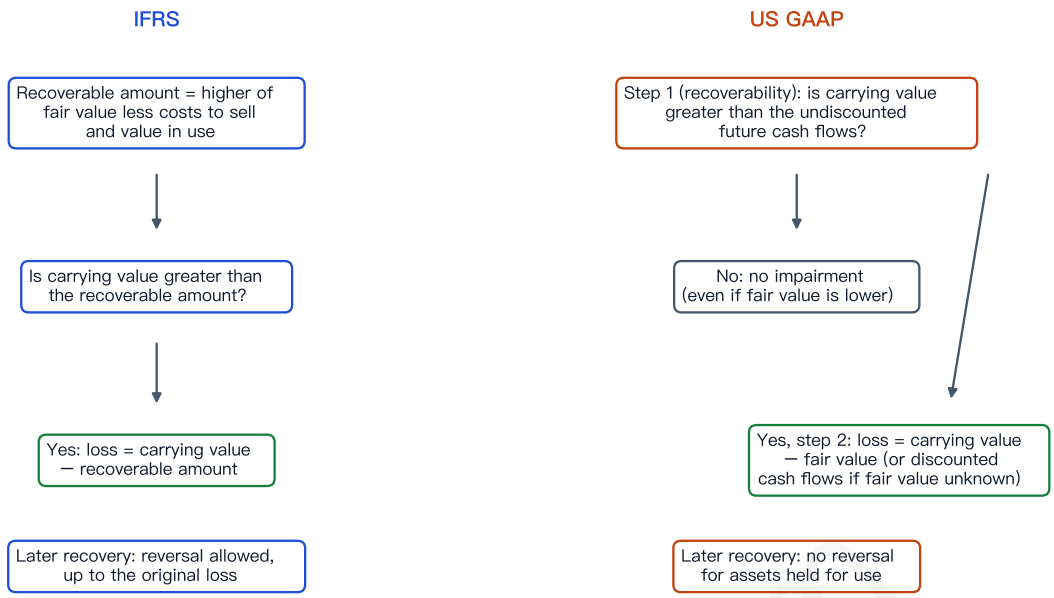
Depreciation and **amortization** allocate an asset's cost over the periods that benefit from it. An **impairment** is an unexpected decline in value that pushes the asset's value below its **carrying value** (cost − accumulated depreciation − any earlier impairment). **Derecognition** removes the asset when it is sold, exchanged, abandoned or distributed.

Impairment of assets held for use: IFRS and US GAAP

These rules cover long-lived assets, tangible or intangible, that have finite lives and are still in use. Both frameworks recognize an impairment as a loss in the income statement.

KEY CONCEPT

	IFRS	US GAAP
When tested	Assess every year whether there are indicators of impairment	Only when events or circumstances suggest the carrying value may not be recoverable
Test	Carrying value > **recoverable amount**; the same comparison measures the loss	Step 1, **recoverability test**: carrying value > **undiscounted future cash flows**
Recoverable amount	Greater of **fair value less costs to sell** and **value in use** (PV of future cash flows)	—
Loss measured as	Carrying value − recoverable amount	Step 2: carrying value − **fair value** (or discounted future cash flows if fair value is unknown)
Later reversal	Allowed, limited to the original loss (carrying value cannot exceed what it would have been without the impairment)	Not permitted for assets held for use



Impairment test for an asset held for use: IFRS versus US GAAP

The recoverable amount is the higher of the two measures because the owner can either sell the asset or keep using it and would pick whichever brings in more cash. The asset is impaired only if even the better of the two cannot recover its carrying value.

KEY CONCEPT

$$\text{IFRS loss} = CV - \max(\text{FV} - \text{costs to sell},\ \text{VIU})$$

$$\text{US GAAP loss} = CV - \text{FV} \ \ (\text{only if } CV > \text{undiscounted CF})$$

Example. A machine cost \$2,000,000 and has accumulated depreciation of \$800,000, so $CV = \$1{,}200{,}000$. Fair value is \$1,105,000, costs to sell are \$15,000, value in use is \$1,130,000 and undiscounted expected cash flows are \$1,150,000.

- IFRS: recoverable amount $= \max(1{,}090{,}000,\ 1{,}130{,}000) = 1{,}130{,}000$; loss $= 1{,}200{,}000 - 1{,}130{,}000 = \$70{,}000$.
- US GAAP: in step 1, undiscounted cash flows of \$1,150,000 are below \$1,200,000, so the asset fails recoverability; in step 2, the loss is $1{,}200{,}000 - 1{,}105{,}000 = \$95{,}000$.

If undiscounted cash flows had exceeded the carrying value, US GAAP would record no loss even if fair value were lower. Because the test uses undiscounted flows, rising interest rates alone do not trigger impairments. The US GAAP loss is measured against fair value itself, not fair value less costs to sell; the discounted cash flows used when fair value is unknown correspond to value in use under IFRS. Both value in use and the undiscounted cash flow estimate depend heavily on management's forecasts (and, for value in use, on the choice of discount rate).

Effects of an impairment on the statements and ratios

KEY CONCEPT

Item	Year of impairment	Later years
Net income	Lower (loss; shown as an unusual or infrequent item if material)	Higher (smaller depreciation base)
Total assets, equity	Lower	Lower
ROA, ROE	Lower	Higher (higher NI, smaller denominators)
Asset turnover	Higher	Higher
Debt-to-equity, debt-to-assets	Higher (debt unchanged, equity/assets lower)	Higher
Cash flow	No effect; the loss is not tax-deductible until disposal	No effect
Deferred tax liability	The tax base is unchanged while the book value falls, so any DTL on the asset decreases	Stays below the no-impairment amount; the gap narrows as the lower book depreciation catches up

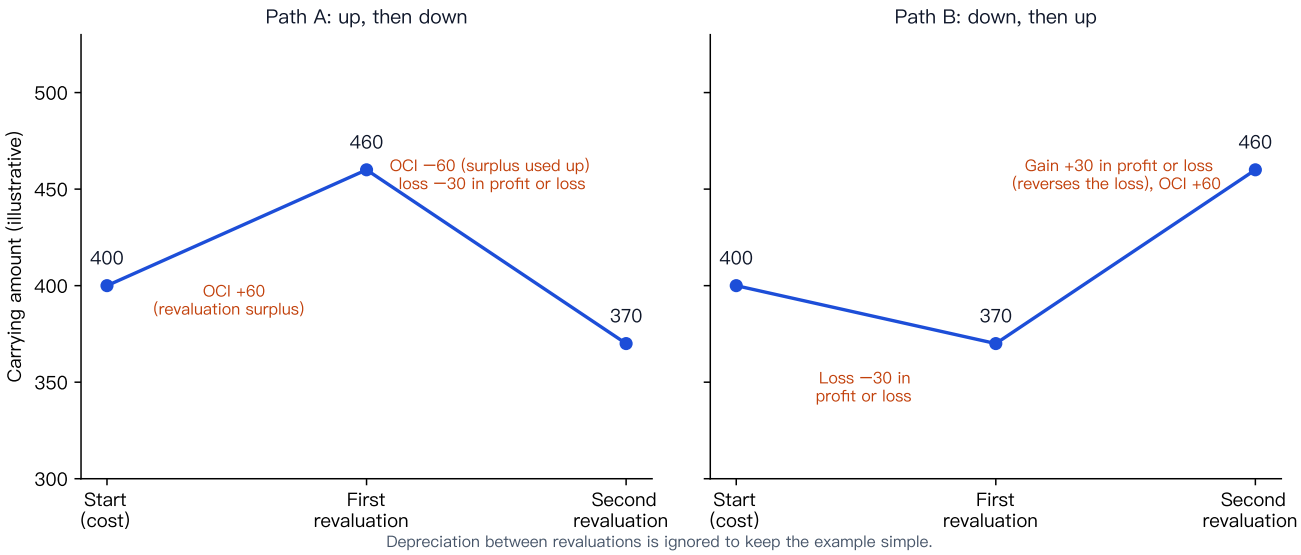
An impairment signals that past depreciation was too low, so past earnings were overstated. Because impairments require judgment, managers can time them. Delaying a loss until a year of strong earnings smooths reported results. Alternatively, management may take a "big bath" when results are already poor for external reasons or when new management arrives, so the low earnings are not blamed on them; the smaller asset and equity base then boosts future ROA and ROE.

Revaluation model (IFRS only)

IFRS lets a firm carry a class of PP&E at fair value (**revaluation model**); US GAAP requires the **cost model** and prohibits upward revaluation of assets held for use.

- An increase goes to other comprehensive income as a **revaluation surplus**. The exception is the part that reverses an impairment previously charged to the income statement, which is recognized as a gain in the income statement.
- A decrease is charged to profit or loss, except to the extent it reverses an existing revaluation surplus (then it reduces OCI).

The order of the changes decides where each one goes. In the figure, the same asset ends at different amounts after two revaluations. Each step first undoes whatever the previous step recorded, in the same place, and only the rest goes to the other place.



Revaluation model: where each change is recognized depends on the order (illustrative values)

- After an upward revaluation, assets and equity are higher, so solvency ratios (debt-to-equity, debt-to-assets) are lower and asset turnover is lower. The larger depreciable base raises future depreciation, so future net income, ROA and ROE are lower.

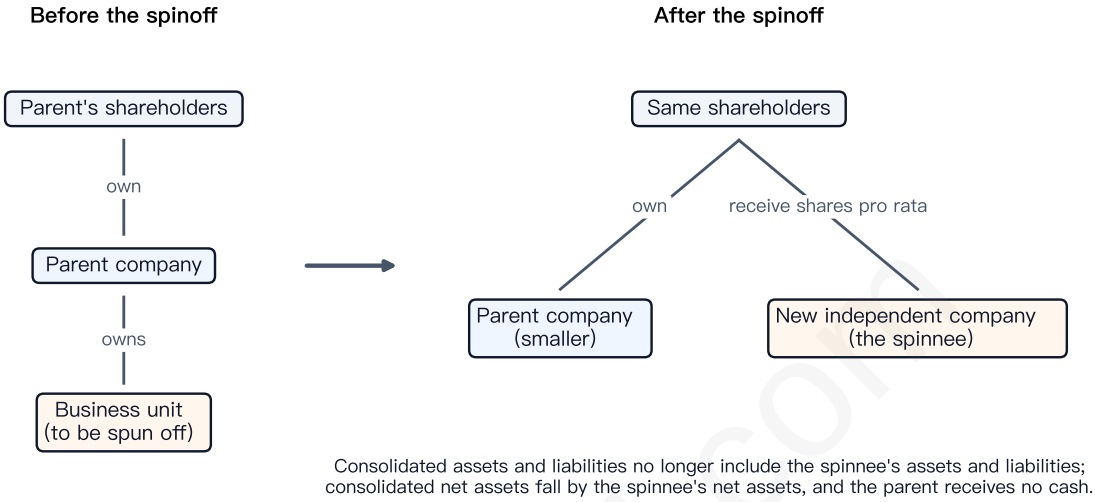
Indefinite-lived intangibles and assets held for sale

- **Indefinite-lived intangibles** are not amortized and are tested for impairment at least annually. Exam convention: the asset is impaired when its carrying amount is above its fair value, and the loss equals the difference; there is no undiscounted cash flow screen. Current practice: US GAAP applies this fair-value comparison, while IFRS (IAS 36) compares the carrying amount with the recoverable amount, the greater of fair value less costs to sell and value in use. With a carrying amount of 100, fair value of 80 and value in use of 110, US GAAP records a loss of 20 and IFRS records none.
- For an asset **held for sale** (intent to sell, sale probable, asset available immediately), depreciation stops, and the asset is impaired if carrying value exceeds fair value less costs to sell; it is then written down to that amount, with the loss in the income statement. Under both IFRS and US GAAP the loss can be reversed, but not above the original carrying value.

Derecognition

Event	Accounting
Sale	Gain or loss = proceeds − carrying value (cost − accumulated depreciation − impairments), reported with other gains and losses or separately if material; with the indirect method the gain/loss is removed from net income in CFO, and the proceeds are an investing inflow
Abandonment	Loss equal to the carrying value (no proceeds)
Exchange	Gain or loss = fair value of old asset (or of new asset if more clearly evident) − carrying value of old asset; new asset recorded at fair value (at the old asset's carrying value if no reliable fair value)

Event	Accounting
Spinoff	A division or subsidiary is placed in a new legal entity whose shares go to the parent's shareholders, who give up no parent shares; the spinnee then leaves the parent's consolidated accounts. Classification before the distribution and any gain or loss: see below



Ownership before and after a spinoff

Exam convention: once a spinoff is probable, the spinnee's assets and liabilities are reclassified as held for sale (also called held for distribution), and no gain or loss is recorded on the distribution. Current practice: IFRS 5 classifies them as held for distribution once the distribution is highly probable, and IFRIC 17 measures a pro rata non-cash distribution to owners at fair value, so a gain or loss can arise unless the entity is under common control. US GAAP keeps them as held and used until the distribution and records the spinoff at carrying amount, with no gain or loss.

Example. Equipment costing $500,000 with $320,000 accumulated depreciation is sold for $150,000: loss $= 150{,}000 - 180{,}000 = -\$30{,}000$.

Example. A delivery van that cost $70,000 and has accumulated depreciation of $44,000 (carrying value $26,000) is traded for a forklift. The van's fair value is $31,000. The van is removed from the balance sheet, the forklift is recorded at $31,000 and a gain of $31{,}000 - 26{,}000 = \$5{,}000$ is reported. If neither fair value could be measured reliably, the forklift would be recorded at $26,000 and no gain or loss would arise.

COMMON EXAM TRAPS

- US GAAP uses undiscounted cash flows to test for impairment but fair value to measure the loss. For the machine, measuring the loss against the undiscounted cash flows gives $50,000 instead of $95,000.

- Under IFRS, the recoverable amount is the higher of two measures: fair value less costs to sell, and value in use. For the machine, using the lower measure, fair value less costs to sell, gives a loss of $110,000 instead of $70,000.
- Impairment raises future ROA and ROE and raises leverage ratios; it has no cash flow effect.
- US GAAP allows no reversal and no upward revaluation for assets in use, so a higher appraisal changes nothing.
- An upward revaluation lowers future profitability (more depreciation) and lowers solvency ratios.

EXAM SHORTCUTS

- Under US GAAP, when undiscounted future cash flows are at or above carrying value there is no impairment, whatever the fair value, so step 2 is not needed.
- Because the US GAAP recoverability test uses undiscounted cash flows, a rise in interest rates alone does not trigger an impairment.

BOTTOM LINE

- Under IFRS, a long-lived asset held for use is impaired when its carrying value exceeds its recoverable amount, the greater of fair value less costs to sell and value in use; the loss is the difference, and a later reversal is allowed up to the original loss.
- Under US GAAP, an asset held for use is impaired only if its carrying value exceeds its undiscounted future cash flows; the loss is carrying value minus fair value (or discounted cash flows if fair value is unknown), and no reversal is permitted.
- In the year of an impairment, net income, assets and equity fall and leverage ratios rise; in later years net income, ROA and ROE are higher; the impairment itself has no cash flow effect.
- Under the IFRS revaluation model, an increase goes to OCI as a revaluation surplus unless it reverses an impairment previously charged to the income statement, and a decrease is charged to profit or loss unless it reverses an existing revaluation surplus.
- An asset held for sale is no longer depreciated and is written down if its carrying value exceeds fair value less costs to sell; under both IFRS and US GAAP the loss can be reversed, but not above the original carrying value.
- On a sale, the gain or loss is proceeds minus carrying value; on an exchange, it is the fair value of the old asset (or of the new asset if that is more clearly evident) minus the carrying value of the old asset.

QUICK CHECK

Question 2

Tamarind Beverages reports under US GAAP. It owns a beverage brand that it acquired several years ago, and management has concluded that the brand has an indefinite useful life. The annual impairment review of the brand produces the following estimates.

Tamarind Beverages: impairment review of an indefinite-lived brand

Item	$ millions
Carrying amount	64
Undiscounted expected future cash flows	90
Present value of expected future cash flows (value in use)	58
Fair value	52
Costs to sell	3

The impairment loss that Tamarind recognizes on the brand is *closest* to:

A $12 million.

B $6 million.

C $0.

Answer: A

An intangible asset with an indefinite useful life is not amortized. It is tested for impairment at least annually, and an impairment loss is recognized when its carrying amount exceeds its fair value. Under US GAAP there is no recoverability screen based on undiscounted cash flows for such an asset, so the $64 million carrying amount is compared directly with the $52 million fair value. The loss of $12 million is measured against fair value itself, with no deduction for costs to sell.

$$\text{Impairment loss} = \text{carrying amount} - \text{fair value} = 64 - 52 = \$12 \text{ million}$$

The brand is written down to $52 million and the $12 million loss is reported in the income statement.

For comparison, the two tests that do not apply here:

Test	Calculation	Result
US GAAP recoverability test for PP&E held for use	undiscounted cash flows 90 > carrying amount 64	no impairment
IFRS recoverable amount	$\max(52 - 3,\ 58) = 58$; loss $= 64 - 58$	loss of 6

Why the other options are wrong

- **B.** $6 million uses the IFRS recoverable amount, the higher of fair value less costs to sell ($49 million) and value in use ($58 million). Tamarind reports under US GAAP, which measures the loss against fair value.
- **C.** $0 applies the US GAAP recoverability test used for PP&E and finite-lived assets held for use: undiscounted cash flows of $90 million exceed the carrying amount, so that test would stop there. An indefinite-lived intangible asset is compared directly with its fair value, so the undiscounted cash flows do not matter.

Key takeaway Under US GAAP, PP&E held for use passes an undiscounted cash flow test before any loss is measured. An indefinite-lived intangible asset skips that screen: whenever its carrying amount exceeds fair value, the excess is an impairment loss, even if undiscounted cash flows are well above the carrying amount.

MODULE 33.3

Long-Term Asset Disclosures

LOS 33.c — Disclosures about PP&E and intangible assets, and what an analyst does with them

Required disclosures

	IFRS	US GAAP
PP&E, per class	Measurement basis, **depreciation method**, useful lives or rates, depreciation expense, gross carrying amount and **accumulated depreciation** at the start and end of the period, reconciliation of opening to closing carrying amount	Depreciation expense for the period, balances of major classes of assets (land, buildings, machinery …), accumulated depreciation by class or in total, general description of depreciation methods
Other PP&E items	**Title restrictions and assets pledged as collateral**, agreements to acquire PP&E	Not listed separately, but title restrictions and assets pledged as collateral are generally disclosed under both frameworks
Revalued assets (IFRS only)	Revaluation date, how fair value was determined, carrying amount under the cost model, **revaluation surplus** in OCI	Not applicable (no revaluation)
Intangible assets	Like PP&E, plus whether useful lives are **finite or indefinite**	Like PP&E, plus **estimated amortization expense for each of the next five years**
Impairments	Losses and **reversals** by class, where they sit in the income statement, circumstances causing them	Description of the impaired asset, circumstances, how fair value was determined, amount of loss, where it sits in the income statement (no reversals to disclose for assets **held for use**, because US GAAP does not permit them)

Presentation. IFRS income statements "by nature" show depreciation and amortization on the face; statements "by function" include them in cost of sales and SG&A. Under the indirect method, depreciation and amortization are added back as noncash charges in CFO; purchases and disposal proceeds of long-lived assets are investing cash flows. US GAAP firms using the direct method must still disclose the indirect reconciliation in the notes. Average age is not a required disclosure; the analyst estimates it.

Review of assumed knowledge: depreciation methods

The reading also assumes the common depreciation methods are known. Depreciation spreads the depreciable cost (cost less residual value) over the periods that use the asset.

Three depreciation methods, with a first-year example (illustrative values: cost $50,000, residual value $5,000, 5-year life, capacity 90,000 units, 22,000 units produced in Year 1)

Method	Annual depreciation	Year 1 example	Pattern
Straight-line	(Cost − residual value) ÷ useful life	(50,000 − 5,000) ÷ 5 = $9,000	Same amount every year
Double-declining balance	(2 ÷ useful life) × carrying value at the start of the year, never taking carrying value below residual value	(2 ÷ 5) × 50,000 = $20,000	High early, falling later
Units-of-production	(Cost − residual value) × units produced ÷ total expected units	45,000 × 22,000 ÷ 90,000 = $11,000	Follows usage

Two details are often tested. Double-declining balance applies its rate to the whole carrying value, not to cost less residual value, but it stops once the carrying value reaches the residual value, so the last year's charge is often a plug. Choosing a longer useful life or a higher residual value lowers depreciation expense, which raises net income and the carrying value of the assets and lowers asset turnover.

Ratios built from the disclosures

$$\text{Fixed asset turnover} = \frac{\text{Revenue}}{\text{Average fixed assets}}$$

Fixed asset turnover measures revenue generated per dollar of fixed assets; a higher ratio indicates more efficient use of long-term assets.

Assuming straight-line depreciation and zero salvage value:

KEY CONCEPT

$$\text{Average age} \approx \frac{\text{Accumulated depreciation}}{\text{Annual depreciation expense}}$$

$$\text{Total useful life} \approx \frac{\text{Gross (historical cost) PP\&E}}{\text{Annual depreciation expense}}$$

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$$\text{Remaining useful life} \approx \frac{\text{Net PP\&E}}{\text{Annual depreciation expense}} = \text{Total useful life} - \text{Average age}$$

KEY CONCEPT

Ratio	Numerator	What it tells the analyst
Average age	Accumulated depreciation	How old the asset base is (see the two uses below)
Total useful life	Gross PP&E (historical cost)	Typical life of the asset base when new
Remaining useful life	Net PP&E	Roughly when major replacement spending will be needed
Capex / depreciation	Capital expenditures	Whether the firm is replacing capacity as fast as it wears out (≈ 1 or more)

Average age is useful in two ways. A company running older, less efficient assets may be at a competitive disadvantage, and the estimate helps predict the timing of major capital expenditures and therefore the company's future (near-term) financing requirements.

Example. Gross PP&E is $1,200 million, accumulated depreciation $450 million and depreciation expense $75 million.

- Average age $= 450/75 = 6$ years
- Total useful life $= 1{,}200/75 = 16$ years
- Remaining useful life $= (1{,}200 - 450)/75 = 750/75 = 10$ years $(= 16 - 6)$

Limitations. The estimates are rough: assets are grouped in broad classes, accelerated methods and material salvage values distort them, and the asset mix matters.

COMMON EXAM TRAPS

- Matching the numerator to the question: accumulated depreciation gives average age, gross PP&E gives total life, and net PP&E gives remaining life. In the example, dividing net PP&E by depreciation gives an "average age" of 10 years instead of 6, and dividing gross PP&E gives a "remaining life" of 16 years instead of 10.
- The ratios use end-of-year balances with the year's depreciation expense.
- Average age is used to anticipate capital spending and financing needs, not to measure earnings potential.
- Only US GAAP requires the five-year amortization forecast; only IFRS has revaluation disclosures, and only IFRS permits (and so discloses) impairment reversals for assets held for use.

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EXAM SHORTCUTS

- Remaining useful life equals total useful life minus average age, so once two of the three estimates are known the third needs no further division.

BOTTOM LINE

- Assuming straight-line depreciation and zero salvage value, average age is about accumulated depreciation divided by annual depreciation expense, total useful life is about gross PP&E divided by it, and remaining useful life is about net PP&E divided by it.
- Fixed asset turnover is revenue divided by average fixed assets, and a higher ratio indicates more efficient use of long-term assets.
- Average age helps show whether a company runs older, less efficient assets and helps predict the timing of major capital spending and the related financing needs.
- Only US GAAP requires estimated amortization expense for each of the next five years, while only IFRS has revaluation disclosures and permits, and so discloses, impairment reversals for assets held for use.
- A longer useful life or a higher residual value lowers depreciation expense, which raises net income and the carrying value of the assets and lowers asset turnover.

QUICK CHECK

Question 3

Harrowgate Freight uses straight-line depreciation with negligible salvage values. Selected data from its PP&E note are shown below.

Harrowgate Freight — PP&E note, latest fiscal year (USD millions)

Item	Amount
Gross PP&E, end of year	840
Accumulated depreciation, end of year	336
Depreciation expense for the year	48

The estimated remaining useful life of Harrowgate's PP&E is *closest* to:

A 7.0 years.

B 10.5 years.

C 17.5 years.

Answer: B

Remaining useful life is net PP&E (gross PP&E minus accumulated depreciation) divided by annual depreciation expense.

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$$\text{Net PP\&E} = 840 - 336 = 504$$

$$\text{Remaining useful life} = \frac{504}{48} = 10.5 \text{ years}$$

Check: total useful life $= 840/48 = 17.5$; average age $= 336/48 = 7.0$; $17.5 - 7.0 = 10.5$.

Why the other options are wrong

- **A.** 7.0 years is the average age (accumulated depreciation / depreciation expense = 336 / 48).
- **C.** 17.5 years is the total useful life (gross PP&E / depreciation expense = 840 / 48).

Key takeaway Remaining life = total life − average age; computing all three is a quick consistency check.

Practice questions

Question 4

Two otherwise identical companies each spend the same amount on a long-lived asset in Year 1. Brackley Tools capitalizes the outlay and depreciates it; Ostrander Tools expenses it immediately. For the years that follow Year 1, Brackley, compared with Ostrander, will *most likely* report:

A Lower net income and higher return on assets.

B Lower net income and lower return on assets.

C Higher net income and lower return on assets.

Answers: 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/straight-line-depreciation/. The full bank has 25 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 34

Finance Lease vs Operating Lease

What you'll learn

LOS 34.a Explain how lessees and lessors report finance and operating leases under IFRS and US GAAP.
LOS 34.b Explain the reporting of defined contribution, defined benefit and share-based compensation plans.
LOS 34.c Describe the presentation and disclosure of leases, pensions and share-based compensation.

MODULE 34.1

Leases

This reading covers lease accounting by lessees and lessors, defined contribution and defined benefit pension plans, share-based compensation, and the disclosures required for each. A candidate must be able to classify a lease, measure the lessee's lease liability and right-of-use asset over the lease term, compute a pension plan's funded status and measure the expense of a share-based award.

LOS 34.a — Financial reporting of leases by lessees and lessors

What a lease is and why firms lease

In a **lease**, the **lessee** buys the right to use a specific asset owned by the **lessor** for a period, in exchange for periodic payments. A contract is a lease only if it (1) refers to a specific asset, (2) gives the lessee effectively all of the asset's economic benefits during the term, and (3) gives the lessee the right to direct how the asset is used.

Leasing is an alternative to borrowing and buying. Typical advantages for the lessee:

- Less initial cash outflow. A lease needs little or no down payment.
- Less costly financing. The lease is effectively secured by the asset, so the implicit rate can be below the rate on a loan to buy it.
- Less risk of obsolescence. The asset usually goes back to the lessor, which bears the residual-value risk and prices it into the implicit rate. If the lessee guarantees a residual value, that risk stays with the lessee.

What the lessee gives up is ownership. It normally does not benefit if the asset rises in value, unless it holds a **purchase option**.

Classification: finance lease vs. operating lease

KEY CONCEPT

A **finance lease** transfers substantially all the **benefits and risks of ownership** to the lessee. If either the benefits or the risks are not substantially transferred, the lease is an **operating lease**. Lessee and lessor classify a given lease the same way. A lease is a finance lease if any one of these conditions holds:

1. Ownership transfers to the lessee by the end of the term.
2. The lessee holds a purchase option that it is expected to exercise.
3. The lease covers most of the asset's useful life.
4. The **present value of the lease payments** is at least equal to the asset's fair value.

5. The asset is so specialized that the lessor has no other use for it.

A long term alone does not decide the classification, because the test compares the term with the asset's useful life. The payment test uses the present value of the payments, not their undiscounted sum.

Exam convention: the purchase option counts when the lessee is expected to exercise it, and the payment test is met when the lease payments have a present value at least equal to the asset's fair value. Current practice: IFRS 16 and ASC 842 require exercise of the purchase option to be reasonably certain, include any residual value guaranteed by the lessee in the payments, and ask whether their present value amounts to substantially all of the fair value. A present value at or above fair value always passes, and one slightly below fair value can also pass.

Lessee accounting

At commencement the lessee records a **right-of-use (ROU) asset** and a **lease liability**. Both equal the present value of the lease payments, discounted at the rate implicit in the lease. The ROU asset is the lessee's right to use the underlying asset; the physical asset stays with the lessor. The lease liability is then carried like an amortizing loan: each payment is split into interest (liability × rate) and principal.

Exam convention: the ROU asset is an intangible asset, not PP&E. Current practice: IFRS 16 lets the lessee present it as a separate line item or within the line where the same kind of asset would appear if owned (for example, PP&E), with disclosure of which lines include ROU assets.

KEY CONCEPT

	IFRS (all leases) and US GAAP finance lease	US GAAP operating lease	Short-term leases (both) and low-value leases (IFRS)
Balance sheet	ROU asset + lease liability	ROU asset + lease liability	Nothing recognized
ROU asset	Amortized straight-line	Reduced by the principal repayment each period, so ROU asset = liability throughout	n/a
Income statement	**Amortization** + **interest expense** reported separately	Single **lease expense** = the lease payment (straight-line)	**Rent expense**, straight-line
Cash flow statement	Principal → **CFF**; interest → CFO (US GAAP) or CFO/CFF (IFRS)	Entire payment → **CFO**	Entire payment → CFO

Exam convention: an IFRS lessee may report the interest part of a lease payment in CFO or CFF (the IAS 7 choice). Current practice: IFRS 18, effective for annual periods beginning on or after 1 January 2027, removes that choice for companies whose main business is not financing; they report interest paid in CFF.

Under IFRS, a short-term lease is one of 12 months or less, and a low-value lease is roughly USD 5,000 or less. Every other IFRS lease, whatever its classification, is accounted for by the lessee like a finance lease.

Exam convention: lease payments are made at the end of each period (in arrears). Current practice: most leases call for payment at the start of each period.

At inception the lease liability is the present value of the remaining lease payments, discounted at the rate implicit in the lease. With equal payments PMT at the end of each of N periods and a periodic rate r:

$$\text{Lease liability}_0 = \sum_{t=1}^{N} \frac{PMT}{(1+r)^t} = PMT \times \frac{1-(1+r)^{-N}}{r}$$

Each period, interest expense is r times the opening liability, and the rest of the payment reduces the liability.

Example. A lessee leases a machine for 3 years, paying $40,000 at the end of each year. The implicit rate is 8%.

$$PV = 40{,}000 \times \frac{1-1.08^{-3}}{0.08} = \$103{,}084$$

On the calculator: N = 3, I/Y = 8, PMT = −40,000, FV = 0, CPT PV. Straight-line amortization is $103{,}084/3 = \$34{,}361$ a year.

Year	Interest (8%)	Principal	Ending liability	Finance lease: ROU asset	Finance lease: total expense	US GAAP operating: expense
1	8,247	31,753	71,331	68,723	42,608	40,000
2	5,706	34,294	37,037	34,361	40,068	40,000
3	2,963	37,037	0	0	37,324	40,000

Under the finance lease method the ROU asset falls faster than the liability in the early years, because the principal repaid early on is smaller than the straight-line amortization. Total expense is front-loaded. Over the whole lease both methods expense the same $120,000.

The cash paid is $40,000 a year under either method; only its classification differs. In Year 1 a US GAAP finance-lease lessee reports the $8,247 of interest in CFO and the $31,753 of principal in CFF. A US GAAP operating-lease lessee reports the whole $40,000 in CFO.

KEY CONCEPT

Effect (lessee)	Finance lease	US GAAP operating lease
ROU asset	Lower	Higher
Lease liability	Same	Same
Net income, early years / later years	Lower / higher	Higher / lower
EBIT	Higher (interest is below EBIT)	Lower
Interest expense	Higher	Lower (none reported)
CFO	Higher	Lower

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Effect (lessee)	Finance lease	US GAAP operating lease
CFF	Lower	Higher

Lessor accounting

In a finance lease, the lessor derecognizes the leased asset and recognizes a **lease receivable**. Its initial amount is the **net investment in the lease**: the present value of the lease payments plus the present value of the expected residual value. The receivable is amortized with the effective interest method, and the lessor reports **interest income**; for a manufacturer or dealer this interest income is part of revenue. A profit or loss arises at inception only if the lease's value differs from the asset's carrying amount.

The net investment in the lease equals the asset's fair value, because the rate implicit in the lease is defined as the discount rate at which the payments plus the residual value are worth the asset's fair value. By the end of the term the receivable has fallen to the expected residual value. If the lessor later disposes of the asset for a different amount, it reports a further gain or loss.

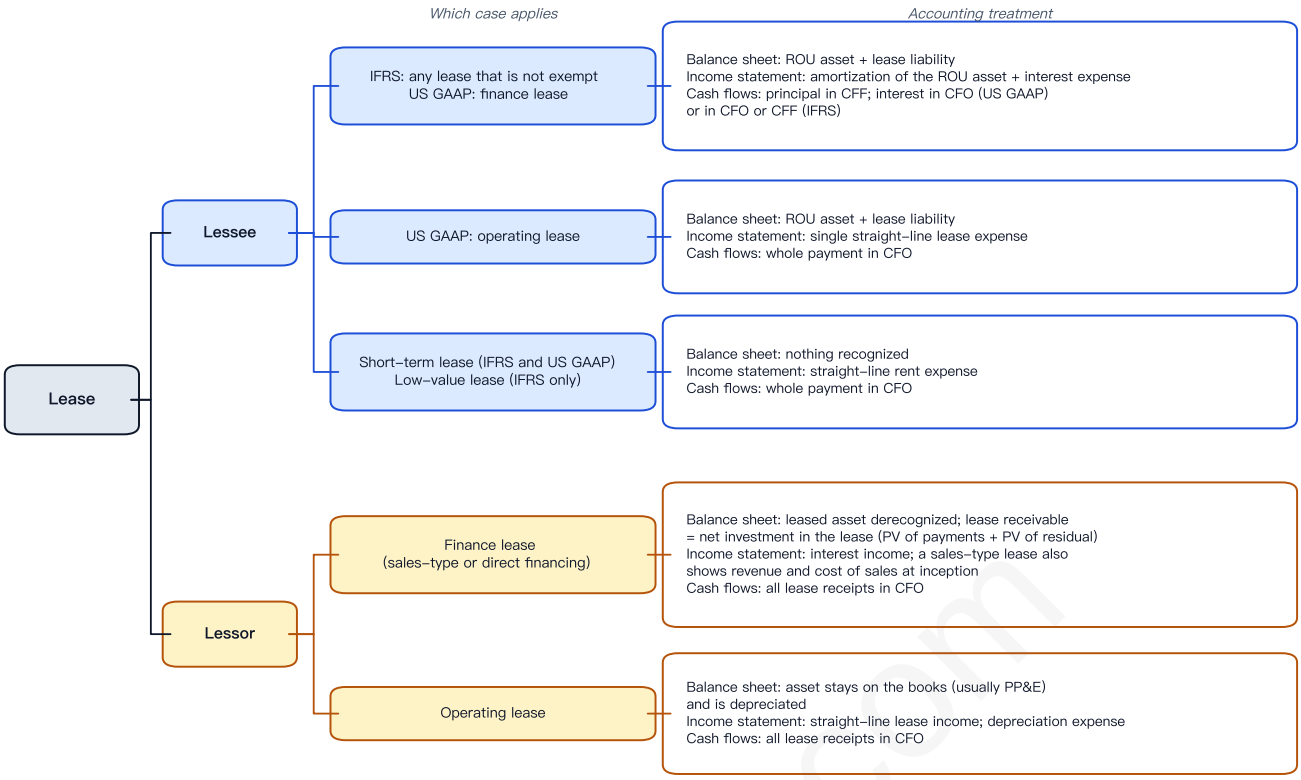
- In a **sales-type lease** the lessor is a manufacturer or dealer. Revenue is the present value of the lease payments, and cost of sales is the carrying value minus the present value of the residual. The selling profit is recognized at inception, as with a sale of inventory.
- In a **direct financing lease** the lessor is a financing company. Exam convention: no gain or loss is recognized at inception; a gain is deferred and recognized as interest income over the term, and a loss is deferred and recognized as an expense over the term. Current practice: under ASC 842 a selling profit on a direct financing lease is deferred and earned through interest income, but a selling loss is recognized at commencement. IFRS 16 does not use the term direct financing lease, and it allows selling profit or loss at commencement only for manufacturer or dealer lessors.

In an operating lease, the lessor keeps the asset on its balance sheet (usually in PP&E) and continues to depreciate it. It reports the lease payments as income on a straight-line basis, with depreciation and other costs as expenses. No receivable is created.

For the lessor, all lease cash inflows are CFO under either classification. Total income over the lease is the same under both methods; only the timing differs.

The figure brings both sides together. For the lessee, the model depends on the reporting framework as well as the classification; for the lessor, it depends on the classification alone.

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Lease accounting models for lessees and lessors

COMMON EXAM TRAPS

- At commencement the lessee's ROU asset equals the lease liability. Both equal the present value of the payments, which is neither the fair value nor the undiscounted total. For the machine lease, this is $103,084, not the undiscounted $120,000.
- Lessees with operating leases still recognize an ROU asset and a lease liability under IFRS and US GAAP. The exceptions are short-term leases and, under IFRS, low-value leases.
- In an operating lease the physical asset stays on the lessor's books. The lessee shows only an ROU asset.
- The principal portion of a lessee's finance lease payment is a financing outflow. For a US GAAP operating lease the whole payment is an operating outflow.
- Sales-type and direct financing leases are both kinds of finance lease. In both, the lessor derecognizes the asset and earns interest income.

EXAM SHORTCUTS

- For a US GAAP operating lease, the lessee's single lease expense equals the lease payment and the ROU asset equals the lease liability throughout, so the expense needs no split into interest and amortization.

BOTTOM LINE

- Exam convention: a lease is a finance lease if any one of these holds: title passes to the lessee by the end of the term, the lessee has a purchase option it is expected to exercise, the term covers most of the asset's useful life, the lease payments have a present value at least equal to the asset's fair value, or the asset is so specialized that the lessor has no other use for it.
- Current practice under IFRS 16 and ASC 842 requires exercise of the purchase option to be reasonably certain, includes any lessee-guaranteed residual value in the lease payments, and asks whether their present value amounts to substantially all of the asset's fair value.
- At commencement a lessee records a right-of-use asset and a lease liability, each measured as the lease payments discounted at the rate implicit in the lease, except for short-term leases and, under IFRS, low-value leases.
- Under IFRS (other than for short-term and low-value leases) and for US GAAP finance leases, the lessee reports straight-line amortization of the ROU asset and interest expense separately, with the principal repaid in CFF; for a US GAAP operating lease it reports one straight-line lease expense and the whole payment in CFO.
- In a finance lease the lessor derecognizes the asset, records a lease receivable equal to the net investment in the lease (present value of the lease payments plus present value of the expected residual value) and reports interest income; a sales-type lessor also recognizes the selling profit at inception.
- In an operating lease the lessor keeps the asset on its balance sheet, continues to depreciate it and reports the lease payments as income on a straight-line basis.

QUICK CHECK

Question 1

Which of the following is the *least likely* reason for Northgate Printing to lease a new press rather than purchase it?

A To lower its financing cost.

B To reduce the cash it pays out at the start.

C To share in any increase in the press's value.

Answer: C

A lessee usually returns the asset at the end of the lease, so it does not benefit from any appreciation (unless the lease includes a purchase option). Capturing gains in the asset's value is a reason to own the asset rather than lease it.

Why the other options are wrong

- **A.** Since the leased asset effectively serves as collateral, the implicit lease rate may be below what a lender would charge to finance a purchase. This is a genuine reason to lease.
- **B.** Leases typically require a small initial payment, if any, so leasing conserves cash at inception compared with a purchase. This is also a genuine reason to lease.

Key takeaway Advantages of leasing: smaller initial outflow, cheaper financing, less obsolescence

risk. Appreciation belongs to the owner (the lessor).

MODULE 34.2

Deferred Compensation and Disclosures

LOS 34.b — Defined contribution, defined benefit and share-based compensation plans

Pensions and **share-based compensation** are forms of **deferred compensation**: employees earn them now but receive them later, so their accounting relies on management estimates.

Defined contribution vs. defined benefit plans

KEY CONCEPT

	Defined contribution plan	Defined benefit plan
Employer's promise	Contribute an agreed amount each period (e.g., a % of salary)	Pay a defined benefit after retirement (e.g., 1.5% × final salary × years of service)
Who bears investment risk	**Employee**	**Employer**
Income statement	**Pension expense = employer's contribution** for the period	Components of the change in funded status (see below)
Balance sheet	Nothing once the contribution is paid	**Net pension asset or liability (funded status)**

A defined benefit plan creates a balance sheet item because the employer has promised the retirement benefit itself rather than a fixed contribution. Each year of service adds to that promise, and the obligation remains until the benefits are paid.

The **funded status** of a defined benefit plan is

KEY CONCEPT

$$\text{Funded status} = \text{Fair value of plan assets} - \text{PV of the defined benefit obligation}$$

A positive amount means the plan is **overfunded** and the sponsor reports a **net pension asset**. A negative amount means it is **underfunded** and the sponsor reports a **net pension liability**.

The sponsor usually funds a defined benefit plan by transferring assets to a separate legal entity, typically a trust, which invests them to meet the benefits as they fall due. The **fair value of plan assets** is the current value of that pool. The obligation is the present value, at the balance sheet date, of the benefits earned to date that will be paid from retirement until death. IFRS calls it the **present value of the defined benefit obligation (PVDBO)**; US GAAP calls it the **projected benefit obligation (PBO)**. Both terms describe the same obligation. Estimating the obligation requires external actuaries and assumptions about future compensation (salary growth), employee

turnover, retirement age, mortality and the discount rate. Post-retirement health care plans are typically not prefunded with plan assets, so they appear as a liability. Analysts treat a net pension liability like debt when computing leverage.

Example. Plan assets of $780 million and a benefit obligation of $860 million give a funded status of −$80 million, which is a net pension liability of $80 million. If that is the net liability at the start of a year and the discount rate is 5%, IFRS net interest expense for the year is $0.05 \times 80 = \$4$ million.

Where the change in funded status goes

KEY CONCEPT

Component	IFRS	US GAAP
Service cost (current period)	Income statement	Income statement
Past service cost	Income statement (part of service cost)	OCI, then amortized into income over the service period
Interest	**Net interest** = discount rate × net pension asset/liability, in the income statement (income if the plan starts the year overfunded, expense if underfunded)	**Interest cost** (growth of the obligation with the passage of time), in the income statement
Return on plan assets	Netted against interest cost inside net interest; the difference between actual and expected return is a remeasurement	**Expected return on plan assets** shown separately (it can use a different rate) and reduces expense in the income statement
Remeasurements / actuarial gains and losses	OCI (never recycled)	OCI, generally amortized into income (immediate recognition allowed)

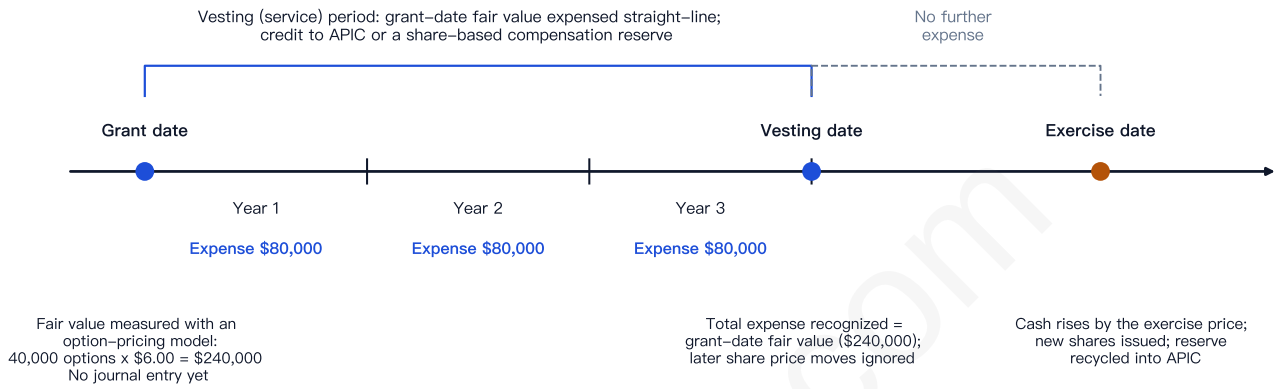
US GAAP therefore has five components: three in the income statement and two in OCI. Given the same actuarial assumptions, the **total periodic cost** of the plan is the same under both standards. The standards differ only in how that cost is divided between the income statement and OCI. Manufacturers allocate pension expense to inventory (and so to COGS) and to SG&A, so the full amount may be visible only in the financial statement notes.

Share-based compensation

Share-based pay aligns managers' and shareholders' interests without an immediate cash outflow. Newly issued shares, however, **dilute** existing owners and reduce EPS. The main criticisms are that employees have limited control over the share price, that share grants can make managers too **risk-averse** while options (with their asymmetric payoff) can encourage **excessive risk taking**, and that performance shares based on accounting metrics can invite manipulation.

For equity-settled awards (stock grants, restricted stock units, performance shares, employee stock options), both IFRS and US GAAP require the company to measure the **fair value at the grant date** and to expense it over the **vesting period** (service period). The vesting period runs from the grant date until the employee receives the shares or is first able to exercise the option. The grant-date value is not updated afterward, so a rise or fall in the share price after the grant does not change the expense. If vesting is immediate, the whole fair value is expensed at grant, and common stock and APIC rise by the same amount.

The timeline follows an award of 40,000 employee stock options with a grant-date fair value of $6.00 each and a three-year vesting period.



Timeline of an equity-settled employee stock option award

Instrument	Fair value at grant	Accounting
Stock grants (awarded outright, with restrictions, or contingent on performance); **restricted stock units** (grants that vest only after service or performance criteria are met); **performance shares** (grants tied to a target such as ROE rather than the share price)	Share price on the grant date	Expense straight-line over the service period; credit an equity reserve or APIC; at vesting the reserve is recycled into common stock and APIC
Employee stock options	Option-pricing model (e.g., Black-Scholes-Merton, binomial); inputs such as volatility are subjective	No entry at grant; expense straight-line over the vesting period with a credit to APIC or a share-based compensation reserve; on exercise the company issues new shares, cash rises by the exercise price, equity rises by the same amount (common stock at par and APIC), and any reserve is recycled into APIC; if vested options expire unexercised, no adjustment is made

Instrument	Fair value at grant	Accounting
Stock appreciation rights (SARs) / **phantom stock** (usually **cash-settled**)	Payoff linked to the change in the share price; the holder does not need to own the stock	Paid in cash, so there is no dilution but there is a cash outflow when the stock does well; option-like payoffs avoid the risk-aversion bias of share grants; phantom stock, used by firms whose shares are not exchange traded, pays according to a hypothetical stock

Exam convention: share-based pay is measured at grant-date fair value and expensed over the service period; the expense lowers retained earnings and the offsetting credit to APIC or a reserve leaves total equity unchanged. Current practice: this holds for **equity-settled** awards, while cash-settled awards such as SARs and phantom stock are carried as a liability that is remeasured to fair value at each reporting date until settlement, with the changes in profit or loss, so total equity falls by the expense. Both kinds of award are measured at fair value. Intrinsic value (share price − exercise price) is not used, and an at-the-money or out-of-the-money option still has a positive grant-date fair value.

> *Example.* A company grants 20,000 restricted stock units when the share price is $30, vesting after three years. The expense is $20{,}000 \times 30/3 = \$200{,}000$ per year.

LOS 34.c — Presentation and disclosures for leases, pensions and share-based pay

Lease disclosures (IFRS 16)

Lease disclosures help users assess the effect of leases on the entity's financial position, its performance and its cash flows. Lessees and lessors provide both qualitative and quantitative information. IFRS and US GAAP set the objectives; companies have discretion over how to meet them, within the guidance the standards give.

Lessee	Lessor, finance leases	Lessor, operating leases
Carrying amount of ROU assets by class; additions to ROU assets	**Selling profit or loss** on derecognition	**Lease income**, variable payments shown separately
Interest expense on lease liabilities; ROU amortization by class	**Finance income** on the net investment in the lease	**Maturity analysis** of lease payments receivable (each of next 5 years + thereafter)
Total lease cash outflows; expenses for short-term and low-value leases and for variable payments	Income from variable payments not in the net investment	Leased asset stays on the books, so PP&E disclosures (IAS 16) by class and **impairment** disclosures (IAS 36) apply
Maturity analysis of lease liabilities; split into current and noncurrent	Explanation of significant changes in the net investment	
Nature of leasing activities, residual value guarantees, **restrictions and covenants** imposed by leases, sale and leaseback transactions	**Maturity analysis** of lease payments receivable; reconciliation of undiscounted payments to the net investment	

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A **maturity analysis** is the item common to lessees and both types of lessor. In the lessee's balance sheet, the principal due within the next year is a current liability and the remaining principal is long-term.

Variable lease payments come in two types. Payments linked to an index or rate (such as an inflation index or a market reference rate) are included in the lease liability and ROU asset at the current index level. The liability and the ROU asset are remeasured when the index changes. Payments that depend on the lessee's future sales or on how much the asset is used are left out of the liability and expensed as incurred.

Pension disclosures (IAS 19)

- Defined contribution plans. The only requirement is to disclose separately the **employer's contribution expensed** in the income statement.
- Defined benefit plans. The disclosures have three objectives: (1) explain the **characteristics and risks** of the plan; (2) identify the amounts in the financial statements arising from it; (3) describe how it affects the **amount, timing and uncertainty of future cash flows**, which mainly means employer contributions. Minimum disclosures include the plan's nature, governance and risks; reconciliations of plan assets, obligation and funded status; a **sensitivity analysis** of the obligation to key actuarial assumptions; the composition of plan assets; **expected employer contributions** for the next period; and the maturity profile of the obligation.

Share-based compensation disclosures

These disclosures help users understand the nature and extent of the arrangements and their effect on current and future cash flows. Companies must disclose the nature and key terms of the plans (grant date, vesting date, service period, settlement in shares or cash), **how grant-date fair value was determined**, and the **effect on earnings and financial position** (income statement and balance sheet).

COMMON EXAM TRAPS

- Only defined benefit plans create a net pension asset or liability on the balance sheet.
- Under IFRS, remeasurements (actuarial gains and losses) go to OCI and never pass through the income statement.
- IAS 19's third objective concerns future cash flows. Future net income is not one of the objectives.
- Equity-settled share-based pay is measured at grant-date fair value and expensed over the vesting period under both IFRS and US GAAP. Cash-settled SARs and phantom stock are instead a liability remeasured each period. The share price at the reporting date is not a required disclosure.
- Lessor operating lease disclosures include impairment of the leased asset. Selling profit and finance income are finance lease disclosures.

BOTTOM LINE

- In a defined contribution plan the employee bears the investment risk and pension expense equals the employer's contribution; in a defined benefit plan the employer bears the investment risk and reports the plan's funded status on the balance sheet.
- Funded status equals the fair value of plan assets minus the PVDBO (called the PBO under US GAAP); a positive amount is a net pension asset and a negative amount a net pension liability.
- Under IFRS, service cost (including past service cost) and net interest, the discount rate times the net pension asset or liability, go to the income statement, while remeasurements go to OCI and are never recycled.

- Given the same actuarial assumptions, IFRS and US GAAP give the same total periodic cost for a defined benefit plan and differ only in how it is split between the income statement and OCI.
- Equity-settled share-based pay is measured at fair value on the grant date and expensed over the vesting period under both IFRS and US GAAP, and later share price changes do not change the expense.
- IAS 19 disclosures for a defined benefit plan explain its characteristics and risks, identify the amounts it creates in the financial statements, and describe its effect on the amount, timing and uncertainty of future cash flows.

QUICK CHECK

Question 2

Marrick Semiconductor grants employee stock options on the terms below and expects all of them to vest.

Marrick Semiconductor — employee stock option grant (1 January)

Item	Value
Options granted	60,000
Share price on grant date	$38.00
Exercise price	$40.00
Fair value per option on grant date (option-pricing model)	$7.50
Vesting period	4 years (all options vest together)

The compensation expense Marrick recognizes in the first year is *closest* to:

A $450,000.

B $112,500.

C $0.

Answer: B

Both IFRS and US GAAP measure an employee stock option at its fair value on the grant date, estimated with an option-pricing model, and recognize that amount straight-line over the vesting period as compensation expense (with a matching credit to equity).

$$\text{Total fair value} = 60{,}000 \times 7.50 = \$450{,}000$$

$$\text{Annual expense} = \frac{450{,}000}{4} = \$112{,}500$$

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Each year: compensation expense $112,500; equity (APIC or a share-based compensation reserve) +$112,500, so total equity is unchanged.

Why the other options are wrong

- **A.** $450,000 is the total grant-date fair value ($60{,}000 \times 7.50$), which would be expensed at once only if the options vested immediately. Here it is spread over four years.
- **C.** $0 uses the intrinsic value at grant (the share price of $38 is below the $40 exercise price). The expense is based on the options' grant-date fair value, which is positive even though they are out of the money.

Key takeaway Option expense equals grant-date fair value divided by the vesting years. It is not based on intrinsic value and is not remeasured for later share price changes.

Practice questions

Question 3

Aerolinea Sur leases a regional jet from Skyward Capital under an arrangement classified as an operating lease. The jet itself is reported as a physical asset on the balance sheet of:

A neither the lessor nor the lessee.

B the lessee.

C the lessor.

Question 4

Which of the following items is *least likely* to be a required disclosure about Tavener Systems' share-based compensation plans?

A How the fair value of the awards at the grant date was determined.

B The effect of share-based payment transactions on the company's balance sheet.

C Tavener's share price at the reporting date.

Answers: 3. C · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/finance-lease-vs-operating-lease/. The full bank has 31 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 35

Deferred Tax Liability

What you'll learn

LOS 35.a Contrast accounting profit, taxable income, taxes payable and income tax expense, and temporary versus permanent differences between accounting profit and taxable income.

LOS 35.b Explain how deferred tax liabilities and assets are created and how analysts should treat them (liability, equity, realizability) for financial analysis.

LOS 35.c Calculate, interpret and contrast an issuer's effective tax rate, statutory tax rate and cash tax rate.

LOS 35.d Analyze deferred tax disclosures and the effective tax rate reconciliation, and explain how they affect financial statements and ratios.

MODULE 35.1

Differences Between Accounting Profit and Taxable Income

This reading explains why tax reporting and financial reporting differ, how temporary differences create deferred tax liabilities and assets, and how analysts treat those balances and the related disclosures. A candidate must be able to compute income tax expense from taxes payable and the changes in deferred taxes, remeasure deferred taxes after a change in the tax rate, and calculate and interpret the statutory, effective and cash tax rates.

LOS 35.a — Two sets of books: tax return versus financial statements

Companies prepare their financial statements under IFRS or US GAAP, but they compute the tax they owe under the tax law of each jurisdiction. The two sets of rules often disagree about when an item counts, and sometimes about whether it counts at all. As a result, the tax charge in the income statement rarely equals the tax on the return.

Differences between tax reporting and financial reporting arise when:

- revenues and expenses are recognized in different periods in the income statement and on the tax return;
- some items count for the income statement only, or for the tax return only;
- assets or liabilities have carrying amounts that differ from their tax bases;
- gains or losses are recognized differently in the income statement and on the tax return;
- tax losses from prior periods offset future taxable income;
- adjustments made in the financial statements do not affect the tax return, or affect it in a different period.

Terms on each side

Tax return side	Financial reporting side
Taxable income: income subject to tax according to the tax return	**Accounting profit**: pretax income under accounting standards (also called income before tax or earnings before tax)
Taxes payable: the tax liability produced by taxable income (also called the *current tax expense*); the term also names the balance sheet liability for taxes due but not yet paid	**Income tax expense**: the tax charge in the income statement, equal to taxes payable adjusted for the changes in deferred tax liabilities and assets

Tax return side	Financial reporting side
Income tax paid: the actual cash paid for income taxes, which can include payments or refunds relating to other years	**Deferred tax liabilities (DTLs)**: balance sheet amounts arising when income tax expense exceeds taxes payable, expected to cause future cash outflows
Tax loss carryforward: a current or past taxable loss that can be used to reduce taxable income (and taxes payable) in future periods; it can create a deferred tax asset	**Deferred tax assets (DTAs)**: balance sheet amounts arising when taxes payable exceed income tax expense, expected to be recovered from future operations; tax loss carryforwards also create them
Tax base: the net amount of an asset or liability used for tax reporting	**Carrying value**: the net balance sheet amount of an asset or liability

A **valuation allowance** is a reduction of deferred tax assets that reflects the likelihood that they will not be realized. Under US GAAP it is a contra account.

Temporary versus permanent differences

- A **temporary difference** arises when an asset's or liability's carrying value differs from its tax base, and the gap will lead to taxable or deductible amounts in later periods. Over the life of the item the same total passes through both the income statement and the tax return; only the timing differs. Temporary differences create deferred tax assets or deferred tax liabilities.
- A **permanent difference** is a gap between pretax income and taxable income that never reverses. Examples are revenue that is never taxable (such as tax-exempt municipal bond interest in the United States), expenses that are never deductible (such as premiums on life insurance for key officers, or fines and donations where the law disallows them), and tax credits. No DTA or DTL results from a permanent difference. They make the effective tax rate differ from the statutory tax rate.

Taxes payable are driven by taxable income × tax rate. Pretax accounting profit × tax rate does not determine them. Temporary differences therefore make taxes payable differ from income tax expense, and the change in deferred taxes bridges the two. Income tax expense can in turn differ from pretax income × the statutory rate, for the reasons listed under LOS 35.c.

When is a DTL created?

> **KEY CONCEPT**
>
> A **deferred tax liability (DTL)** is created when
>
> - revenues or gains appear in the income statement *before* they are taxed, or
> - expenses or losses are deducted on the tax return *before* they are expensed in the income statement.

The classic case is accelerated depreciation on the tax return with straight-line depreciation in the accounts. These are called **taxable temporary differences**: the firm pays less tax now and more tax when the difference reverses.

When is a DTA created?

A **deferred tax asset (DTA)** is created when

- revenues or gains are taxed in an *earlier* period than they reach the income statement (for example, **unearned revenue** taxed on receipt),

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- expenses or losses are expensed in the income statement *before* they are deductible (for example, warranty provisions, bad-debt provisions, post-employment benefits, restructuring accruals, or R&D expensed in the accounts but capitalized for tax), or
- **tax loss carryforwards** are available to reduce future taxable income.

These are **deductible temporary differences**: they provide future tax savings.

A quick test: if a temporary difference first makes taxable income lower than pretax accounting profit, a DTL is created; if it first makes taxable income higher than pretax accounting profit, a DTA is created.

The balance sheet test (carrying value vs. tax base)

KEY CONCEPT

Item	Relationship	Result
Asset	Carrying value > tax base	Deferred tax liability
Asset	Carrying value < tax base	Deferred tax asset
Liability	Carrying value > tax base	Deferred tax asset
Liability	Carrying value < tax base	Deferred tax liability

The DTA or DTL balance is the absolute value of (carrying value − tax base) × tax rate. For a liability such as a warranty provision, the tax base is the carrying value minus the amounts that will be deductible in future tax returns. If the whole provision will be deductible when claims are paid, the tax base is zero.

The income tax expense equation

Income tax expense is also called the **tax provision**. The current period's transactions created or changed the deferred tax balances, so accrual accounting records those changes in this period's tax expense, on top of the taxes payable on this year's taxable income:

KEY CONCEPT

$$\text{income tax expense} = \text{taxes payable} + \Delta DTL - \Delta DTA$$

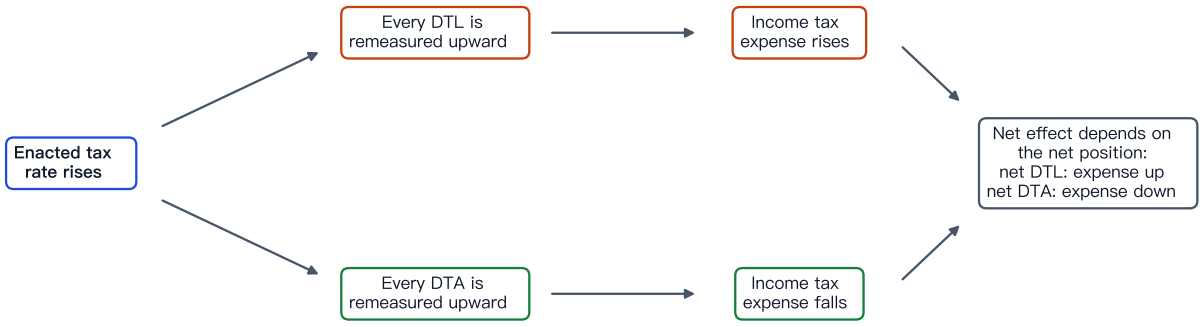
Any rearrangement must preserve the signs, e.g. $\Delta DTL = \text{income tax expense} - \text{taxes payable} + \Delta DTA$ and $\Delta DTA = \text{taxes payable} - \text{income tax expense} + \Delta DTL$. Creating or increasing a DTL raises tax expense; creating or increasing a DTA lowers it.

Example. Kestrel Tools reports taxes payable of \$1,150. During the year its DTL rose by \$90 and its DTA rose by \$40. Income tax expense is $1{,}150 + 90 - 40 = \$1{,}200$.

Changes in the enacted tax rate

DTAs and DTLs are measured at the tax rate expected to apply when they reverse. If the enacted rate changes, existing balances are remeasured. An increase in the rate raises both DTAs and DTLs, and a decrease lowers both. The remeasurement flows through income tax expense. Whether a rate increase raises or lowers that

expense depends on the net position. A firm with a net DTL sees its DTL rise by more than its DTA, so tax expense rises. A firm with a net DTA sees its DTA rise by more, so tax expense falls.



A rate cut works the same way with every arrow reversed.

How a change in the enacted tax rate flows through deferred taxes to income tax expense

Example. Orla Freight has a DTL of $64,000 and a DTA of $16,000, both measured at 20%. The rate rises to 25%. The underlying differences are 64,000/0.20 = 320,000 and 16,000/0.20 = 80,000. The new DTL is 80,000 (up 16,000) and the new DTA is 20,000 (up 4,000). Income tax expense rises by 16,000 − 4,000 = $12,000, so net income falls by $12,000.

When new temporary differences arise in the same year as a rate change, work in this order:

1. Taxes payable = taxable income × the rate in force this year.
2. Cumulative temporary difference at year-end = beginning difference (beginning DTL ÷ old rate) + the difference created this year.
3. Ending DTL = cumulative difference × the rate enacted for the years in which it will reverse.
4. Income tax expense = taxes payable + (ending DTL − beginning DTL).

Example. Harlan Mills begins the year with a DTL of $90 from accelerated tax depreciation, measured at 30%. This year pretax income is $1,000 and taxable income is $800; the $200 gap is a new taxable temporary difference. The rate for this year is 30%, and a cut to 25%, effective from next year, is enacted during the year. Taxes payable are $800 \times 0.30 = 240$. The cumulative difference is $90/0.30 + 200 = 500$, so the ending DTL is $500 \times 0.25 = 125$. Income tax expense is $240 + (125 - 90) = \$275$. Without the rate cut it would have been $240 + (150 - 90) = \$300$, which is 30% of pretax income.

COMMON EXAM TRAPS

- Confusing taxes payable (tax return) with income tax expense (income statement).
- Treating a permanent difference as if it created a DTA or DTL. It changes the effective tax rate instead.
- Getting the direction wrong. A tax deduction taken earlier than the book expense creates a DTL; a book expense recognized earlier than the tax deduction creates a DTA.
- A tax rate change remeasures existing deferred balances. It does not by itself create a new DTA.

- After a rate change, compute the changes in the DTL and the DTA separately. The net effect on tax expense is ΔDTL − ΔDTA. For Orla Freight, adding the two increases gives a $20,000 rise in tax expense instead of $12,000.

EXAM SHORTCUTS

- If a temporary difference first makes taxable income lower than pretax accounting profit, it creates a DTL; if it first makes taxable income higher, it creates a DTA.
- After an increase in the enacted tax rate, the direction of the change in income tax expense follows from the net position alone: it rises for a firm with a net DTL and falls for a firm with a net DTA.

BOTTOM LINE

- Taxes payable are computed from taxable income on the tax return, while income tax expense, the income statement charge, equals taxes payable + ΔDTL − ΔDTA.
- A DTL arises when revenues or gains reach the income statement before they are taxed, or when expenses or losses are deducted for tax before they are expensed; a DTA arises in the reverse cases and from tax loss carryforwards.
- An asset whose carrying value exceeds its tax base, or a liability whose carrying value is below its tax base, gives a DTL; the opposite cases give a DTA, equal to the absolute value of (carrying value − tax base) × tax rate.
- Permanent differences never reverse, create no DTA or DTL, and make the effective tax rate differ from the statutory rate.
- When the enacted tax rate changes, existing DTAs and DTLs are remeasured at the new rate, and the remeasurement flows through income tax expense.

QUICK CHECK

Question 1

When pretax income reported in the financial statements differs from taxable income because of a temporary difference, the *most likely* consequence is:

A the recognition of a deferred tax asset or deferred tax liability.

B a gain or loss recognized in other comprehensive income.

C a permanent gap between the firm's effective and statutory tax rates.

Answer: A

A temporary difference creates a deferred tax item: a DTL if income tax expense (financial reporting) exceeds taxes payable (tax reporting), or a DTA if taxes payable exceed income tax expense. Permanent differences, rather than temporary ones, are what make the effective tax rate differ from the statutory rate.

Why the other options are wrong

- **B.** Neither temporary nor permanent differences generate a gain or loss; they affect tax balances and tax expense.

- **C.** Temporary differences typically do not make the effective and statutory rates differ, because the deferred tax expense adjusts income tax expense to the statutory rate on pretax income. Permanent differences cause that gap.

Key takeaway A temporary difference creates a deferred tax item. A permanent difference makes the effective rate differ from the statutory rate.

MODULE 35.2

Deferred Tax Assets and Liabilities

LOS 35.b — Creating deferred tax balances and treating them in analysis

How the balances build up and reverse

Measuring a deferred tax balance at any date depends on precise definitions of carrying value and tax base.

For a depreciable asset, the **carrying value** is cost less accumulated book depreciation. The **tax base** is cost less accumulated tax depreciation, which equals the tax depreciation still to be deducted in future returns. Liabilities follow the tax-base rule given under LOS 35.a, with one exception: revenue received in advance is taxed on receipt, so its tax base is the carrying value less amounts that will *not* be taxable in future.

Example. Linnet Packaging buys a machine for $48,000 with a four-year life and no residual value. The books use straight-line depreciation ($12,000 a year). For tax, the machine is depreciated straight-line over two years ($24,000 a year). The tax rate is 20%.

End of year	Carrying value	Tax base	Difference	DTL (20%)	Change in DTL
1	36,000	24,000	12,000	2,400	+2,400
2	24,000	0	24,000	4,800	+2,400
3	12,000	0	12,000	2,400	−2,400
4	0	0	0	0	−2,400

The DTL builds while tax depreciation runs ahead of book depreciation. It then reverses to zero once tax deductions stop but book depreciation continues. In Years 1–2 income tax expense exceeds taxes payable by $2,400 a year; in Years 3–4 it is $2,400 lower.

Warranties. The provision is recognized at the time of sale under the matching principle, while the tax deduction waits until repair costs are incurred. The liability's tax base is therefore zero (LOS 35.a).

Receivables. A receivable of $50,000 carries a bad-debt allowance of $4,000, so its carrying value is $46,000. If bad debts are deductible only when they are written off, the tax base is the full $50,000. The asset's carrying value is below its tax base, so at a 25% tax rate there is a DTA of $4{,}000 \times 0.25 = \$1{,}000$.

Realizability of deferred tax assets

Deferred tax balances are not discounted to present value. A DTA has value only if the firm will earn enough future taxable income to use it, so DTAs are reviewed at every balance sheet date.

KEY CONCEPT

	IFRS	US GAAP
Doubt about recovering a DTA	The DTA is reduced directly (a smaller DTA is reported)	The full DTA is kept, and a **valuation allowance** is deducted if it is *more likely than not* that some of it will not be realized
Effect on earnings	Reduction increases income tax expense	Creating or increasing the allowance increases income tax expense and reduces net income; reducing it does the opposite

Management must be able to support the carrying amount of every DTA. Evidence against realization includes cumulative recent losses and a history of tax loss carryforwards expiring unused. Evidence for realization includes order backlogs and existing profitable contracts. The judgement is subjective, so changes in the valuation allowance are a potential tool of **earnings management**: a surprise reduction in the allowance boosts earnings.

Example. A US GAAP firm reports taxes payable of $260, an increase in its DTL of $18, an increase in its gross DTA of $12 and an increase in its valuation allowance of $5. The net DTA rises by only 12 − 5 = 7, so income tax expense is $260 + 18 - 7 = \$271$.

Treatment for analytical purposes

The central question is whether, and when, the DTL will reverse.

KEY CONCEPT

Expectation	Analytical treatment
DTL expected to reverse (e.g., capital spending flat or falling)	Treat as a **liability**
DTL not expected to reverse in the foreseeable future (typically because capital expenditures keep growing, so new timing differences replace the reversing ones)	Treat as **equity**: reduce liabilities and increase equity by the same amount

In practice treatments vary, so the analyst decides **case by case** whether, and when, each DTL will reverse.

Reclassifying a DTL as equity leaves total assets unchanged, lowers liabilities and raises equity. The **debt-to-equity ratio** and the financial leverage ratio (assets/equity) therefore fall. Return on equity also falls because its denominator is larger. **Return on assets** is unaffected, since neither net income nor total assets changes.

COMMON EXAM TRAPS

- A DTL that is expected not to reverse is treated as equity. It is neither ignored nor offset by a valuation allowance.
- A valuation allowance applies to DTAs only. Increasing it raises tax expense. In the example, ignoring the $5 increase in the allowance gives tax expense of $266 instead of $271.
- Under IFRS there is no separate valuation allowance account; the DTA is reduced directly.
- Deferred taxes are not discounted.

- Reclassifying a DTL to equity lowers leverage ratios, while ROA does not change.
- The DTL at a date is (carrying value − tax base) × rate, using the balances at that date. The change during the year is a different number. For Linnet at the end of Year 2, reporting the year's change gives $2,400 instead of the DTL of $4,800.

EXAM SHORTCUTS

- Reclassifying a DTL as equity changes neither net income nor total assets, so an answer in which ROA changes can be ruled out.

BOTTOM LINE

- For a depreciable asset, the carrying value is cost less accumulated book depreciation and the tax base is cost less accumulated tax depreciation.
- Deferred tax assets and liabilities are not discounted to present value.
- Under US GAAP a DTA is kept in full and reduced by a valuation allowance when it is more likely than not that part of it will not be realized, while IFRS reduces the DTA directly; either reduction raises income tax expense.
- Because realizing a DTA is a subjective judgment, changes in the valuation allowance are a potential tool of earnings management, and a surprise reduction boosts earnings.
- A DTL expected to reverse is treated as a liability, while one not expected to reverse in the foreseeable future is treated as equity, and the analyst decides case by case.
- Reclassifying a DTL as equity lowers the debt-to-equity ratio, the financial leverage ratio and ROE, and leaves ROA unchanged.

QUICK CHECK

Question 2

An analyst reclassifies a company's deferred tax liability from liabilities to equity. Which of the following ratios is *least likely* to be affected by this reclassification?

A Return on assets.

B Debt-to-equity ratio.

C Return on equity.

Answer: A

Reclassifying a DTL moves an amount from liabilities to equity; total assets and net income are unchanged, so return on assets is unaffected. Equity rises, which changes every ratio that uses equity in the numerator or denominator.

Why the other options are wrong

- **B.** The debt-to-equity ratio falls because liabilities decrease and equity increases.
- **C.** Return on equity falls because the equity denominator becomes larger while net income is unchanged.

Key takeaway Reclassifying a DTL as equity lowers leverage ratios and ROE but leaves ROA unchanged.

MODULE 35.3

Tax Rates and Disclosures

LOS 35.c — Statutory, effective and cash tax rates

Three tax rates measure different things:

KEY CONCEPT

Rate	Definition	What it shows
Statutory tax rate	The corporate income tax rate of the jurisdiction where the company is domiciled	The "headline" rate set by law
Effective tax rate	$\frac{\text{income tax expense}}{\text{pretax income}}$	The tax charge actually reported per unit of accounting profit; the most useful rate for forecasting earnings
Cash tax rate	$\frac{\text{cash taxes paid}}{\text{pretax income}}$	The cash burden of taxes; useful for forecasting cash flows

The effective and cash tax rates both use pretax (accounting) income as the denominator. The statutory rate is not computed from the financial statements at all; it is set by tax law and applied to taxable income. When only net income is given, add income tax expense back first: pretax income = net income + income tax expense.

Why the effective rate differs from the statutory rate

- Different tax rates in different jurisdictions (foreign operations)
- **Permanent differences**: tax credits, tax-exempt income, nondeductible expenses, and different tax treatment of capital gains versus operating income
- New tax legislation or changed tax rates
- Tax holidays granted in some countries

Temporary differences typically do *not* make the effective and statutory rates differ, because deferred tax expense adjusts income tax expense back toward statutory rate × pretax income. They do, however, make the cash tax rate differ from the effective rate.

Example. Rennick Foods earns pretax income of $500 at home (tax rate 28%) and $300 in a subsidiary abroad (tax rate 16%). Income tax expense is $0.28(500) + 0.16(300) = 140 + 48 = 188$. The effective tax rate is $188/800 = 23.5\%$, below the 28% statutory rate because of the low-tax foreign operations.

Example. Pretax income is $400, including $20 of tax-exempt interest. The statutory rate is 30% and there are no other differences. Income tax expense is $0.30 \times (400 - 20) = 114$, and the effective rate is $114/400 = 28.5\%$.

Using the rate reconciliation

The footnotes must reconcile the statutory rate to the effective rate. Understanding why reported income tax expense differs from tax at the statutory rate helps the analyst forecast earnings and cash flows. For each reconciling item, look at its relative size, its trend so far and the direction it is expected to take; the footnotes and management's discussion and analysis are the sources. Percentages can be converted into amounts when that helps. When studying trends and forecasting, include only reconciliation items that are **continuous** in nature and set aside those that are **sporadic**:

Usually continuous	Usually sporadic
Different tax rates in different countries	Large asset sales
Tax-exempt income	Tax holiday savings
Nondeductible expenses	Other one-off "special items"

For a **tax holiday** (a temporary exemption from tax in a jurisdiction), find out when it ends and whether accumulated taxes become payable at that point. Volatile reconciliation items make the effective rate hard to forecast and reduce comparability with peers.

LOS 35.d — Deferred tax disclosures and their analytical use

Common temporary differences

KEY CONCEPT

Source	Result	Why
Accelerated tax depreciation, straight-line in the books	DTL	Tax deductions come first; consider the firm's growth and capex before assuming reversal
Impairments	DTA	Write-down expensed now; tax deduction usually only on sale/disposal
Restructuring charges	DTA	Expensed when announced; deducted when paid (large cash outflows follow, net of tax savings)
Inventory cost flow method	Usually none in the US (LIFO conformity); can arise elsewhere	Book and tax methods may differ outside the US
Post-employment benefits, deferred compensation	DTA	Expensed when earned by employees; deducted when paid

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Source	Result	Why
Unrealized gains/losses on available-for-sale securities	Deferred tax adjustment in equity	Gains go directly to equity; tax only when realized

Exam convention: the future tax on unrealized gains and losses from available-for-sale securities is recorded as a deferred tax adjustment to stockholders' equity, and the balance sheet shows no DTL for it. Current practice: for available-for-sale debt securities (US GAAP) and FVOCI assets (IFRS 9), ASC 740 and IAS 12 recognize the deferred tax liability (or asset) on the balance sheet and charge the related tax to other comprehensive income, so the income statement tax expense is unaffected. Unrealized gains on these securities therefore appear among deferred tax liabilities in disclosure tables.

On the balance sheet, deferred tax assets and liabilities are classified as **noncurrent** under both IFRS and US GAAP. They are not split into current and noncurrent portions by expected reversal date.

What is typically disclosed

- DTLs, DTAs, any **valuation allowance** and the net change in the allowance during the period
- Any unrecognized deferred tax liability (DTL) that relates to earnings retained by subsidiaries or joint ventures
- For each kind of temporary difference, its tax effect in the current year
- The components of income tax expense (current and deferred)
- A reconciliation between reported income tax expense and tax computed at the statutory rate
- Tax loss carryforwards and credits

Reading the disclosures

- If income tax expense keeps exceeding taxes payable, DTLs are growing faster than DTAs (deferred tax expense is positive).
- A decrease in the valuation allowance lowers deferred tax expense and raises reported earnings. It signals that management now expects more future taxable income, so check that it is justified.
- With a straight-line book schedule and an accelerated tax schedule, taxes payable can be computed year by year. Taxable income is pretax income before depreciation minus tax depreciation, and taxes payable are taxable income × rate. When only temporary differences exist and the tax rate does not change, income tax expense is pretax accounting income × rate.

Example. Book depreciation is 40 a year and tax depreciation is 60 in Year 1. Income before depreciation is 250 and the tax rate is 20%. Taxes payable are $(250 - 60) \times 0.20 = 38$. Income tax expense is $(250 - 40) \times 0.20 = 42$. The DTL rises by $42 - 38 = 4 = (60 - 40) \times 0.20$.

COMMON EXAM TRAPS

- Effective and cash tax rates divide by pretax income. Net income and taxable income are the wrong denominators. For Rennick, dividing by net income of $612 gives an effective rate of 30.7% instead of 23.5%.
- Taxes payable use tax depreciation; income tax expense uses book depreciation. In the depreciation example, swapping the two gives taxes payable of 42 and income tax expense of 38 instead of 38 and 42.

- Changes in DTAs and DTLs from temporary differences at an unchanged tax rate, and higher cash taxes, do not by themselves make the effective rate differ from the statutory rate. Permanent differences and foreign rates do.
- Tax holidays are sporadic items; identify the end date.

EXAM SHORTCUTS

- When only temporary differences exist and the tax rate does not change, income tax expense is pretax accounting income × the tax rate, so no deferred tax schedule is needed to find it.

BOTTOM LINE

- The effective tax rate is income tax expense divided by pretax income, the cash tax rate is cash taxes paid divided by pretax income, and the statutory rate is set by law in the jurisdiction where the company is domiciled.
- The effective rate differs from the statutory rate because of different rates in foreign jurisdictions, permanent differences, new legislation or rate changes, and tax holidays; temporary differences typically do not cause the gap but make the cash tax rate differ from the effective rate.
- When forecasting from the rate reconciliation, an analyst includes continuous items such as foreign rate differences, tax-exempt income and nondeductible expenses, and sets aside sporadic items, for example tax holiday savings or a large asset sale.
- Accelerated tax depreciation with straight-line book depreciation creates a DTL, while impairments, restructuring charges and post-employment benefits create DTAs.
- Deferred tax assets and liabilities are classified as noncurrent under both IFRS and US GAAP.
- Income tax expense that keeps exceeding taxes payable means DTLs are growing faster than DTAs, and a decrease in the valuation allowance raises reported earnings.

QUICK CHECK

Question 3

An analyst reviewing Kittredge Pharma notes that its effective tax rate is 6 percentage points below its statutory rate. Almost all of the gap comes from a tax holiday granted for a new plant in Malaysia. This reconciliation item is *most likely*:

A continuous in nature, so the date on which the holiday ends does not matter.

B sporadic in nature, but its effect is normally cancelled out by higher home-country tax when the profits are repatriated.

C sporadic in nature, and the analyst should find out when the holiday ends and whether taxes will become payable at that point.

Answer: C

A tax holiday is a temporary exemption from tax in a particular jurisdiction, so it is a sporadic reconciliation item. The key analytical task is to establish when the holiday will expire and how its expiry (including any requirement to pay accumulated taxes) will affect future taxes payable and the effective rate.

Why the other options are wrong

- **A.** A tax holiday is temporary, so it is a sporadic item. Its termination date is the main thing the analyst needs to find out.
- **B.** Nothing suggests the holiday's benefit is routinely offset by home-country taxes. The analyst's focus is the end date and any taxes that become payable then.

Key takeaway A tax holiday is a sporadic item. Find its end date and any catch-up tax.

Practice questions

Question 4

Which of the following is *most likely* to cause a deferred tax asset to be created?

A Accelerated tax depreciation.

B Unearned revenue.

C A decrease in the enacted tax rate.

Question 5

Lochmere Outfitters has built up large tax loss carryforwards. Its deferred tax balances at the start of the year, measured at the 24% income tax rate then enacted, are shown below.

Lochmere Outfitters: deferred tax balances at the start of the year (measured at 24%)

Deferred tax item	Balance ($ thousands)
Deferred tax asset (tax loss carryforwards and warranty provisions)	360
Deferred tax liability (accelerated tax depreciation)	150

During the year the legislature enacts an increase in the income tax rate to 28%, applying from next year onward, when all of these differences are expected to reverse. No temporary differences arise or reverse during the year. As a result of the rate change, Lochmere's income tax expense for the year will:

A increase by $25,000.

B remain unchanged.

C decrease by $35,000.

Answers: 4. B · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/deferred-tax-liability/. The full bank has 41 questions on this reading.

FINANCIAL STATEMENT ANALYSIS · READING 36

Financial Reporting Quality

What you'll learn

LOS 36.a Contrast financial reporting quality with the quality of reported results (earnings, cash flows and balance sheet items).

LOS 36.b Describe a spectrum that ranks financial reports from highest to lowest quality.

LOS 36.c Distinguish conservative accounting from aggressive accounting.

LOS 36.d Describe the motivations and conditions that lead managers to issue low-quality or fraudulent financial reports.

LOS 36.e Describe the mechanisms that discipline financial reporting quality and their limitations.

LOS 36.f Describe presentation choices, including non-GAAP measures, that can be used to sway an analyst's view.

LOS 36.g Describe accounting choices and estimates that can be used to manage earnings, cash flows and balance sheet items.

LOS 36.h Describe accounting warning signs and ways to detect manipulation of financial report information.

MODULE 36.1

Reporting Quality

This reading separates financial reporting quality from earnings quality, ranks reports on a quality spectrum, and covers conservative and aggressive accounting, the conditions behind low-quality reporting, the mechanisms that discipline it and the use of non-GAAP measures. It then shows the choices and estimates management can use to manage earnings, cash flows and balance sheet items, and the warning signs an analyst looks for.

LOS 36.a — Reporting quality versus earnings quality

Two different questions are asked about any set of financial statements.

1. **Financial reporting quality** asks how good the reports themselves are. The first test is compliance with **generally accepted accounting principles (GAAP)** in the firm's jurisdiction. Compliance alone is not enough, because GAAP leave room for choices of methods and estimates. High-quality reporting must also be **decision useful**, which requires two characteristics:
   - **Relevance**: the information could change a user's decision. Relevant items must also be **material**.
   - **Faithful representation**: the information is **complete**, neutral (unbiased) and **free from error**.
2. **Quality of earnings** asks how good the performance described in the reports is. More broadly, it covers the quality of reported results: earnings, cash flows and balance sheet values. It is judged on two dimensions:
   - **Sustainability**: how much of this period's earnings can be expected to recur. Profit from higher efficiency or a growing market share tends to persist; a one-time gain on selling a long-held asset or a favorable currency move does not.
   - **Level (adequacy)**: earnings must be high enough to keep the business going and to give investors an **adequate return** on the capital they supplied.

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Sustainable earnings are expected to continue, so one dollar of high-quality earnings adds more to firm value (the present value of expected future earnings) than one dollar of a one-off gain. Balance sheet and cash flow items feed into earnings quality too. Under-accrued liabilities or overstated asset values inflate current earnings and make them less sustainable.

KEY CONCEPT

Question asked	Financial reporting quality	Quality of earnings
What is judged?	The financial reports	The results reported
Main criteria	GAAP compliance; relevance; faithful representation	Sustainability; adequate level

Reporting quality and earnings quality combined

	Earnings quality high	Earnings quality low
Reporting quality high	Best case: useful reports of sustainable, adequate earnings	Useful reports of earnings that are not sustainable or not adequate
Reporting quality low	Earnings quality cannot be judged reliably	Earnings quality cannot be judged reliably

The two dimensions are linked in one direction. If the reports are of low quality, a user cannot reliably assess earnings, cash flows or balance sheet values. The reverse does not hold: a firm can report a mostly one-time gain transparently, which gives high reporting quality and low earnings quality.

Example. Tallis Marine reports net income of $25 million, of which $18 million is a disclosed gain from selling a warehouse. The reports are compliant and transparent (high reporting quality), but only about $7 million looks recurring (low earnings quality).

Common exam traps

- Relevance, completeness and neutrality describe reporting quality. Sustainability and adequacy describe earnings quality.
- Earnings that are lower than last year are not automatically "low quality". The tests are sustainability and adequacy.
- Non-compliance with GAAP does not make earnings low quality by definition. It makes their quality hard or impossible to evaluate.

LOS 36.b — The quality spectrum

Combining both dimensions gives a **spectrum** of financial report quality, from best to worst:

KEY CONCEPT

Rank	GAAP?	Description
1 (best)	Compliant	Decision useful; earnings **sustainable and adequate**
2	Compliant	Decision useful, but earnings quality is low (not sustainable or not adequate)
3	Compliant	Earnings quality low and reporting choices and estimates are **biased**
4	Compliant	Earnings are **actively managed** (to increase, decrease or smooth them)
5	Not compliant	Numbers still reflect the firm's actual economic activities
6 (worst)	Not compliant	Numbers are **fictitious or fraudulent**

Departures from GAAP make earnings quality harder to judge, and the analyst must rely more on the underlying economic activity. Only at the bottom of the spectrum, where the numbers are fictitious or fraudulent, does an assessment of earnings quality become impossible.

Common exam traps

- Compliant, decision-useful reporting of unsustainable earnings (rank 2) still ranks above compliant reporting with biased choices (rank 3).

LOS 36.c — Conservative versus aggressive accounting

Ideally reporting is **neutral** (unbiased). Biased choices within GAAP are of two kinds:

- **Conservative accounting** consists of choices that decrease reported earnings and the financial position for the current period. They tend to raise later-period earnings.
- **Aggressive accounting** consists of choices that increase reported earnings, revenues, operating cash flows or the financial position for the current period. They tend to reduce later-period earnings.

KEY CONCEPT

Aggressive choice	Conservative choice
Capitalize current-period costs	Expense current-period costs
Longer estimated useful lives	Shorter estimated useful lives
Higher salvage value estimates	Lower salvage value estimates
Straight-line depreciation	Accelerated depreciation
Delay recognition of impairments	Recognize impairments early
Smaller reserves for bad debt	Larger reserves for bad debt

Aggressive choice	Conservative choice
Smaller valuation allowances on deferred tax assets	Larger valuation allowances on deferred tax assets

The effect on later periods runs the other way because most of these choices shift an expense between periods rather than change its total. Capitalizing a cost or using a longer useful life, for example, moves expense out of the current year and into later years.

Both biases can be used, in different periods, for **earnings smoothing**, because volatile earnings tend to lower share value. In a strong year management over-accrues a liability or reserve (conservative) to hold back earnings. In a weak year it releases the excess (aggressive) to meet targets. A reserve used to store earnings this way is called a **cookie jar reserve**.

Bias can also appear in presentation. Statements can be transparent, or they can give minimal disclosure that highlights good news and hides bad news.

Conservatism is not good by default. Any bias departs from neutrality and faithful representation and makes reports less decision useful.

Some standards are conservative in themselves because they demand stronger evidence for gains than for losses:

- **Research costs** are expensed as incurred, while related revenues come later.
- **Litigation losses** are accrued once probable; gains face a stricter test.
- Under US GAAP, inventory is written down when impaired, but increases are not recognized until sale.

Conservatism can have benefits: lower litigation risk, lower current taxes (where tax and book deductions coincide), and protection for less-informed parties such as lenders.

Common exam traps

- "Aggressive" does not mean "non-GAAP". Aggressive choices usually comply with GAAP but are biased and less decision useful.

LOS 36.d — Motivations and conditions for low-quality reporting

Managers may bias reports for several reasons:

- Meeting or beating an earnings benchmark: prior management **guidance**, **consensus analyst expectations**, or earnings for the **same period of the prior year**.
- **Career concerns** and reputation.
- **Incentive compensation** tied to the share price or earnings.
- Credibility with equity investors, and the way customers and suppliers view the firm.
- Avoiding a breach of **debt covenants**, especially at highly leveraged, unprofitable firms.
- Earnings that are running above benchmarks motivate the conservative, cookie-jar choices described under LOS 36.c.

Three conditions are typically present when reporting quality is low:

Element	Meaning	Examples
Motivation	An incentive to misreport	Earnings targets, bonuses, stock options, debt covenants
Opportunity	Circumstances that make misreporting possible	**Weak internal controls**, inadequate **board oversight**, wide range of acceptable accounting treatments, weak penalties for fraud
Rationalization	A mindset that justifies the behavior	"I'll reverse it next quarter", "Everyone does it"

Common exam traps

- Weak controls are an opportunity, not a motivation.
- Pressure to meet expectations is a motivation, not an opportunity.

LOS 36.e — Mechanisms that discipline reporting quality

Three mechanisms discipline financial reporting quality: securities regulators, auditors and private contracts.

1. **Securities regulators**, such as the SEC in the United States and the **FCA** in the UK, coordinate internationally through **IOSCO**. IOSCO has more than 200 members, drawn from stock exchanges, regional authorities and national securities regulators. **ESMA** is a regional member; it coordinates the securities regulators of EU countries. Typical requirements everywhere are:
   - registration of new public securities and regulatory review;
   - disclosure and periodic financial statements with notes;
   - an **independent audit**;
   - management commentary;
   - a **signed statement** by the person responsible for preparing the reports.

   Enforcement tools include fines, suspension from issuing or trading, public disclosure and criminal prosecution.
2. For US-traded securities, management must also report its own evaluation of how well the firm's internal controls work, on top of the auditor's opinion.
3. **Auditors.** A clean (**unqualified**) **audit opinion** offers only **reasonable assurance** that the statements are fairly presented under GAAP. The opinion is no guarantee that the statements are free of error or fraud. Independence is also imperfect, because the audited company chooses and pays its own auditor.
4. **Private contracts.** Lenders and other **counterparties** specify how covenant measures are calculated and have an incentive to insist on high-quality reports. Covenants therefore cut both ways: they give the borrower's managers a motivation to manipulate, and they give the lender a reason to police the reports.

Accounting **standard-setting bodies** (IASB, FASB) write standards but do not enforce compliance. Securities regulators enforce them, and auditors and contract counterparties add further discipline.

Common exam traps

- Independent audits and signed preparer statements are typical requirements of securities regulators generally. The management assessment of internal control is an extra requirement for US-traded securities.

LOS 36.f — Presentation choices and non-GAAP measures

Firms often present **non-GAAP measures**, such as "adjusted EBITDA" or **pro forma** earnings, that exclude items described as one-time, nonoperating, non-cash, or "for comparability". These exclusions usually make performance look better. Firms that stress such measures are trying to steer which metrics they are judged on and to draw attention away from the GAAP figures, so a strong management focus on them is a warning sign.

US (SEC) requirements for non-GAAP measures	IFRS requirements for non-IFRS measures
Show the most comparable GAAP measure with **equal prominence**	**Define** the measure and **explain its relevance**
Explain why management thinks the measure is useful	**Reconcile** it to the most comparable IFRS measure
Reconcile it to the most comparable GAAP measure	
Disclose other purposes for which the firm uses it	
Treatment of items likely to recur (see below)	

Exam convention: a US non-GAAP measure must include items that are likely to recur, even if the financial statements treat them as nonrecurring, unusual or infrequent. Current practice: in SEC filings, Item 10(e) of Regulation S-K prohibits adjusting a performance measure to remove an item labeled "nonrecurring", "infrequent" or "unusual" when a similar charge or gain is reasonably likely within two years or occurred within the prior two years. Regulation G, which covers all public disclosures, prohibits non-GAAP measures that are misleading and requires a reconciliation to the most comparable GAAP measure.

Exam convention: the two IFRS requirements in the table are the only ones, so showing the measure for every period presented is not required. Current practice: IFRS 18, effective for annual periods beginning on or after 1 January 2027, treats subtotals of income and expenses that management uses in public communications as management-defined performance measures. They are disclosed together in one note, which explains why each measure is useful and how it is calculated, reconciles it to the most comparable IFRS subtotal and gives comparative information.

Common exam traps

- Growth in operating cash flow or asset turnover is not in itself a warning sign. Heavy emphasis on pro forma earnings is.
- The SEC rule on recurring items concerns labeling. Calling a recurring charge "nonrecurring" and removing it is prohibited, but an adjustment for a recurring item is not banned as long as the measure is not misleading.

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BOTTOM LINE

- Financial reporting quality concerns the reports themselves: compliance with GAAP plus decision usefulness, which requires relevance (including materiality) and faithful representation (complete, neutral and free from error).
- Earnings quality concerns the results reported and is judged by their sustainability and by whether their level is adequate to keep the business going and give investors an adequate return.
- Low reporting quality means earnings, cash flows and balance sheet values cannot be reliably assessed, but high reporting quality can go together with low earnings quality.
- Conservative choices decrease current-period earnings and financial position and tend to raise later earnings, while aggressive choices increase current-period earnings, revenues, operating cash flows or financial position and tend to reduce later earnings; either bias departs from neutrality.
- Low-quality reporting typically involves motivation, such as earnings targets or debt covenants, opportunity, such as weak internal controls or inadequate board oversight, and rationalization.
- For a non-GAAP measure, the SEC requires the closest GAAP measure to be shown with equal prominence and reconciled to it, while IFRS requires the measure to be defined, its relevance explained and a reconciliation to the most comparable IFRS measure.

QUICK CHECK

Question 1

An analyst concludes that Fenwick Brewing's financial reports are of high quality. Fenwick's reported earnings *could* still be:

A of low quality, because they are not sustainable.

B smoothed through management's choice of estimates.

C overstated because asset impairments were recognized late.

Answer: A

Reporting quality is about whether the reports give decision-useful information that faithfully represents what happened. Earnings quality is about the underlying results: whether earnings are sustainable and adequate. A company can report a weak or one-off year accurately, so high-quality reporting can sit alongside low-quality earnings.

Why the other options are wrong

- **B.** Earnings smoothed through chosen estimates do not faithfully represent performance, so the reports would not be high quality.
- **C.** Delaying impairments overstates assets and earnings; that is a bias in the reporting, which rules out high reporting quality.

Key takeaway High reporting quality says the reports are faithful and useful; it does not guarantee that the earnings they show are sustainable or adequate.

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MODULE 36.2

Accounting Choices and Estimates

LOS 36.g — Choices and estimates used to manage earnings, cash flows and balance sheet items

Revenue recognition

- Shipping terms. Under **FOB shipping point**, title passes at the seller's dock, so revenue is recognized earlier than under **FOB destination**.
- Timing of shipments. A firm can accelerate shipments (with discounts or special financing) to boost the current period, or hold them back when the period is already strong.
- **Channel stuffing** means loading distributors with more goods than they would normally sell in the period.
- In a **bill-and-hold** transaction, the customer is invoiced but the goods stay at the seller. Fictitious bill-and-hold sales inflate current revenue, since the goods are really still inventory, and reduce future revenue when real orders are filled.

Reserves and allowances (contra accounts)

- **Allowance for uncollectible accounts**. Lowering the estimate raises net receivables and net income; raising it does the reverse. If the estimate is too low, a later catch-up expense reduces income.
- **Warranty reserves** work the same way.
- **Valuation allowance** on deferred tax assets. A smaller allowance raises the net deferred tax asset and net income; a larger one lowers both. Understating the allowance overstates the asset, and later adjustments can smooth earnings.
- All of these reserves can serve the earnings smoothing described under LOS 36.c.

Depreciation, amortization and impairment

- **Accelerated depreciation** gives higher expense and lower income early in the asset's life than **straight-line** depreciation, and a lower carrying value.
- A longer **useful life** or a higher **salvage value** lowers depreciation expense and raises net income and carrying value. If the salvage value is set above the eventual sale price, a loss appears when the asset is sold.
- Amortization of purchased intangible assets involves the same choices of method, life and residual value as depreciation of tangible assets.
- **Goodwill** is not amortized but is tested for **impairment**. Delaying an impairment charge raises current earnings.
- Under IFRS, an upward **revaluation** of PP&E above depreciated cost goes to a revaluation surplus in other comprehensive income. It reaches profit or loss only to the extent it reverses a loss previously taken to profit or loss. The higher carrying amount then raises later depreciation.

KEY CONCEPT

$$\text{Annual depreciation (straight-line)} = \frac{\text{cost} - \text{salvage value}}{\text{useful life}}$$

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Example. Equipment costs $900,000 with a 6-year life and $60,000 salvage value. Depreciation is $(900{,}000 - 60{,}000)/6 = \$140{,}000$. Raising the salvage estimate to $150,000 cuts it to $(900{,}000 - 150{,}000)/6 = \$125{,}000$, adding $15,000 to pretax income each year.

Inventory

- When prices are rising, **FIFO** gives lower COGS and higher gross profit and inventory than **weighted-average cost**; the opposite holds when prices are falling. FIFO inventory is closer to current replacement cost, so FIFO gives the more relevant balance sheet value. Weighted-average COGS is closer to current cost, so gross profit and margins under weighted-average cost better reflect economic reality; FIFO gross profit includes holding gains when prices rise (or losses when they fall). Reports that explain clearly how the chosen method affects the income statement and balance sheet are of higher quality.
- LIFO is permitted under US GAAP only. Selling more units than are bought or produced (**LIFO liquidation**) runs old, low costs through COGS when prices are rising and raises current earnings unsustainably.

Related-party transactions

Prices charged by a management-controlled private supplier can shift profit into or out of the public company.

Capitalization versus expensing

Capitalizing a cost creates an asset and spreads the expense through depreciation or amortization, so current earnings rise and future earnings fall. Capitalizing more research and development spending likewise pulls income forward into the current period.

Example. A $2.4 million software implementation cost amortized over 4 years reduces this year's pretax income by only $0.6 million instead of $2.4 million, which lifts current pretax income by $1.8 million. A $1.8 million asset remains to be amortized over the next three years.

Capitalization also moves the cash outflow from **cash flow from operations (CFO)** to **cash flow from investing (CFI)**, so reported CFO is higher by the full amount. Capitalized interest has the same effect.

Other cash flow management

- **Stretching payables**, that is, delaying payments to suppliers past period-end, raises current CFO and lowers next period's CFO. It has no effect on reported earnings because the expense was already recognized.
- Exam convention: under IAS 7, a company may report interest and dividends paid in either CFO or CFF, and interest and dividends received in either CFO or CFI, which gives management another lever on reported CFO. Current practice: IFRS 18, effective for annual periods beginning on or after 1 January 2027, removes most of this choice for companies whose main business is not investing or financing.

Covenant-driven manipulation

An **interest coverage ratio** covenant (EBIT / interest expense) is most easily met by overstating earnings. Asset values do not enter the ratio.

Common exam traps

- Capitalizing an expense raises CFO and lowers CFI; total cash is unchanged.

- A LIFO liquidation raises earnings only when prices are rising; with falling prices the old layers carry higher costs and earnings fall.

EXAM SHORTCUTS

- An interest coverage covenant (EBIT / interest expense) is most easily met by overstating earnings; asset values do not enter the ratio, so manipulation of asset values can be ruled out as the route.

BOTTOM LINE

- Revenue is recognized earlier under FOB shipping point than under FOB destination, and channel stuffing and fictitious bill-and-hold sales inflate current revenue.
- A lower allowance for uncollectible accounts, a smaller warranty reserve or a smaller valuation allowance on deferred tax assets raises net income, and such reserves can be used to smooth earnings.
- A longer useful life or a higher salvage value lowers depreciation and raises net income and carrying value, and delaying a goodwill impairment raises current earnings.
- Capitalizing a cost raises current earnings and lowers future earnings, and it moves the outflow from CFO to CFI, so reported CFO is higher by the full amount.
- Stretching payables raises current CFO and lowers next period's CFO, with no effect on reported earnings.
- When prices are rising, a LIFO liquidation (possible under US GAAP only) runs old, low costs through COGS and raises current earnings unsustainably.

QUICK CHECK

Question 2

The management of Orsini Fabrication wants to use its accounting discretion to raise operating income in the coming period. Which of the following actions is it *most likely* to take?

A Raise the estimated residual values of its plant and equipment.

B Shorten the estimated useful lives of its plant and equipment.

C Revalue its plant and equipment upward from depreciated cost to fair value.

Answer: A

A higher residual (salvage) value reduces the depreciable base, so annual depreciation expense falls and operating income rises.

$$\text{Depreciation} = \frac{\text{cost} - \text{residual value}}{\text{useful life}}$$

A higher residual value shrinks the numerator, which lowers expense. A shorter life shrinks the denominator, which raises expense.

Why the other options are wrong

- **B.** Shorter useful lives spread the cost over fewer periods, which increases depreciation expense and lowers operating income.

- **C.** Under IFRS, a revaluation above depreciated cost is not recognized in profit (it goes to a revaluation surplus) unless it reverses a loss previously recognized. The higher carrying amount also increases depreciation in later periods, which lowers operating income.

Key takeaway Aggressive depreciation choices: longer lives and higher salvage values. Both lower depreciation and raise income.

MODULE 36.3

Warning Signs

LOS 36.h — Accounting warning signs and detection of manipulation

A **warning sign** is not proof of fraud or manipulation. It calls for more analysis to find out whether a real business reason exists. If satisfactory answers are not available, especially when several signs appear together, avoiding the investment is a reasonable choice.

KEY CONCEPT

Area	Warning signs
Revenue recognition	Changes in revenue recognition methods; bill-and-hold, **barter** or rebate programs that need estimates; little transparency on how parts of an order become revenue; revenue growth out of line with peers; **receivables turnover** falling over several periods; falling **total asset turnover**, especially with growth by acquisition; nonoperating or one-time sales included in revenue
Inventories	Declining **inventory turnover**; LIFO liquidations (US GAAP only; see LOS 36.g)
Capitalization	Capitalizing costs that industry peers normally expense
Earnings vs. cash flow	Ratio of **operating cash flow to net income** persistently below one or falling
Other	Depreciation methods, useful lives or salvage values out of line with peers; a fourth-quarter earnings pattern (unusually high or low) not explained by industry or company seasonality; significant **related-party transactions** (with entities controlled by management); expenses labeled **nonrecurring** that appear regularly; gross or operating margins well above peers; minimal disclosure; emphasis on non-GAAP measures and aggressive use of special or nonrecurring designations for charges; heavy growth by acquisition (many chances to set asset values and future depreciation/amortization, and prior-period comparisons become unreliable)

Linking the signs to what is being manipulated

- Aggressive estimates (longer lives, higher salvage values), aggressive revenue recognition and LIFO liquidation change net income. Apart from any tax effects (a LIFO liquidation, for example, also raises taxable income), they leave the amount and classification of cash flows unchanged.

- Capitalizing costs that peers expense raises current earnings and also raises CFO, because the outlay moves to investing activities (LOS 36.g). Stretching payables changes CFO alone. A jump in **days payables** well above historical levels suggests stretched payables. CFO is then higher but not sustainably so, which puts the **quality of the cash flow statement** in doubt. Earnings are unaffected, and higher payables reduce net working capital, other things equal.

$$\text{Days payables} = \frac{\text{accounts payable}}{\text{COGS (or purchases)}/365}$$

Cash flow versus earnings

KEY CONCEPT

$$\text{CFO-to-net-income ratio} = \frac{\text{cash flow from operations}}{\text{net income}}$$

A ratio persistently below 1.0 means reported earnings are not being converted into cash, which suggests that accruals are inflating income. A ratio above one is not, by itself, a warning sign.

Example. Net income of $80 million and CFO of $52 million give a ratio of 0.65. If this is repeated year after year, the earnings deserve scrutiny.

Restructuring and impairment charges

Large restructuring or impairment charges partly correct past understated expenses and overstated assets. Analysts should not treat them as good news. Instead, they should consider spreading the charges over prior periods and restating earlier earnings to see the true trend.

Common exam traps

- Inappropriate capitalization understates current expenses. It does not overstate revenue or understate liabilities.
- Channel stuffing shows up as rising **days of sales outstanding** (falling receivables turnover). Payables are not involved.

EXAM SHORTCUTS

- Channel stuffing shows up as rising days of sales outstanding (falling receivables turnover), so an answer pointing to payables can be ruled out.

BOTTOM LINE

- A warning sign is not proof of manipulation; it calls for more analysis to find out whether there is a real business reason.
- Revenue warning signs include changes in revenue recognition methods, revenue growth out of line with peers, receivables turnover falling over several periods, falling total asset turnover and nonoperating or one-time sales included in revenue.

- A ratio of CFO to net income persistently below 1.0 suggests that accruals are inflating income, while a ratio above one is not by itself a warning sign.
- Aggressive estimates, aggressive revenue recognition and LIFO liquidation change net income but, apart from tax effects, leave the amount and classification of cash flows unchanged; capitalizing costs that peers expense raises both earnings and CFO.
- A jump in days payables well above historical levels suggests stretched payables, which make CFO higher but not sustainably so while leaving earnings unaffected.
- Large restructuring or impairment charges partly correct past understated expenses and overstated assets, so analysts consider spreading them over prior periods to see the true trend.

QUICK CHECK

Question 3

Which of the following observations about Sandhurst Industrial, a diversified manufacturer, is *most likely* to signal that management is manipulating how its results are perceived?

A Sandhurst adopted a new lease accounting standard in the year it became mandatory for all companies.

B Sandhurst's ratio of operating cash flow to net income has stayed between 1.1 and 1.3 for five years.

C Sandhurst has reported large restructuring charges labeled nonrecurring in five of the last six years.

Answer: C

Expenses that are classified as nonrecurring but appear year after year are a warning sign. Labeling ordinary costs as one-time charges again and again lowers the credibility of the category and invites investors to exclude real, recurring costs when they judge sustainable profitability.

Why the other options are wrong

- **A.** Adopting a new standard when it becomes mandatory is required of every company. It reflects no management choice, so it does not by itself signal manipulation.
- **B.** The warning sign is a ratio of operating cash flow to net income that is persistently below one or falling. A stable ratio above one suggests that earnings are well backed by cash.

Key takeaway Watch for nonrecurring charges that recur, heavy emphasis on non-GAAP earnings and aggressive use of special-charge labels. They can distort the picture of sustainable earnings.

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Practice questions

Question 4

A review of an accounting scandal concludes that the company's managers had both the motivation and the opportunity to misreport. Which third factor typically completes the set of conditions behind low-quality financial reporting?

A Weak internal controls.

B Rationalization of the actions.

C Pressure to beat earnings expectations.

Question 5

The management of Pellworth Chemicals is worried that the company will breach a bank-loan covenant that sets a minimum interest coverage ratio. If management manipulates its financial reporting to avoid the breach, it is *most likely* to:

A overstate earnings.

B understate assets.

C capitalize leases.

Answers: 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/financial-reporting-quality/. The full bank has 38 questions on this reading.

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FINANCIAL STATEMENT ANALYSIS · READING 37

Dupont Analysis

What you'll learn

LOS 37.a Describe the tools and techniques of financial analysis (ratio, common-size, graphical and regression analysis) and their uses and limitations.

LOS 37.b Calculate and interpret activity, liquidity, solvency and profitability ratios.

LOS 37.c Describe relationships among ratios and evaluate a company using ratio analysis.

LOS 37.d Apply DuPont analysis to decompose return on equity and interpret the effects of changes in its components.

LOS 37.e Describe the uses of industry-specific ratios, including ratios for financial institutions and measures of business risk.

LOS 37.f Describe how ratio analysis and other techniques are used to model and forecast earnings.

MODULE 37.1

Introduction to Financial Ratios

This reading covers the tools of financial analysis, the activity, liquidity, solvency and profitability ratios and the relationships among them, DuPont analysis of return on equity, industry-specific ratios, and the use of ratios in modeling and forecasting. A candidate must be able to calculate and interpret each ratio, decompose ROE into three or five parts, and judge a company's position from a set of ratios.

LOS 37.a — Tools and techniques of financial analysis

Raw financial statement numbers are hard to compare. A large firm has larger receivables, larger debt and larger profits than a small one simply because of its size. Step 3 of the financial statement analysis framework therefore converts the (adjusted) data into comparable forms, such as ratios, common-size statements and graphs, and may add statistical work such as regression. The four tools are **ratio analysis**, **common-size analysis**, **graphical analysis** and **regression analysis**.

Uses of ratio analysis

A ratio relates one statement item to another (for example, net income to revenue), which removes the effect of size. Ratios are most useful for raising questions that the analyst then investigates; a ratio seldom answers a question by itself. Ratios help an analyst to:

- forecast future earnings and cash flow;
- judge the firm's flexibility, meaning its ability to grow and to meet obligations when something unexpected happens;
- rate how well management has performed;
- track how the firm and its industry change over time;
- compare the firm with its industry competitors.

Comparisons run in two directions. **Cross-sectional analysis** compares a firm's ratios with those of other firms (peers, industry averages) for the same period. **Time-series analysis** (trend analysis) compares a firm's ratios with its own past values.

Limitations of ratio analysis

- A ratio viewed in isolation says little. It is informative only against peers or the firm's own history.
- Different accounting treatments (methods, estimates, IFRS versus US GAAP) distort comparisons, so the analyst may need to adjust the data first.
- Comparable industry ratios are hard to find for a company that operates in multiple industries (a conglomerate).
- No conclusion should rest on a single ratio; ratios must be read together.
- Deciding on the target or comparison value takes judgment. The benchmark is usually a range of acceptable values rather than one number.
- Ratios must be read in context: prior results, stated strategy, analyst expectations, the stage of the business cycle and peers.
- Definitions vary between analysts and data providers (for example, total liabilities versus interest-bearing debt in leverage ratios). What matters is using a definition consistently and knowing how published ratios were built.
- Reasonable values differ across industries. A grocer's asset turnover, for example, is far higher than a utility's.

Common-size analysis

Common-size statements express every line item relative to a common base so that firms of different size, or one firm across years, can be compared.

KEY CONCEPT

Format	Base (divisor)	Typical use
Vertical common-size income statement	Revenue (sales) of the same year	Trends in costs and margins
Vertical common-size balance sheet	Total assets of the same date	Asset mix and financing mix
Horizontal common-size statement	Each item's own value in a **base year** (set to 1.0 or 100%)	Growth of each item over time

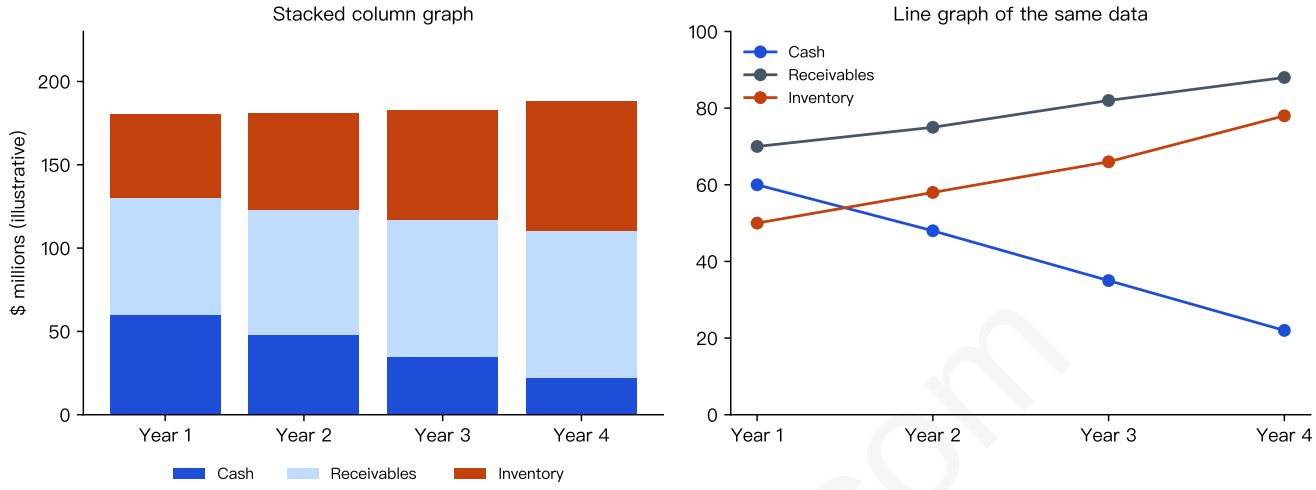
Every income statement line is divided by revenue, so a vertical common-size income statement shows the **gross profit margin**, **operating profit margin** and **net profit margin** directly. It also shows which cost lines drive a change in margin. A vertical common-size balance sheet shows ratios such as long-term debt to total assets at a glance.

Example. Kestrel Outfitters reports revenue of \$640, cost of goods sold of \$416, SG&A of \$128 and net income of \$48. Vertically, COGS is $416/640 = 65\%$ of revenue, gross margin is 35%, SG&A is 20% and the net profit margin is $48/640 = 7.5\%$. Horizontally, if inventory was \$90 in the base year and \$117 two years later, it is shown as $117/90 = 1.30$, a 30% increase since the base year.

Check the column order in any exhibit: some put the most recent year on the left, others on the right.

Graphical analysis

Graphs present comparisons and the composition of statement items over time. A **stacked column graph** (also called a **stacked bar graph**) shows how each component, such as cash, receivables or inventory, contributes to a total in each year. A **line graph** plots the same items as lines and makes trends easy to see, for example rising trade payables together with falling cash, which is a possible liquidity warning.



The same current-asset data as a stacked column graph and as a line graph (illustrative values)

Regression analysis

Regression analysis identifies relationships between variables and is mainly used for forecasting. An analyst might, for example, estimate next year's sales from the historical relationship between the firm's sales and GDP.

Common exam traps

- A vertical common-size statement divides income statement items by sales and balance sheet items by total assets. Dividing by a base-year value is the horizontal format.
- Common-size analysis is a way of scaling the data, not a type of comparison. Setting a firm's ratios beside its rivals' ratios for the same year is cross-sectional analysis whatever scaling is used.
- "Calculating ratios is highly subjective" is not a valid limitation. The formulas are mechanical; judgment enters when the comparison value is chosen and the result interpreted.

BOTTOM LINE

- Financial analysis uses four tools: ratios, common-size statements, graphs and regression.
- Cross-sectional analysis compares a firm's ratios with those of other firms for the same period, while time-series analysis compares them with the firm's own past values.
- Ratios are informative only against peers or the firm's history, they are distorted by different accounting treatments, comparable industry ratios are hard to find for a conglomerate, and no conclusion should rest on a single ratio.
- A vertical common-size income statement divides each line by revenue and a vertical common-size balance sheet divides each line by total assets, while a horizontal common-size statement divides each item by its own base-year value.
- Regression analysis identifies relationships between variables and is mainly used for forecasting.

QUICK CHECK

Question 1

Leon Varga places this year's gross profit margin, inventory turnover, and debt-to-equity ratio for Pallister Grocers next to the same ratios, for the same year, of four rival supermarket chains. Varga's approach is best described as:

A cross-sectional analysis.

B time-series analysis.

C common-size analysis.

Answer: A

Setting a firm's ratios beside the same ratios of rival firms (or an industry average) for the same period is cross-sectional analysis. It shows how the firm performs relative to peers.

Why the other options are wrong

- **B.** Time-series analysis compares the same company's ratios across several periods, not across different firms in one period.
- **C.** Common-size analysis restates line items as percentages of a base (sales, total assets, or a base year). Varga is comparing ratios across firms, which is a different exercise.

Key takeaway Comparing several firms at one date is cross-sectional analysis. Following one firm across dates is time-series analysis.

MODULE 37.2

Financial Ratios, Part 1

LOS 37.b — Activity, liquidity, solvency and profitability ratios

Ratios are grouped by the question they answer. The groups overlap: **payables turnover**, for example, is an activity ratio that also says something about liquidity.

KEY CONCEPT

Category	Question it answers	Examples
Activity ratios (asset utilization, operating efficiency, turnover ratios)	How efficiently does the firm use its assets?	receivables, inventory, payables, total asset, fixed asset and working capital turnover; the "days" ratios
Liquidity ratios	Can the firm pay its short-term obligations as they come due?	current, quick and cash ratios; defensive interval; cash conversion cycle

Category	Question it answers	Examples
Solvency ratios	Can the firm meet its long-term obligations; how much financial leverage does it use?	debt-to-equity, debt-to-capital, debt-to-assets, financial leverage, interest coverage, fixed charge coverage
Profitability ratios	How well does the firm turn sales, assets and equity into profit?	gross, operating and net profit margins; pretax margin; ROA; operating ROA; ROIC; ROE

When a ratio combines an income statement flow with a balance sheet stock, it usually uses the average balance (beginning plus ending, divided by 2), unless a question specifies year-end figures. For seasonal businesses, averages of more data points (for example, quarterly) are better.

Activity ratios

$$\text{receivables turnover} = \frac{\text{revenue}}{\text{average receivables}}$$

$$\text{days of sales outstanding (DSO)} = \frac{365}{\text{receivables turnover}}$$

$$\text{inventory turnover} = \frac{\text{cost of goods sold}}{\text{average inventory}} \qquad \text{days of inventory on hand (DOH)} = \frac{365}{\text{inventory turnover}}$$

$$\text{payables turnover} = \frac{\text{cost of goods sold}}{\text{average trade payables}}$$

$$\text{number of days of payables (DPO)} = \frac{365}{\text{payables turnover}}$$

Many analysts put **purchases** in the numerator of payables turnover instead of COGS, because payables arise from purchases. Purchases are rarely reported as a line item, but the inventory relationship gives them:

$$\text{purchases} = \text{ending inventory} - \text{beginning inventory} + \text{COGS}$$

When a question gives purchases, use them; otherwise use COGS.

$$\text{total asset turnover} = \frac{\text{revenue}}{\text{average total assets}} \qquad \text{fixed asset turnover} = \frac{\text{revenue}}{\text{average net fixed assets}}$$

$$\text{working capital turnover} = \frac{\text{revenue}}{\text{average working capital}}$$

DSO is also called the **average collection period**, DOH the **average inventory processing period**, and days of payables the **payables payment period**. For quarterly turnover ratios, the days in the quarter replace 365.

Interpretation:

- A high **receivables turnover** points to efficient credit and collection. It can also reflect strict credit terms, a large discount for early payment or heavy penalties for late payment, and credit terms that are too strict cost sales. Comparing revenue growth with peers shows which: slower growth suggests terms are too tight, while

growth at or above the peer average suggests good credit management.

- **Inventory turnover** uses COGS, not sales. Inventory is carried at cost and COGS is also measured at cost, while revenue includes the markup and would overstate how often inventory turns over. A high value suggests good inventory management, or stock levels so low that sales are lost. A low value may indicate obsolete, slow-moving inventory.
- A high payables turnover means the firm pays quickly, perhaps without using supplier credit fully or in order to take early-payment discounts. A low value may signal cash-flow trouble or simply lenient supplier terms; liquidity ratios help decide.
- **Total asset turnover** differs greatly by industry, from around 1 for capital-intensive manufacturers to around 10 for some retailers, so it is judged against the industry norm. A value that is too low means too much capital is tied up in assets; a value that is too high suggests too few assets for potential sales, or an outdated asset base.
- **Fixed asset turnover** uses net fixed assets (net of accumulated depreciation), so firms with more recently acquired assets typically show lower turnover and firms with older, heavily depreciated assets show higher turnover. A low value suggests too much capital in fixed assets or inefficient use of them; a very high value may point to obsolete equipment or to capital spending that will soon be needed to support growing revenue.
- **Working capital turnover** shows revenue generated per dollar of working capital. When payables are as large as inventory and receivables combined, working capital is close to zero and the ratio becomes very large, unstable from period to period and less informative about efficiency.

Liquidity ratios

$$\text{current ratio} = \frac{\text{current assets}}{\text{current liabilities}}$$

$$\text{quick ratio} = \frac{\text{cash} + \text{marketable securities} + \text{receivables}}{\text{current liabilities}}$$

$$\text{cash ratio} = \frac{\text{cash} + \text{marketable securities}}{\text{current liabilities}}$$

$$\text{defensive interval} = \frac{\text{cash} + \text{marketable securities} + \text{receivables}}{\text{average daily expenditures}}$$

$$\text{average daily expenditures} = \frac{\text{COGS} + \text{SG\&A} + \text{R\&D} - \text{noncash charges (e.g., depreciation, amortization)}}{365}$$

KEY CONCEPT

$$\text{cash conversion cycle} = \text{DSO} + \text{DOH} - \text{number of days of payables}$$

- The three balance sheet ratios differ only in which current assets they treat as available. The **quick ratio** (acid-test ratio) excludes inventory and other less-liquid current assets such as prepaid expenses. The **cash ratio** also excludes receivables. Higher values mean more liquidity. A current ratio below 1 means negative **working capital** (current assets – current liabilities), which usually signals liquidity trouble.

- The **defensive interval** ratio is the number of days the firm could pay its cash expenses from liquid assets alone. Expenditures are cash costs (COGS, SG&A, R&D). When they are estimated from income statement expense lines, noncash charges included in those lines, such as depreciation and amortization, are subtracted. Reported expenses of 100 that include depreciation of 20 mean cash expenditures of 80. Depreciation is added back only when the starting point is net income, as in the indirect-method cash flow statement.
- The **cash conversion cycle** (net operating cycle) is the time between paying suppliers and collecting from customers. DOH + DSO, the time from acquiring inventory to collecting cash for it, is the **operating cycle**; the cash conversion cycle subtracts the days of supplier credit from it. A long cycle ties up capital and may require short-term financing. A firm that collects before it pays (a negative cycle) can operate with a current ratio below 1.

Effect of transactions on a liquidity ratio. Consider a ratio with a positive numerator and denominator. If it is above 1, an equal decrease in numerator and denominator (for example, paying payables with cash) raises it, and an equal increase lowers it. If the ratio is below 1, the effects are reversed. Equal increases therefore pull the ratio toward 1, and equal decreases push it away from 1. Moving value between two items inside the numerator, such as collecting receivables into cash, leaves the current ratio unchanged but raises the cash ratio.

Solvency ratios

Here **total debt** means interest-bearing short-term and long-term debt, including the current portion of long-term debt. Exam convention: unless told otherwise, lease liabilities are left out of total debt. Current practice: lease liabilities bear interest and are recognized on the balance sheet under IFRS 16 and ASC 842, and many analysts and rating agencies include them in debt. Leaving them out makes leverage look low in lease-heavy industries such as airlines. Some analysts also count trade payables and other non-interest-bearing current liabilities as debt, while others subtract cash, cash equivalents and marketable securities to get **net debt**.

$$\text{debt-to-equity} = \frac{\text{total debt}}{\text{total shareholders' equity}}$$

$$\text{debt-to-capital} = \frac{\text{total debt}}{\text{total debt} + \text{total shareholders' equity}} \qquad \text{debt-to-assets} = \frac{\text{total debt}}{\text{total assets}}$$

$$\text{financial leverage ratio} = \frac{\text{average total assets}}{\text{average total equity}} \qquad \text{interest coverage} = \frac{\text{EBIT}}{\text{interest payments}}$$

$$\text{debt-to-EBITDA} = \frac{\text{total debt}}{\text{EBITDA}}$$

KEY CONCEPT

$$\text{fixed charge coverage} = \frac{\text{EBIT} + \text{lease payments}}{\text{interest payments} + \text{lease payments}}$$

Higher debt ratios mean more reliance on debt. The **financial leverage ratio** is also called the **equity multiplier**.

Profitability ratios

$$\text{gross profit margin} = \frac{\text{gross profit}}{\text{revenue}} \qquad \text{operating profit margin} = \frac{\text{operating income}}{\text{revenue}} \text{ or } \frac{\text{EBIT}}{\text{revenue}}$$

$$\text{pretax margin} = \frac{\text{EBT}}{\text{revenue}} \qquad \text{net profit margin} = \frac{\text{net income}}{\text{revenue}}$$

$$\text{ROA} = \frac{\text{net income}}{\text{average total assets}} \qquad \text{adjusted ROA} = \frac{\text{net income} + \text{interest expense}(1 - \text{tax rate})}{\text{average total assets}}$$

$$\text{operating ROA} = \frac{\text{operating income (or EBIT)}}{\text{average total assets}}$$

$$\text{ROE} = \frac{\text{net income}}{\text{average total equity}} \qquad \text{return on common equity} = \frac{\text{net income} - \text{preferred dividends}}{\text{average common equity}}$$

Operating profit (gross profit − SG&A and other operating costs) is usually approximated by **earnings before interest and taxes (EBIT)**. **Earnings before taxes (EBT)** equals EBIT − interest, and gross profit equals revenue − COGS. Ratios that compare sales, at the top of the income statement, with a profit figure further down are **operating profitability ratios**; each margin isolates a different group of costs.

From revenue to net income: the profit line behind each margin

Income statement line	Margin that uses it (÷ revenue)
Revenue	—
− Cost of goods sold	
= Gross profit	Gross profit margin
− Operating expenses (SG&A and other)	
= Operating profit (approximated by EBIT)	Operating profit margin
− Interest expense	
= Earnings before taxes (EBT)	Pretax margin
− Income taxes	
= Net income	Net profit margin
− Preferred dividends	
= Income available to common shareholders	(Used in return on common equity)

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Example. Pellworth Hardware has revenue of $900, COGS of $540, operating expenses of $198, interest expense of $27 and a 25% tax rate. Average receivables are $75 and average inventory is $90. Gross margin is $360/900 = 40\%$. EBIT is 162, so the operating margin is 18%. EBT is 135 and net income 101.25, a net margin of 11.25%. Interest coverage is $162/27 = 6.0$. Receivables turnover is $900/75 = 12$, giving a DSO of 30.4 days, and inventory turnover is $540/90 = 6$, giving a DOH of 60.8 days.

WORKED EXAMPLE: CASH CONVERSION CYCLE AND DEFENSIVE INTERVAL

A distributor reports revenue of $7,300, COGS of $4,380 and SG&A of $1,095; the expense lines include depreciation of $365. Average receivables are $600, average inventory $720 and average trade payables $420. At year-end it holds cash of $150, marketable securities of $90 and receivables of $600. Using COGS for payables turnover and a 365-day year, find the operating cycle, the cash conversion cycle and the defensive interval.

Step 1. Receivables turnover $= 7{,}300/600 = 12.17$, so DSO $= 365 \times 600/7{,}300 = 30.0$ days.

Step 2. Inventory turnover $= 4{,}380/720 = 6.08$, so DOH $= 365 \times 720/4{,}380 = 60.0$ days.

Step 3. Payables turnover $= 4{,}380/420 = 10.43$, so days of payables $= 365 \times 420/4{,}380 = 35.0$ days.

Step 4. Operating cycle $= 30.0 + 60.0 = 90.0$ days; cash conversion cycle $= 30.0 + 60.0 - 35.0 = 55.0$ days.

Step 5. Average daily expenditures $= (4{,}380 + 1{,}095 - 365)/365 = 5{,}110/365 = 14.0$. The depreciation is subtracted because it is a noncash charge inside the expense lines.

Result. Defensive interval $= (150 + 90 + 600)/14.0 = 60.0$ days. Inventory is left out of the numerator.

Common exam traps

- Receivables turnover uses revenue, not COGS, because receivables are recorded at selling prices. Days ratios are 365 ÷ turnover, so the turnover itself is not a number of days. For Pellworth, using COGS gives a turnover of 7.2 and a DSO of 50.7 days instead of 12 and 30.4 days.
- In the cash conversion cycle, days of payables are subtracted.
- Inventory is not in the numerator of the defensive interval.
- Marketable securities stay in the quick ratio and in the cash ratio.
- Interest coverage uses EBIT. EBT and net income are the wrong numerators. For Pellworth, EBT gives 5.0 and net income gives 3.75 instead of 6.0.
- When only common-size percentages are given, convert them to amounts (percentage × total assets or × sales) before combining items from different statements.

EXAM SHORTCUTS

- For a ratio with a positive numerator and denominator, an equal decrease in both raises the ratio if it is above 1 and lowers it if it is below 1, and an equal increase does the opposite, so paying payables with cash raises a current ratio that is above 1.
- Collecting receivables into cash moves value inside the numerator of the current ratio, so the current ratio is unchanged while the cash ratio rises.

BOTTOM LINE

- Receivables turnover is revenue divided by average receivables, inventory turnover is COGS divided by average inventory, payables turnover is COGS (or purchases) divided by average trade payables, and each days ratio is 365 divided by the turnover.
- The cash conversion cycle equals DSO + DOH − days of payables, and the operating cycle equals DOH + DSO.
- The defensive interval equals cash, marketable securities and receivables divided by average daily cash expenditures, from which noncash charges in the expense lines are removed.
- Interest coverage is EBIT divided by interest payments, and fixed charge coverage is (EBIT + lease payments) divided by (interest payments + lease payments).
- Exam convention: total debt is interest-bearing short- and long-term debt with lease liabilities left out unless stated otherwise. Current practice: lease liabilities are on the balance sheet under IFRS 16 and ASC 842, and many analysts and rating agencies include them in debt.
- ROA is net income divided by average total assets, adjusted ROA adds interest expense × (1 − tax rate) to the numerator, and ROE is net income divided by average total equity.

QUICK CHECK

Question 2

An analyst notes that the inventory turnover of Fenmore Outdoor dropped sharply this year, although its sales and cost of goods sold were roughly unchanged. Which explanation is *most* consistent with the decline?

A At the start of the year, Fenmore wrote a large amount of obsolete stock down to net realizable value.

B A fault in a new ordering system placed duplicate orders with suppliers and built up surplus stock.

C Fenmore switched to a just-in-time system in which suppliers deliver goods only as they are needed.

Answer: B

Inventory turnover is cost of goods sold divided by average inventory. With cost of goods sold roughly unchanged, a sharp fall in turnover means that average inventory rose. Duplicate orders add surplus stock, which increases average inventory and lowers turnover.

Why the other options are wrong

- **A.** A write-down at the start of the year reduces the carrying amount of inventory for the whole year. Average inventory is lower, so turnover rises.
- **C.** A just-in-time system is designed to keep inventory low. Lower average inventory raises turnover.

Key takeaway $\text{Inventory turnover} = \text{COGS}/\text{average inventory}$. Low turnover relative to peers can signal slow-moving or obsolete stock; high turnover can mean efficient management or inventory kept too low to meet demand, so compare revenue growth with peers before concluding.

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MODULE 37.3

Financial Ratios, Part 2

LOS 37.b — Solvency and profitability ratios: interpretation details

The formulas are in Module 37.2. This module adds interpretation points and then turns to reading ratios together (LOS 37.c).

Solvency.

- A financial leverage ratio close to 1 means the assets are financed mostly with equity; the further it rises above 1, the more debt is used, which typically raises risk for both shareholders and bondholders. If "debt" is taken as all liabilities, $A/E = 1 + L/E$.
- The lower interest coverage is, the harder it is to meet interest payments. **Debt-to-EBITDA** indicates roughly how many years of operating cash flow, with EBITDA as a proxy, would be needed to repay the debt.
- **Fixed charge coverage** is the more meaningful measure for firms that lease many of their assets, such as some airlines. It is also useful for US GAAP reporters with operating leases, because an operating lease records a single lease cost and no interest expense in the income statement, so interest coverage alone misses the lease burden. Because lease payments are added to both the numerator and the denominator, they pull the ratio toward 1. When interest coverage is above 1 (the normal case), fixed charge coverage is lower than interest coverage, and the gap widens as lease payments grow. The two are equal at 1. If interest coverage is below 1, fixed charge coverage is higher; with EBIT of 5, interest of 10 and leases of 10, it is $15/20 = 0.75$ against interest coverage of $5/10 = 0.50$.
- Firms with stable cash flows can carry more debt, so solvency ratios are judged against cash-flow variability.

Profitability.

- The **net profit margin** should be based on income from **continuing operations**, because discontinued operations will not recur.
- The **gross profit margin** rises with higher prices or lower production costs. Competition limits pricing.
- The EBIT used for the **operating profit margin** can include some nonoperating items, and some analysts add back depreciation and amortization to use EBITDA. Whichever measure is chosen should be used consistently.
- The adjusted ROA adds back after-tax interest, so the numerator reflects returns to both debt and equity holders. The add-back uses gross interest expense, because lenders earn the full interest; interest income is not netted against it.
- **Return on invested capital (ROIC)** is after-tax operating profit ÷ **average long-term capital**, where long-term capital is long-term (interest-bearing) debt + preferred equity + common equity. Exam convention: long-term capital excludes working capital, meaning non-interest-bearing current liabilities such as accounts payable are not counted as capital. Current practice: invested capital is usually total interest-bearing debt, including short-term borrowings and debt due within one year, plus equity, often net of excess cash. Analysts should be concerned if ROIC is too low.
- **ROE** (return on total equity) uses total equity including preferred stock. **Return on common equity** removes preferred dividends from the numerator and preferred equity from the denominator.

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LOS 37.c — Relationships among ratios and evaluating a company

No single ratio tells the story. An analyst compares each ratio with the firm's history (time-series) and with peers (cross-sectional), then asks which combination of changes explains the picture.

Useful relationships

KEY CONCEPT

Observation	Likely explanation to test
Current ratio rising while quick ratio falls	Inventory building up (check days of inventory on hand)
DSO falling while DOH rising	Faster collections may be offsetting a cash drain from slow-moving inventory
ROE rising while net margin and asset turnover fall	Higher **financial leverage**: more risk, not better operations
ROA falling but ROE rising	Leverage has increased ($\text{ROE} = \text{ROA} \times \text{assets/equity}$)
Gross margin rising but net margin falling	Costs below gross profit (operating costs, interest, taxes) grew faster than sales
Interest coverage far below a peer's	Higher financial risk, even if margins are similar

Higher is not always better. Values should be close to the industry norm. An activity ratio far better than the norm, such as a collection period well below the industry's, is a question to investigate rather than a sure strength; the possible explanations for each turnover ratio are in Module 37.2. Because DSO $= 365/\text{turnover}$, a turnover below the industry average implies a collection period longer than the industry's.

Liquidity signals. Better liquidity shows up as higher current, quick and cash ratios and as faster conversion of receivables and inventory into cash (higher turnovers, shorter days). A lower balance of trade payables is ambiguous: it may reflect strong liquidity or suppliers demanding cash on delivery.

Example. Three years of data for Ferrand Glass:

	Year 1	Year 2	Year 3
ROA	9.5%	8.5%	7.5%
Average liabilities / average equity	0.90	1.40	2.00
Net profit margin	6.0%	5.5%	5.0%

ROE $= \text{ROA} \times (1 + L/E)$: Year 1 $9.5\% \times 1.9 = 18.05\%$; Year 2 $8.5\% \times 2.4 = 20.4\%$; Year 3 $7.5\% \times 3.0 = 22.5\%$. ROE improved even though ROA and margins deteriorated. The gain comes entirely from more leverage, so shareholders face more risk.

Evaluating a company against its industry. Each group of ratios is read in turn: liquidity (current ratio above or below the industry's), efficiency (asset turnover), profitability (margins, ROE) and leverage (debt ratios). A firm with a higher current ratio, lower margins and ROE, and a lower debt-to-capital ratio than its industry, for

instance, is more liquid, less profitable and less leveraged than its peers.

Common exam traps

- A firm whose margins and ROE are below the industry's is less profitable, even if its coverage ratio is higher. The higher coverage indicates lower financial risk.
- When comparing interest coverage between firms, compute both ratios before judging how much lower one is than the other.

EXAM SHORTCUTS

- When interest coverage is above 1, fixed charge coverage is lower than interest coverage, because adding lease payments to both numerator and denominator pulls the ratio toward 1.

BOTTOM LINE

- A financial leverage ratio close to 1 means assets are financed mostly with equity, and if all liabilities are counted as debt, A/E = 1 + L/E.
- Fixed charge coverage is the more meaningful coverage measure for firms that lease many of their assets.
- Income from continuing operations is the right basis for the net profit margin, because discontinued operations will not recur.
- ROE uses total equity including preferred stock, while return on common equity removes preferred dividends from the numerator and preferred equity from the denominator.
- ROE rising while net margin and asset turnover fall, or ROA falling while ROE rises, points to higher financial leverage and more risk rather than better operations.
- Ratios should be close to the industry norm, so an activity ratio far better than the norm is a question to investigate rather than a sure strength.

MODULE 37.4

DuPont Analysis

LOS 37.d — DuPont analysis of return on equity

DuPont analysis breaks **return on equity (ROE)** into component ratios so the analyst can see why ROE is high, low or changing. It decomposes ROE; it is not a different way to measure it. When net income and equity are known, ROE is simply one divided by the other. Average or year-end equity can be used, as long as the choice is consistent.

Two-part decomposition

$$\text{ROE} = \frac{\text{net income}}{\text{average total equity}} = \underbrace{\frac{\text{net income}}{\text{average total assets}}}_{\text{ROA}} \times \underbrace{\frac{\text{average total assets}}{\text{average total equity}}}_{\text{financial leverage}}$$

Original (three-part) DuPont equation

Multiplying ROA by revenue/revenue splits it into margin and turnover:

KEY CONCEPT

$$\text{ROE} = \underbrace{\frac{\text{net income}}{\text{revenue}}}_{\text{net profit margin}} \times \underbrace{\frac{\text{revenue}}{\text{average total assets}}}_{\text{total asset turnover}} \times \underbrace{\frac{\text{average total assets}}{\text{average total equity}}}_{\text{financial leverage}}$$

The third term is the financial leverage ratio (equity multiplier, or leverage ratio) from Module 37.2. A low ROE must come from a poor **net profit margin**, poor **total asset turnover**, too little leverage, or a mix of these. Because revenue/equity is **equity turnover**, ROE also equals net profit margin × equity turnover.

Extended (five-part) DuPont equation

The net profit margin is split into three pieces:

KEY CONCEPT

$$\text{ROE} = \underbrace{\frac{\text{net income}}{\text{EBT}}}_{\text{tax burden}} \times \underbrace{\frac{\text{EBT}}{\text{EBIT}}}_{\text{interest burden}} \times \underbrace{\frac{\text{EBIT}}{\text{revenue}}}_{\text{EBIT margin}} \times \underbrace{\frac{\text{revenue}}{\text{average total assets}}}_{\text{total asset turnover}} \times \underbrace{\frac{\text{average total assets}}{\text{average total equity}}}_{\text{financial leverage}}$$

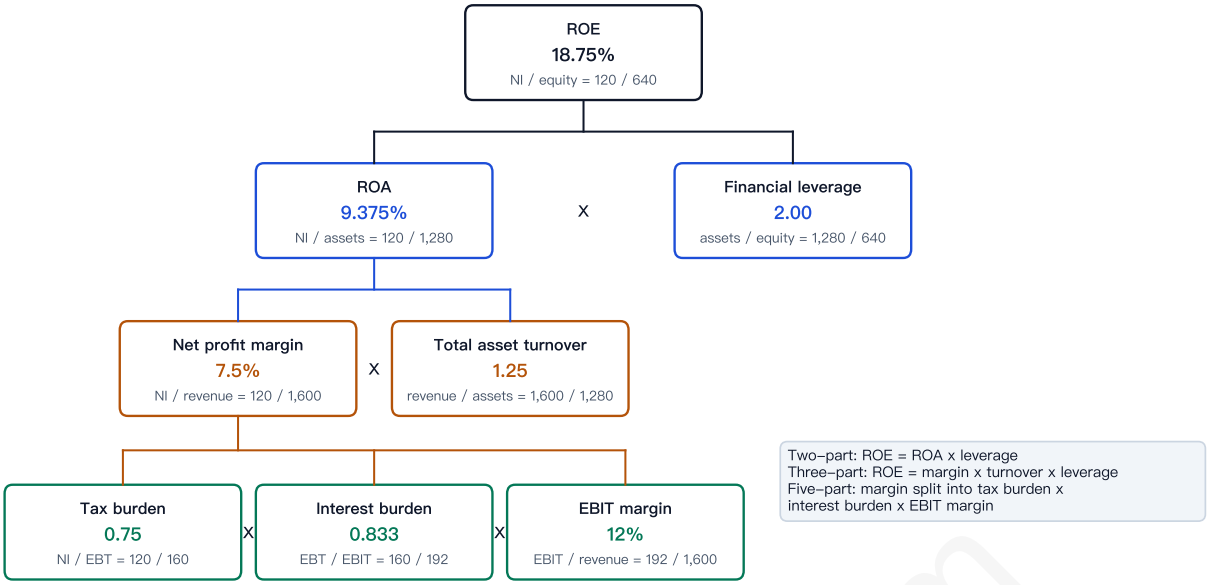
- **Tax burden** = NI/EBT = $1 - \text{tax rate}$. A lower ratio means heavier taxes; with positive pretax income, higher taxes reduce ROE.
- **Interest burden** = EBT/EBIT. More interest relative to EBIT lowers the ratio, which is a heavier interest burden.
- **EBIT margin** = EBIT/revenue. If operating income replaces EBIT, the term becomes the operating margin and the interest burden term also captures nonoperating income.
- Tax burden × interest burden × EBIT margin = net profit margin, so the five-part and three-part results must agree.

Leverage cuts both ways. Borrowing helps shareholders only when the assets it funds earn more than the debt costs. Higher leverage raises the equity multiplier but also raises interest expense, which lowers the interest burden ratio. ROE rises with leverage only if the multiplier effect outweighs the fall in the interest burden ratio. For a profitable firm (positive net income and positive equity), holding all other components constant, a higher asset turnover or a higher net profit margin raises ROE. The condition matters. With a net loss, a higher asset turnover makes ROE more negative, and with zero net income it leaves ROE unchanged.

WORKED EXAMPLE

Example. Arbutus Freight reports revenue of $1,600, EBIT of $192, interest expense of $32, taxes of $40, average total assets of $1,280 and average total equity of $640.

- Net income $= 192 - 32 - 40 = 120$; EBT $= 160$; ROE $= 120/640 = 18.75\%$.
- Three-part: $\frac{120}{1{,}600} \times \frac{1{,}600}{1{,}280} \times \frac{1{,}280}{640} = 0.075 \times 1.25 \times 2.0 = 18.75\%$.
- Five-part: $\frac{120}{160} \times \frac{160}{192} \times \frac{192}{1{,}600} \times 1.25 \times 2.0 = 0.75 \times 0.8333 \times 0.12 \times 1.25 \times 2.0 = 18.75\%$.



DuPont tree for Arbutus Freight

Comparing two firms. A firm can earn a higher ROE with less leverage if its EBIT margin and asset turnover are clearly better. The five-part decomposition shows whether tax, interest, operating margin, efficiency or leverage explains the gap.

Solving for a missing component. Rearrange the identity. For example, net profit margin $= \text{ROE}/(\text{asset turnover} \times \text{leverage})$, then net income $= \text{margin} \times \text{revenue}$, and equity $= \text{net income}/\text{ROE}$.

Example. Oriel Stores and Paxton Goods both earn an ROE of 12%. Oriel has revenue of $450, average total assets of $300 and average total liabilities of $120, so its average equity is $180. Paxton has revenue of $900, average total assets of $900 and average total liabilities of $600, so its average equity is $300.

- Oriel: asset turnover $= 450/300 = 1.50$, leverage $= 300/180 = 1.667$, net profit margin $= 0.12/(1.50 \times 1.667) = 0.12/2.50 = 4.8\%$.
- Paxton: asset turnover $= 900/900 = 1.00$, leverage $= 900/300 = 3.00$, net profit margin $= 0.12/(1.00 \times 3.00) = 0.12/3.00 = 4.0\%$.

The same ROE comes from different sources. Oriel earns it with a higher margin and faster turnover; Paxton relies on more leverage.

Common exam traps

- The three components are net profit margin, total asset turnover and the equity multiplier (assets/equity). Gross margin and debt-to-equity are not components.
- EBT/EBIT is the interest burden; NI/EBT is the tax burden.
- ROE divides by equity, and net income = EBIT − interest − taxes. For Arbutus, dividing by average total assets gives 9.375%, which is ROA, instead of an ROE of 18.75%.
- When a question gives beginning and ending balances, it expects average assets and average equity; year-end balances give a different answer.

EXAM SHORTCUTS

- Tax burden × interest burden × EBIT margin equals the net profit margin, so a missing component can be solved from the identity and the five-part and three-part results must agree.

BOTTOM LINE

- ROE equals ROA times financial leverage, where financial leverage is average total assets divided by average total equity.
- In the three-part DuPont equation, ROE is the product of net profit margin, asset turnover (revenue / average total assets) and the equity multiplier (average total assets / average total equity).
- The five-part DuPont equation is ROE = tax burden (NI/EBT) × interest burden (EBT/EBIT) × EBIT margin × total asset turnover × financial leverage.
- Higher leverage raises the equity multiplier but lowers the interest burden ratio, so ROE rises with leverage only if the multiplier effect outweighs the fall in the interest burden ratio.
- For a profitable firm with positive equity, holding the other components constant, a higher asset turnover or a higher net profit margin raises ROE; with a net loss, higher asset turnover makes ROE more negative.

QUICK CHECK

Question 3

A company has positive net income and positive shareholders' equity. Holding its net profit margin and its equity multiplier constant, if the company improves its total asset turnover, its return on equity will:

A decrease.

B increase.

C increase, decrease, or stay the same.

Answer: B

In the original DuPont equation, ROE is the product of net profit margin, total asset turnover and financial leverage. Because net income and equity are positive, the net profit margin and the equity multiplier are both positive, so with those two components fixed ROE moves in the same direction as asset turnover.

Why the other options are wrong

- **A.** Asset turnover enters ROE as a multiplier. With a positive margin and a positive equity multiplier held constant, a higher turnover cannot lower ROE. It would lower ROE only for a loss-making firm, which the stem rules out.
- **C.** The outcome is not ambiguous here: the other two components are held constant and are positive. The result would be indeterminate only without the positive-income premise (a zero margin leaves ROE unchanged; a negative margin makes ROE fall).

Key takeaway ROE = net profit margin × total asset turnover × financial leverage; for a profitable firm, raising any one component with the others fixed raises ROE.

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MODULE 37.5

Industry-Specific Financial Ratios

LOS 37.e — Industry-specific ratios

Every industry has its own **key performance indicators**, so part of an analyst's skill is choosing the ratios that matter for the business being analyzed.

KEY CONCEPT

Industry	Ratio	What it shows
Service and consulting firms	**Sales per employee**, **net income per employee**	Productivity of the people who are the main "asset"
Retail, restaurants	**Growth in same-store sales**	Growth from existing locations only (excludes new openings): how well the firm attracts and keeps customers; a decline can mean new stores are taking customers from old ones
Retail	**Sales per square foot**	Productivity of selling space; useful for comparing retailers with each other
Hotels	**Average daily rate** = room revenue ÷ number of rooms sold	Pricing / profitability indicator
Hotels	**Occupancy rate** = rooms sold ÷ rooms available	Activity (utilization) indicator
Subscription services (e.g., streaming)	**Average revenue per user**	Monetization of the customer base
Lending institutions (banks)	**Exam convention:** net interest margin = interest income ÷ interest-earning assets. **Current practice:** net interest margin = net interest income (interest income − interest expense) ÷ average interest-earning assets. Exam questions follow the exam convention unless they name the current-practice definition.	Performance of the lending business

Regulators often require **financial institutions** to keep certain ratios above minimums or below maximums:

- **Capital adequacy** is a monetary measure of the firm's operational and financial risk relative to its equity capital. Regulators monitor it so that the firm has a buffer to absorb losses and to limit contagion across the banking system.

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- **Value at risk (VaR)** is a common measure of capital risk. It estimates the size of loss that the firm will exceed only a specified percentage of the time over a specified period. This minimum loss in the worst outcomes is equivalent to the maximum loss at the stated confidence level: a 5% one-month VaR is the minimum loss in the worst 5% of months, which is also the maximum loss with 95% confidence. VaR is not the worst possible loss, because larger losses can still occur.
- **Reserve requirements** set a minimum level of eligible reserves (such as balances at the central bank) relative to a bank's reservable liabilities. Exam convention: banks are subject to minimum reserve requirements. Current practice: some central banks now set the minimum at zero.
- A **liquid asset requirement** requires a bank's liquid assets to be at least a stated proportion of certain of its liabilities.

Business risk and the coefficient of variation

The standard deviations of revenue, operating income and net income measure how variable (uncertain) a firm's results are, but they grow with firm size. The **coefficient of variation (CV)** is the size-adjusted version:

KEY CONCEPT

$$\text{CV} = \frac{\text{standard deviation of the item}}{\text{mean (expected value) of the item}}$$

A CV is variation per unit of the item, so it says nothing about the level of sales or income. Comparing CVs over time or across peers helps assess **business risk**:

- The CV of sales measures revenue stability (sales risk).
- The CV of operating income combines sales risk with the cost structure (operating leverage), which together make up business risk. On its own it cannot show whose operating costs are more volatile, because it also depends on how large costs are relative to sales.
- The CV of net income adds the effect of financial leverage (interest), taxes and nonoperating items.

Example. Operating income of Delford Mills averaged $50 million with a standard deviation of $12 million, so its CV is $12/50 = 0.24$. A peer averaging $200 million with a standard deviation of $36 million has a CV of 0.18. The peer's swings are larger in dollars but smaller in relative terms.

LOS 37.f — Ratio analysis in modeling and forecasting earnings

Pro forma (projected) statements start from a forecast of next-period revenue. Other items are then tied to revenue with ratios the analyst expects to hold:

- If COGS as a percentage of sales is expected to stay constant, the common-size COGS percentage is applied to forecast sales to get forecast COGS (and gross profit).
- Without information suggesting a change, the prior-period operating profit margin can be applied to forecast sales to estimate operating profit.
- Ratios can also be moved in a chosen direction when the analyst expects a change.

Expense items are forecast as percentages of sales, the base of the common-size income statement, and not as percentages of net income.

Instead of relying on single point estimates, the analyst can study the range of outcomes:

KEY CONCEPT

Technique	How it works
Sensitivity analysis	"What if" analysis: change one input at a time (e.g., sales growth of 3% instead of 5%) and observe the effect on the outcome
Scenario analysis	Specify a coherent set of values for several key variables at once (a scenario) and compute the outcome for each scenario
Simulation	Assign **probability distributions** to key variables; a computer draws values at random many times and produces a distribution of outcomes

Example. Revenue of $80 million is forecast to grow 15%. COGS has been 55% of sales and the operating margin 12%. Forecast revenue is 92, COGS 50.6, gross profit 41.4 and operating profit $0.12 \times 92 = 11.04$ million.

Common exam traps

- Net interest margin is a ratio for lending institutions such as banks, not for insurers or subscription businesses. Exam convention: interest income ÷ interest-earning assets. Current practice: net interest income ÷ average interest-earning assets.
- CV = standard deviation ÷ mean, not mean ÷ standard deviation. For Delford, mean ÷ standard deviation gives 4.17 instead of a CV of 0.24.
- The CV of operating income relates to business risk. Financial risk comes from the capital structure, which operating income excludes.

BOTTOM LINE

- Growth in same-store sales excludes new openings; a hotel's average daily rate is room revenue divided by rooms sold, and its occupancy rate is rooms sold divided by rooms available.
- Exam convention: net interest margin is interest income divided by interest-earning assets. Current practice: it is net interest income divided by average interest-earning assets.
- Value at risk is the loss a firm will exceed only a specified percentage of the time over a specified period, and it is not the worst possible loss.
- The coefficient of variation is the standard deviation of an item divided by its mean; the CV of sales measures sales risk, the CV of operating income measures business risk, and the CV of net income adds financial leverage, taxes and nonoperating items.
- Pro forma statements start from forecast revenue and tie other items to it with ratios such as the common-size COGS percentage and the operating profit margin.
- Sensitivity analysis changes one input at a time, scenario analysis sets a coherent set of values for several variables at once, and simulation draws values from probability distributions many times.

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QUICK CHECK

Question 4

Carraway Outfitters reported sales of $200 million for 20X5 and expects sales to grow by 9.5% in 20X6. Cost of goods sold is expected to stay at 50% of sales, and management targets an average of 50 days of inventory on hand in 20X6. Using a 365-day year, Carraway's forecast average inventory for 20X6 is *closest* to:

A $13.70 million.

B $15.00 million.

C $30.00 million.

Answer: B

Forecast next year's sales, apply the common-size COGS percentage, and convert the target days of inventory on hand into an inventory balance: average inventory = COGS/365 × days of inventory on hand.

$$\text{Sales}_{20X6} = 200 \times 1.095 = 219 \qquad COGS = 0.50 \times 219 = 109.5$$

$$\text{Average inventory} = \frac{109.5}{365} \times 50 = 15.00 \quad (\$ \text{ millions})$$

Why the other options are wrong

- **A.** $13.70 million ignores the expected 9.5% sales growth and uses 20X5 COGS of $100 million.
- **C.** $30.00 million applies the inventory days to sales instead of cost of goods sold.

Key takeaway Days of inventory on hand is based on COGS: inventory = (COGS ÷ 365) × DOH.

Practice questions

Question 5

An analyst wants to assess how well a manufacturer converts its sales into operating profits. Which of the following ratios is *least likely* to be one of the operating profitability ratios she would use?

A Gross profit / net sales.

B Net sales / total assets.

C Net income / net sales.

Answers: 5. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/dupont-analysis/. The full bank has 106 questions on this reading.

FINANCIAL STATEMENT ANALYSIS · READING 38

Pro Forma Financial Statements

What you'll learn

LOS 38.a Demonstrate the development of a sales-based pro forma company model.
LOS 38.b Explain how behavioral biases affect analyst forecasts and recommend remedies.
LOS 38.c Explain how a company's competitive position under Porter's five forces affects its prices and costs.
LOS 38.d Explain how to forecast industry and company sales and costs under price inflation or deflation.
LOS 38.e Explain the choice of an explicit forecast horizon and how to project results beyond the short term.

MODULE 38.1

Financial Statement Modeling

This reading shows how to build a sales-based pro forma model, how behavioral biases distort forecasts, and how competitive forces, input-price inflation or deflation and the choice of forecast horizon shape long-term projections. A candidate must be able to project an income statement from a revenue forecast and explain the effect of each bias and each competitive force on the forecast.

LOS 38.a — Building a sales-based pro forma model

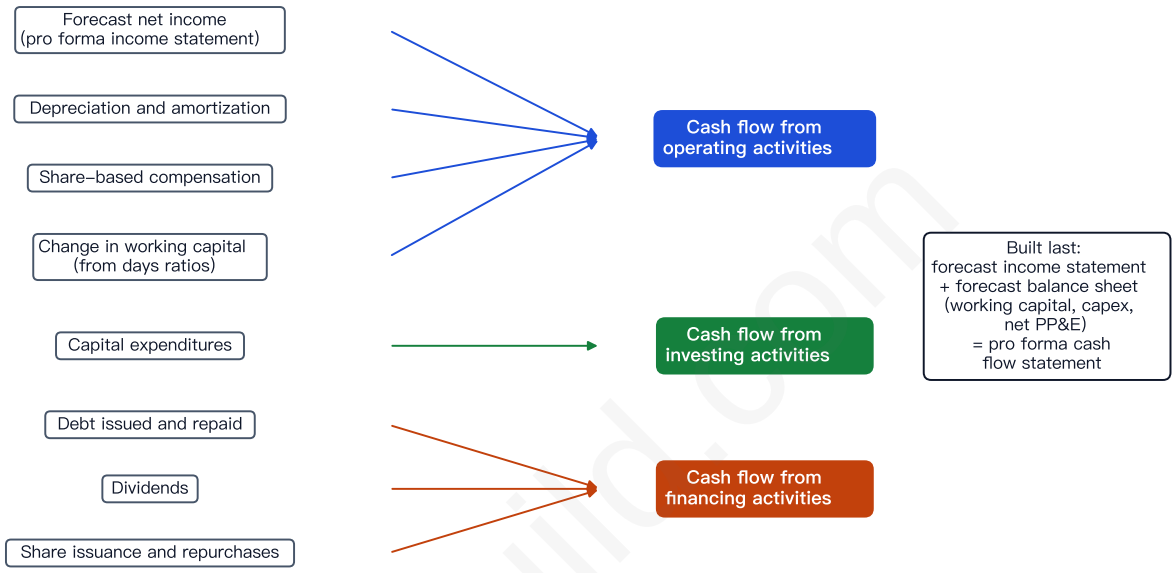
Financial statement analysis studies a company's current and past statements. Financial statement modeling projects its future statements, which the analyst then uses to value the company and its securities. A **sales-based pro forma company model** is a set of projected financial statements driven by the analyst's forecast of future revenue. The sequence matters because each statement feeds the next:

KEY CONCEPT

Step	Task	Notes
1	Estimate **revenue growth** and future revenue	From market growth and market share, a trend growth rate, or growth relative to GDP
2	Estimate **COGS**	As a percentage of sales, or in more detail from strategy and competition; COGS need not grow exactly with sales because of fixed elements and input-price changes
3	Estimate **SG&A**	Fixed, growing with revenue (e.g., sales-related labor), or another method
4	Estimate **financing costs**	Interest rates, debt levels, planned capex and changes in financial structure
5	Estimate **income tax expense and cash taxes**	Historical effective rates and trends, segment/jurisdiction mix, changes in deferred tax items
6	Model the **balance sheet**	Working capital accounts that flow from the income statement (receivables, inventory, payables)

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Step	Task	Notes
7	Estimate **capital expenditures and net PP&E**	From depreciation and capital expenditures (for maintenance and for growth)
8	Build the **pro forma cash flow statement**	Last: it is derived from the pro forma income statement and balance sheet



Building the pro forma statement of cash flows

Example. Last year Corliss Tools had revenue of $400 million. The analyst expects 5% revenue growth, a 38% gross margin, SG&A of $30 million fixed plus 10% of revenue, debt of $150 million at 6% interest and a 25% effective tax rate.

1. Revenue $= 400 \times 1.05 = 420$.
2. COGS $= (1 - 0.38) \times 420 = 260.4$, so gross profit is 159.6.
3. SG&A $= 30 + 0.10 \times 420 = 72$, so operating profit is 87.6.
4. Interest expense $= 0.06 \times 150 = 9$, so pretax income is 78.6.
5. Income tax expense $= 0.25 \times 78.6 = 19.65$, so forecast net income is $58.95 million.

A change in expected unit sales mainly moves the working capital accounts tied to sales: accounts receivable, because credit sales change with volume, and inventory, because the number of units the firm needs on hand changes (and stock builds up or runs down if purchasing does not adjust). Prepaid expenses are payments for future-period expenses and are unrelated to purchases for resale, so they are largely unaffected.

LOS 38.b — Behavioral biases in forecasting

KEY CONCEPT

Bias	What it looks like	Remedy
Overconfidence bias	Too much faith in one's own forecasts; confidence intervals too narrow (more common in analysts who go against the consensus)	Share forecasts and invite critique; review past forecast errors; use **scenario analysis** to produce a range
Illusion of control bias	Overestimating what one can control: seeking "expert" opinions to justify a view, adding more and more variables (overfitting); overfitted models forecast poorly out of sample and can hide assumptions that need updating	Use only variables with known explanatory power; seek outside opinions only from those with a relevant perspective
Conservatism bias (also called **anchoring**)	Only small adjustments to prior forecasts when new information arrives; slow to reflect news (usually negative, sometimes positive)	Periodically evaluate forecast errors; use simpler models that are easy to update
Representativeness bias	Relying on known classifications; one form is **base-rate neglect** — ignoring the rate of incidence in the wider population in favor of company-specific facts	Consider both the **inside view** (company-specific) and the **outside view** (the base rate for the industry or peer group)
Confirmation bias	Seeking data that support one's view, discounting contrary evidence (including taking management's upbeat comments at face value)	Read research from analysts with opposite views; ask colleagues with no emotional stake

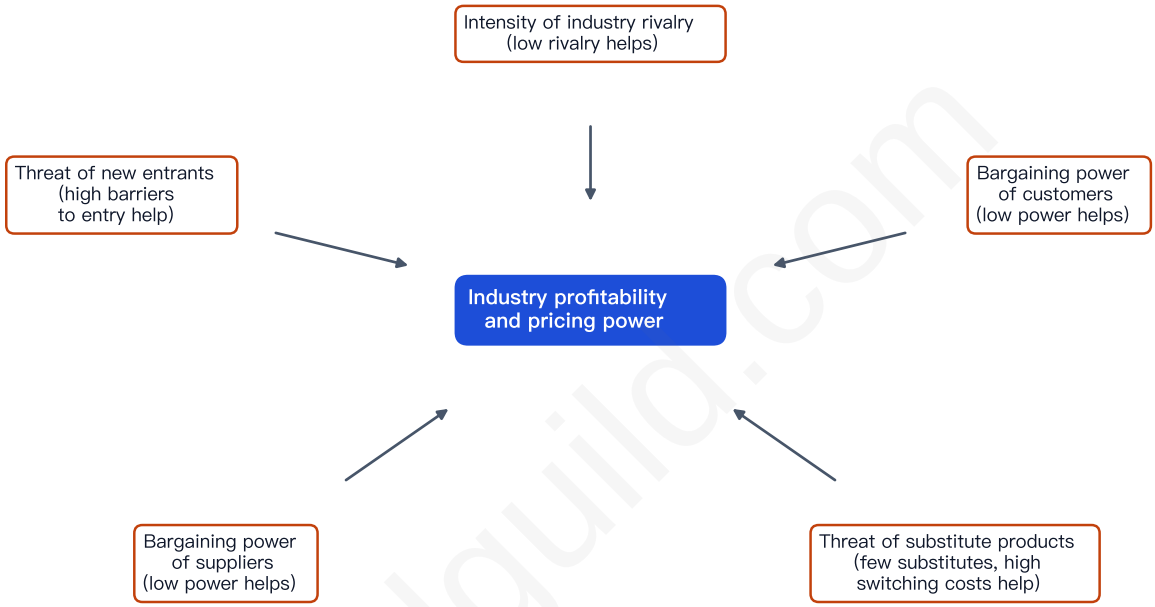
LOS 38.c — Porter's five forces, prices and costs

A firm's competitive position is probably the most important driver of its future revenue and profitability. **Porter's five forces**:

KEY CONCEPT

Force	Pricing power / margins are higher when…
Threat of substitute products	the threat is low and switching costs are high
Intensity of industry rivalry	rivalry is low (rivalry is intense with many competitors, high fixed costs and exit barriers, slow or negative growth, undifferentiated products)

Force	Pricing power / margins are higher when…
Bargaining power of suppliers	supplier power is low (less pressure on input costs; when suppliers are few, they can capture more of the value added)
Bargaining power of customers	customer power is low (not a few large buyers; high switching costs)
Threat of new entrants	the threat is low, i.e., **barriers to entry** are high, which lets incumbents sustain economic profits



The weaker each of the five forces, the stronger the pricing power and margins.

Porter's five forces

Favorable forces support optimistic revenue and margin forecasts.

LOS 38.d — Forecasting under input-price inflation or deflation

- Hedging with derivatives or fixed-price contracts delays and smooths the effect of input-price changes; **vertical integration** reduces exposure to input costs.
- Without either, the analyst must judge how fast and how far cost increases can be passed on to customers, and the effect of higher prices on volume. Production costs should be tracked by product category and geographic location. Input prices in turn respond to weather, to taxes and other government rules, to tariffs and to the way each input market is structured. The firm's hedging activities and its vertical structure belong in the same review.
- Firms may switch to a substitute input when one input's price rises.
- If input-cost increases are temporary (short term), cutting other costs (e.g., advertising) to protect operating margins can be sensible; for long-term increases this is not appropriate.

- **Elasticity of demand**: with elastic demand, the percentage fall in units exceeds the percentage rise in price, so a price increase reduces revenue. The main influence on elasticity is the availability of substitute products.
- A first mover on price increases loses more volume than a follower; a firm that delays may gain share but suffers lower gross margins in the meantime.

Example. A firm sells units at \$60 with unit COGS of \$36 (gross margin $24/60 = 40\%$). Unit input cost rises by \$3 and the firm raises its price by exactly \$3, with no change in volume. Gross profit per unit stays at \$24, but gross margin falls to $24/63 = 38.1\%$. Passing through the money amount of a cost increase preserves gross and operating profit if volume holds (which is unlikely), but it lowers the gross, operating and net margins because revenue is larger.

LOS 38.e — Forecast horizon and long-term projections

- **Buy-side**: the horizon can be the expected holding period. Average holding period ≈ 1 / annual portfolio turnover (a turnover of 20% gives about 5 years).
- **Cyclical companies**: choose a horizon that reaches at least the middle of the business cycle, so the forecast reflects midcycle sales and profits rather than the current phase. **Normalized earnings** = expected midcycle earnings (or earnings free of temporary effects).
- Acquisitions, mergers and restructurings: the horizon should be long enough for their benefits to be realized (or not). A manager may also dictate the horizon.
- Beyond the short term, a common approach assumes the prior cycle's trend revenue growth continues.
- **Terminal value**: by a multiples approach or a DCF approach. A multiple must be consistent with the expected growth rate and required return. Using a 10-year average P/E assumes the next period's growth and required return will match the past 10 years on average. In a DCF, the earnings or cash flow used should be normalized to a midcycle level free of temporary effects. The terminal value is a growing perpetuity, so small changes in the long-term growth rate have large effects on value.
- **Inflection points** are times when the future will not resemble the past: a shift in the economic environment or in the stage of the business cycle, government regulation, or technology.

COMMON EXAM TRAPS

- Conservatism bias and confirmation bias can both look like sticking to a view. Conservatism is under-reaction to news that has been acknowledged; confirmation bias is selective attention to evidence that agrees with the view.
- Seeking expert opinions to justify a forecast is illusion of control, not a remedy; the remedy is to consult only people with a directly relevant perspective.
- For a cyclical firm, the test is whether the horizon reaches midcycle, not whether it covers one or more full cycles.

BOTTOM LINE

- A sales-based pro forma model starts from the revenue forecast, estimates COGS, SG&A, financing costs and taxes, then the balance sheet and capital expenditures, and derives the pro forma cash flow statement last from the projected income statement and balance sheet.
- Overconfidence bias shows up as confidence intervals that are too narrow and is countered by inviting critique of the forecasts, reviewing past forecast errors and using scenario analysis to produce a range.

- Conservatism bias, also called anchoring, is making only small adjustments to prior forecasts when new information arrives, while confirmation bias is seeking data that support one's view and discounting contrary evidence.
- Illusion of control bias includes seeking expert opinions to justify a view and overfitting a model with ever more variables, and representativeness bias includes base-rate neglect, which is countered by weighing the outside view with the inside view.
- Under Porter's five forces, pricing power and margins are higher when the threat of substitutes, the intensity of rivalry, the bargaining power of suppliers and of customers, and the threat of new entrants are low.
- With elastic demand the percentage fall in units sold exceeds the percentage rise in price, so a price increase reduces revenue, and the main influence on elasticity is the availability of substitutes.
- Passing through the money amount of an input-cost increase preserves gross and operating profit if volume holds, but it lowers the gross, operating and net margins because revenue is larger.
- For a cyclical company the forecast horizon should reach at least the middle of the business cycle, and a terminal value is very sensitive to the long-term growth rate because it is a growing perpetuity.

QUICK CHECK

Question 1

An analyst building sales-based pro forma financial statements will *most likely* carry out which of these tasks first?

A Building the pro forma cash flow statement.

B Estimating SG&A expenses.

C Forecasting capital expenditures and net PP&E.

Answer: B

The income statement is modeled first: revenue, then COGS, then SG&A, financing costs and taxes. Balance sheet items such as capital expenditures and net PP&E come after, and the cash flow statement is built last because it is derived from the pro forma income statement and balance sheet.

Why the other options are wrong

- **A.** The pro forma cash flow statement is the final step; it cannot be constructed until the income statement and balance sheet are complete.
- **C.** Capital expenditures and net PP&E are balance sheet items estimated after the income statement has been modeled.

Key takeaway The order is revenue, COGS, SG&A, financing costs and taxes, then the balance sheet (working capital, capex and PP&E), and finally the cash flow statement.

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Practice questions

Question 2

Since finishing her earnings forecast for Brenton Mining, Clara Voss has seen the company announce a new product line and a reorganization of its divisions. She had anticipated some of these developments in her forecast but not others. Nonetheless, she is unwilling to revise her forecast. Voss is *most likely* displaying:

A confirmation bias.

B conservatism bias.

C representativeness bias.

Question 3

Sorrel Audio sells 2,000 speakers a year at $80 each; unit COGS is $50 and SG&A is a fixed $30,000. Next year the input cost per speaker will rise by $4. Sorrel plans to raise its selling price by exactly $4 and expects unit sales to be unaffected. Sorrel's gross margin next year is *closest* to:

A 35.7%.

B 37.5%.

C 40.5%.

Question 4

When choosing an explicit forecast horizon for a company whose earnings swing sharply with the economy, an analyst should *most appropriately* extend the forecast:

A over one year only.

B far enough to include the middle of the business cycle.

C far enough to include two complete business cycles.

Answers: 2. B · 3. A · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/financial-statement-analysis/pro-forma-financial-statements/. The full bank has 23 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Equities

Exam weight 11–14% · 12 readings

How shares are issued, traded and voted, where equity returns come from, and how analysts value a company with discounted cash flow models and multiples. It also covers industry and company analysis, research reports and factor models such as the CAPM.

Reading 39	Common Stock vs Preferred Stock
Reading 40	Equity Jurisdictions, Classes, and the Voting Process
Reading 41	Equity Issuance and Trading
Reading 42	Sources of Equity Returns
Reading 43	Introduction to Equity Valuation
Reading 44	Discounted Cash Flow (DCF) and Growth Models
Reading 45	Relative Value Equity Valuation Approaches
Reading 46	Financial Statement Forecasting in Equity Valuation
Reading 47	Industry and Competitive Analysis
Reading 48	Company Analysis: Past, Present, and Future
Reading 49	Equity Analyst Research Reports
Reading 50	The Capital Asset Pricing Model, Market Model, and Other Factor-Based Equity Models

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EQUITIES · READING 39

Common Stock vs Preferred Stock

What you'll learn

LOS 39.a Describe what equity instruments are, how they differ from debt, and the features of common shares and the main types of preference shares.

LOS 39.b Contrast private equity with publicly listed equity and describe venture capital, growth equity, buyout equity and special situations.

MODULE 39.1

Public and Private Equity Investments

This reading defines an equity instrument, compares common shares with preference shares and their features, and contrasts public equity with private equity and its four main types.

LOS 39.a — What an equity instrument is, and the main types

An **equity instrument** is a security that gives its holder ownership of part of a corporation. The owners (**shareholders**) share in whatever is left of the company's assets after higher-ranking claims (debts, taxes) are met, and they may be able to vote on company policies and decisions. Their liability is limited. If the company goes bankrupt they can lose what they invested, but they are not otherwise liable for the company's obligations.

Equity versus debt

KEY CONCEPT

Feature	Equity instrument	Debt instrument (loan, bond)
Relationship with the company	Ownership interest	Creditor relationship
Life	Indefinite; exists until a corporate event (e.g., acquisition or liquidation) ends it	Finite; has a maturity
Payments	**Dividends** are paid at the company's discretion	Interest and principal are contractual obligations
Claim in liquidation	Residual, after debt and other higher-priority claims	Ranks ahead of equity

Dividends and the company life cycle

A company's willingness to pay dividends usually changes as it moves through four phases:

- **Start-up phase**: little or no revenue, negative cash flow as invested capital is spent building the business; nothing to distribute.
- **Growth phase**: revenue rising and cash flow may turn from negative to positive, but earnings are mostly reinvested to grow the business.

- **Mature phase**: stable cash flows and fewer growth opportunities, so shareholders prefer that earnings be distributed regularly as dividends. This is the phase most associated with dividend payments.
- **Decline phase**: falling revenue and cash flow; future dividends become doubtful.

Rights disclosed in an equity prospectus

When equity is issued, the **equity prospectus** describes the security's features and how the money raised will be used. It states whether the security carries:

- **voting rights** on company matters;
- **dividend rights** (regular or special dividends);
- **conversion rights**: to change the security into another type, such as preferred into common shares;
- **liquidation rights**: what the holder receives if the company is dissolved;
- **split, subdivision, or combination rights**: how the holder's position is treated if shares are split, divided or combined;
- **preemptive rights**: a first opportunity to buy any new shares the company issues in the future (protecting the holder's proportional stake).

Common shares

Common shares are the most widely issued form of equity. Common shareholders:

- have a **residual claim** on the firm's assets in liquidation: they are paid only after debtholders, other creditors and preferred shareholders;
- govern the company through voting rights: electing the board of directors, approving mergers, selecting auditors. A shareholder who cannot attend the annual meeting can **vote by proxy** (someone else casts the vote as instructed);
- receive **variable dividends** that the company has no legal obligation to pay.

The company benefits because it raises capital while retaining control of its operations; investors benefit because they can own a fraction of a large company with a small outlay.

Preference shares (preferred stock)

Preference shares (preferred stock) are a hybrid of common stock and debt:

Like common stock	Like debt
Dividends are not a contractual obligation	Payments are usually fixed
Usually no maturity	Usually no voting rights

Preferred shares have a stated **par value** and pay a dividend equal to a fixed percentage of par. If the firm is liquidated, preferred holders may receive a set **liquidation preference payout** before common shareholders receive anything.

KEY CONCEPT

$$\text{Annual preferred dividend} = \text{dividend rate} \times \text{par value}$$

Types of preference shares:

Type	Key feature	Whose right?
Callable preference shares	Firm may repurchase the shares at a set **call price**	Issuer
Putable preference shares	Holder may sell the shares back to the issuer at a set price (the **put contingency price**)	Holder
Cumulative preference shares	Unpaid dividends accumulate (**dividends in arrears**) and must be paid before any common dividend	Holder
Noncumulative preference shares	Missed dividends are lost; but in any period, the current preferred dividend still comes before common dividends	—
Participating preference shares	Extra dividends if profits exceed a preset level; may receive more than par in liquidation	Holder
Nonparticipating preference shares	Claim limited to par value in liquidation; no extra dividends	—
Convertible preference shares	Can be exchanged for common stock at a **conversion ratio** fixed at issuance	Holder

For the embedded options in the table, whose right it is tells which way the option moves value and risk. A holder's put or conversion right makes the shares more valuable than otherwise identical shares without it, while an issuer's call right makes them less valuable. Preferred shares are less risky than the same company's common shares because preferred distributions, when declared, and preferred liquidation claims rank ahead of common claims. Cumulative shares are less risky than noncumulative shares because a skipped cumulative dividend must be made up before any common dividend can be paid. Reading 44 applies the option-value relationship when valuing preferred stock.

Example. A preferred share with a \$25 par value and a 5.5% dividend rate pays $0.055 \times 25 = \$1.375$ per year. If the shares are cumulative and last year's dividend was skipped, the company must pay $2 \times 1.375 = \$2.75$ per preferred share this year before any common dividend is allowed. If they are noncumulative, only this year's \$1.375 must be paid first.

Participating preferred stock is often issued by smaller, riskier firms, whose investors want a share of the upside. Convertible preferred shares are common in financing risky early-stage firms: the conversion option compensates investors for risk and lets the firm raise capital more cheaply than by issuing common shares.

Common exam traps

- A fixed preferred dividend is still discretionary; interest and principal on debt are contractual obligations.
- "Fixed dividend, no maturity" describes typical preferred stock; debt has a maturity.
- A call is the issuer's right to buy the shares back; a put is the holder's right to sell them back.
- Missed dividends are owed only on cumulative shares. A participating or convertible feature gives no right to arrears.
- Common shareholders do not need to attend the meeting to vote; they can vote by proxy.

- A statement that puts common shareholders ahead of any liability or of preferred shares reverses the priority of claims.

LOS 39.b — Public versus private equity

Private equity is usually issued to institutional investors through **private placements**, and there is no public market for it. Both private firms and publicly listed firms can issue private equity (a listed company selling shares privately is an example).

How private equity differs from public equity

	Company-specific characteristics of private equity	Stock-specific characteristics of private equity
1	Typically smaller companies	**Less liquidity**: no public market to trade the shares; no quoted price, so a seller must negotiate with other investors
2	Earlier (start-up, growth) or later (decline) in the **company life cycle**	**More concentrated control** of the company
3	**More limited financial disclosure** (not subject to public-company reporting rules, so lower reporting costs)	**Restrictions on the sale** of shares by owners
4	**Greater overlap between ownership and management**	
5	**More concentrated ownership**	

Public equity, by contrast, tends to feature larger, mature-stage companies with dispersed ownership, liquid secondary markets and public reporting requirements.

The four main types of private equity investment

1. **Venture capital**: capital for firms early in their life cycle (start-up, development, early growth). Providers include family, friends, wealthy individuals and private equity funds. Investments are illiquid; investors' money is usually tied up for several years before an **exit** is possible.
2. **Growth equity**: capital a firm raises during its growth stage (e.g., to expand). Bringing in growth equity investors can dilute the concentrated ownership typical of the venture capital stage.
3. **Buyout equity**: focuses on mature-stage companies that investors believe are underperforming and could be improved by restructuring operations or capital.
   - **Take-private transaction**: a buyout of a publicly traded company in which the investing group assumes full control (the shares are delisted).
   - **Leveraged buyout (LBO)**: buying all of a firm's equity using debt financing (leverage); targets usually have enough cash flow to service the debt, or undervalued assets whose sale can pay it down over time.
   - **Management buyout (MBO)**: an LBO in which the buyers are the firm's current managers.
4. **Special situations**: investments in distressed firms (e.g., decline stage) or firms that are restructuring or reorganizing. One form is a **private investment in public equity (PIPE)**: a listed firm that needs capital quickly sells equity privately to a select group of investors.

Common exam traps

- Buyout equity targets mature companies, not start-ups; venture capital stakes cannot be sold readily in a secondary market.
- A take-private deal is defined by full control and delisting; an MBO is defined by who buys (current management); a PIPE is a public company raising private capital.
- When asked how private equity differs from public equity, an option such as a more liquid market or more dispersed ownership describes public equity. Lower reporting costs do describe private equity.
- A firm that is already restructuring or reorganizing points to special situations. Buyout investors instead buy an underperforming mature firm and restructure it after taking control.

EXAM SHORTCUTS

- For a preferred share's embedded option, ask whose right it is: a holder's put or conversion right adds value, and an issuer's call right reduces it.

BOTTOM LINE

- Equity is an ownership claim with a residual claim, an indefinite life and discretionary dividends; debt is a creditor claim with contractual payments and a maturity.
- Dividend payments are most associated with the mature phase of the company life cycle.
- Common shareholders have a residual claim, voting rights (which they can exercise by proxy) and variable dividends the company has no obligation to pay.
- Preferred shares pay a fixed percentage of par, rank ahead of common shares and usually carry no voting rights.
- Dividends in arrears on cumulative preferred shares must be paid before any common dividend; missed dividends on noncumulative shares are lost.
- Private equity is less liquid than public equity, with more concentrated ownership and control and more limited financial disclosure.
- Private equity investments fall into four types: venture capital, growth equity, special situations and buyout equity.

QUICK CHECK

Question 1

A trainee asks how an equity instrument differs from the other securities a company may issue. An equity instrument is *best* described as a security that:

A establishes a creditor relationship with a company.

B represents an ownership interest in a company.

C can represent either an ownership interest or a creditor position, depending on its terms.

Answer: B

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An equity instrument gives its holder ownership of a portion of a corporation. Debt instruments such as loans and bonds, by contrast, create a creditor relationship between the investor and the company.

Why the other options are wrong

- **A.** Loans, bonds and other debt instruments create a creditor relationship.
- **C.** An equity instrument is by definition an ownership interest; it does not switch between ownership and creditor status.

Key takeaway Equity is ownership; debt is a creditor relationship.

Practice questions

Question 2

Under the terms in its equity prospectus, holders of Qarth Energy's shares will be offered any new shares the company issues before those shares are offered to other investors. This feature is known as:

A conversion rights.

B preemptive rights.

C split, subdivision, or combination rights.

Question 3

Brackley Utilities has three series of preference shares outstanding, identical apart from the features below. For the investors who hold them, which series is the *most* risky?

A Putable, cumulative preference shares.

B Noncallable, cumulative preference shares.

C Callable, noncumulative preference shares.

Question 4

An analyst is preparing a briefing on the main categories of private equity investment. Which of her statements is *most accurate*?

A Buyout equity concentrates on companies at the start-up and early stages of their development.

B Venture capital stakes can readily be sold in active, liquid secondary markets.

C Growth equity is capital a company raises during its growth stage, for example to finance expansion.

Answers: 2. B · 3. C · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/common-stock-vs-preferred-stock/. The full bank has 25 questions on this reading.

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EQUITIES · READING 40

Equity Jurisdictions, Classes, and the Voting Process

What you'll learn

LOS 40.a Describe how economic and voting rights differ across jurisdictions (direct investing, depository receipts, dual listing) and across share classes (dual-class structures).

LOS 40.b Describe the voting process and the roles of management, the board, custodians, proxy advisors, asset managers and asset owners.

MODULE 40.1

Rights of Equity Shareholders

This reading explains how shareholder rights vary with the jurisdiction and the class of shares: direct and indirect foreign investing, depository receipts, dual listing and dual-class structures. It then describes how shareholder voting works and what management, the board, shareholders, custodians, asset managers and proxy advisors each do in it.

LOS 40.a — Rights across jurisdictions and share classes

The economic and voting rights of shareholders (Reading 39) can differ when a company has more than one class of shares, or when the investor holds shares of a foreign company.

Investing in foreign shares: direct versus indirect

Investors often show **home asset bias**: they overweight assets from their own country. Buying foreign shares counters this bias, and there are two routes.

Direct investing means buying a foreign company's securities in that company's own (foreign) market. Obstacles include:

- extra cost of opening and running securities accounts abroad;
- unfamiliar market procedures (e.g., clearing and settlement);
- different tax rules, trading hours and reporting requirements, with disclosures possibly in a foreign language;
- **foreign exchange risk** and extra transaction costs, because the investment and its return are in a foreign currency;
- possible restrictions or taxes on moving capital back home;
- a foreign stock exchange that may be illiquid.

Indirect investing means owning an instrument that represents the foreign shares instead of buying the shares on their home exchange. Such vehicles reduce some of the obstacles to direct investing. A common one is the **depository receipt (DR)**.

Depository receipts

KEY CONCEPT

A **depository receipt** represents ownership of shares in a foreign company and trades in another country's market, in that market's currency. A domestic **depository bank** holds the deposited foreign shares, issues receipts each representing a set number of those shares, and acts as **custodian**, handling dividends, stock

splits and other corporate events.

- The investor does not have to exchange currency to buy a DR, but the DR's value is still affected by exchange rate changes, as well as by the firm's fundamentals and economic events. So a DR holder's return can differ from that of a home-market holder of the same shares.
- To rule out arbitrage, the DR price should equal the foreign share price converted at the current exchange rate (times the number of shares per DR), although this can break down.

Example. One DR represents 3 shares of a eurozone company whose shares trade at €40, and the exchange rate is $1.10 per €. The DR should trade near $3 \times 40 \times 1.10 = \$132$. If the euro weakens to $1.05 while the share price is unchanged, the DR falls to $3 \times 40 \times 1.05 = \$126$. The dollar-based holder loses $6 per DR, a loss that a euro-based holder does not suffer.

Type	Where issued / traded	Currency	Notes
American depository receipt (ADR)	United States	US dollars	
European depository receipt (EDR)	Europe	Typically euros	
Global depository receipt (GDR)	Outside the United States and outside the issuer's home country	Usually US dollars	Not listed on US exchanges but can be sold to US institutional investors. Usually listed where investors know the firm

Exam convention: GDRs are not subject to the capital flow restrictions that governments impose, so they give the firm and investors greater opportunities for foreign investment. Current practice: because GDRs are sold outside the issuer's home country, they avoid the home government's limits on direct foreign purchases, but DR programs still have to follow the foreign-investment and DR rules that apply to them.

KEY CONCEPT

	Sponsored DR	Unsponsored DR
Foreign company involved in the issue?	Yes	No; a bank issues it on its own
Voting rights	Passed to the DR investor	Retained by the depository bank
Disclosure requirements	Usually greater	Usually lower

Dual listing

Some large multinationals make their shares easier for nondomestic investors to own through **dual listing**: the shares are listed on a home-market exchange and also on at least one exchange abroad. The investor owns the actual shares rather than a receipt.

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Dual-class share structures

A company may have more than one class of common stock (e.g., **Class A and Class B shares**). In a **dual-class share structure**, some classes carry more **voting rights** than others (e.g., 10 votes per share versus 1). This lets a group such as the founders keep effective control while owning less than a majority of the shares. Classes may also differ in **economic rights**, such as dividends or priority in a liquidation. Economic rights are sometimes in the same proportion as votes; in other cases one class has more economic rights and another has more votes. The rights attached to each class appear in the company's regulatory filings (in the United States, those made with the SEC).

Differences in dividends, votes or liquidation priority between two holders of the same company's common shares are therefore often explained by different share classes. Cumulative/noncumulative and participating/nonparticipating are features of preferred shares; they do not distinguish classes of common stock. Preferred shares usually carry no vote, and under an unsponsored DR the depository bank keeps the vote, so of these three instruments a common share is the one that normally gives its holder a vote.

Common exam traps

- GDRs: issued outside the US *and* outside the home country, but usually denominated in US dollars.
- Sponsored and unsponsored are easily swapped. The investor gets the votes only when the foreign company takes part in the issue (sponsored).
- DR holders still bear currency risk even though they never exchange currency themselves.
- Dual listing (same shares on several exchanges) is not the same as a dual-class share structure (share classes with unequal votes).
- Buying a foreign company's shares on its home exchange is direct investing; buying DRs is indirect investing.

LOS 40.b — The voting process and who takes part in it

Voting rights are how owners exercise authority over a company. Participants and their roles:

KEY CONCEPT

Participant	Role in the voting process
Management and board	Put forward routine matters for a vote (e.g., selection of the **auditor**, board candidates); decide which matters go to shareholders; solicit shareholder proposals; prepare the **proxy statement**, often with voting recommendations
Shareholders (asset owners)	Vote in person at the **annual meeting** or **by proxy** (proxy cutoff is usually slightly earlier); may submit **shareholder proposals**; may delegate their votes to asset managers
Custodian (broker or bank)	Holds shares on behalf of the **beneficial owner**; distributes proxy statements; determines the **shareholder of record** eligible to vote
Asset managers	Vote proxies delegated to them; CFA Institute members and candidates must vote proxies on relevant issues in an informed, responsible way and in the beneficial owners' interests (Standard III(A) Loyalty, Prudence, and Care)

Participant	Role in the voting process
Proxy advisors	Specialist firms that recommend to institutional investors (e.g., mutual funds holding many companies across markets) how to vote proxies, based on current information and good corporate governance principles

Exam convention: a custodian holds the shares in the name of the beneficial owner. Current practice: shares held through a broker are often registered in the name of a nominee ("street name"); the custodian's own records show the investor as beneficial owner, and the economic and voting rights stay with the investor.

Shareholder proposals

A **shareholder proposal** recommends (or requires) that the company or its board take a specific action. Proposals are typically **nonbinding**, but strong support signals issues shareholders expect management to address. A company need not put every shareholder proposal to a vote, although regulations may require management to explain why it rejected one.

Proxy statement and record date

The **proxy statement** sets out the agenda, including every proposal that shareholders will vote on, frequently with a recommendation from management or the board. It also states the voting requirements and deadlines. Only the **shareholder of record** on the **record date** may vote; the record date is often 30 days or more before the meeting. A holder who sells between the record date and the meeting keeps the right to vote those shares, and someone who buys after the record date cannot vote them at that meeting.

Common exam traps

- Custodians distribute proxy materials and determine the holder of record; they do not decide how shares are voted or draft proposals. Voting power stays with the beneficial owner.
- Strong support does not usually make a shareholder proposal binding, and the company may leave some proposals off the ballot (regulations may require it to give reasons).
- Buying shares shortly before the meeting does not confer a vote: eligibility is fixed on the record date.
- Proxy advisors recommend; asset managers (when delegated) vote; custodians administer.

EXAM SHORTCUTS

- When two holders of the same company's common shares get different votes, dividends or liquidation priority, the likely explanation is different share classes; cumulative, noncumulative, participating and nonparticipating are preferred-share features and can be eliminated.

BOTTOM LINE

- Direct investing means buying a foreign company's securities in its home market, with obstacles such as account costs, unfamiliar procedures, different tax and reporting rules, foreign exchange risk and a possibly illiquid exchange; indirect vehicles such as depository receipts reduce some of these obstacles.
- A depository receipt represents a set number of a foreign company's shares held by a depository bank and trades in another market in that market's currency, yet its value still changes with the exchange rate, so the holder bears currency risk without exchanging currency.

- To rule out arbitrage, a DR should be priced at the foreign share price converted at the current exchange rate, times the number of shares per DR, although this relationship can break down.
- A GDR is issued neither in the United States nor in the issuer's home country; it is usually denominated in US dollars and, although not listed on US exchanges, can be sold to US institutional investors.
- In a sponsored DR the foreign company takes part in the issue and the voting rights pass to the investor; in an unsponsored DR a bank issues it alone and the depository bank keeps the votes.
- Dual listing puts the same shares on a home exchange and at least one foreign exchange, whereas a dual-class structure gives some classes of common stock more votes (and possibly different economic rights), which can let a group such as the founders keep effective control with less than a majority of the shares.
- Shareholder proposals are typically nonbinding, and a company need not put every one to a vote, although regulations may require management to give reasons for rejecting one.
- Only the shareholder of record on the record date, often 30 days or more before the meeting, may vote, so a holder who sells between the record date and the meeting can still vote and a buyer after the record date cannot vote those shares at that meeting.

QUICK CHECK

Question 1

A Chilean pension fund opens a brokerage account in Tokyo and buys the shares of a Japanese manufacturer on that company's home stock exchange. This is an example of:

A indirect investing.

B direct investing.

C dual listing.

Answer: B

Direct investing in foreign companies means buying the foreign firm's securities in its own (foreign) market. The pension fund does this when it buys on the Tokyo exchange.

Why the other options are wrong

- **A.** Indirect investing means holding an instrument that represents the company's shares, such as a depository receipt, instead of buying the shares on their home exchange.
- **C.** Dual listing is something a company arranges: its shares trade on one or more foreign exchanges in addition to its home exchange. The pension fund is simply buying the shares on the company's home exchange, which is direct investing.

Key takeaway Buying foreign shares on their home exchange is direct investing; buying DRs is indirect investing; dual listing is a company's decision to list its shares abroad.

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Practice questions

Question 2

The founders of Veridian Apps own 15% of its shares but control a majority of the votes, because each of their shares carries ten votes while every other share carries one. This arrangement is an example of a:

A cumulative preference share issue.

B participating preference share issue.

C dual-class share structure.

Question 3

Kavanagh Dairy set the record date for voting at its annual meeting 35 days before the meeting. Ingrid Solheim, who owned 800 Kavanagh shares on the record date, sells them to Paulo Mendes 5 days before the meeting. Who is entitled to vote those 800 shares at the meeting?

A Mendes, because he owns the shares on the meeting date.

B Solheim, because she was the shareholder of record on the record date.

C Neither of them, because the shares changed hands after the record date.

Answers: 2. C · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/equity-jurisdictions-classes-and-the-voting-process/. The full bank has 15 questions on this reading.

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EQUITIES · READING 41

Equity Issuance and Trading

What you'll learn

LOS 41.a Describe primary and secondary equity markets: IPOs and other listing routes, seasoned offerings, and the functions of secondary trading.

LOS 41.b Compare exchange, off-exchange and over-the-counter equity trading, including quotes, spreads, order types, the limit order book and VWAP.

LOS 41.c Describe liquidity measures for a listed share and calculate market float, average daily volume and turnover.

LOS 41.d Describe the main types of equity indexes and match them to benchmarking needs.

MODULE 41.1

Equity Markets and Exchanges

This reading covers how shares are first issued in the primary market and then traded on exchanges, off-exchange and over the counter, including the bid-ask spread, order types and VWAP. It then shows how to calculate market float, average daily volume and the turnover ratio, and describes the main types of equity index.

LOS 41.a — Primary and secondary equity markets

Primary equity markets are where newly issued shares are sold to investors for the first time; the issuer receives the proceeds. **Secondary equity markets** are where those shares change hands afterward, between investors.

KEY CONCEPT

	Primary market	Secondary market
What trades	New issues (IPOs, primary follow-on offerings, private placements)	Shares already issued
Who receives the cash	Issuer	The selling investor
Sensitivity to market direction	Most active when share prices are rising overall; slower when prices fall	Active in rising and falling markets
Main function	Raising capital, changing ownership structure	Liquidity for holders, price discovery, feedback on performance

Why firms issue equity in the primary market

Typical situations: an early-stage company outgrows **venture capital** funding; a public company **spins off** a division as a separately listed company; a company goes public after several years of restructuring by private owners; or a government sells all or part of a **state-owned enterprise**, which is a **privatization**: the business

moves from government to private ownership. In market language, "publicly held" means that a company's shares trade publicly; it does not mean that the state owns the company.

The IPO process

In an **initial public offering (IPO)** a company that has never had publicly traded shares lists on an exchange and sells shares, usually with an investment bank acting as **equity underwriter**. The underwriter prices the offer, handles compliance and may guarantee that the whole issue is bought; underwriters play the same role in later (secondary) offerings. Before the offer, the company must meet listing and other regulatory requirements and market the shares. The marketing introduces the company to analysts and investors, tests how the market will receive it, measures demand and leads to a price range for the issue. An approximate timeline:

Time before issuance	Regulatory track	Marketing track
About 6 months	Issuer and bank propose the listing and timeline to regulators	Issuer and bank meet **equity analysts** and gauge market acceptance
About 3 months	Issuer files a **confidential prospectus** with regulators	Issuer and bank look for potential investors; analysts publish research
Issuance date	Regulators approve; prospectus released to investors	Bank sets the **price range** and starts **book building**

The **equity prospectus** covers the business model, the main risks facing the company and its industry, audited financial statements, the offer terms (share count, expected price range) and the use of proceeds. The issuer and its advisers, including the underwriters, prepare it, and the relevant market authority must approve it before the issuer releases it. Regulatory approval does not mean the regulator endorses the offer or vouches for the prospectus's accuracy.

IPO underpricing

Underwriters and the issuer's owners have a conflict of interest. Owners want the highest price, and a higher offer price also earns the underwriters higher fees, but over time underwriters may gain more by placing underpriced shares with institutional clients (strong demand, investors keen to join future deals). This conflict helps explain why IPOs are often underpriced and jump on the first trading day. That benefits the underwriters and the investors who were allocated shares; it costs the existing owners, who raise less than the shares are worth.

Other routes to a listing

Route	How it works	New capital raised?
Direct listing	A private company's existing shares are admitted to trading; no underwriter, price set by the market	No
Special purpose acquisition company (SPAC)	A listed shell raises cash in its own IPO, holds it in a trust account, and must buy a private company (approved by shareholders) within a set period (the **de-SPAC transaction**) or return the cash	Yes, at the SPAC IPO
Back-door listing	A private company acquires a company that is already listed and uses that listing	No (for the acquirer)

Exam convention: a direct listing creates no new shares and raises no new capital. Current practice: this describes the traditional (secondary) direct listing; some exchanges, the NYSE among them, also permit a primary direct listing in which the company sells newly issued shares.

Shareholders vote on the de-SPAC target, but an individual who voted against it can still end up owning shares in the acquired business.

Seasoned equity offerings

KEY CONCEPT

A company that already has listed equity can sell more shares in a **seasoned equity offering** (a follow-on offering); a prospectus is still required. Whether the offering dilutes existing holders depends on whose shares are sold:

- **Primary follow-on offering**: the company sells new shares. Each existing share's proportional claim on net income and assets falls, so the offering is **dilutive**. Used to raise equity or pay down debt.
- **Secondary follow-on offering**: existing privately held shares (founders, insiders, private equity funds) are sold to the public for the first time. No new shares are created, so it is **nondilutive**, and the company receives no proceeds; the sellers diversify their concentrated holdings.

A **private placement** sells shares directly to qualified investors rather than to the public.

What the secondary market does

It gives holders liquidity and lets new investors buy in. Its prices let investors and analysts measure returns, volatilities and correlations and judge a company's performance, and they feed back into the issuer's decisions on financing, payouts and compensation (e.g., firms that pay in **restricted stock units (RSUs)** use share price changes to evaluate performance).

Common exam traps

- A back-door listing does not raise new capital for a listed company. It is a way for a *private* firm to become listed.
- "Seasoned" does not mean dilutive: only a *primary* follow-on is necessarily dilutive.
- An IPO is a first public *offering*, usually underwritten, and it typically includes new shares. Any selling shareholders receive proceeds only for the existing shares they sell; those shares are the secondary part of the offer. A direct listing, which uses no underwriter, is a different route.
- Analyst meetings start roughly six months before issuance; the price range is set at issuance.

LOS 41.b — Exchange, off-exchange and over-the-counter trading

Equity exchanges

An **equity exchange** is a rules-based venue with regular trading hours; its rules promote liquidity and transparency. Investors trade through **broker-dealers**, who route orders to a centralized **order matching system** that pairs buy and sell orders. A broker-dealer acts either as an *agent*, arranging the client's trade with another party, or as a *principal*, taking the other side of the client's trade itself (the counterparty). The exchange usually

appoints one **designated market maker** per listed stock, which stands ready to buy, hold or sell the shares so that the stock trades continuously. It supports liquidity and **price discovery** during normal trading hours and at times of order imbalance or high volatility.

Quotes and the spread

The **bid price** is what a dealer will pay; the **ask price** (**offer price**) is what a dealer will sell at. The **bid-ask spread** (bid-offer spread) is the dealer's compensation and the investor's round-trip cost:

KEY CONCEPT

$$\text{bid-ask spread} = \text{lowest ask} - \text{highest bid} \qquad \text{bid-ask spread}(\%) = \frac{\text{lowest ask} - \text{highest bid}}{\text{lowest ask}}$$

$$\text{mid-market price} = \frac{\text{lowest ask} + \text{highest bid}}{2}$$

An investor always trades on the less favorable side of the spread, buying at the ask and selling at the bid.

Example. The best quotes for a stock are a bid of 24.90 and an ask of 25.00. Spread $= 25.00 - 24.90 = 0.10$, or $0.10/25.00 = 0.40\%$; mid-market price $= 24.95$.

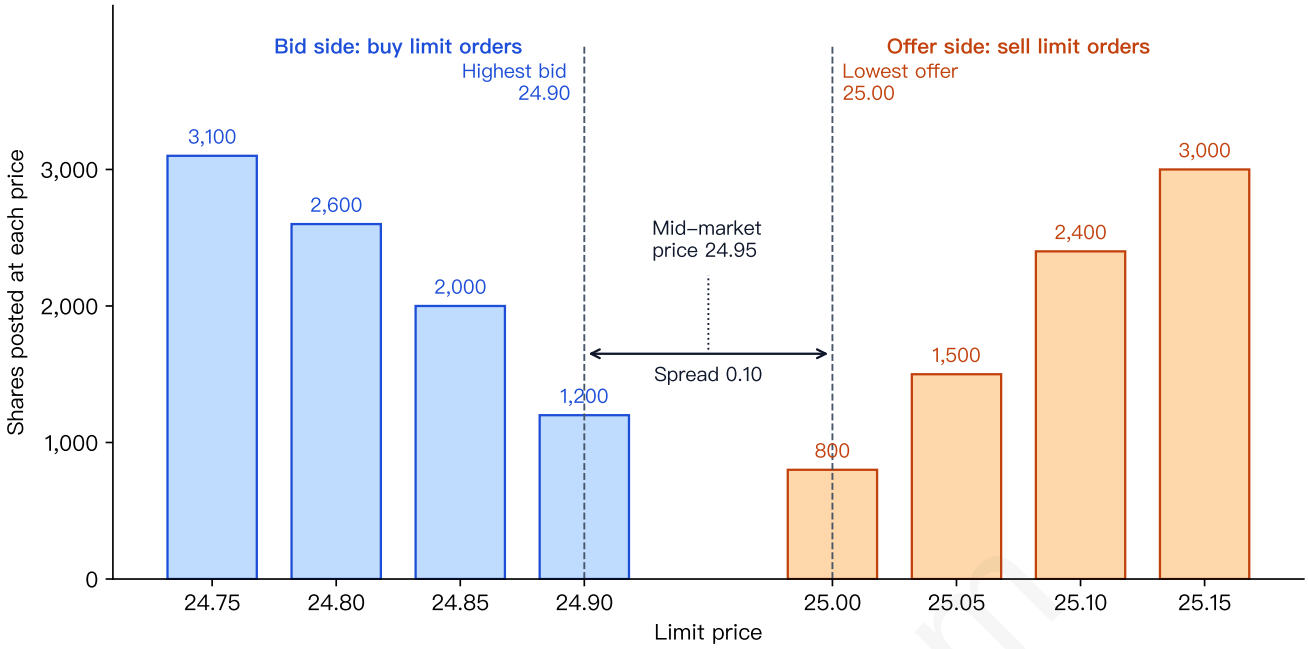
Order types

A **market order** executes immediately at the best available price. A **limit order** sets a boundary: a *maximum* price for a buy, a *minimum* price for a sell. The limit can be above, at, or below the current market price; a sell limit above the market simply waits until the price rises to it.

The limit order book

In a **limit order book**, buy limit orders (the bid side) sit at prices below the mid-market price and sell limit orders (the offer side) sit above it; the gap between the highest bid and the lowest offer is the spread.

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A limit order book: shares posted at each limit price

Market depth is the relative *size* of the orders at or near the best bid and ask (or the mid-market price). **Market breadth** is the *number* of orders at price levels close to the best bid and ask. Both measure how much can be traded without moving the price, and they are compared with market resiliency under LOS 41.c.

> *Example.* The book above lies behind the quotes in the spread example: the highest bid is 24.90 and the lowest offer is 25.00. Depth at the best prices is 1,200 shares bid and 800 shares offered. A market order to buy 1,000 shares therefore fills 800 shares at 25.00 and the other 200 at the next price, 25.05, an average of $(800 \times 25.00 + 200 \times 25.05)/1{,}000 = 25.01$. The average is worse than the best offer because the order is larger than the depth at that price. A price that moves against a large order in this way is market impact, one of the implicit costs of trading described below. A buy limit order at 24.95 would not trade at once. It would become the new highest bid and narrow the spread to 0.05.

VWAP

The **volume-weighted average price (VWAP)** tracks intraday trading:

$$\text{VWAP} = \frac{\sum_i (\text{price}_i \times \text{volume}_i)}{\sum_i \text{volume}_i}$$

> *Example.* Trades of 2,000 shares at 10.00, 3,000 at 10.20 and 5,000 at 10.10 give $\text{VWAP} = (20{,}000 + 30{,}600 + 50{,}500)/10{,}000 = 10.11$.

Off-exchange and OTC trading

Off-exchange trading is used for illiquid, thinly traded shares and for **block trades** (usually 10,000 shares or more). Its purpose is to reduce the **implicit costs of trading**, the indirect and unobservable costs that come from the market impact of trading: **market impact** (the price moves against a large order executed at once), **delay**

costs or **slippage** (the trade cannot be completed immediately), and **opportunity costs** (profit forgone because the trade was not done quickly).

Over-the-counter (OTC) trading is **quote-driven**: broker-dealers in a network act as market makers and post bid and ask prices that are **firm quotes**, meaning they must trade at those prices. Dealers hold inventory and offset trades with one another. Quote-driven markets are also called **dealer markets** or **price-driven markets**; most non-equity securities trade this way, often electronically.

	Exchange	Off-exchange	OTC (quote-driven)
Matching	Centralized order matching system	Negotiated / alternative venues	Dealers trade from inventory
Typical use	Liquid listed shares	Block trades, illiquid shares	Dealer network; most non-equity securities
Transparency	Highest	Lower	Lower than exchanges; best price not guaranteed

Common exam traps

- The spread is the *lowest* ask minus the *highest* bid. The extremes of the book are irrelevant.
- A sell limit is a *minimum* price, but it need not be below the market price.
- OTC quotes are firm, yet OTC markets are *less* transparent than exchanges.

EXAM SHORTCUTS

- Of the two follow-on offerings, only the primary one creates new shares, so a secondary follow-on can be ruled out whenever a question asks which offering is dilutive or brings the company cash.

BOTTOM LINE

- In the primary market newly issued shares are sold and the issuer receives the cash, and it is most active when share prices are rising overall; in the secondary market investors trade shares already issued, and it is active whether prices rise or fall.
- Underwriters may gain more over time by placing underpriced shares with institutional clients than from the higher fees of a higher offer price, a conflict with the issuer's owners that helps explain why IPOs are often underpriced and jump on the first trading day.
- Exam convention: a direct listing admits a private company's existing shares to trading without an underwriter and raises no new capital; current practice: that describes the traditional direct listing, and some exchanges, the NYSE among them, also permit a primary direct listing of new shares.
- A SPAC raises cash in its own IPO, holds it in a trust account and must buy a shareholder-approved private company within a set period or return the cash, while a back-door listing has a private company acquire an already listed company to use its listing.
- A primary follow-on offering sells new shares and dilutes each existing share's claim on net income and assets; a secondary follow-on sells existing privately held shares, creates no new shares and brings the company no proceeds.

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- The bid-ask spread is the lowest ask minus the highest bid (as a percentage, divided by the lowest ask), and an investor buys at the ask and sells at the bid; OTC dealers post firm quotes, yet OTC markets are less transparent than exchanges.

QUICK CHECK

Question 1

An analyst lists three recent transactions. Which one took place in the primary market?

A Two pension funds trade a block of listed shares directly with each other.

B A bank buys investment-grade corporate bonds from a dealer's inventory.

C A utility sells newly issued common shares to investors to fund a new plant.

Answer: C

The primary market is the market for new issues: the securities are sold for the first time and the issuer receives the proceeds. The utility's sale of newly issued shares to finance a plant is a primary market transaction.

Why the other options are wrong

- **A.** Institutions trading existing listed shares with each other is a secondary market trade; the issuer receives nothing.
- **B.** Buying bonds that already sit in a dealer's inventory is a secondary market purchase. Credit quality (investment grade or not) has nothing to do with whether a market is primary or secondary.

Key takeaway A primary market sale is the first sale of a new security, with proceeds going to the issuer.

MODULE 41.2

Equity Indexes and Liquidity Measures

LOS 41.c — Float, trading volume and other liquidity measures

Market float is the portion of a company's shares available to the investing public; it is often quoted as the market value of those shares. Shares held by controlling shareholders, restricted shares (e.g., executives' shares under selling restrictions) and other shares not available for trading are excluded. Institutional investors such as mutual funds, pension funds and ETFs *are* part of the investing public, so their holdings stay in the float.

KEY CONCEPT

$$\text{market float} = \text{shares outstanding} - \text{restricted shares or shares not available for trading}$$

Average daily volume (ADV) is the average number of shares traded per day over a chosen period:

$$\text{average daily volume} = \frac{\sum_{i=1}^{n}(\text{shares traded on day } i)}{n}$$

The **turnover ratio** shows what fraction of the float changes hands on a typical day:

KEY CONCEPT

$$\text{turnover ratio} = \frac{\text{average daily volume}}{\text{market float}}$$

These measures tell an investor how quickly a position can be built or unwound without moving the price.

Example. A company has 50 million shares outstanding, of which 8 million are held by its controlling family. Float $= 50 - 8 = 42$ million. Over the last month ADV was 1.26 million shares, so turnover $= 1.26/42 = 3.0\%$ per day. A fund holding 900,000 shares may trade no more than 15% of ADV per day, i.e. $0.15 \times 1{,}260{,}000 = 189{,}000$ shares; it needs $900{,}000/189{,}000 = 4.76$, so 5 trading days to exit fully. A fractional result is rounded *up*.

Order-book liquidity measures show how easily shares can be traded without a big adverse price move. Depth and breadth were introduced with the limit order book under LOS 41.b; the table compares them with resiliency:

Measure	What it captures	Falls when…
Market depth	Relative *size* of orders posted at or near the best bid and offer	Orders near the best prices get smaller
Market breadth	*Number* of orders at prices just above and below the best bid and offer	Fewer separate orders near the best prices
Market resiliency	Ability of the shares to hold a steady price that clears every buy and sell order; after a shock, a more resilient market settles at its new clearing price sooner	Prices take longer to stabilize

Common exam traps

- Institutional holdings stay in the float. Only controlling, restricted or otherwise untradeable shares are subtracted.
- The turnover ratio uses float in the denominator rather than total shares outstanding. In the example, dividing ADV of 1.26 million by the 50 million shares outstanding gives 2.52% instead of 3.0%.
- Smaller orders with the same number of orders mean lower depth and unchanged breadth.
- Days-to-liquidate: divide by the *daily* allowed volume and round up. In the example, dividing 900,000 by the full ADV of 1,260,000 gives 0.71 days and rounding 4.76 down gives 4 days, but the fund needs 5.

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LOS 41.d — Types of equity indexes

A **security market index** represents the performance of an asset class, market or market segment. It is built as a portfolio of **constituent securities**, and its value is computed from their prices at a point in time. Indexes are used as **benchmarks** for managers and as the basis for index funds.

KEY CONCEPT

Index type	What it holds / how weighted	Typical use
Broad-based equity index (often called a broad market index)	The largest listed companies, **market-capitalization-weighted** and adjusted for **market float**; may be tiered by size. A **composite market index** measures the overall market and usually covers more than 90% of its total value	Measuring the overall market
Sector index	Companies in one industry sector (e.g., health care, financials), national or global	Benchmarking sector managers; building index portfolios; business-cycle (cyclical) analysis, since sectors lead or lag in different phases
Investment theme index	Companies selected by an investor trend that cuts across industries and sizes (e.g., ESG-screened firms, an aging-population or clean-energy theme)	Thematic funds
Alternatively weighted index	Same constituents as a broad-based equity index but equal-weighted, or weighted by fundamental factors (**factor-based** indexes, e.g., quality, value)	Reducing large-cap concentration; factor investing
Multi-market index	Combines national indexes for a region, a development stage (e.g., emerging markets), or the world	Global and regional benchmarks

The weighting method decides which companies drive an index. In a market-capitalization-weighted index of three companies with market capitalizations of 600, 300 and 100, the largest carries 60% of the weight and the smallest 10%. An equal-weighted index of the same three gives each one-third, so the smallest company's influence more than triples. When an equal-weighted index outperforms a cap-weighted index with the same constituents, the smaller constituents have therefore done better, on average, than the larger ones.

Choosing a benchmark

Match the index to the mandate. A fund confined to one industry is benchmarked against a sector index, a fund built around a cross-industry trend against an investment theme index, and a fund that picks stocks by value or quality characteristics against a factor-based index.

Common exam traps

- Broad-based equity indexes are typically float-adjusted and cap-weighted. Equal weighting and factor weighting are alternative methods.
- A theme spanning several industries is *not* a sector index.
- A factor-based index selects on stock characteristics such as value, quality or dividend yield; a theme index selects on a trend.

EXAM SHORTCUTS

- When an equal-weighted index outperforms a cap-weighted index with the same constituents, the smaller constituents have done better on average than the larger ones, so no weight calculation is needed to answer which group drove the gap.

BOTTOM LINE

- Market float is shares outstanding less restricted or otherwise untradeable shares, so shares of controlling shareholders and restricted shares are excluded while holdings of institutions such as mutual funds, pension funds and ETFs stay in.
- Average daily volume is total shares traded over a period divided by the number of days, and the turnover ratio is average daily volume divided by market float.
- The days needed to exit a position equal the position divided by the volume allowed per day (the permitted share of ADV times ADV), with a fractional result rounded up.
- Market depth is the relative size of orders at or near the best bid and offer, market breadth is the number of orders at prices close to them, and market resiliency is the ability to hold a stable market-clearing price, a more resilient market reaching its new clearing price sooner after a shock.
- Broad-based equity indexes hold the largest listed companies, weighted by float-adjusted market capitalization, and a composite market index usually covers over 90% of the total value of the market.
- A fund confined to one industry is benchmarked against a sector index, a fund built on a cross-industry trend against an investment theme index, and a fund selecting on value or quality against a factor-based index; multi-market indexes combine national indexes for a region, a development stage or the world.

QUICK CHECK

Question 2

An index provider designing a broad-based equity index for a national stock market would *most typically* weight the constituent stocks:

A by float-adjusted market capitalization.

B in proportion to their share prices.

C equally, regardless of company size.

Answer: A

Broad-based equity indexes include the largest listed firms and are calculated on a market-capitalization-weighted basis, adjusted for market float. Larger companies therefore carry larger weights.

Why the other options are wrong

- **B.** Price weighting is not the typical method for broad-based equity indexes.
- **C.** Equal weighting is an alternative weighting method used to reduce the dominance of the largest companies in a broad-based equity index; it is not the standard approach.

Key takeaway Broad-based equity indexes are float-adjusted and cap-weighted; equal-weighted and factor-weighted versions are the alternatives.

Practice questions

Question 3

An analyst makes three statements about methods of issuing equity. Which statement is *least accurate*?

A A private company can obtain listed status through either a back-door listing or a direct listing.

B A SPAC shareholder can end up owning part of an acquired company even after voting against that target.

C Every seasoned equity offering reduces the proportional ownership of the company's current shareholders.

Question 4

Owen Achebe owns 2,000 shares of Kilnsey Water and enters a limit order to sell all of them at 27.35. Just before his order arrives, the limit order book looks like this:

Limit order book for Kilnsey Water shares

Bid size (shares)	Limit price	Offer size (shares)
	27.60	800
	27.55	1,200
900	27.45	
600	27.40	
1,100	27.30	

Any shares that cannot be sold at once remain in the book as an offer. The average price of the shares he sells immediately is *closest* to:

A 27.35.

B 27.40.

C 27.43.

Answers: 3. C · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/equity-issuance-and-trading/. The full bank has 32 questions on this reading.

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EQUITIES · READING 42

Sources of Equity Returns

What you'll learn

LOS 42.a Describe the features and uses of cash dividends, share repurchases, stock dividends, stock splits and reverse splits.

LOS 42.b Describe the dividend payment chronology and the share price behavior on the ex-dividend date.

LOS 42.c Calculate price return and total return on equity, including splits and reinvested dividends, and relate price to the present value of expected cash flows.

MODULE 42.1

Dividends, Stock Splits, and Share Repurchases

This reading describes cash dividends, share repurchases, stock dividends, stock splits and reverse splits, and the chronology of a dividend from declaration to payment. It then shows how to calculate price return and total return, including returns with reinvested dividends, and how a share's value follows from discounting its expected price and dividend.

LOS 42.a — Dividends, share repurchases, stock splits and reverse splits

Cash dividends are cash payments to shareholders. They are subject to income tax in the period received. Paying them is at the discretion of the board of directors. Mature companies with few growth opportunities tend to pay out cash; fast-growing companies tend to retain it for reinvestment.

KEY CONCEPT

Distribution	What happens	Typical use
Regular cash dividend	A portion of profits paid on a consistent schedule (e.g., quarterly)	A stable or rising record signals financial stability; firms often hold dividend per share steady to avoid a negative signal from a cut
Special cash dividend (extraordinary dividend)	One-time cash payment on top of any regular dividend	Cyclical firms share profits in good years without committing to higher payments in bad years
Liquidating dividend	Paid by a company that is ceasing operations and has positive net assets	Viewed more as a **return of capital** than a distribution of earnings
Share repurchase (buyback)	The company buys its own outstanding shares	Alternative way to distribute cash; each holder decides whether to sell
Stock dividend (bonus issue)	New shares instead of cash, stated as a percentage (a 10% stock dividend gives 10 more shares per 100 held)	Share count rises, price per share falls, total equity unchanged

Distribution	What happens	Typical use
Stock split (forward split)	Each share becomes several shares, stated as a ratio (3-for-1: each old share becomes three)	Makes a high-priced share more affordable for retail investors, which can improve liquidity and broaden the shareholder base. Today stock splits are used more often than stock dividends
Reverse stock split	Several shares become one (1-for-5: five old shares become one)	Raises the price per share, e.g., to stay above an exchange's minimum price and avoid delisting

Share repurchases in more detail

A repurchase has the same effect on total shareholder wealth as a cash dividend of the same amount. Reasons to buy back shares: to support the share price or signal that management thinks the shares are undervalued; to offset the increase in shares outstanding when employees exercise stock options; to raise reported EPS by shrinking the share count; to keep flexibility, since a company that wants to hold cash for growth opportunities as they arise may prefer buybacks to regular dividends (a buyback creates no expectation of repetition); and, where capital gains are taxed more lightly than dividends, because shareholders prefer it.

Splits and stock dividends do not create value

The company's total value is unchanged, so the price per share adjusts in proportion to the change in the share count. A shareholder's wealth and proportional ownership are unchanged. A 3-for-2 split is economically identical to a 50% stock dividend: 50% more shares, each worth one-third less.

Example. An investor holds 200 shares at 90. After a 3-for-1 split she holds 600 shares; other things equal the price falls to 30, and the position is still worth $600 \times 30 = 18{,}000 = 200 \times 90$.

Common exam traps

- Among cash distributions, only a share repurchase reduces the number of shares outstanding; regular and special cash dividends leave it unchanged. A reverse split also cuts the share count, but it rescales every holding proportionally and distributes no cash. It therefore cannot offset dilution from option exercises the way a buyback can.
- A forward split or a stock dividend *lowers* the price per share; a reverse split raises it.

LOS 42.b — Dividend payment chronology

In time order, a dividend passes through its declaration date, its ex-dividend date, its record date and finally its payment date. The ex-dividend and record dates can fall on the same day, as explained below.

KEY CONCEPT

Date	What happens
Declaration date	The board approves the dividend and announces the amount per share, the record date and the payment date

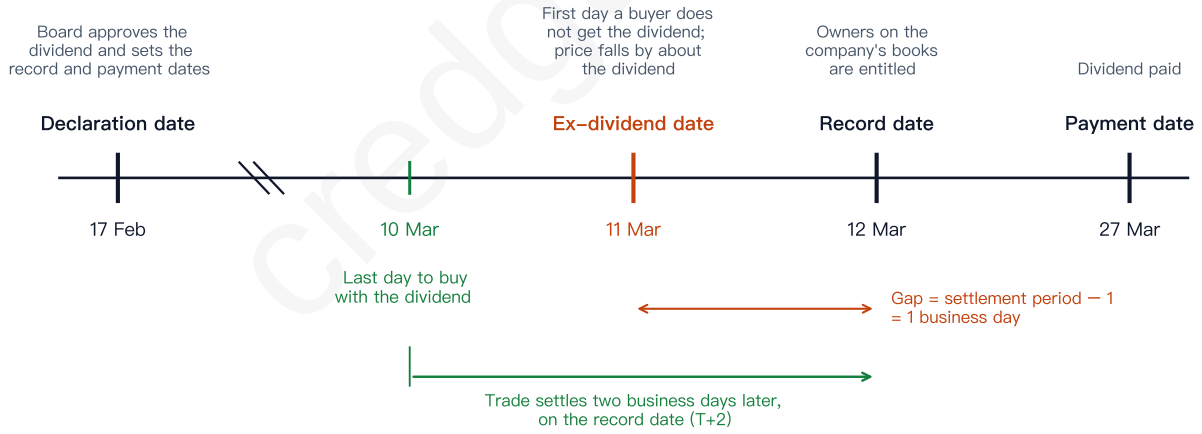
Date	What happens
Ex-dividend date	The first day on which a buyer of the shares will not receive the declared dividend. Its position depends on the **settlement period** for share trades
Record date (holder-of-record date)	The company identifies the owners entitled to the dividend. This dividend record date is separate from the record date used for proxy voting
Payment date	The dividend is paid (checks mailed or electronic transfer)

How the ex-dividend date is set relative to the record date

A trade takes time to settle. The last purchase that carries the dividend is the one made the business day before the ex-dividend date; it settles exactly on the record date. So the gap between the ex-dividend date and the record date is one business day less than the settlement period: with T+3 settlement the gap is two business days, with T+2 it is one, and with T+1 the two dates coincide. Anyone who buys *before* the ex-dividend date receives the dividend. Anyone who sells *on or after* the ex-dividend date still receives it, because they were the owner at the cutoff, and their buyer does not.

Exam convention: the ex-dividend date comes one or two business days ahead of the record date, with the exact gap set by the settlement cycle. Current practice: US equity trades have settled on a T+1 basis since May 2024, so a US company's ex-dividend date and record date are now normally the same day.

The timeline shows the exam-convention case of T+2 settlement, with a one-day gap, for a dividend whose record date is 12 March.



Dividend payment chronology under T+2 settlement (exam convention)

Price behavior

Other things equal, the share price drops by the amount of the dividend (relative to the prior close) when the stock goes ex-dividend, because buyers no longer get the payment.

Separating the causes of a price change on the ex-dividend date

The day's percentage change equals the negative dividend effect plus the market effect plus the company-specific effect. If the stock's risk equals the market's, its expected market-driven move equals the index move.

Example. A share closes at 60.00 the day before going ex-dividend; the dividend is 1.20 (2.0% of 60). On the ex-dividend date it closes at 59.40, while the market index rises 0.8%.

- Actual change $= (59.40 - 60.00)/60.00 = -1.0\%$
- Change not caused by the dividend $= -1.0\% - (-2.0\%) = +1.0\%$
- Company-specific part $= 1.0\% - 0.8\% = +0.2\%$

In price terms the share fell 0.60, even though the dividend alone would have taken off 1.20: the other +0.60 came from the market ($0.8\% \times 60 = 0.48$) and company news (0.12).

Common exam traps

- The cutoff for buyers is the ex-dividend date, not the record date.
- A seller between the ex-dividend date and the record date still gets the dividend.
- The price drops on the ex-dividend date. The declaration, record and payment dates bring no such drop.
- When isolating the company-specific move, *add back* the dividend drop and *subtract* the market move. In the example, subtracting the 0.8% market move from the −1.0% actual change without adding back the 2.0% dividend drop gives −1.8% instead of +0.2%.

EXAM SHORTCUTS

- A 3-for-2 split and a 50% stock dividend are economically identical (50% more shares, each worth one-third less), so other things equal neither lowers the share price more than the other.
- The ex-dividend date comes one business day less than the settlement period before the record date, so T+2 settlement gives a one-day gap and T+1 settlement puts the two dates on the same day.

BOTTOM LINE

- A share repurchase has the same effect on total shareholder wealth as a cash dividend of the same amount, but each holder decides whether to sell, and among cash distributions only a repurchase reduces the number of shares outstanding.
- A special cash dividend is a one-time payment on top of any regular dividend, which lets cyclical firms share profits in good years without committing to them in bad years, while a liquidating dividend is viewed more as a return of capital.
- Stock dividends and forward splits raise the share count and lower the price per share in proportion, and reverse splits do the opposite, so the company's total value, a shareholder's wealth and proportional ownership are unchanged.
- In time order a dividend is declared, the stock goes ex-dividend, the record date passes and the dividend is paid; other things equal, the share price falls by the dividend amount when the stock goes ex-dividend, the first day a buyer no longer receives it.
- Exam convention: the ex-dividend date precedes the record date by one or two business days, with the gap set by the settlement cycle; current practice: US equity trades have settled T+1 since May 2024, so a US company's two dates are now normally the same day.

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- On the ex-dividend date the percentage price change equals the negative dividend effect plus the market effect plus the company-specific effect, so for a stock as risky as the market the company-specific part is the actual change plus the dividend percentage minus the index move.

QUICK CHECK

Question 1

Torvald Steel's profits rise and fall with the business cycle. After an exceptionally profitable year, its board wants to share the windfall with shareholders without committing to larger payments in future years. The type of distribution the board is *most likely* to declare is a:

A special dividend.

B regular dividend.

C stock dividend.

Answer: A

A special (extraordinary) cash dividend is a one-time payment made when favorable circumstances allow, in addition to any regular dividend. Many cyclical companies use special dividends to share profits in good years while keeping the flexibility to conserve cash when profits fall.

Why the other options are wrong

- **B.** Raising the regular dividend creates an expectation that the higher payment will continue; cutting it later would send an adverse signal, and the board wants to avoid both.
- **C.** A stock dividend distributes new shares rather than cash and does not share any profits: total equity and shareholder wealth are unchanged.

Key takeaway A cyclical firm that wants to share a good year without a lasting commitment pays a special dividend.

MODULE 42.2

Return Calculations for Equities

LOS 42.c — Price return and total return on equity

A **price return** uses only the change in the share price (adjusted for stock splits). A **total return** adds income, i.e., cash dividends received.

KEY CONCEPT

$$\text{price return} = \frac{P_1 - P_0}{P_0} \qquad \text{total return} = \frac{P_1 - P_0 + D}{P_0}$$

Example. Buy at 30.00, receive a 0.60 dividend just before selling at 33.00. Price return $= 3.00/30.00 = 10.0\%$; total return $= 3.60/30.00 = 12.0\%$.

Stock splits and stock dividends do not change returns

Adjust the beginning price, ending price and dividend per share to the same share basis (or simply work with the value of the whole position). An investor who ends up with more shares after a split receives the same total dividend and holds a position of the same value, so the rate of return is unchanged. The same is true of stock dividends and reverse splits. The common mistake is to compare a *post-split* ending price with a *pre-split* purchase price.

Reinvested dividends

When a dividend received part-way through the holding period is used to buy more of the same stock (**reinvested dividends**), the investor acquires **fractional shares** of D/P_{PD} per share originally held, where P_{PD} is the price on the dividend payment date. The holding period return is:

KEY CONCEPT

$$r_E = \frac{P_1\left(1 + \frac{D}{P_{PD}}\right) - P_0}{P_0}$$

With several reinvested dividends, track the share count step by step: each dividend adds (shares held × dividend per share) / price on that payment date.

Example. Buy at 40.00; halfway through the period a 1.00 dividend is paid when the price is 50.00 and reinvested; the ending price is 45.00. Extra shares $= 1.00/50.00 = 0.02$ per share, so $r_E = [45.00(1.02) - 40.00]/40.00 = 14.75\%$. Had the dividend instead been deposited in an account paying 4% over the whole holding period, it would earn interest only for the second half and grow to $1.00 \times 1.04^{0.5} = 1.0198$, and $r_E = (45.00 - 40.00 + 1.0198)/40.00 = 15.05\%$.

Summary: how a dividend enters the holding period return

KEY CONCEPT

Treatment of the dividend	How it enters the holding period return
Received just before the sale	Add it once to the price change: $(P_1 - P_0 + D)/P_0$
Reinvested in the same stock	Convert it into extra shares at P_{PD}; value all shares at P_1
Invested elsewhere (e.g., a bank deposit)	Add the dividend *plus* what it earns until the end of the period to the price change
Paid in shares (stock dividend) or a split	Not income; once prices are on the same share basis the return is unchanged

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The holding period used in these calculations may be shorter or longer than a year; read the question to see whether an annualized figure is wanted.

Equity value as a present value

Rearranging the return formula for a stock with no dividend, $1 + r_E = P_1/P_0$, gives $P_0 = P_1/(1 + r_E)$. Because the ending price is not known in advance, use expectations, and include a dividend expected at the end of the period:

$$P_0 = \frac{E(P_1) + E(D_1)}{1 + r_E}$$

Here r_E is investors' **required rate of return** on the equity. This is the foundation of **discounted cash flow models**: a share is worth the present value of its expected future cash flows.

Example. Expected price in one year 52.50, expected dividend 1.50, required return 8%: $P_0 = 54.00/1.08 = 50.00$.

Common exam traps

- When dividends are reinvested, do not also add the cash dividend; it has already been turned into shares. In the reinvestment example, adding the 1.00 dividend on top of the extra shares gives 17.25% instead of 14.75%.
- Use the price on the *payment date* to compute the shares bought with each dividend. In the same example, buying the extra shares at the 40.00 purchase price gives 15.31% instead of 14.75%.
- After a split, put both prices (and the dividend) on the same share basis before computing returns.
- In the present value formula, discount the expected dividend as well as the expected price. In the example, adding the 1.50 dividend undiscounted to $52.50/1.08$ gives 50.11 instead of 50.00.

EXAM SHORTCUTS

- Splits, stock dividends and reverse splits leave the rate of return unchanged, so working with the value of the whole position avoids restating every per-share figure on a new share basis.

BOTTOM LINE

- Price return is $(P_1 - P_0)/P_0$, using only the split-adjusted price change, and total return is $(P_1 - P_0 + D)/P_0$, adding the cash dividends received.
- Stock splits, stock dividends and reverse splits do not change the rate of return once the beginning price, ending price and dividend per share are on the same share basis.
- A dividend reinvested in the same stock buys D/P_{PD} extra shares per share held at the price on the payment date, and the return is $r_E = [P_1(1 + D/P_{PD}) - P_0]/P_0$, with the cash dividend not added again.
- A dividend invested elsewhere, such as in a bank deposit, enters the holding period return as the dividend plus what it earns until the end of the period.

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- A share's value is $P_0 = [E(P_1) + E(D_1)]/(1 + r_E)$, where r_E is the required rate of return; this is the basis of discounted cash flow models, in which a share is worth the present value of expected future cash flows.
- A holding period may be shorter or longer than a year, so a return question has to be read for whether an annualized figure is wanted.

QUICK CHECK

Question 2

An analyst expects shares of Marlow Utilities to trade at $66.00 one year from now and to pay a dividend of $2.20 per share at the end of the year. Investors' required rate of return on Marlow's equity is 11%. The value of a Marlow share today is *closest* to:

A $59.46.

B $61.44.

C $61.66.

Answer: B

The value of an equity security today is the present value of the cash flows the investor expects to receive, discounted at the required rate of return. Over one period, those cash flows are the expected dividend and the expected ending price, both received at the end of the year.

$$P_0 = \frac{E(P_1) + E(D_1)}{1 + r_E} = \frac{66.00 + 2.20}{1.11} = \frac{68.20}{1.11} = \$61.44$$

Why the other options are wrong

- **A.** $59.46 $(66.00/1.11)$ discounts only the expected price and leaves out the expected dividend.
- **C.** $61.66 $(66.00/1.11 + 2.20)$ discounts the expected price but adds the dividend undiscounted, even though it is also received a year from now.

Key takeaway $P_0 = [E(P_1) + E(D_1)]/(1 + r_E)$: discount every future cash flow, including the dividend.

Practice questions

Question 3

Galloway Brewing pays a dividend of $0.90 per share. Its shares close at $48.00 on the day before the ex-dividend date and at $45.60 on the ex-dividend date. Assume Galloway has the same risk as the overall market and there is no company news. The fraction of Galloway's ex-dividend-date price drop that is attributable to the market is *closest* to:

A 62.5%.

B 37.5%.

C 3.3%.

Answers: 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/sources-of-equity-returns/. The full bank has 17 questions on this reading.

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EQUITIES · READING 43

Introduction to Equity Valuation

What you'll learn

LOS 43.a Contrast market price with intrinsic and perceived value, and the naïve, relative and absolute approaches to valuation.

LOS 43.b Contrast book value of equity, market capitalization and enterprise value as indicators of intrinsic value, including P/B and EV/EBITDA.

LOS 43.c Describe present value, asset-based and multiplier valuation models and their advantages and disadvantages.

MODULE 43.1

Measures and Models of Equity Value

This reading separates market price from intrinsic value and describes naïve, relative and absolute ways of forming a perceived value. It shows how to calculate book value, market capitalization, P/B and enterprise value, and how to tell the categories of valuation model apart.

LOS 43.a — Price versus value

Market price is observable: it is what the shares trade at. **Intrinsic value** (**fundamental value**) is the value a rational investor would place on the stock with full knowledge of its characteristics; it cannot be observed, so analysts estimate it with **valuation models**.

Each investor's own estimate, based on his or her assumptions, is a **perceived value**, and market prices aggregate the perceived values of active participants. Compare perceived value with price:

Perceived value vs. market price	Investor's view	Likely action
Higher	**Undervalued**	Buy / overweight
Lower	**Overvalued**	Sell / underweight
Equal	**Fairly valued**	Hold

Three ways investors form a perceived value

KEY CONCEPT

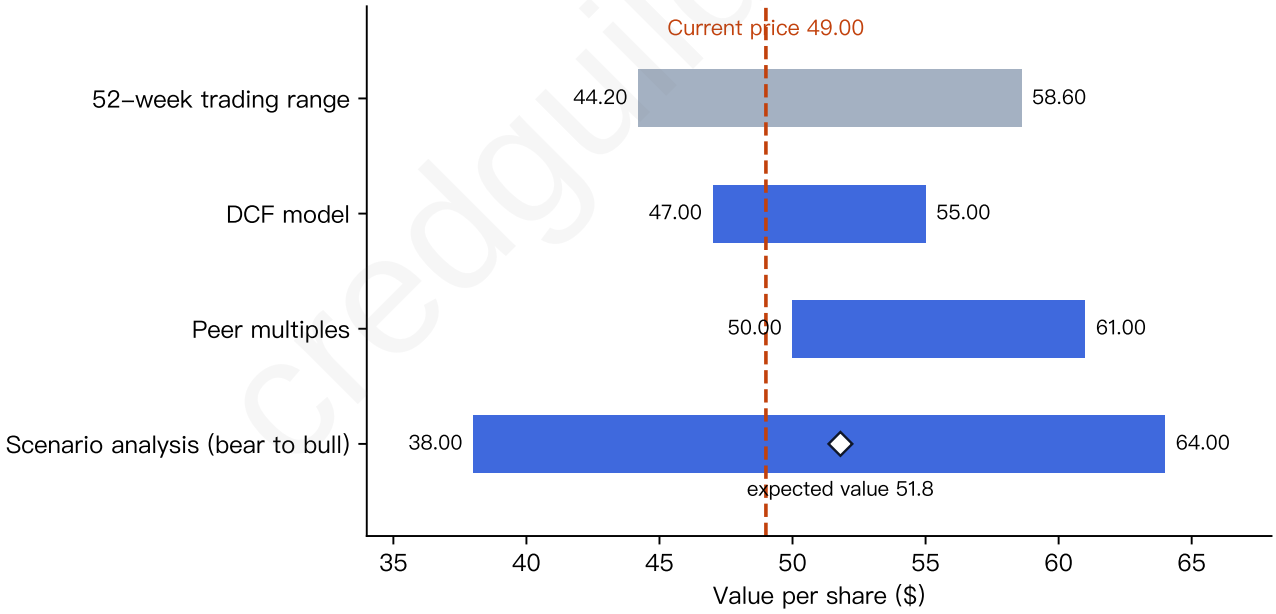
Approach	Basis	Example
Naïve or **story-based valuation**	An informal story with no link to the company's finances or fundamentals	**Meme stocks** driven by social-media buzz; **cult stocks** whose value rests on devotion to a founder or product

Approach	Basis	Example
Relative valuation	Compare the company's multiples with a peer group; similar companies should trade at similar multiples (e.g., P/E, P/B, EV/EBITDA)	"Its P/E is below the sector average, so it looks cheap"
Absolute valuation (fundamental valuation)	The company's own characteristics only, without a benchmark — e.g., present value of expected future cash flows at a risk-adjusted discount rate	A dividend discount or free cash flow model

Ranges and scenarios

Analysts often express intrinsic value as a range (from different inputs or different models) and compare it with the stock's recent trading range, sometimes side by side in a **floating bar valuation chart**. In **scenario analysis** the analyst values the stock under a **base case**, a **bullish** (more favorable) case and a **bearish** (less favorable) case; the expected value is the probability-weighted average of the three.

Example. Base, bull and bear values of 50, 64 and 38 with probabilities 0.5, 0.3 and 0.2 give an expected value of $0.5(50) + 0.3(64) + 0.2(38) = 51.8$.



Floating bar valuation chart: estimated value ranges compared with the recent trading range (illustrative figures)

Common exam traps

- A story with no financial analysis is naïve valuation, even if the story is plausible.
- P/E comparisons with peers are *relative* valuation.

LOS 43.b — Book value, market capitalization and enterprise value

Book value of equity is balance sheet assets minus liabilities (and minus minority, i.e., noncontrolling, interest). It rises when the company earns net income and retains it; a company that pays no dividends retains all of its net income, so positive net income raises its book value, other things equal. Equivalently, add up the components of stockholders' equity, excluding noncontrolling interest:

$$\begin{aligned}\text{book value} = {} & \text{share capital} + \text{additional paid-in capital} + \text{retained earnings} + \\ & \text{accumulated other comprehensive income (and other reserves)} - \text{treasury stock}\end{aligned}$$

Exam convention: book value = share capital + additional paid-in capital + retained earnings − treasury stock. Current practice: stockholders' equity also contains **accumulated other comprehensive income (AOCI)** and other reserves, so they are added as in the formula above; the shorter version gives the same answer only when those items are zero. For instance, paid-in capital of 100, retained earnings of 0 and AOCI of +10 give a book value of 110.

$$\text{book value per share (BVPS)} = \frac{\text{book value of equity}}{\text{shares outstanding}}$$

> *Example.* Common stock 400, additional paid-in capital 1,100, retained earnings 650, treasury stock (250) and noncontrolling interest 90 (all in millions). Book value $= 400 + 1{,}100 + 650 - 250 = 1{,}900$ (total equity of 1,990 less the 90 of noncontrolling interest). With 50 million shares, BVPS $= 38.00$.

Limits of book value

Assets may be tangible (equipment, inventory) or intangible (patents, copyrights, goodwill). Carrying values of tangible assets depend on when they were bought and on the depreciation method, and accounting choices differ across companies, which affects BVPS comparisons. Internally generated intangibles (brands, know-how) are usually expensed and never appear on the balance sheet; purchased intangibles are amortized over their useful lives or impairment-tested. **Goodwill** arises when an acquirer pays more than the fair value of the target's net identifiable assets. Book value is therefore most useful for companies with substantial tangible assets (e.g., manufacturers) and least useful for early-stage or asset-light companies and those whose value lies in intangibles.

Negative book value means liabilities exceed assets. It is common in financial distress, but it can also occur at profitable, valuable companies, such as those growing rapidly or running aggressive share repurchases, because treasury stock is deducted from equity. Negative book value alone does *not* mean the shares cannot provide value to investors.

Market capitalization $= \text{share price} \times \text{shares outstanding}$. It reflects the fair value of the company as a **going concern**, including investors' expectations about future earnings. It does not measure what the assets would fetch if sold piecemeal. Because it depends on expectations, net income that disappoints investors can *lower* market capitalization even as it raises book value.

- Market cap > book value: the firm is creating **excess value**.
- Market cap < book value: a **value shortfall**; the analyst should investigate whether the stock is undervalued or the company is uncompetitive.

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The **price-to-book ratio (P/B)** expresses the same comparison as a ratio, so a P/B above 1 corresponds to excess value and a P/B below 1 to a value shortfall:

KEY CONCEPT

$$\text{P/B} = \frac{\text{market price per share}}{\text{BVPS}} = \frac{\text{market capitalization}}{\text{book value of equity}}$$

Relative to a peer benchmark, a P/B well above (below) the benchmark suggests the stock is relatively overvalued (undervalued), absent a justifying reason. *Example:* at a price of 57 and BVPS of 38, P/B $= 1.5$.

Enterprise value (EV) measures the value of the whole firm, roughly what it would cost to acquire it:

KEY CONCEPT

$$\text{EV} = \text{market value of common and preferred stock} + \text{market value of debt} - \text{cash and short-term investments}$$

Net debt equals the market value of debt less cash and short-term investments; cash is subtracted because the acquirer takes over the target's liquid assets along with its debt. EV is useful for comparing companies with very different capital structures (convert to a common currency first for cross-border comparisons). It is usually divided by **EBITDA** (or EBIT, or revenue).

The balance sheet usually carries debt at book value, so its market value often has to be estimated. For long-term debt the analyst uses financial market data, such as current prices of the firm's bonds. Short-term debt can be taken at book value if the firm is not in financial distress, because a debt instrument's market value moves toward its face value as maturity approaches.

Example. A firm has equity market value 2,850, no preferred stock, cash 150 and EBITDA 425. Its long-term debt is valued at 520 from bond prices and its short-term debt is taken at its book value of 180, so the market value of debt is 700. Then $\text{EV} = 2{,}850 + 700 - 150 = 3{,}400$; $\text{EV/EBITDA} = 8.0\times$. If peers trade at 9×, the firm looks relatively undervalued.

Measure	Based on	Viewpoint
Book value of equity	Balance sheet carrying values	Equity holders, accounting
Market capitalization	Share price × shares	Equity holders, market expectations
Enterprise value	Market value of equity and debt, less cash	All capital providers

Common exam traps

- Additional paid-in capital and accumulated other comprehensive income are *part of* book value; treasury stock is *subtracted*; noncontrolling interest is *excluded*. In the example, keeping the 90 of noncontrolling interest gives a book value of 1,990 and a BVPS of 39.80 instead of 1,900 and 38.00.
- Do not forget to subtract cash when computing EV, or to include preferred stock. In the example, leaving out the 150 of cash gives an EV of 3,550 and an EV/EBITDA of 8.35× instead of 3,400 and 8.0×.

- P/B is price divided by BVPS; dividing BVPS by price inverts it. With a price of 57 and BVPS of 38, the inverted ratio is 0.67 instead of 1.5.

LOS 43.c — Categories of valuation models

Absolute valuation models use only company-specific information.

1. **Present value models** (**discounted cash flow models**) value a share as the present value of expected future cash flows. Inputs: forecast cash flows over a forecast period, a **terminal value**, and a discount rate.

Model	Cash flow	Discount rate	Best suited to
Dividend discount model	Dividends	Required return on equity	Mature firms with stable, predictable dividends
Free cash flow to equity (FCFE)	Cash available to shareholders after capital expenditures and working capital needs	Required return on equity	Non-dividend payers with stable capital structures
Free cash flow to the firm (FCFF)	Cash available to *both* debt and equity holders; gives firm value	**Weighted average cost of capital (WACC)**	Negative FCFE or unstable capital structure (heavy debt, big buybacks)
Residual income	Current book value + PV of future residual income (net income minus an equity charge on beginning book value)	Required return on equity	Non-payers or firms with negative free cash flow

2. **Asset-based models** estimate equity value as the fair value of assets minus liabilities and preferred stock (book values adjusted to fair values). They suit firms whose assets have readily available market values: mainly **tangible, short-term (liquid) assets**, resource companies with known reserves, real estate worth more than its cash flow potential, divisions being sold, and firms being liquidated. An asset-based model can give a **floor value** for the stock.

Relative valuation models (**multiplier models**) compare a ratio with peers:

- **Equity multiples**: the share price divided by a per-share measure such as EPS, sales, book value or cash flow. A **trailing P/E** uses the past year's EPS, a **leading P/E** uses next year's forecast EPS; P/E loses its usefulness when EPS is negative or very small, and reported earnings are usually adjusted for nonrecurring items.
- **Enterprise value multiples**: EV divided by EBITDA, EBIT or revenue. This second type of multiplier model looks at value from the viewpoint of both debt and equity holders.

Method of comparables example. A share trades at 45 with trailing EPS of 3.00, a P/E of 15; the peer average is 18, so the share is relatively undervalued (unless firm-specific factors justify the discount).

The table sets the three families side by side.

Equity valuation model families: advantages and disadvantages

Model family	Value is based on	Advantages	Disadvantages
Present value (discounted cash flow) models	PV of expected dividends, FCFE or FCFF; for residual income, current book value plus PV of future residual income	Grounded in finance theory; assumptions such as growth and margins can be changed to see their effect on value	Inputs must be estimated; the value is highly sensitive to them
Asset-based models	Fair value of assets minus liabilities and preferred stock	Give a floor value; useful for divisions being sold and firms being liquidated; most reliable when assets are mainly tangible and short-term or have ready market values; more practical as firms report fair values	Ignore the firm as a going concern (margins, efficient use of assets, competition); ignore taxes; unreliable when intangibles are significant
Multiplier (relative) models	Price or EV relative to earnings, sales, book value, cash flow or EBITDA	Compare the company directly with a peer benchmark	A multiple far from the benchmark may be justified by firm-specific factors; P/E loses its usefulness when EPS is negative or very small

Analysts usually apply more than one model, with several sets of inputs, and arrive at a range of values. Different models can give inconsistent results. The analyst reconciles them by checking that the assumptions behind each approach are consistent and can combine them in a weighted average that puts more weight on the models whose inputs they trust most.

Common exam traps

- FCFF is discounted at the WACC; dividends, FCFE and residual income are discounted at the required return on equity.
- Fair values of assets and liabilities are the inputs of an asset-based model. A present value model needs forecast cash flows, a terminal value and a discount rate.
- Liquidation, or assets that are mostly liquid and tangible, points to an asset-based model.

- An EV/EBITDA comparison is a multiplier (relative valuation) model even though EV includes debt. It is not a discounted cash flow model.

EXAM SHORTCUTS

- Pick the model from the facts given: stable, predictable dividends point to a dividend discount model, a non-payer with a stable capital structure to FCFE, negative FCFE or an unstable capital structure to FCFF, and liquidation or mostly liquid tangible assets to an asset-based model.

BOTTOM LINE

- Intrinsic value cannot be observed and is estimated with valuation models; a perceived value above the market price means the stock looks undervalued, below it overvalued, and equal to it fairly valued.
- Naïve or story-based valuation rests on a story unconnected to the company's finances, relative valuation compares multiples with a peer group, and absolute valuation uses only the company's own characteristics, such as the present value of expected cash flows.
- Exam convention: book value of equity = share capital + additional paid-in capital + retained earnings − treasury stock; current practice also adds AOCI and other reserves, so the two agree only when those items are zero, and noncontrolling interest is excluded in both.
- Book value is most useful for companies with substantial tangible assets and least useful for early-stage or asset-light companies and those whose value lies in intangibles, and a negative book value alone does not mean the shares cannot provide value to investors.
- Market capitalization (share price × shares outstanding) reflects the company's fair value as a going concern, so a P/B above 1 signals excess value and a P/B below 1 a value shortfall to investigate.
- Enterprise value is the market value of the common and preferred stock and of the debt, less cash and short-term investments, and it is usually divided by EBITDA, EBIT or revenue to compare companies with different capital structures.
- In present value models, dividends, FCFE and residual income are discounted at the required return on equity, while FCFF, the cash available to both debt and equity holders, is discounted at the WACC to give firm value.
- An asset-based model values equity at the fair value of assets minus liabilities and preferred stock and can give a floor value, whereas multiplier models compare equity multiples or EV multiples with peers.

QUICK CHECK

Question 1

Shares of Delvin Pharmacy trade at 14.0 times analysts' consensus forecast of next year's EPS, which is $3.20. Last year Delvin earned $2.90 per share. Using her own models, an analyst estimates the intrinsic value of a Delvin share at $40.00. From her point of view, Delvin shares are:

A undervalued by $4.80.

B overvalued by $0.60.

C overvalued by $4.80.

Answer: C

The market price follows from the multiple and the earnings figure it is based on, here next year's forecast EPS. A stock whose market price is above the investor's estimate of intrinsic value is overvalued from that investor's point of view. The analyst's $40.00 is below the $44.80 price.

$$P_0 = 14.0 \times 3.20 = \$44.80$$

$$\text{intrinsic value} - \text{price} = 40.00 - 44.80 = -\$4.80$$

The shares are overvalued by $4.80.

Why the other options are wrong

- **A.** The size of the gap is right, but the direction is reversed. Undervalued would require intrinsic value above the market price.
- **B.** $0.60 applies the multiple to the EPS of the year just ended, $14.0 \times 2.90 = 40.60$. The multiple is stated on next year's forecast, so the price is 14.0×3.20.

Key takeaway Intrinsic value above price means undervalued; below price means overvalued. Pair each multiple with the earnings figure it is based on.

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Practice questions

Question 2

An analyst gathers the following market and financial data for Tamsin Ridge Appliances (amounts in millions except the share price).

Tamsin Ridge Appliances: selected data

Item	Value
Share price	$28.50
Common shares outstanding	12.0 million
Market value of preferred stock	$18 million
Market value of debt	$95 million
Cash and short-term investments	$31 million
EBITDA	$52 million

Tamsin Ridge's EV/EBITDA multiple is *closest* to:

A 7.8×.

B 8.2×.

C 8.8×.

Question 3

An analyst compares Tolliver Freight's ratio of enterprise value to EBITDA with the average ratio for other trucking companies. The analyst is using which type of valuation model?

A A multiplier model.

B An asset-based model.

C A discounted cash flow model.

Answers: 2. B · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/introduction-to-equity-valuation/. The full bank has 19 questions on this reading.

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EQUITIES · READING 44

Discounted Cash Flow (DCF) and Growth Models

What you'll learn

LOS 44.a Contrast the inputs of dividend, FCFE, FCFF and residual income models (and EBITDA as a proxy) and explain the five-step process for valuing equity with present value models.

LOS 44.b Calculate and interpret intrinsic value with constant growth and multistage present value models, including sustainable growth, implied growth, and FCFF discounted at the WACC.

LOS 44.c Explain the shortcomings of constant and multistage growth assumptions in present value models.

LOS 44.d Value non-callable, non-convertible preferred stock (finite and perpetual) and explain how callable, putable and convertible features change its value.

MODULE 44.1

Cash Flow Metrics

This reading covers the cash flow measures used in present value models (dividends, FCFE, FCFF, EBITDA and residual income) and the steps of the valuation process. It shows how to value shares with constant growth and multistage models, how to estimate growth and the WACC, where growth assumptions fall short, and how to value preferred stock and judge the effect of call, put and conversion features.

LOS 44.a — Cash flow measures for present value models and the valuation process

A **present value model** (also called a **discounted cash flow (DCF) model**) values equity as the present value of the cash flows shareholders can expect. Unlike a fixed-coupon bond, a common share has no promised cash flows and no maturity date, so the analyst must (1) forecast a cash flow measure over a **forecast horizon** and (2) estimate a **terminal value** that captures everything beyond the horizon. Value today is the present value of both:

$$V_0 = \sum_{t=1}^{n} \frac{E(CF_t)}{(1+r)^t} + \frac{E(TV_n)}{(1+r)^n}$$

where n is the number of periods in the forecast horizon and r is the discount rate that matches the cash flow measure.

Five measures are used. Two of them, EBITDA and residual income, are not cash flows, yet the same discounting framework still applies to them.

KEY CONCEPT

Measure	What it represents	Perspective	Discount rate	Works best when
Dividends	Cash actually paid to shareholders	**Minority shareholder** (cannot set dividend policy)	**Required rate of return on equity**	Positive, stable/growing earnings; high, consistent payout; stable capital structure and share count

Measure	What it represents	Perspective	Discount rate	Works best when
Free cash flow to equity (FCFE)	Cash that *could* be paid to common shareholders: the firm's **capacity to pay dividends**	**Controlling shareholder**	Required rate of return on equity	Non-payers or firms whose payout differs from capacity; stable capital structure
Free cash flow to the firm (FCFF)	Cash from operations, before interest, left after the investment needed to sustain capacity; available to debt and equity holders	Whole firm / all capital providers (pre-leverage)	**Weighted average cost of capital (WACC)**	High capital spending (negative FCFE) or a volatile capital structure
EBITDA	Pre-tax, pre-leverage earnings	Pre-leverage	WACC, as for FCFF	Rough proxy only; needs adjusting to reach FCFF
Residual income	Earnings in excess of the cost of equity (economic profit)	Equity	Required rate of return on equity	FCFE and FCFF negative with no clear date for turning positive

Dividends

Dividend discount models (DDMs) use the dividends shareholders expect to receive. They are **theoretically justified** because dividends are the cash equity investors actually get. The model assumes earnings are either paid out or retained to fund growth; if the firm borrows to pay dividends or buy back stock (i.e., changes its capital structure), that assumption breaks and a DDM is a poor choice. DDMs suit mature payers and industries where high payouts are required by law or regulation (e.g., REITs, regulated utilities). Drawbacks: hard to apply to **non-dividend-paying** firms, and they reflect only a minority holder's view.

Free cash flow to equity

FCFE is the cash left for common shareholders after the firm pays its debt obligations and funds the working capital and capital expenditures needed to maintain existing assets and grow:

KEY CONCEPT

$$FCFE = \text{net income} + \text{depreciation} - WCInv - CapEx + \text{net borrowing}$$

$$FCFE = CFO - FCInv + \text{net borrowing}$$

WCInv is investment in working capital and CapEx is capital expenditure, which the second form calls fixed capital investment (FCInv). Net income plus depreciation minus WCInv approximates cash flow from operations (CFO), which gives the second form. **Net borrowing** = new debt borrowed − debt repaid; it is the part of capital

spending and working capital investment financed with debt. From the cash flow statement, FCFE can also be read as its uses: $FCFE = \Delta\text{cash} + \text{dividends} - \Delta\text{equity}$, where Δequity = shares issued − shares repurchased. In other words, FCFE can be used to add to cash, pay dividends or buy back shares.

If the firm keeps a **target debt-to-assets ratio (DR)**, it borrows DR of every dollar of **net investment**:

$$\text{net investment} = CapEx - \text{depreciation} + WCInv$$

$$FCFE = \text{net income} - \text{net investment} + \text{net borrowing}, \qquad \text{net borrowing} = DR \times \text{net investment}$$

$$FCFE = \text{net income} - \text{net investment} + DR \times \text{net investment} = \text{net income} - (1 - DR)(\text{net investment})$$

Free cash flow to the firm

FCFE is less useful when heavy capital spending makes it negative or when the capital structure is volatile (future borrowing is hard to forecast). Then use **FCFF**, a **pre-leverage** cash flow:

$$FCFF = CFO - CapEx + \text{interest}(1 - t)$$

$$FCFF = FCFE + \text{interest}(1 - t) - \text{net borrowing}$$

Here t is the tax rate. Only *after-tax* interest is added back because interest reduces taxes. For a firm with no debt, FCFF = FCFE.

EBITDA

EBITDA differs from FCFF because it is **before tax**, ignores the **tax savings on depreciation**, and ignores investment in fixed and working capital:

$$FCFF = EBITDA(1 - t) + (\text{depreciation} \times t) - FCInv - WCInv$$

"Adjusted EBITDA" figures remove items management calls non-recurring; the adjustments are subjective and not an accounting-standard measure.

Residual income

Residual income = net income minus a charge for the cost of equity capital. Per share:

$$RI_t = EPS_t - r \times BVPS_{t-1}$$

It is discounted at the required return on equity. The model requires **clean surplus accounting**: book value of equity changes only through the income statement (retained earnings) and transactions with shareholders (dividends, issuance, buybacks). Items that bypass the income statement (e.g., foreign currency translation gains/losses) must be adjusted for.

The valuation process

1. Choose the cash flow measure.
2. Forecast it over a horizon suited to the company.

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3. Choose the discount rate consistent with that measure.
4. Estimate a **terminal value** for the period beyond the horizon.
5. Compute the present values of the forecast cash flows and terminal value and add them.

WORKED EXAMPLE

Lindqvist Marine reports net income 80, depreciation 25, working capital investment 10, capital expenditures 45, net borrowing 12, interest expense 15, tax rate 30% (all in millions).

- $FCFE = 80 + 25 - 10 - 45 + 12 = 62$
- $FCFF = 62 + 15(1 - 0.30) - 12 = 60.5$

A share with EPS of 3.00 this year, beginning book value per share of 20.00 and a 10% required return earns $RI = 3.00 - 0.10 \times 20.00 = 1.00$ per share.

COMMON EXAM TRAPS

- Match measure and rate: the cost of equity is the rate for dividends, FCFE and residual income, and the WACC is the rate for FCFF. FCFE is never discounted at a cost of debt, nor residual income at the WACC.
- A DDM takes a minority-shareholder view; FCFE takes a controlling-shareholder view, while FCFF takes the whole-firm view. FCFE is the *capacity* to pay; dividends are what is *actually* paid.
- Net borrowing is *added* in FCFE and *subtracted* when moving from FCFE to FCFF. In the worked example, adding the 12 of net borrowing again when moving to FCFF gives 84.5 instead of 60.5.
- With a target DR, multiply net investment by $(1 - DR)$ rather than by DR.
- In the EBITDA bridge, the depreciation tax shield is *added*.
- Calculating the terminal value is step 4; summing present values is the final step.

EXAM SHORTCUTS

- For a firm with no debt, FCFF equals FCFE, so there is no interest or net borrowing adjustment to make.

BOTTOM LINE

- A present value model values equity as the present value of a cash flow measure forecast over a horizon plus the present value of a terminal value, using a discount rate that matches the measure.
- A DDM takes the minority shareholder's view and suits mature firms with high, consistent payouts, while FCFE, the capacity to pay dividends, takes a controlling shareholder's view; the discount rate for both is the required return on equity.
- $FCFE = \text{net income} + \text{depreciation} - WCInv - CapEx + \text{net borrowing}$, and with a target debt-to-assets ratio DR it becomes $\text{net income} - (1 - DR)(\text{net investment})$.
- FCFF is a pre-leverage cash flow, $FCFF = FCFE + \text{interest}(1 - t) - \text{net borrowing}$, discounted at the WACC and used when heavy capital spending makes FCFE negative or the capital structure is volatile.
- Residual income per share is $EPS_t - r \times BVPS_{t-1}$, discounted at the required return on equity, and the model requires clean surplus accounting.

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- The valuation process chooses the cash flow measure, forecasts it, chooses a matching discount rate, estimates the terminal value and finally sums the present values.

QUICK CHECK

Question 1

Which of the following statements *most accurately* contrasts a dividend discount model (DDM) with a free cash flow to equity (FCFE) model?

A A DDM reflects the firm's capacity to pay dividends, while an FCFE model uses the cash shareholders actually receive.

B A DDM takes the view of a minority shareholder, while an FCFE model takes the view of a controlling shareholder.

C A DDM assumes a volatile capital structure, while an FCFE model assumes the firm borrows aggressively.

Answer: B

Dividends are what every shareholder receives and reflect the position of an investor who cannot influence dividend policy, which is a **minority shareholder** perspective. An FCFE valuation reflects a **controlling shareholder**, who can decide how much of the free cash flow is paid out.

Why the other options are wrong

- **A.** This reverses the two models: the DDM uses the cash actually received (dividends), while FCFE represents the firm's theoretical capacity to pay dividends.
- **C.** Neither model assumes a volatile capital structure or heavy borrowing; each is built on the premise that the firm's financing mix stays steady. When the mix is volatile, FCFF is the preferred measure.

Key takeaway A DDM takes the minority view of cash actually paid; FCFE takes the controlling view of cash that could be paid. Both assume a stable capital structure.

MODULE 44.2

Valuation Models

LOS 44.b — Constant growth and multistage present value models

Growth and the company life cycle

- **Start-up**: earnings negative and many firms fail, so cash flow models are unsuitable.
- **Growth**: earnings turn positive and grow rapidly; the analyst estimates how long this phase lasts.
- **Mature**: growth settles at a long-run rate in line with the overall economy (multinationals may exceed domestic growth). A constant growth assumption is most plausible here.
- **Decline**: obsolete products or technology.

The best candidates for a constant growth model are (1) **mature** firms (2) with **stable leverage** near the industry average and (3) a **high dividend payout ratio**. A constant rate is also used for the **terminal value** after a high-growth period.

Constant growth (Gordon growth) model

A share is worth the present value of all its future dividends. If the dividend grows at the same rate g every year and g is below the required return r, that endless sum collapses into one short fraction:

KEY CONCEPT

$$V_0 = \frac{D_1}{r - g} = \frac{D_0(1+g)}{r - g}$$

The same form works for FCFE ($V_0 = FCFE_1/(r-g)$) and FCFF (firm value $= FCFF_1/(WACC - g)$). Conditions: $g < r$, and g should be less than the long-run growth rate of the economy. The first condition keeps the value finite. Each dividend's present value is the previous one's multiplied by $(1+g)/(1+r)$, so the present values shrink toward zero only when g is below r; with $g \geq r$ they never shrink and the sum has no finite value.

- **Past vs expected dividend**: "just paid", "last paid" or "currently pays" means D_0; grow it one period. "Will pay next year" or "is expected to pay" means D_1; do not grow it.
- **Value at a future date**: $V_t = D_{t+1}/(r-g) = V_0(1+g)^t$. The model always gives value **one period before** the dividend in the numerator.
- **Sensitivity**: value rises when r falls or g rises, because the spread $r - g$ in the denominator shrinks.
- **One-period model**: for a one-year holding, the terminal value is the expected price in one year, so $V_0 = (D_1 + P_1)/(1+r)$. If the market price exceeds this value, the share is **overvalued**; if it is below, **undervalued**.

Implied growth rate. If the constant growth model fits the company, rearranging it with the market price gives the growth rate the market is pricing in (the same works with $FCFE_1$ in place of D_1):

$$g = r - \frac{D_1}{P_0}$$

Multistage (two-stage) models

Two types: (1) growth of g_S for n periods, then a perpetual rate g_L (usually $g_S > g_L$); (2) cash flows forecast individually for n periods (spreadsheet), then a constant-growth terminal value.

KEY CONCEPT

$$V_0 = \sum_{t=1}^{n} \frac{D_t}{(1+r)^t} + \frac{V_n}{(1+r)^n}, \qquad V_n = \frac{D_{n+1}}{r - g_L}$$

Steps: (1) forecast each high-growth dividend; (2) find the first constant-growth dividend D_{n+1}; (3) compute the terminal value V_n; (4) discount every dividend and V_n at r.

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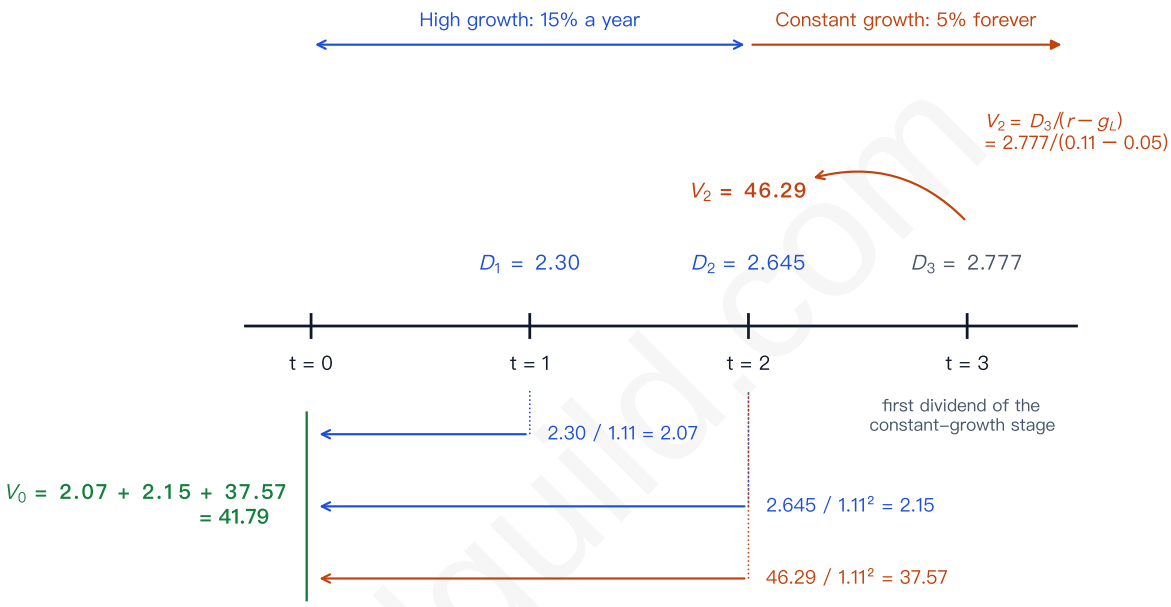
First dividend in the future: if the first dividend D_k arrives at the end of year k, the Gordon model gives $V_{k-1} = D_k/(r-g)$, which is discounted back k – 1 years.

Worked example. $D_0 = 2.00$; growth 15% for two years, then 5% forever; $r = 11\%$.

- $D_1 = 2.30$, $D_2 = 2.645$, $D_3 = 2.645 \times 1.05 = 2.777$
- $V_2 = 2.777/(0.11 - 0.05) = 46.29$
- $V_0 = 2.30/1.11 + (2.645 + 46.29)/1.11^2 = 2.07 + 39.72 = 41.79$

Calculator: CF0 = 0; C01 = 2.30; C02 = 48.93; [NPV] I = 11, [↓] [CPT] = 41.79.

The timeline shows the same cash flows and their present values.



Two-stage dividend discount model on a timeline (dividend just paid 2.00; 15% growth for two years, then 5%; required return 11%)

Estimating the constant growth rate

Either use **historical growth**, which is unreliable if capital structure, margins, competitive position or technology change, or estimate the **sustainable growth rate** from fundamentals, assuming constant ROE, constant payout and no new equity:

KEY CONCEPT

$$g = (1 - \text{dividend payout ratio}) \times ROE = b \times ROE$$

where the **retention ratio** $b = 1 - \text{payout ratio}$ and ROE is measured on beginning book value, so equivalently $g = (1 - \text{dividends/net income}) \times (\text{net income/beginning book value})$. The required return and beta are *not* inputs. Example: with ROE of 14% and a payout of 60%, $g = 0.40 \times 14\% = 5.6\%$. A result far above long-run economic growth should be questioned.

Putting the pieces together, each input of the constant growth model comes from a different place. The table uses one set of illustrative figures.

Where the inputs of the constant growth model come from (illustrative figures)

Input	How it is estimated	Illustration
Dividend just paid, D_0	Payout ratio × EPS	$0.50 \times 3.00 = 1.50$
Growth rate, g	Sustainable growth: retention ratio × ROE	$0.50 \times 10\% = 5\%$
Next dividend, D_1	$D_0(1+g)$	$1.50 \times 1.05 = 1.575$
Required return, r	CAPM: $R_f + \beta[E(R_m) - R_f]$ (Reading 50)	$3\% + 1.2 \times 5\% = 9\%$
Value, V_0	$D_1/(r-g)$	$1.575/(0.09-0.05) = 39.38$

For FCFE and FCFF:

$$SGR_{FCFE} = \left(1 - \frac{FCFE}{\text{net income}}\right)\left(\frac{\text{net income}}{\text{beginning book value}}\right)$$

$$SGR_{FCFF} = \left(1 - \frac{FCFF}{NOPAT}\right)\left(\frac{NOPAT}{\text{beginning book value of total capital}}\right)$$

with **NOPAT** = operating income × (1 – t). In theory all three rates agree; in practice they differ and the assumptions must be reconciled.

Weighted average cost of capital

FCFF goes to all capital providers, so its discount rate is the **weighted average cost of capital (WACC)**: the after-tax cost of debt and the cost of equity weighted by their market values.

$$WACC = \frac{MV_d}{MV_d + MV_e} r_d(1-t) + \frac{MV_e}{MV_d + MV_e} r_e$$

Debt has priority, so $r_d < r_e$. Present value of FCFF = **firm value**; equity value = firm value – market value of debt; divide by shares outstanding for a per-share value.

COMMON EXAM TRAPS

- Using D_0 in the numerator instead of D_1 (or growing an already-expected D_1 again).
- Terminal value: built from the **next** period's cash flow (D_{n+1}, $FCFF_{n+1}$) and discounted n periods. In the worked example, building V_2 from D_2 gives $V_0 = 40.00$, and discounting V_2 three periods gives 38.06, against 41.79.

- Forgetting to discount the terminal value to today, or forgetting the last high-growth dividend. In the worked example, leaving out $D_2 = 2.645$ gives 39.64 instead of 41.79.
- Sustainable growth uses the retention ratio. Using the payout ratio is a common error. With ROE of 14% and a payout of 60%, the payout ratio gives 8.4% instead of 5.6%.
- Discounting FCFF at the cost of equity, or reporting firm value as equity value.

EXAM SHORTCUTS

- Under constant growth, $V_t = V_0(1+g)^t$, so a value at a future date follows from today's value without forecasting the later dividend.

BOTTOM LINE

- The constant growth model $V_0 = D_1/(r-g) = D_0(1+g)/(r-g)$ requires g below r and below the long-run growth rate of the economy, and it fits mature firms with stable leverage and a high payout ratio best.
- The constant growth model gives value one period before the cash flow in its numerator, and the same form values FCFE at the cost of equity and FCFF at the WACC.
- In a two-stage model each high-growth dividend is discounted at r, and the terminal value $V_n = D_{n+1}/(r-g_L)$ is discounted n periods.
- If the constant growth model fits the company, the growth rate implied by the market price is $g = r - D_1/P_0$.
- Sustainable growth is $g = b \times ROE$, where b is the retention ratio, assuming constant ROE, constant payout and no new equity; the required return and beta are not inputs.
- The WACC weights the after-tax cost of debt and the cost of equity by market values, the present value of FCFF at the WACC is firm value, and equity value is firm value minus the market value of debt.

QUICK CHECK

Question 2

Colm Brennan projects that shares of Ardent Furnishings will trade at $63.00 one year from now and that Ardent will pay a $1.80 dividend per share at the end of the year. Brennan discounts Ardent's cash flows at 9%. Ardent currently trades at $60.00. Based on Brennan's analysis, Ardent's shares are:

A Overvalued.

B Fairly priced.

C Undervalued.

Answer: A

Compare the market price with the value implied by the analyst's forecasts. The one-period value is the present value of the year-end dividend plus the projected price.

$$V_0 = \frac{D_1 + P_1}{1+r} = \frac{1.80 + 63.00}{1.09} = \frac{64.80}{1.09} = \$59.45$$

The market price of $60.00 is above the estimated value of $59.45, so the shares are overvalued.

Why the other options are wrong

- **B.** Fairly priced would require the market price to equal the $59.45 value; it is $0.55 higher.
- **C.** Undervalued would require the price to be below the estimated value. Comparing $60.00 with the undiscounted $64.80 might suggest this, but the cash flows arrive in one year and must be discounted.

Key takeaway A price above estimated value means the shares are overvalued; a price below it means they are undervalued.

MODULE 44.3

Model Limitations and Preferred Stock Valuation

LOS 44.c — Shortcomings of constant and multistage growth assumptions

A present value model is only as good as its inputs. The choice of cash flow measure, its size, the length of the **forecast horizon** and the **discount rate** are all estimated and can all be wrong.

Weak point	Why it matters
Discount rate	Both the cost of equity and the cost of debt depend on the **risk-free rate**, which fluctuates; small changes in r (especially in $r - g$) move value a lot
Historical growth rates	Past dividend growth is a poor guide for a firm whose competitive environment is deteriorating
DDMs and payout form	DDMs capture cash dividends only and **ignore stock buybacks**, now a major way of returning cash
Constant growth	Most plausible for mature firms; a rapidly growing firm needs a **multistage model** with a finite high-growth period
Time horizon	The length of the forecast horizon is itself an estimate and can be wrong

WORKED EXAMPLE: HOW FAR A CONSTANT GROWTH VALUE MOVES WITH R AND G

A mature company is expected to pay a dividend of $2.16 per share next year, and the dividend is expected to grow at 5.0% a year forever. The required return on equity is 9.0%. Find the value per share, then the value if the required return is 10.0% with growth unchanged, and the value if growth is 4.0% with the required return unchanged at 9.0%.

Step 1. Base case: $V_0 = 2.16/(0.09 - 0.05) = 2.16/0.04 = \$54.00$.

Step 2. Required return of 10.0%: $V_0 = 2.16/(0.10 - 0.05) = 2.16/0.05 = \$43.20$, which is 20% below the base value.

Step 3. Growth of 4.0%: $V_0 = 2.16/(0.09 - 0.04) = 2.16/0.05 = \$43.20$, the same 20% fall.

Result. A one-percentage-point change in either r or g widens $r - g$ from 4 to 5 percentage points and lowers the value from $54.00 to $43.20, a fall of 20%.

Analysts should cross-check inputs against the firm's industry position, competitive advantages, technological change and the macroeconomic environment.

COMMON EXAM TRAPS

- A DDM does not require debt financing; it works for all-equity firms.
- Multistage models still need a terminal value.

LOS 44.d — Valuing preferred stock and the effect of contingency features

Preferred stock ranks ahead of common equity for dividends and in liquidation, so its required return is **lower** than that of common equity. It pays a fixed dividend, usually stated as a percentage of par (face) value: a 7% preferred with $40 par pays $2.80 a year. It may also have features such as **cumulative** dividends (missed dividends accumulate) or **participating** dividends (extra dividends when profits are high), and contingency features such as a call, a put or a conversion right.

Non-callable, non-convertible preferred

Finite-maturity preferred (economically a bond; often classified as debt):

$$V_0 = \sum_{t=1}^{N} \frac{D_p}{(1+r_p)^t} + \frac{FV}{(1+r_p)^N}$$

Perpetual preferred (the constant growth model with g = 0):

KEY CONCEPT

$$V_0 = \frac{D_p}{r_p}$$

The same zero-growth formula values a common share whose dividend is expected to stay constant forever. The required yield r_p can be quoted as a spread over (or under) another rate, e.g., a reference rate or the issuer's bond yield. Rearranged, $r_p = D_p / P_0$.

Worked examples.

- Perpetual: $80 par and a 5% dividend give $D_p = 4.00$; with $r_p = 6.25\%$, $V_0 = 4.00 / 0.0625 = \$64.00$. Value is below par because the required yield exceeds the dividend rate.
- Finite: 3 years, $100 face, $6 dividend, $r_p = 7\%$. Calculator (END mode, P/Y = C/Y = 1): N = 3, I/Y = 7, PMT = 6, FV = 100, CPT PV = −97.38 (negative because it is the price paid).

Contingency features (embedded options)

Contingency features are **embedded options**: rights that the holder may exercise but need not. An option has non-negative value to its holder, so its value is **subtracted** from the option-free value when the **issuer** holds it and **added** when the **investor** holds it.

KEY CONCEPT

Type	Who holds the right	Value vs identical option-free preferred	Typical dividend
Callable preferred	Issuer (redeem at a fixed price)	Lower	Higher
Putable preferred	Investor (sell back at a fixed price)	Higher	Lower
Convertible preferred	Investor (convert into common at a set ratio)	Higher	—

A convertible preferred mixes debt-like traits (fixed dividend, priority) with an equity-like conversion option. Because required return and value move inversely, callable preferred carries a **higher required return** than putable or convertible preferred.

COMMON EXAM TRAPS

- Use the dividend in dollars: a "6% preferred, $50 par" pays $3.00.
- Do not grow a preferred dividend; g = 0.
- Value differs from par whenever r_p differs from the dividend rate.
- A finite-maturity preferred needs the PV of the face value as well as the dividends. In the 3-year example, discounting the dividends alone gives 15.75 instead of 97.38.
- A call right belongs to the issuer and lowers value; a put or conversion right belongs to the investor and raises value.

EXAM SHORTCUTS

- For a perpetual preferred, a required yield above the dividend rate means a value below par, so a price above par can be ruled out without calculation.

BOTTOM LINE

- The cash flow measure and its size, the length of the forecast horizon and the discount rate are all estimates in a present value model, and small changes in r, especially in $r - g$, move the value a lot.
- DDMs ignore stock buybacks, past dividend growth is a poor guide for a firm whose competitive environment is deteriorating, and a rapidly growing firm needs a multistage model with a finite high-growth period.
- Preferred stock ranks ahead of common equity for dividends and in liquidation, so its required return is lower than that of common equity, and its dividend is the stated rate times par.
- A perpetual preferred is worth D_p/r_p, the constant growth model with g = 0, while a finite-maturity preferred is valued like a bond as the present value of its dividends and its face value.
- An embedded option's value is subtracted when the issuer holds the right and added when the investor holds it, so a callable preferred is worth less and pays a higher dividend, while putable and convertible preferreds are worth more than identical option-free preferreds.

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QUICK CHECK

Question 3

Compared with an otherwise identical option-free preferred stock, a preferred stock with an embedded option *most likely* has a lower intrinsic value when the right to exercise the option is held by:

A the investor, as with a conversion feature.

B either party, because an embedded option always lowers value.

C the issuer, as with a call feature.

Answer: C

An embedded option is a right, not an obligation, so it has a non-negative value to the party that holds it. When the issuer holds the right, its value is subtracted from the option-free value of the preferred stock. A call feature lets the issuer redeem the shares at a predetermined price, so a callable preferred is worth less than a comparable non-callable preferred.

Why the other options are wrong

- **A.** When the investor holds the right, the option's value is added to the option-free value. A conversion feature lets the investor exchange the preferred for common shares at a set ratio, so a convertible preferred is worth more, not less.
- **B.** The effect depends on who holds the right. An option held by the issuer lowers the preferred's value; one held by the investor, such as a put or a conversion right, raises it.

Key takeaway Embedded option held by the issuer (call): subtract its value. Held by the investor (put, conversion): add its value.

Practice questions

Question 4

Maren Solberg has concluded that both free cash flow measures for Halden Robotics are negative and are unlikely to turn positive for several years, so she will value the company's equity with a residual income model. Which condition must hold for her to use Halden's reported book value of equity without adjustment?

A The company follows clean surplus accounting.

B The company holds its debt-to-assets ratio constant.

C The company adjusts earnings for depreciation tax savings.

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Question 5

Brookmere Clinics is expected to generate free cash flow to the firm (FCFF) of $50 million in Year 1, $62 million in Year 2 and $80 million in Year 3. After Year 3, FCFF is expected to grow at 4% a year. Brookmere's weighted average cost of capital (WACC) is 8.5%. The value of the firm today is *closest* to:

A $1,609 million.

B $1,553 million.

C $1,495 million.

Answers: 4. A · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/discounted-cash-flow-dcf-and-growth-models/. The full bank has 63 questions on this reading.

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EQUITIES · READING 45

Relative Value Equity Valuation Approaches

What you'll learn

LOS 45.a Explain how the method of comparables and the method of forecasted fundamentals set a benchmark multiple, and how PEG ratios and regression reconcile them.

LOS 45.b Calculate and interpret price multiples (P/E, P/B, P/S, dividend yield, P/FCFE) and enterprise value multiples (EV/EBITDA, EV/FCFF, EV-to-book), and describe when each is appropriate.

LOS 45.c Determine a peer group using industry classification or statistical factor exposures, and recognize the limitations of each.

LOS 45.d Describe how multiples based on past, current and projected values are used, including historical comparisons and terminal values.

MODULE 45.1

Price Multiples and Equity Valuation

This reading compares the method of comparables with the method of forecasted fundamentals and shows how to calculate justified P/E and P/B ratios, the PEG ratio and regression-based multiples. It then covers the calculation and use of price, enterprise value and cash flow multiples, the choice of an industry-based or factor-based peer group, and multiples built on past or forecast values, including a terminal value from a justified multiple.

LOS 45.a — Comparables versus forecasted fundamentals

Relative valuation judges a stock by comparing what investors pay for it with some measure of value. Two families of ratios are used:

- **Price multiples** put the share price over a per-share measure: earnings (P/E), book value (P/B), sales (P/S), dividends or free cash flow to equity.
- **Enterprise value (EV) multiples** put EV over a firm-level measure that belongs to all capital providers: EBITDA, EBIT, free cash flow to the firm (FCFF) or revenue.

The timing of the denominator matters:

Label	Denominator	Example
Trailing multiple	value for the most recent past period	P_0/E_0
Leading multiple (also **forward multiple**)	forecast for the next period	P_0/E_1
Justified multiple	the multiple a fairly priced stock should carry, estimated by the analyst	from peers or from a DCF model

The multiple must make economic sense for the company. A P/E is meaningless when earnings are negative, and a dividend-based multiple does not work for a peer group that includes non-payers.

The decision rule for price multiples: a stock whose multiple is above the benchmark looks overvalued, and one whose multiple is below the benchmark looks undervalued.

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Method of comparables

The **method of comparables** takes the benchmark from a **peer group**, usually the mean or median multiple of similar companies. Its economic rationale is the **law of one price**: similar assets should sell at similar multiples.

- Peers must genuinely be similar in competitive position, risk and earnings growth. Different accounting methods, nonrecurring items and different fiscal year-ends all weaken comparability.
- When the peers' multiples are skewed by a few extreme values (often caused by temporarily depressed earnings), the median is the better benchmark because outliers do not pull it. The alternative is to remove the outliers before computing the mean.

Applying the method:

1. Compute the benchmark multiple from the peer group.
2. Compare the subject company's multiple with the benchmark.
3. For a value estimate, multiply the benchmark by the subject company's own per-share measure for the same period: forecast EPS for a leading P/E, trailing EPS for a trailing P/E.

Example. Five peers trade at P/Es of 11, 13, 14, 15 and 37. The median is 14, while the single high multiple pulls the mean up to 18. The subject company earns $2.50 per share and trades at $40.00, a P/E of 16. Against the median it looks overvalued, with an implied value of $14 \times 2.50 = \$35.00$. Against the mean it would wrongly look undervalued, with an implied value of $18 \times 2.50 = \$45.00$.

Method of forecasted fundamentals

The **method of forecasted fundamentals** derives the benchmark from a **discounted cash flow model**, typically the constant growth (Gordon) model $P_0 = D_1/(r - g)$. Dividing by next year's earnings gives the **justified leading P/E**:

KEY CONCEPT

$$\frac{P_0}{E_1} = \frac{D_1/E_1}{r - g} = \frac{\text{payout ratio}}{r - g}$$

The trailing version multiplies by $(1 + g)$: $P_0/E_0 = \text{payout} \times (1 + g)/(r - g)$.

The justified P/E rises with the **earnings growth rate** (g) and falls as the **required rate of return** (r, which reflects the company's risk) rises. With r and g held constant, a higher payout ratio raises it.

Dividing the same model by book value per share, and substituting that growth relation, gives the **justified P/B ratio**:

KEY CONCEPT

$$\frac{P_0}{B_0} = \frac{ROE - g}{r - g}$$

The P/B therefore depends on the spread between ROE and the required rate of return on equity (the cost of equity). With $r > g$, the justified P/B is above 1.0 if ROE is above r, exactly 1.0 if ROE equals r, and below 1.0 if ROE is below r.

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Example. A firm pays out 50% of earnings, r = 12% and g = 7%. Its justified leading P/E is $0.50/0.05 = 10.0$. A second firm has ROE = 15%, r = 10% and g = 6%, so its justified P/B is $(0.15 - 0.06)/(0.10 - 0.06) = 2.25$.

Reconciling the two methods

The two methods can disagree. A firm may deserve a P/E above its peer average because it grows faster, for example. Two tools adjust for fundamentals:

1. The **price/earnings-to-growth ratio (PEG)** adjusts for one fundamental, growth:

$$PEG = \frac{P/E}{g} \quad (g \text{ in whole percentage points})$$

A PEG higher than the peers' benchmark means the P/E is high relative to growth, so the stock looks overvalued. A lower PEG points to undervaluation. *Example.* A P/E of 18 and growth of 9% give a PEG of 2.0. Against a peer median of 1.6, the stock looks overvalued.

2. The **regression approach** adjusts for several fundamentals at once. The observed multiple (the dependent variable) is regressed on fundamentals such as ROE, growth, payout and risk (the independent variables) across the peer group. The fitted equation then gives the subject company's justified multiple:

$$\widehat{P/B}_i = b_0 + b_1\, ROE_i + b_2\, g_i$$

Example. With $b_0 = 0.60$, $b_1 = 8.0$, $b_2 = 5.0$, ROE = 10% and g = 4%, the justified P/B is $0.60 + 0.80 + 0.20 = 1.60$. The intercept is part of the estimate, and the ROE input is the firm's actual ROE rather than its required return. At Level I, candidates interpret regression output; they are not asked to estimate the regression.

KEY CONCEPT

Tool	Controls for	Weakness
Peer mean/median	nothing (assumes peers are alike)	peers may differ, or be mispriced
PEG ratio	growth only	ignores risk, ROE and payout differences
Regression	several fundamentals simultaneously	needs a peer sample; relationships can change

Terminal values from multiples

A justified multiple estimated for the end of the forecast horizon can also supply the terminal value in a present value model. LOS 45.d gives the formula and an example.

COMMON EXAM TRAPS

- Using the mean of a skewed peer distribution instead of the median. In the example, the mean of 18 gives an implied value of $45.00 instead of $35.00 from the median of 14.

- Mixing up the rationales. The law of one price underlies comparables; a DCF model underlies forecasted fundamentals.
- Calculating PEG as g/(P/E), or entering g as a decimal (0.12 instead of 12). In the PEG example, $g/(P/E)$ gives 0.5 and entering growth as 0.09 gives 200, instead of 2.0.
- Reading a low PEG as overvalued. For PEG, as for P/E, a value below the peers' signals undervaluation.
- In the justified leading P/E, using the retention ratio instead of the payout ratio, or dividing by r instead of r − g. In the example, dividing the 50% payout by r = 12% gives 4.17 instead of 10.0.
- Leaving out the intercept in a regression-based justified multiple. In the example, dropping $b_0 = 0.60$ gives 1.00 instead of 1.60.

EXAM SHORTCUTS

- With r above g, the justified P/B is above 1.0 exactly when ROE is above r, so the side of 1.0 can be read from ROE and r before any calculation.

BOTTOM LINE

- Price multiples put the share price over a per-share measure, EV multiples put EV over a firm-level measure belonging to all capital providers, a trailing multiple uses the most recent past period and a leading multiple uses next period's forecast.
- The method of comparables takes its benchmark from the mean or median multiple of a peer group and rests on the law of one price, and when a few extreme values skew the peers' multiples the median, or the mean after removing outliers, is the better benchmark.
- A stock whose price multiple is above the benchmark looks overvalued, and one whose multiple is below the benchmark looks undervalued.
- The justified leading P/E from the constant growth model is $\text{payout ratio}/(r-g)$, the trailing version multiplies it by $(1+g)$, and it rises with g, falls as r rises and, with r and g constant, rises with the payout ratio.
- The justified P/B is $(ROE-g)/(r-g)$, so with r above g it is above 1.0 if ROE exceeds r, equal to 1.0 if ROE equals r and below 1.0 if ROE is below r.
- The PEG ratio, $(P/E)/g$ with g in whole percentage points, adjusts for growth only and a PEG above the peers' benchmark looks overvalued, while the regression approach adjusts for several fundamentals at once and its fitted equation, intercept included, gives the justified multiple.

QUICK CHECK

Question 1

Beatriz Almeida is valuing Penhallow Instruments with the method of comparables. The leading P/E ratios of the five companies in her peer group are 13.5, 14.5, 16.0, 20.0 and 48.0. The 48.0 multiple belongs to a peer whose earnings next year are expected to be temporarily depressed by a plant refit. Penhallow earned $2.60 per share over the last 12 months, and the consensus forecast of its EPS for next year is $2.95. Almeida's estimate of Penhallow's value per share is *closest* to:

A $41.60.

B $47.20.

C $66.08.

Answer: B

The method of comparables values a stock at the benchmark multiple of its peer group, applied to the stock's own value driver. One peer's multiple is inflated by temporarily depressed earnings, so the median, which a single extreme value does not pull, is the appropriate benchmark. A leading P/E has next year's forecast EPS in its denominator, so the benchmark is applied to Penhallow's forecast EPS of $2.95. Its trailing EPS would pair with a trailing multiple.

Sort the peers' leading P/Es: 13.5, 14.5, 16.0, 20.0, 48.0. With five values, the median is the third one:

$$\text{median leading } P/E = 16.0$$

Apply the benchmark to next year's EPS:

$$V_0 = \text{benchmark } \frac{P_0}{E_1} \times E_1 = 16.0 \times \$2.95 = \$47.20$$

Dropping the 48.0 outlier and averaging the other four peers gives the same benchmark: $(13.5 + 14.5 + 16.0 + 20.0)/4 = 64.0/4 = 16.0$. The mean of all five, $(13.5 + 14.5 + 16.0 + 20.0 + 48.0)/5 = 112.0/5 = 22.4$, is higher than four of the five peers because of the single 48.0 value.

Why the other options are wrong

- **A.** $16.0 \times \$2.60 = \$41.60$ applies a leading multiple to trailing EPS. The peer multiple and the subject's earnings must cover the same period, so a leading P/E is paired with forecast EPS.
- **C.** $22.4 \times \$2.95 = \$66.08$ uses the mean leading P/E of all five peers. The 48.0 multiple reflects one peer's temporarily depressed earnings and drags the mean well above the typical peer, so the median is the better benchmark.

Key takeaway Comparables-based value = benchmark multiple × the subject company's own value driver. Match the timing (a leading multiple with forecast EPS, a trailing multiple with trailing EPS), and use the median when a few extreme multiples skew the peer group.

MODULE 45.2

Price and Enterprise Value Multiples

LOS 45.b — Price and enterprise value multiples: calculation, interpretation and uses

Value = assets in place + growth opportunities

A share's value can be split into the **value of assets in place** (next year's earnings as a no-growth perpetuity, E_1/r) and the **present value of growth opportunities (PVGO)**:

KEY CONCEPT

$$P_0 = \frac{E_1}{r} + PVGO \qquad \Rightarrow \qquad \frac{PVGO}{P_0} = 1 - \frac{E_1/P_0}{r}$$

The **earnings yield** is E/P, the inverse of the P/E. E_1/P_0 is the **forward earnings yield**, the reciprocal of the leading P/E, so the same split can be written $E_1/P_0 = r(1 - PVGO/P_0)$. A negative PVGO suggests management is investing in negative-NPV projects and should pay out more of its earnings.

Example. With $P_0 = 50$, $E_1 = 3.00$ and $r = 8\%$, assets in place are worth 37.50 and PVGO is 12.50, which is 25% of the price.

Earnings-based multiples: core and normalized earnings

A P/E built on earnings that contain large one-off items is misleading, so analysts use **core earnings** (also called persistent or continuing earnings). Core earnings strip out nonrecurring items, for example discontinued operations, write-downs, gains or losses on asset sales, changes in accounting estimates and provisions for future losses. EPS should also reflect dilution from options, warrants and convertible bonds.

For cyclical firms, analysts estimate **normalized earnings**, the EPS expected in the middle of the business cycle:

KEY CONCEPT

Method	Normalized EPS	Comment
Method of historical average EPS	average EPS over the most recent business cycle	ignores growth in firm size
Average ROE method	average ROE over the cycle × **current book value per share**	captures changes in firm size better

Example. EPS over a four-year cycle of 2.10, 2.50, 1.70 and 2.90 gives a historical average of 2.30. An average ROE of 11% times current BVPS of 22.00 gives 2.42.

Price-to-sales (P/S)

The **price-to-sales (P/S)** multiple is used when earnings are negative or not comparable across firms. It suits mature and cyclical firms with similar revenue recognition. Sales belong to both debt and equity holders while price is an equity measure, so P/S fits firms with little or no leverage. For more leveraged firms, an EV multiple may be more appropriate.

$$\frac{P_0}{S_0} = \frac{P_0}{E_0} \times \frac{E_0}{S_0} = \frac{\text{payout} \times (1+g)}{r-g} \times \text{profit margin}$$

Example. With a payout of 50%, g = 4%, r = 9% and a margin of 6%, the trailing P/E is $0.52/0.05 = 10.4$ and the justified P/S is $10.4 \times 0.06 = 0.624$.

Enterprise value multiples

$$EV = \text{market value of common and preferred stock} + \text{market value of debt} - \text{cash and short-term investments}$$

Firm value is the same figure without subtracting cash. EBIT, EBITDA and FCFF belong to all capital providers, so they are paired with EV.

$$EV = \frac{FCFF_1}{WACC - g}, \qquad FCFF = EBITDA(1-t) + \text{Dep} \times t - \text{CapEx} - \text{WCInv}$$

The justified **EV/EBITDA** multiple is this EV divided by EBITDA. It rises with g and falls with WACC. It is popular in capital-intensive industries with large depreciation.

> *Example.* EBITDA is 500, depreciation 100, CapEx 150, working capital investment 30, t = 25%, WACC 8% and g = 3%. FCFF is $375 + 25 - 150 - 30 = 220$, EV is $220/0.05 = 4{,}400$, and EV/EBITDA is 8.8x.

Balance-sheet multiples

- **P/B** is most useful when the firms compared hold mainly tangible assets and have similar accounting methods, leverage and asset mix. Banks are often valued on **price-to-tangible book value**, which excludes intangible assets such as goodwill from book value.
- The **value-to-book multiple** is (market value of equity + market value of debt) / (book value of equity + book value of debt). It is useful when leverage differs or book equity is negative.
- The **enterprise value-to-book ratio** subtracts cash from both numerator and denominator. It can be written in terms of fundamentals. **Return on capital (ROC)** is $EBIT(1-t)/\text{book value of capital}$. **Reinvestment** is the spending on fixed capital (net of depreciation) and working capital, the part of EBIT that is not available to debt and equity holders. Exam convention: the **reinvestment rate** is reinvestment as a proportion of EBIT, and $FCFF = EBIT(1-t)(1 - \text{reinvestment rate})$. Current practice: that FCFF identity holds exactly only when the rate is measured against after-tax EBIT, $(\text{capital expenditures} - \text{depreciation} + \text{working capital investment})/[EBIT(1-t)]$. When a question states the reinvestment rate, use it as given. Then:

$$\frac{EV}{BV \text{ of capital}} = \frac{ROC \times (1 - \text{reinvestment rate})}{WACC - g}$$

> *Example.* With ROC of 8%, a reinvestment rate of 50%, WACC of 9% and g of 4%, the justified ratio is $0.08(1-0.50)/(0.09-0.04) = 0.8$x. The growth rate matches the reinvestment: $0.50 \times 8\% = 4\%$.

Cash-flow multiples and yields

- The **price-to-dividend multiple** is P_0/D, and its reciprocal is the **dividend yield**. A **trailing dividend yield** uses the dividend paid over the past year. A **leading dividend yield** divides the dividend expected over the next year by the current price.
- The **justified dividend yield** can be computed two ways:

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$$\frac{D_0}{P_0} = \frac{D_0}{E_0} \times \frac{E_0}{P_0} \quad \text{(payout} \times \text{earnings yield)} \qquad \text{or} \qquad \frac{D_0}{P_0} = \frac{r-g}{1+g}$$

Example. A payout of 60% and a justified trailing P/E of 15 give an earnings yield of 6.67% and a dividend yield of 4.0%. For a different firm with $r = 9\%$ and $g = 3\%$, the second formula gives $0.06/1.03 = 5.83\%$.

- Yields work in reverse. An actual yield higher than the justified yield signals an undervalued stock; a lower one signals an overvalued stock. As with multiples, a yield is compared with peers only after allowing for differences in fundamentals such as growth and the required return on equity.
- Justified $P_0/FCFE_1 = 1/(r-g)$ and $EV/FCFF_1 = 1/(WACC-g)$. **FCFE** matches price (equity) and **FCFF** matches EV (all capital). Cash flow from operations can also serve as a denominator, but FCFE measures the cash actually available to shareholders more accurately. The main concern with free-cash-flow multiples is that capital expenditures and working capital investment are cyclical, so free cash flow can swing over the cycle.
- Industry-specific multiples (e.g., EV per unit of reserves in mining or oil) cannot be compared across industries and can let a whole industry become mispriced.

Which multiple when?

KEY CONCEPT

Situation	Preferred measure
Negative or non-comparable earnings, mature/cyclical, low leverage	P/S
Capital-intensive, leverage differs across peers	EV/EBITDA
Mostly tangible assets, similar accounting and leverage; banks	P/B; price-to-tangible book for banks
Book equity negative or leverage differs	value-to-book or EV-to-book
Peer group includes non-dividend payers	avoid dividend-based multiples

COMMON EXAM TRAPS

- For the PVGO share, subtract assets in place from price and then divide by price. E_1/P_0 is the earnings yield, a different quantity. In the example, E_1/P_0 is 6%, while the PVGO share is 25%.
- In FCFF, add the depreciation tax shield $\text{Dep} \times t$. EV/EBIT is a different multiple from EV/EBITDA. In the example, leaving out the 25 shield gives FCFF of 195, EV of 3,900 and EV/EBITDA of 7.8x instead of 220, 4,400 and 8.8x.
- The justified P/S uses the trailing P/E, which includes $1+g$. In the example, the leading P/E of 10.0 gives 0.60 instead of 0.624.
- The average ROE method multiplies by current BVPS. Next year's EPS plays no part.
- For dividend and earnings yields, a value above the justified level means undervalued.

EXAM SHORTCUTS

- Match the multiple to the situation: negative or non-comparable earnings point to P/S, a capital-intensive industry with different leverage across peers to EV/EBITDA, negative book equity to value-to-book or EV-to-book, and a peer group with non-payers rules out dividend-based multiples.

BOTTOM LINE

- A share's value is $P_0 = E_1/r + PVGO$, so the PVGO share of the price is $1 - (E_1/P_0)/r$, and a negative PVGO suggests management is investing in negative-NPV projects.
- Core earnings strip out nonrecurring items, and for cyclical firms normalized EPS is either the average EPS over the most recent cycle, which ignores growth in firm size, or the average ROE over the cycle times current BVPS, which captures it better.
- P/S is used when earnings are negative or not comparable, suits mature and cyclical firms with similar revenue recognition and little or no leverage, and its justified value is the justified trailing P/E times the profit margin.
- EBIT, EBITDA and FCFF belong to all capital providers and are paired with EV; the justified EV/EBITDA is $FCFF_1/(WACC - g)$ divided by EBITDA, rising with g and falling with WACC.
- P/B works best for firms with mainly tangible assets and similar accounting, leverage and asset mix, banks are often valued on price-to-tangible book, and value-to-book or EV-to-book helps when leverage differs or book equity is negative.
- The justified dividend yield is $(r - g)/(1 + g)$ or payout times earnings yield, and for yields the rule is reversed: an actual yield above the justified yield signals an undervalued stock.

QUICK CHECK

Question 2

Over its most recent business cycle, Dunmore Engineering earned an average return on equity of 16%. Its current book value per share is $31.50, its expected EPS for next year is $4.80, and its EPS averaged $4.35 over the same cycle. Using the average ROE method, Dunmore's normalized EPS is *closest* to:

A $4.35.

B $4.80.

C $5.04.

Answer: C

Normalized earnings estimate mid-cycle EPS for a cyclical company. Under the average ROE method, normalized EPS equals the average ROE over the most recent cycle multiplied by the current book value per share.

$$\text{Normalized EPS} = \overline{ROE} \times BVPS_{\text{current}} = 0.16 \times 31.50 = \$5.04$$

Why the other options are wrong

- **A.** $4.35 is normalized EPS under the method of historical average EPS. That method does not reflect changes in firm size as well as the average ROE method does.
- **B.** $4.80 is simply next year's expected EPS, which is not adjusted for where the firm is in the business cycle.

Key takeaway Average ROE method uses current BVPS, so it captures growth in the firm's size; historical average EPS does not.

MODULE 45.3

Peer Groups and Multiples-Based Valuation

LOS 45.c — Choosing a peer group

Comparables-based valuation benchmarks a company against the mean or median multiple of a peer group. There are two ways to form that group.

Industry-based peer group

An **industry-based peer group** contains firms in the same principal business activity. Because they face much the same competitive forces, supply chains and technology, their cash flows should behave similarly. Industry classification systems are the usual starting point. Good peers should also match on cyclicality, life cycle stage and market structure, and the analyst still has to check fundamentals such as leverage.

Industry peers are hard to find for:

- unique firms with few or no genuinely similar companies;
- industries whose members are at widely different life cycle stages (start-ups next to mature incumbents);
- firms exposed to different tax or regulatory jurisdictions;
- **multi-industry companies** (conglomerates), whose blend of businesses, markets and fundamental factors (risk, growth, leverage) may leave them with no close peers at all.

Statistical factor-based peer group

A **statistical factor-based peer group** groups companies whose exposures to risk-return factors are the same or close, regardless of industry. The **CAPM** is a single-factor model that measures only systematic risk (beta). **Multifactor models** are the usual tool for finding factor-based peers.

- The advantage is independence from industry labels, which helps with firms that industry lists handle badly.
- The disadvantage is that the statistical relationships are estimated from past data and may not persist, so the model can be misspecified going forward.

KEY CONCEPT

Feature	Industry-based	Statistical factor-based
Grouping criterion	same business activity	similar factor exposures
Typical tool	industry classification system	multifactor model

Feature	Industry-based	Statistical factor-based
Main weakness	conglomerates, unique firms, mixed life cycles, jurisdictions	historical relationships may not hold in future

LOS 45.d — Multiples based on past, current and future values

Weaknesses of current-value comparables

1. Mispriced peers. If the whole peer group is overvalued or undervalued, a stock that looks cheap or expensive relative to its peers may not be cheap or expensive in absolute terms.
2. Distorted current figures. Multiples built on current, unadjusted numbers can be skewed by accounting choices, nonrecurring items or cyclicality. The remedies are core or normalized earnings, or a comparison with multiples from past periods (below), which span different points in the cycle.

Using historical values

- **Historical values of the subject company's multiples**: today's multiple is compared with the company's own past multiples. Changes can be linked to changes in its fundamentals, and the analysis does not rely on the current pricing of peers.
- **Historical averages of peer group multiples**: the company is compared with its peers over a whole period, such as a business cycle. This reduces the risk of benchmarking against a peer group that is mispriced at the moment.

Fundamental differences such as growth and required return still have to be controlled for. Historical data do not remove that need.

Using projected future values

A **forecast multiple** can be applied to a forecast metric to estimate a **terminal value** for a present value model:

KEY CONCEPT

$$TV_n = \text{justified leading P/E}_n \times E_{n+1}, \qquad \text{justified leading P/E}_n = \frac{\text{payout}}{r - g}$$

Example. EPS in year 5 is expected to be 4.00, after which growth settles at 3% with a 60% payout and r = 9%. The justified leading P/E is $0.60/0.06 = 10.0$ and $E_6 = 4.00 \times 1.03 = 4.12$, so $TV_5 = 41.20$. The justified trailing P/E of 10.3 applied to E_5 gives the same value.

COMMON EXAM TRAPS

- The problem with a conglomerate is the lack of close industry peers. A factor-based peer group built with a multifactor model can still be formed.
- Factor-based peers usually come from multifactor models. Industry codes describe industry-based peers, and the single-factor CAPM captures beta only.
- A mispriced peer group is handled with historical multiples (the company's own or peer averages). Forecasts built on current peer prices carry the same mispricing.

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- A leading P/E is applied to next-period earnings (E_{n+1}). In the example, applying 10.0 to $E_5 = 4.00$ gives 40.00 instead of 41.20.

EXAM SHORTCUTS

- When next-period earnings are $E_n(1 + g)$, the justified trailing P/E applied to E_n gives the same terminal value as the justified leading P/E applied to E_{n+1}, so either pairing can be used.

BOTTOM LINE

- An industry-based peer group holds firms with the same principal business activity, and suitable peers are hard to find for unique firms, industries with members at widely different life cycle stages, firms in different tax or regulatory jurisdictions and conglomerates.
- A statistical factor-based peer group, usually formed with a multifactor model, holds firms with the same or close exposures to risk-return factors regardless of industry, but its relationships are estimated from past data and may not persist.
- Comparables can mislead when the whole peer group is overvalued or undervalued, and multiples built on current unadjusted figures can be distorted by accounting choices, nonrecurring items or cyclicality.
- Comparing a stock with its own past multiples or with peer averages over a whole business cycle reduces the risk of benchmarking against a currently mispriced peer group, though differences in growth and required return still have to be controlled for.
- A terminal value can be estimated as $TV_n = \text{justified leading P/E}_n \times E_{n+1}$, with the justified leading P/E equal to $\text{payout}/(r - g)$.

QUICK CHECK

Question 3

Which of the following is *most likely* a limitation of relying on an industry-based peer group?

A Industry peers are assumed to carry the same financial leverage.

B Some companies operate across several distinct industries and markets.

C The law of one price requires factor models for firms operating abroad.

Answer: B

Multi-industry companies are a recognized problem for industry-based peer groups. A conglomerate's mix of businesses gives it a blended risk and growth profile, and a single set of industry peers is unlikely to match that profile.

Why the other options are wrong

- **A.** Industry-based peer groups make no automatic assumption that all members have identical leverage. Analysts are told to check leverage differences among industry peers.
- **C.** The law of one price is the rationale for comparables. It does not require factor models for firms with international operations.

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Key takeaway Industry-peer problems: unique firms, mixed life cycle stages, different tax/regulatory jurisdictions, and multi-industry companies.

Practice questions

Question 4

Carys Morgan forecasts that Norrland Timber will earn $6.00 per share in Year 7. From then on she expects earnings and dividends to grow at 5% a year indefinitely, with 40% of earnings paid out, and she requires a 10% return. Using a justified leading P/E ratio, Norrland's terminal value per share at the end of Year 7 is *closest* to:

A $48.00.

B $50.40.

C $75.60.

Answers: 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/relative-value-equity-valuation-approaches/. The full bank has 34 questions on this reading.

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EQUITIES · READING 46

Financial Statement Forecasting in Equity Valuation

What you'll learn

LOS 46.a Explain why and how analysts forecast revenues, expenses, assets and financing needs in a disaggregated (financial statement) valuation model.

LOS 46.b Evaluate how a company's life-cycle stage (start-up, growth, mature, decline) shapes the construction of its equity valuation model.

LOS 46.c Estimate and interpret the value of an equity security from the outputs of a financial statement forecast model.

MODULE 46.1

Financial Statement Forecast Models

This reading explains how analysts build a disaggregated financial statement forecast from revenue, expense, asset and financing models, including top-down revenue growth and financing needs. It then matches the valuation approach to the company's life-cycle stage, with venture capital pre-money and post-money values, growth-stage terminal values and probability-weighted values in decline, and values equity from the model's FCFF and FCFE.

LOS 46.a — Why and how analysts build financial statement forecast models

Single-stage and multistage present value models compress a company's future into one or a few growth rates. A **disaggregated valuation model** instead forecasts full financial statements line by line. Revenues, expenses, assets and financing can then each follow their own assumptions, and the analyst can bring in industry projections, the competitive environment and company-specific information. The model improves the estimate of future cash flows. It does not change how the discount rate is estimated, and it still needs a terminal value.

A financial statement forecast has four building blocks: **revenue modeling**, **expense modeling** (or profit modeling), **asset modeling** and **financing modeling**.

Revenue modeling

KEY CONCEPT

Approach	Starting point	Best suited to	Typical inputs
Historical approach	The company's own past revenue growth (total or by operating segment)	Mature firms with long records and stable business models	Past growth rates, segment disclosures
Bottom-up approach	Company-specific business drivers	Firms whose revenue can be built from units, prices, capacity or outlets	Unit volumes × average selling prices; same-store sales growth plus planned new store openings; capacity (e.g., **available seat kilometers**) × utilization × price per unit of capacity

Approach	Starting point	Best suited to	Typical inputs
Top-down approach	A macroeconomic variable, usually expected **nominal GDP** growth	Firms whose sales track the economy or an industry	GDP growth and the expected relationship between it and company sales; industry sales growth and **market share**

Two common top-down calculations:

- Growth relative to GDP. When company revenue is expected to grow $x\%$ faster than nominal GDP, with the premium stated as a percentage of GDP growth, the company's growth rate is $g_{GDP}(1 + x)$. A premium stated in percentage points (for example, "2 percentage points above GDP growth") is added instead.
- Market growth and market share. Start from the forecast growth of industry sales, then apply the company's expected market share. Company revenue growth is market growth plus the proportional change in market share:

KEY CONCEPT

$$g_{revenue} = g_{market} + \left(\frac{\text{new share}}{\text{old share}} - 1\right)$$

Example. An industry is expected to grow 4%, and a firm's share rises from 20% to 22%. Forecast revenue growth is $4\% + (22/20 - 1) = 4\% + 10\% = 14\%$. If the firm's current sales are \$30 million, industry sales are $30/0.20 = \$150$ million now and $150 \times 1.04 = \$156$ million next year, and firm sales are $0.22 \times 156 = \$34.32$ million, growth of 14.4%.

Exam convention: the market growth and market share method adds market growth to the proportional change in share, which gives 14% here; use this figure when a question asks for the forecast growth. Current practice: computing next year's sales in levels gives the exact growth, 14.4% here. The gap is the product of the two changes ($4\% \times 10\% = 0.4\%$), so it matters only when both changes are large.

Expense (profit) modeling

- **Historical approach.** Each expense is linked to revenue using past data, often with a regression that separates fixed and variable components:

$$\Delta\text{Expense} = b_0 + b_1 \times \Delta\text{Revenue}$$

The intercept b_0 captures the fixed component and the slope b_1 the variable component. Regressions need a minimum sample size to be reliable, so this approach can be used only for established companies with enough historical data.

- **Itemizing expenses.** Economically significant line items are forecast one by one. This is the approach to use when new information must be reflected, such as a new wage agreement, new supplier terms or a new lease.
- **Direct profit margin approach.** A margin is assumed directly without disaggregating costs. It works well where COGS dominates total cost and moves with revenue:

$$\text{COGS} = \text{forecast revenue} \times (1 - \text{gross margin})$$

Margins are hard to forecast when there are one-off charges such as restructuring costs or impairments. Some items, such as interest expense and interest income, tend not to vary directly with revenues, so they are usually forecast separately.

Example. A regression gives $b_0 = 0.03$ and $b_1 = 0.70$ for the change in SG&A. If revenue grows 8%, SG&A grows $0.03 + 0.70 \times 0.08 = 8.6\%$. In a separate case, a firm has revenue of \$80 million, expected to grow 6%, and a 30% gross margin. Next year's revenue is 84.8, gross profit is $0.30 \times 84.8 = 25.44$, and COGS is $84.8 - 25.44 = \$59.36$ million.

Asset modeling

- A **constant growth rate model** assumes that assets and revenue grow at one common rate, so **total asset turnover** stays constant.
- Alternatively, individual asset categories are forecast separately, possibly with a regression like the one for expenses. Regressions are limited by the timing of capital expenditures, because fixed assets must be in place before they generate revenue.
- Only assets and liabilities with cash flow implications belong in the model. A **goodwill impairment** is a non-cash accounting charge and is left out. Purchases of equipment and changes in inventory involve cash and are included.

Financing modeling

KEY CONCEPT

$$\text{financing needs} = \Delta\text{operating assets} - \Delta\text{non-debt liabilities} - \Delta\text{retained earnings}$$

- **Operating assets** here means all assets other than cash.
- **Non-debt liabilities** are liabilities that carry no interest, for example accounts payable and accrued expenses.
- The change in retained earnings is net income minus dividends.

If Δoperating assets $-$ Δnon-debt liabilities is positive, the company has a funding need. It can be met from existing cash, retained earnings, borrowing or issuing stock. If the difference is negative, the company has a **surplus** that can fund dividends or share buybacks, reduce debt or add to cash. With retained earnings included in the formula, a positive result is the amount of new financing needed and a negative result is the size of the surplus.

The financing assumptions need reasonableness checks. The forecast debt ratio should stay within the company's normal range, so a sudden unexplained drop in leverage is a warning sign. Stock issuance or repurchases should be assumed only if the company's history supports them. Cash should be neither implausibly high nor below the company's minimum or target level.

Example. Operating assets rise \$50,000, non-debt liabilities rise \$18,000, net income is \$20,000 and dividends are \$5,000. Financing needs are $50{,}000 - 18{,}000 - (20{,}000 - 5{,}000) = \$17{,}000$.

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COMMON EXAM TRAPS

- Adding the change in market share in percentage points to market growth, instead of the proportional change in share. In the example, adding the 2-point share gain to 4% market growth gives 6% instead of 14%.
- Adding a premium to GDP growth when the premium is stated as a percentage of GDP growth.
- Reporting gross profit, or this year's COGS, when next year's COGS is asked; applying the gross margin itself as the COGS ratio. In the example, gross profit is $25.44 million and this year's COGS is $56.00 million, while next year's COGS is $59.36 million.
- In financing needs: using the level of retained earnings instead of the change, including the change in cash, or adding the change in non-debt liabilities. In the example, adding the $18,000 rise in non-debt liabilities gives $53,000 instead of $17,000.
- Confusing bottom-up (prices × volumes, outlets) with top-down (GDP, industry growth and share).
- Thinking a constant growth asset model makes asset turnover grow. Turnover stays constant.

EXAM SHORTCUTS

- The exam-convention market-share growth differs from the exact growth computed in levels only by the product of the two changes, so the two answers are close unless both changes are large.

BOTTOM LINE

- A disaggregated valuation model forecasts full financial statements through revenue, expense, asset and financing modeling; it improves the estimate of future cash flows but does not change how the discount rate is estimated and still needs a terminal value.
- Revenue can be forecast with the historical approach (the company's own past growth, for mature firms with stable business models), the bottom-up approach (company drivers such as units × prices, same-store sales plus new stores, or capacity × utilization × price) or the top-down approach (nominal GDP growth, or industry growth and market share).
- A revenue premium stated as a percentage of GDP growth gives growth of $g_{GDP}(1 + x)$, while a premium stated in percentage points is added to GDP growth.
- Exam convention: forecast revenue growth is market growth plus the proportional change in market share, $g_{market} + (\text{new share}/\text{old share} - 1)$; current practice: computing next year's sales in levels gives the exact growth.
- Expenses can be linked to revenue by a regression $\Delta\text{Expense} = b_0 + b_1 \times \Delta\text{Revenue}$ for established companies with enough data, itemized when new information must be reflected, or set by a direct margin, with $\text{COGS} = \text{forecast revenue} \times (1 - \text{gross margin})$.
- A constant growth asset model keeps total asset turnover constant, and financing needs equal the change in operating assets minus the change in non-debt liabilities minus the change in retained earnings, a positive result being new financing needed and a negative one a surplus.

QUICK CHECK

Question 1

Compared with a present value model that applies one or two growth rates to a single cash flow measure, the *main* benefit of a disaggregated valuation model built on forecast financial statements is that it:

A eliminates the need to estimate a terminal value.

B lets each statement component follow its own growth path.

C produces a more precise estimate of the discount rate.

Answer: B

A disaggregated model forecasts revenues, expenses, assets and financing separately, each with its own assumptions. That flexibility lets the analyst reflect industry projections, competition and company-specific information, which improves the cash flow forecast.

Why the other options are wrong

- **A.** A forecast model still covers only a finite horizon, so a terminal value is still needed to capture value beyond it.
- **C.** The model refines the cash flow estimates. The required return is estimated separately.

Key takeaway Disaggregated models improve the numerator (cash flows). They leave the denominator (discount rate) to be estimated separately, and they still need a terminal value.

MODULE 46.2

Models Based on Company Characteristics

LOS 46.b — Matching the valuation model to the company's life-cycle stage

The right model depends on where the company is in its life cycle.

KEY CONCEPT

Stage	Typical characteristics	Typical valuation approach
Start-up stage	Low or no revenue, negative earnings and cash flow, little or no debt (so equity value = firm value), little operating history, high chance of failure, funded by **venture capital (VC)**	A multiple of anticipated future sales (a **price-to-sales (P/S)** multiple), discounted with the VC investor's **ROI multiple**
Growth stage	Some historical data; cash flows fund growth objectives; sales scale up and **profit margins improve**	Forecast margins and growth; most value comes from the **terminal value**, usually an earnings (P/E) multiple on forecast net income

Stage	Typical characteristics	Typical valuation approach
Mature stage	**Stable growth in earnings and cash flow**, ample historical data	**Constant growth model** (e.g., Gordon growth DDM or constant growth FCFF/FCFE)
Decline stage	Declining profit margins, focus on returning capital, possible **financial distress**	Adjusted historical data, higher discount rates, **probability-weighted value** of going-concern and distress outcomes

Start-up stage: venture capital arithmetic

VC investors require very high returns because start-ups have a **high likelihood of business failure**. The required return is expressed as an **ROI multiple** over the investment horizon:

$$\text{ROI multiple} = (1+r)^n \quad \Longrightarrow \quad r = (\text{ROI multiple})^{1/n} - 1$$

where n = years to the VC investor's exit and r = the annual **required rate of return**. An ROI multiple of 3x means a total profit of 200%.

KEY CONCEPT

$$\text{post-money value} = \frac{\text{expected revenue} \times (P/S)}{\text{ROI multiple}}$$

$$\text{pre-money value} = \text{post-money value} - \text{VC investment}$$

$$f = \frac{\text{VC investment}}{\text{post-money value}}$$

The **post-money value** is the firm's value immediately after the VC investment, and the **pre-money value** is its value before the investment. f is the VC investor's **fractional ownership**.

Example. A start-up needs \$5 million. In six years it expects revenue of \$40 million and a P/S of 3x, and the VC investor requires a 10x ROI multiple. The post-money value is $40 \times 3/10 = \$12$ million, the pre-money value is $12 - 5 = \$7$ million, and $f = 5/12 = 41.7\%$. The implied annual return is $10^{1/6} - 1 = 46.8\%$.

Growth stage

Forecast revenue, costs and margins to the end of the high-growth period, estimate net income in that year, and apply an earnings multiple to get the **terminal value**. Discount it to today at the required return, or divide it by the target ROI multiple if the required return is stated as a multiple.

$$TV_n = NI_n \times P/E, \qquad V_0 = \frac{TV_n}{(1+r)^n} \text{ or } V_0 = \frac{TV_n}{\text{ROI multiple}}$$

With no debt, $NI = \text{revenue} \times \text{operating margin} \times (1 - t)$. If revenue and costs grow at different rates, grow each separately before computing profit.

Example. Year-5 revenue is \$25 million and the net margin 16%, so $NI_5 = \$4.0$ million. A P/E of 15 gives $TV_5 = \$60$ million, and at 12% $V_0 = 60/1.12^5 = \$34.05$ million.

Mature stage

Stable growth in earnings and cash flow makes a **constant growth model** appropriate: $V_0 = CF_1/(r - g)$.

Decline stage

Financial distress is the inability to meet financial obligations as they fall due. Analysts adjust historical data for the deteriorating business, and use **higher discount rates** because both debt and equity investors face more risk. Terminal values based on multiples of healthy, stable peers are likely to overvalue a declining firm. When the **going-concern assumption** is in doubt:

$$\text{value} = P(\text{going concern}) \times \text{going-concern value} + P(\text{distress}) \times \text{distress sale value}$$

When equity is being valued, the distress value is what shareholders recover: sale proceeds net of all creditor claims and liquidation costs. If distress means the assets pass to creditors, the equity's distress value is zero.

Example. Going-concern equity value is \$20 million. There is a 40% probability of distress, in which case shareholders would recover \$5 million after creditors are paid. Value $= 0.60 \times 20 + 0.40 \times 5 = \$14$ million.

Enterprise value and EV multiples

$$\text{EV} = \text{market value of common and preferred stock} + \text{market value of debt} - \text{cash and short-term investments}$$

Enterprise value (EV) multiples divide EV by an operating income measure such as **EBITDA** (or by revenue or FCFF). Operating income belongs to all capital providers and is unaffected by capital structure, and EBITDA is less often negative than net income. Common equity value can be backed out as EV less net debt (debt minus cash and short-term investments) and less preferred stock.

COMMON EXAM TRAPS

- Using revenue (instead of post-money value) as the denominator of fractional ownership; switching the P/S and ROI multiples; reporting post-money when pre-money is asked (or adding the investment). In the example, dividing the \$5 million by revenue gives 12.5% instead of 41.7%, and the post-money value of \$12 million is not the \$7 million pre-money value.
- Converting an ROI multiple to an annual return by simple division, or treating 3x as a 300% profit.
- Treating a target ROI multiple (e.g., 8x) as a percentage discount rate (8%).
- Forgetting to discount the terminal value, or discounting it for only one year. In the example, leaving the \$60 million undiscounted, or discounting it one year to \$53.57 million, overstates the \$34.05 million value.

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- In a probability-weighted value: ignoring the distress value, or swapping the probabilities. In the example, ignoring the distress value gives $12 million and swapping the probabilities gives $11 million instead of $14 million.

LOS 46.c — Valuing equity from forecast model outputs

Valuation tools from earlier readings are applied here to the outputs of a forecast model.

- **Free cash flow to the firm (FCFF):** $FCFF = NOPAT - \text{net investment}$, with $NOPAT = EBIT(1 - t)$. Firm value $= FCFF_1/(WACC - g)$; equity value = firm value − market value of debt.
- **Free cash flow to equity (FCFE):** $FCFE = NI - \text{investment in net long-term assets} - \text{increase in working capital} + \text{net borrowing}$. Equity value $= FCFE_1/(r - g)$; divide by shares outstanding and compare with the market price. Intrinsic value below the price means the stock is overvalued.
- **Sustainable growth rate based on FCFF:**

KEY CONCEPT

$$SGR_{FCFF} = \left(1 - \frac{FCFF}{NOPAT}\right)\left(\frac{NOPAT}{\text{beginning book value of total capital}}\right)$$

where the **beginning book value of total capital** = book value of debt + book value of equity. This gives the **sustainable growth rate** of FCFF. The first term is the reinvestment rate and the second is the return on capital.

- **Reading the assumptions.** If revenue is forecast to grow faster than expenses, margins are expected to rise. If most operating expenses are forecast to grow faster than revenue, the **operating profit margin** is expected to fall; the effect on gross and net margins needs more information. A forecast rise in **interest expense** signals more borrowing, so the funding shortfall is at least partly financed with debt.
- Revenue inputs often come from the top-down calculations of LOS 46.a.

Example. EBIT is $90 million and the tax rate 25%, so NOPAT $= 67.5$. Net investment is 22.5, so FCFF $= 45$. With a beginning book value of capital of 750, $SGR = (1 - 45/67.5)(67.5/750) = 0.333 \times 0.09 = 3.0\%$. If next year's FCFF is 45, WACC 8.5% and $g = 3\%$, firm value $= 45/0.055 = 818.2$; with debt of 200, equity $= 618.2$.

COMMON EXAM TRAPS

- Forgetting to subtract debt from firm value, or valuing FCFF with an extra year of growth when it is already a next-year forecast. In the example, reporting firm value gives 818.2 instead of equity of 618.2, and growing the 45 one more year gives equity of 642.7.
- Using $FCFF/NOPAT$ instead of $1 - FCFF/NOPAT$ as the reinvestment rate, or dividing NOPAT by the market value of debt instead of by total book capital. In the example, 0.667×0.09 gives 6.0% instead of 3.0%.
- In FCFE, forgetting (or adding back) the increase in working capital, or ignoring net borrowing.

EXAM SHORTCUTS

- Read margins from the growth assumptions: revenue forecast to grow faster than expenses means rising margins, and most operating expenses growing faster than revenue means a falling operating profit margin, though the effect on gross and net margins needs more information.

BOTTOM LINE

- A start-up is valued with a P/S multiple of anticipated sales discounted by the VC investor's ROI multiple, a growth-stage firm mainly through a terminal value from a P/E on forecast net income, a mature firm with a constant growth model, and a declining firm with adjusted historical data, higher discount rates and a probability-weighted value.
- $\text{ROI multiple} = (1 + r)^n$, post-money value is expected revenue × P/S divided by the ROI multiple, pre-money value is post-money value minus the VC investment, and the VC's fractional ownership is the investment divided by post-money value.
- A growth-stage terminal value is $TV_n = NI_n \times P/E$, discounted at r for n years or divided by the target ROI multiple when the required return is stated as a multiple.
- When the going-concern assumption is in doubt, value is the probability-weighted average of the going-concern value and the distress sale value, where the equity's distress value is what shareholders recover after creditor claims and liquidation costs.
- From a forecast model, $FCFF = NOPAT - \text{net investment}$ gives firm value $FCFF_1/(WACC - g)$, and equity value is firm value minus the market value of debt.
- The sustainable growth rate of FCFF is the reinvestment rate $1 - FCFF/NOPAT$ times the return on capital, NOPAT divided by the beginning book value of total capital (debt plus equity).

QUICK CHECK

Question 2

A venture capital fund targets an ROI multiple of 5x on a start-up investment that it expects to exit after six years. The annual required rate of return implied by this target is *closest* to:

A 30.8%.

B 34.8%.

C 66.7%.

Answer: A

The ROI multiple compounds the annual required return over the investment horizon: ROI multiple $= (1 + r)^n$. Solve for r.

$$5 = (1 + r)^6 \;\Rightarrow\; r = 5^{1/6} - 1 = 1.3077 - 1 = 30.8\%$$

Calculator: 5 [y^x] 6 [1/x] [=] gives 1.3077.

Why the other options are wrong

- **B.** 34.8% treats a 5x multiple as a 500% profit, i.e., a multiple of 6x: $6^{1/6} - 1$. A 5x multiple is a 400% profit.
- **C.** 66.7% divides the 400% total profit by six years ($(5-1)/6$), ignoring compounding.

Key takeaway $r = (\text{ROI multiple})^{1/n} - 1$. A multiple of M means a total profit of $M - 1$.

Practice questions

Question 3

Tomasz Wilk is building a forecast for Halvorsen Aluminum. Halvorsen has just signed a four-year energy supply contract that fixes the electricity price for its smelters at a level well above what it paid in the past. Wilk wants his expense forecast to reflect the new contract terms. Which approach to forecasting expenses is *most* appropriate?

A Itemizing expenses.

B A historical approach.

C A direct profit margin approach.

Question 4

An investor estimates that Wexcombe Textiles has an 85% probability of surviving as a going concern, in which case its equity would be worth $46 million. If it does not survive, the investor expects a distress sale to leave shareholders $10 million after all creditor claims and sale costs are paid. The value of Wexcombe's equity is *closest* to:

A $46.0 million.

B $40.6 million.

C $15.4 million.

Question 5

An analyst's forecast for Carrow Retail assumes that revenue will grow 3% a year, while most of Carrow's operating expenses, including SG&A and depreciation, will grow 5% to 6% a year. Based only on this information, which of the following is *most likely*?

A The gross profit margin is expected to remain steady.

B The net profit margin is expected to increase.

C The operating profit margin is expected to decrease.

Answers: 3. A · 4. B · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/financial-statement-forecasting-in-equity-valuation/. The full bank has 54 questions on this reading.

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EQUITIES · READING 47

Industry and Competitive Analysis

What you'll learn

LOS 47.a Describe the five steps of industry and competitive analysis: define, survey, analyze structure, examine external influences, analyze competitive strategies.

LOS 47.b Describe commercial classification systems (GICS, ICB, TRBC) and their limitations, and grouping by geography and business cycle sensitivity (defensive, cyclical, growth).

LOS 47.c Estimate industry size (top-down vs. bottom-up), judge growth, measure profitability (operating margin, ROIC) and market share and concentration (HHI).

LOS 47.d Analyze industry structure with Porter's five forces and external influences with PESTLE, including the innovation matrix.

MODULE 47.1

Industry Analysis Framework

This reading sets out the five steps of industry and competitive analysis and the ways companies are grouped into industries. It shows how to survey an industry's size, growth, profitability (operating margin and ROIC) and concentration (market share and the HHI), and how to analyze industry structure with Porter's five forces and external influences with PESTLE and the innovation matrix.

LOS 47.a — The five steps of industry and competitive analysis

Industry and competitive analysis studies the environment a company operates in. For this purpose an **industry** is a group of firms offering comparable products or services. Understanding what drives an industry's size and profits lets the analyst justify the inputs of a valuation model and explain the narrative behind them. Being able to do so also helps show that a recommendation has a reasonable basis.

KEY CONCEPT

Step	What the analyst does	Typical content
1. **Define the industry**	Set the scope of the industry	Which products and which geographic regions belong; how to treat companies active in several industries; third-party classification systems
2. **Survey the industry**	Measure the industry	Size, historical growth and growth drivers, profitability, main participants and **market share** trends
3. **Analyze the industry structure**	Judge how competitive the industry is	**Porter's five forces**, giving the degree of competitive rivalry and the key drivers of profitability
4. **Examine external influences**	Look at forces outside the industry	**PESTLE** framework: political, economic, social, technological, legal, environmental

Step	What the analyst does	Typical content
5. **Analyze competitive strategies**	Look at each company in the industry	How each firm positions itself and what competitive advantages it has

COMMON EXAM TRAPS

- PESTLE belongs to step 4 (external influences); Porter's five forces belongs to step 3 (industry structure).
- Deciding which regions or products fall inside the industry is part of defining it. Measuring past growth rates is part of surveying it.
- Reviewing a company's financial statements and estimating its intrinsic value are company analysis and valuation tasks. They are not among the five steps.

LOS 47.b — Grouping companies into industries

Defining the industry comes first. Firms in one industry face the same competitive forces and value drivers, so the classification tells the analyst where to find comparable companies.

Commercial (third-party) classification systems

Global providers classify companies by the **products and services** they offer (their principal business activity) into hierarchical taxonomies. The company sits in the lowest tier; higher tiers aggregate groups.

System	Developer	Coverage	Tiers (broad → narrow)
Global Industry Classification Standard (GICS)	S&P Dow Jones Indices and MSCI	Public companies	Sector → industry group → industry → subindustry
Industry Classification Benchmark (ICB)	FTSE Russell	Public companies	Industry → supersector → sector → subsector
The Refinitiv Business Classification (TRBC)	Refinitiv	Public and private companies, not-for-profits, government entities	Economic sector → business sector → industry group → industry → activity

Regional systems include **NAICS** (North America) and **NACE** (Europe). Providers review classifications at least once a year. Data vendors (Bloomberg, FactSet, S&P Capital IQ and others) and exchanges such as Nasdaq either adopt one of these systems or build a similar taxonomy of their own.

The main advantage of a commercial system is consistency. Its groups can still be too broad or too narrow, so the analyst has to apply judgment. Specific limitations:

- groups may contain companies with different business models or with substitute products;
- a company operating in several industries is often placed according to the product that brings in the most revenue, which can hide a significant other business, and the systems do not all follow the same rules, so the same company can land in different groups;

- industries that serve only local or national markets (e.g., insurance, health care) do not fit global taxonomies well;
- groupings change over time through reclassifications, new entrants and delistings, so historical data may need restating.

Other ways to group companies

- **Geography**, usually the country of domicile or listing (developed, emerging, frontier). It may not match where the revenue is earned or where the assets are.
- **Business cycle sensitivity**:

KEY CONCEPT

Type	Demand / earnings pattern	Typical sectors
Defensive	Relatively stable demand through expansions and contractions	Health care, utilities, telecommunications, consumer staples
Cyclical	Earnings depend heavily on the stage of the business cycle; volatile earnings, usually high operating leverage	Basic materials, consumer discretionary, energy, financial services, industrials and producer durables
Growth	Strong revenue growth through all phases of the cycle; may or may not be profitable yet, depending on its life-cycle phase	Emerging technology, biotechnology

Cyclical products tend to have long lives or high unit values, so purchases can be postponed until the economy improves (cars, appliances, equipment). Cyclical firms also buy inputs, such as commodities, whose demand moves with the cycle, and the combination of swinging revenue and high fixed costs makes their operating profit volatile. For a growth company the analyst estimates how long above-trend growth can last and what profit levels will look like once growth levels off. Geography and cycle sensitivity can be combined when countries are at different points in their cycles.

COMMON EXAM TRAPS

- Commercial systems group companies by products and services. Geography and business cycle sensitivity are other ways analysts can group companies.
- "Cyclical versus defensive" describes how sensitive earnings and demand are to the business cycle. Stock price volatility is a different matter.
- A growth company's revenue growth is largely independent of the cycle, which distinguishes it from a cyclical company in an expansion.

LOS 47.c — Surveying the industry: size, growth, profitability, market share

Industry size

Industry size means one year's total sales of the product or service being analyzed. It can differ from the total sales of the companies in the industry. For a multi-product company, only the relevant part of its sales counts.

Approach	How it works	Advantage	Limitations
Top-down approach	Estimates total industry revenue from government agency or third-party data	Works for industries with many private or small firms that do not publish results	Data may not be granular enough for niche analysis
Bottom-up approach	Adds up the revenues of each company that produces the good or service	Direct and precise when large listed companies dominate	Hard to isolate revenue of multi-product companies; inconsistent segment reporting; poor fit when many firms are private

Growth

Compare industry growth with the economy (life-cycle stage) and across the business cycle (sensitivity).

- **Growth industries** typically grow faster than the economy whatever the phase of the cycle, because they have not yet reached their total addressable market. Estimating when their growth will peak is harder when it rests on an emerging technology.
- **Mature industries** have little or no growth potential left. All industries together grow at the economy's rate, so mature industries grow near or below that rate. Analysts watch for disruptors and signs of decline.
- Industries sensitive to interest rates, discretionary spending or capital goods have more volatile revenue over the cycle; structural shocks (e.g., the pandemic) can also shift demand.

Profitability

$$\text{operating profit margin} = \frac{\text{operating income}}{\text{total revenue}}$$

$$\text{ROIC} = \frac{\text{NOPAT}_t}{\text{average invested capital}}$$

where **NOPAT** = net operating profit after taxes and average invested capital = average of long-term liabilities plus equity over the period. **Return on invested capital (ROIC)** reflects returns to all capital providers and is compared with the **weighted average cost of capital (WACC)**. ROIC above WACC means value is being created. For private companies, analysts use listed peers' returns or government/third-party data.

> *Example.* NOPAT $= \$84$ million and average invested capital $= \$700$ million give ROIC $= 84/700 = 12\%$, above a 9% WACC.

Market share and concentration

$$\text{market share} = \frac{\text{company sales}}{\text{industry size}}$$

Because industry size is hard to pin down, market share is best expressed as a range. Rising share suggests customers favor the firm's products, and a trend of falling shares among incumbents can reveal competition from new entrants. Consolidation can lift a company's share, so the analyst checks share **net of acquisitions**.

The **Herfindahl-Hirschman Index (HHI)** is the sum of the squared market shares (in percent) of all participants:

KEY CONCEPT

$$HHI = \sum_{i=1}^{N} (\text{market share}_i)^2$$

Example. Shares of 40%, 30%, 20% and 10% give $HHI = 1{,}600 + 900 + 400 + 100 = 3{,}000$. The maximum is $100^2 = 10{,}000$ (a monopoly). An HHI above 1,800 is often read as high concentration. Unless the industry is segmented or highly differentiated, higher concentration means less competition, so firms tend to have more pricing power and higher profitability.

COMMON EXAM TRAPS

- Square the shares in percent (e.g., $40^2 = 1{,}600$); using decimals gives a number between 0 and 1. In the example, decimal shares give 0.30 instead of 3,000.
- The top-down approach suits many-private-firm industries; its weakness is lack of granularity. The bottom-up approach sums company revenues; its weakness is separating multi-product revenue.
- ROIC uses after-tax operating profit over average invested capital (long-term liabilities plus equity). It measures the return to all capital providers, which ROE and gross margin do not.

BOTTOM LINE

- Industry and competitive analysis has five steps: define the industry, survey it, analyze its structure with Porter's five forces, examine external influences with PESTLE, and analyze the competitive strategies of its companies.
- Commercial classification systems such as GICS, ICB and TRBC group companies by the products and services they offer into hierarchical tiers; they are consistent, but groups can be too broad or too narrow, and a company in several industries is often placed by the product that earns the most revenue.
- Defensive industries have relatively stable demand through expansions and contractions, cyclical industries have earnings that depend heavily on the stage of the cycle and usually high operating leverage, and growth industries grow revenue strongly through all phases of the cycle.
- Industry size is one year's total sales of the product or service; the top-down approach uses government or third-party data and suits industries with many private or small firms, while the bottom-up approach adds up company revenues and works best when large listed companies dominate.
- ROIC is NOPAT divided by average invested capital (long-term liabilities plus equity), a return to all capital providers, and ROIC above WACC means value is being created.
- The HHI is the sum of squared market shares in percent, with a maximum of 10,000 and a reading above 1,800 often taken as high concentration; unless the industry is segmented or highly differentiated, higher concentration means less competition and more pricing power.

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QUICK CHECK

Question 1

Kalinda Osei is studying how carbon-pricing rules, changing attitudes toward home exercise, and higher interest rates could affect the fitness-equipment industry. She organizes these factors using the PESTLE framework. In the industry and competitive analysis process, this work is best described as:

A examining external influences.

B analyzing the industry structure.

C defining the industry.

Answer: A

The PESTLE framework groups the political, economic, social, technological, legal and environmental factors that act on an industry from outside. Using it is the fourth step of industry and competitive analysis: examining external influences.

Why the other options are wrong

- **B.** Analyzing the industry structure (step 3) uses a framework such as Porter's five forces to judge competitive rivalry and profitability drivers. PESTLE belongs to a later step.
- **C.** Defining the industry (step 1) sets its scope: which products, regions and companies belong in it.

Key takeaway Porter's five forces serve step 3 (industry structure); PESTLE serves step 4 (external influences).

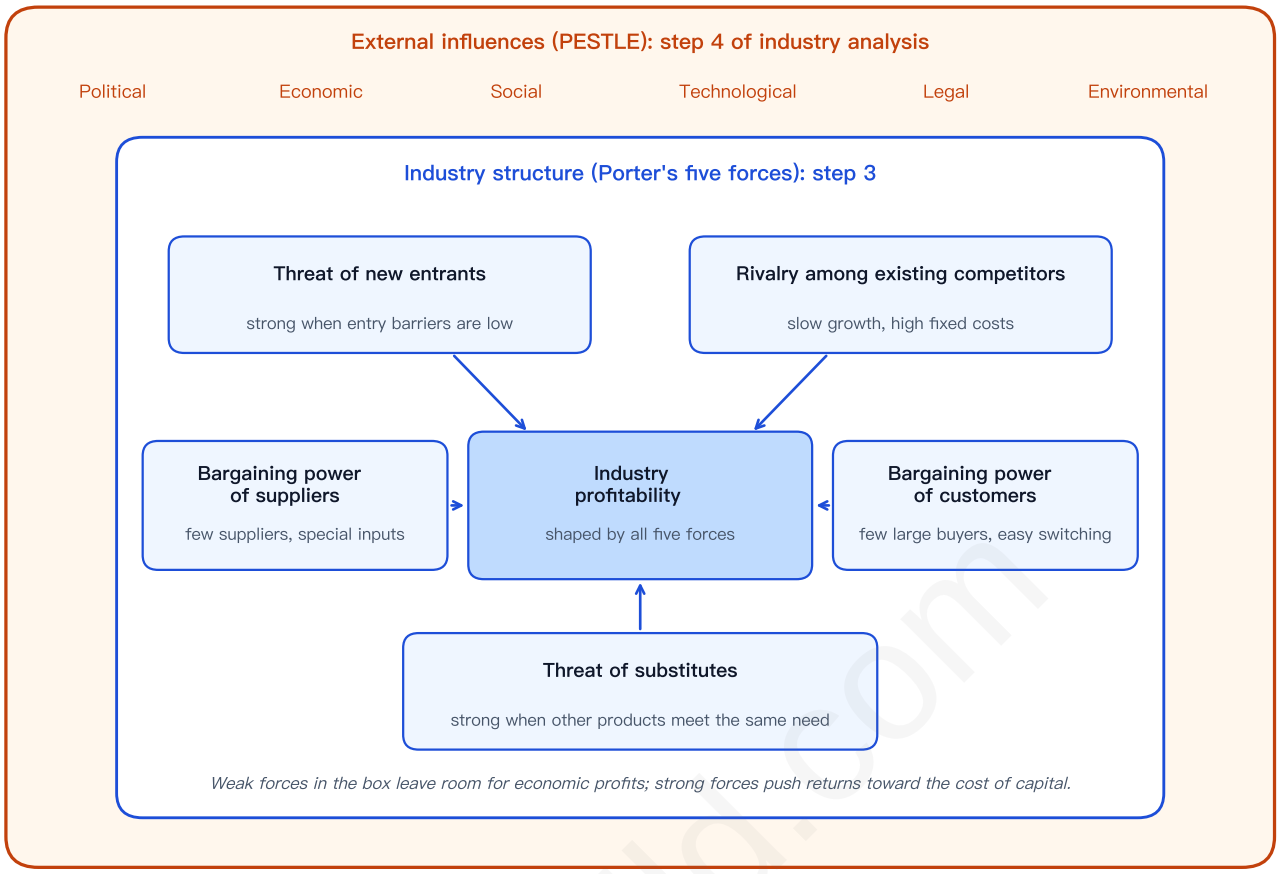
MODULE 47.2

Porter's Five Forces and PESTLE Analysis

LOS 47.d — Industry structure (Porter's five forces) and external influences (PESTLE)

Porter's five forces

The two frameworks of this LOS fit together. The five forces describe the industry's structure: the pressures from rivals, would-be entrants, substitutes, suppliers and customers. PESTLE collects the influences that act on the whole industry from outside.



Porter's five forces inside the industry, PESTLE influences around it

Michael Porter's framework explains the competitive forces that determine an industry's long-run profitability. When the forces are strong, industry returns on capital are unlikely to exceed the cost of capital. An **economic moat** is a durable competitive advantage that protects profits from competitors. Its sources include cost advantages, efficient scale, proprietary intangible assets, network effects and high **switching costs**. The longer the advantage lasts, the wider the moat and the longer profits stay above the cost of capital. Where only a handful of firms compete (an oligopoly, for example), wide moats are typical. Where competition is intense, few firms have one, and most earn less than their cost of capital.

KEY CONCEPT

Force	Stronger (worse for industry profits) when...
Threat of new entrants	Barriers to entry are low: little capital needed, no scale advantages, no licensing or patent protection, weak customer loyalty, low switching costs
Threat of substitutes	Products from other industries meet the same need; this raises the elasticity of demand and caps prices. Differentiated products with high switching costs face a low threat
Bargaining power of suppliers	Few suppliers, scarce or customized inputs, high switching costs, proprietary technology or organized/licensed labor. Weak when many small suppliers sell commodity inputs

Force	Stronger (worse for industry profits) when…
Bargaining power of customers	Buyers are large and concentrated, products are commodity-like, switching costs are low
Rivalry among existing competitors	Slow growth, high fixed costs (firms cut prices to run at full capacity), difficulty exiting or cutting capacity, low concentration (many similar-sized firms), little differentiation

When judging the threat of entry, the analyst looks at both how many firms have entered over time and how much they affected the industry. Ways incumbents defend themselves: partnering with or acquiring entrants, lobbying for protective regulation, subsidies or restrictive licensing, patents, building customer loyalty, raising switching costs (price, opportunity cost, transition risk and lost upgrades from the current provider), network effects, and cost advantages from scale or **economies of scope** (producing a combination of goods and services costs less than producing each separately). In mature industries selling commoditized products with inelastic demand, firms must operate at efficient scale; because a newcomer would push returns below the cost of capital, that scale requirement itself deters entry. To counter supplier or customer power, a firm may pursue **vertical integration**, either **backward integration** (upstream, toward suppliers) or **forward integration** (downstream, toward customers). It may also use joint ventures or exclusive supply agreements, or cut out intermediaries. Against strong customers, firms reward loyalty, bundle services or differentiate their products.

Economic profits are most likely when all the forces are weak: high barriers to entry, few substitutes, weak supplier and customer power, and mild rivalry.

Example. A regional cement industry has high entry barriers (plants are costly, transport limits imports), few substitutes, and powerful energy suppliers; rivalry is intense in downturns because plants have high fixed costs and are hard to close. Profits are therefore strongly cyclical despite the entry barriers.

PESTLE — external influences

KEY CONCEPT

Influence	Examples
Political	Taxes, subsidies, reimbursements; indirect channels (changes in monetary policy, regulation, geopolitics); direct channels (government purchases). Energy, health care and defense are highly exposed
Economic	Cyclical trends in output; structural factors (competitiveness, stage of development); exchange rates (a weaker currency helps exporters, a stronger one makes imports cheaper). Industry forecasts should be consistent with the economic forecast, adjusted for the industry's position in the cycle
Social	How people work, spend and live, which matters most for industries selling to individuals; shift to branded goods as incomes rise; demand for ethical sourcing
Technological	New products that make old ones redundant; drives total factor productivity
Legal	New laws and regulations creating risks (e.g., tobacco restrictions) or opportunities (newly legalized products)

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Influence	Examples
Environmental	Climate change and sustainability, often linked to political, social and legal influences; the **energy transition**, a government-driven structural shift toward lower greenhouse gas emissions through policy, taxes and subsidies

The innovation matrix

Innovations are classified by their **domain** (the products, segments or processes affected) and the **problem** they solve:

KEY CONCEPT

	Indistinct domain	Distinct domain
Distinct problem	**Breakthrough innovation** — a new product, service or business model with a large impact across many industries (e.g., a new battery technology)	**Sustaining innovation** — improves existing products for existing customers (feature enhancements)
Indistinct problem	**Basic research** — deepens scientific understanding with no specific commercial objective	**Disruptive innovation** — creates new markets or redefines existing ones (e.g., streaming replacing cable)

When a market is disrupted, incumbents either change their business model or lose market share.

COMMON EXAM TRAPS

- The five forces are rivalry among existing competitors, the threat of new entrants, the threat of substitutes, and the bargaining power of suppliers and of customers. "Power of regulators" and "rivalry among suppliers" are distractors; regulation is a PESTLE item.
- Specialized inputs give suppliers power; concentrated buyers give customers power.
- Monetary policy and regulation are indirect political influences in PESTLE.
- Sustaining innovation is a better version of an existing product. Breakthrough innovation (distinct problem, indistinct domain) has a large impact across many industries. Disruptive innovation (indistinct problem, distinct domain) creates new markets or redefines existing ones within its domain.

BOTTOM LINE

- Porter's five forces are the threat of new entrants, the threat of substitutes, the bargaining power of suppliers, the bargaining power of customers and rivalry among existing competitors; when they are strong, industry returns on capital are unlikely to exceed the cost of capital.
- An economic moat is a durable competitive advantage, from sources such as cost advantages, efficient scale, proprietary intangible assets, network effects and high switching costs, and the longer it lasts the longer profits stay above the cost of capital.
- The threat of entry is stronger when barriers are low, and rivalry is stronger with slow growth, high fixed costs, difficulty exiting, many similar-sized firms and little differentiation.

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- Suppliers have power when they are few, when inputs are scarce or customized, or when they hold proprietary technology, customers have power when they are large and concentrated and products are commodity-like, and firms may respond with backward or forward vertical integration.
- PESTLE covers political, economic, social, technological, legal and environmental influences on an industry, and monetary policy and regulation are indirect political channels.
- In the innovation matrix, breakthrough innovation solves a distinct problem in an indistinct domain, sustaining innovation a distinct problem in a distinct domain, disruptive innovation an indistinct problem in a distinct domain, and basic research has no specific commercial objective.

QUICK CHECK

Question 2

Fennick Labs is exploring home diagnostics within the clearly defined medical-testing domain. The exact customer problem is not yet distinct, but its new testing model could create a new market or redefine the existing diagnostics market. This strategy is most characteristic of:

A disruptive innovation.

B breakthrough innovation.

C sustaining innovation.

Answer: A

Disruptive innovation addresses an indistinct problem within a distinct domain and creates a new market or redefines an existing one. The domain here is medical testing, the exact problem is not yet distinct, and the model could reshape that market.

Why the other options are wrong

- **B.** Breakthrough innovation addresses a distinct problem across an indistinct domain and can affect many industries. This scenario stays within the distinct domain of medical testing.
- **C.** Sustaining innovation addresses a distinct problem within a distinct domain and improves existing products without redefining the market. Here the problem is indistinct and the market could be redefined.

Key takeaway Disruptive innovation = indistinct problem, distinct domain, and creation or redefinition of a market.

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Practice questions

Question 3

Rosalind Achebe wants to size the market for gluten-free infant cereal. She plans to use a national statistics agency's revenue figures for the packaged-food industry. The main drawback of this approach is *most likely* that:

A it is unsuitable when an industry contains many private firms.

B the data may not be detailed enough for a narrow product niche.

C revenue is hard to isolate for companies with several product lines.

Question 4

The residential elevator industry in a country has five manufacturers with market shares of 50%, 20%, 15%, 10% and 5%. Based on its Herfindahl-Hirschman Index (HHI), the industry's concentration is best described as:

A low.

B moderate.

C high.

Answers: 3. B · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/industry-and-competitive-analysis/. The full bank has 34 questions on this reading.

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EQUITIES · READING 48

Company Analysis: Past, Present, and Future

What you'll learn

LOS 48.a Explain a company's competitive strategy and industry position: business model, intentional vs unintentional strategy, life-cycle stages, market potential (TAM, serviceable, obtainable) and Porter's generic strategies.

LOS 48.b Evaluate revenue drivers (bottom-up vs top-down), operating profitability (EBITDA, EBIT, net income), pricing power and operating leverage.

LOS 48.c Evaluate working capital (cash conversion cycle), capital investments (net capex, ROIC, FCFE) and capital structure (debt vs equity, degree of financial leverage).

MODULE 48.1

Corporate Strategy and Industry Position

This reading moves from industry analysis to the individual company: its business model, competitive strategy, life-cycle stage, market potential and Porter's generic strategies. It then links strategy to the financial statements through revenue drivers, operating margins, pricing power and operating leverage, and through FCFE, the cash conversion cycle, net capital expenditure, ROIC and financial leverage.

LOS 48.a — Competitive strategy and position within an industry

Where company analysis fits

Industry and competitive analysis ends with a look at the individual company. After studying the industry (for example, with **Porter's five forces** and a **PESTLE analysis**), the analyst turns to the firm itself and evaluates its **competitive strategy**: its **competitive advantage**, its **position within its industry**, and the **resources and capabilities** it brings. The goal is to find what drives the company's value creation and then to check those drivers against its financial statements.

Business model versus competitive strategy

- A **business model** explains how a company defines and creates value: whom it serves (existing and potential customers), what it sells (products and services), how it prioritizes resources (key assets, supplier network) and how it protects itself from competitors.
- A **competitive strategy** is one part of the business model: how the firm wins against rivals. Every company has one, whether it chose it deliberately or not.

KEY CONCEPT

	Intentional competitive strategy	Unintentional competitive strategy
Approach	Proactive	Reactive
Direction	Clear, measurable goals, shared with staff and investors	Little clear direction; managers follow their own agendas, keep the status quo or copy industry norms

	Intentional competitive strategy	Unintentional competitive strategy
Process	Performance is monitored; the model is refined through repeated execute-and-evaluate cycles	No formal process for improvement
Long-term value creation	More likely	Less likely

Good strategies are easiest to spot in hindsight, once they have produced consistent positive economic profits. To judge a strategy looking forward, the analyst asks three questions:

1. Can the strategy respond to the threats found with Porter's five forces?
2. Is it neutral to, or helped by, the external influences in the PESTLE analysis?
3. Can the company actually execute it, given its resources, its stage of development and the industry's stage?

Strategy and the company life cycle

KEY CONCEPT

Stage	Revenue and cash flow	Primary strategic focus	Key challenges
Start-up	Little or no revenue; negative cash flow	Innovative product and business-model development; forming partnerships; raising capital for research	Few assets; no operating history; vulnerable to setbacks
Growth	Strong revenue growth; cash flow turns from negative to positive but stays below revenue because of reinvestment	Growth through sales innovation; identifying demand drivers	Operational constraints; regulatory hurdles; competitive challenges
Mature (established)	Revenue growth slows, then plateaus; cash flow rises more slowly or flattens	Efficiency; defending market share; product evolution	Market saturation; risk of a "stuck in the middle" strategy
Decline	Revenue falls or grows more slowly than nominal GDP or rivals; cash flow flattens, then falls	Cost management; restructuring; reinvention; downsizing or exiting	Lower demand; new substitutes; excess industry capacity; technological obsolescence

Changes in consumer preferences and technology breakthroughs can lift a start-up or growth firm to above-trend growth, but the same forces can push a firm that loses market share into decline.

Market potential for start-up and growth companies

For young firms entering established markets, the analyst estimates how big the opportunity is, product by product:

- The **total addressable market (TAM)** is the largest possible market for the goods or services.
- The **serviceable market** is the part of the TAM the company can reach, given its focus, technology or geographic footprint.
- The **obtainable market** is the share of the serviceable market that the company can realistically win given its current scale and strategy, for example as limited by production capacity.

KEY CONCEPT

$$\text{Serviceable market} = \text{TAM} \times \text{technical fit (\%)} \times \text{geographic reach (\%)}$$

Example. A maker of hospital sterilization robots faces a TAM of $6.0 billion a year. Its machines meet the safety certification used in 40% of that market, and it is licensed in countries representing 75% of the certified segment. Its serviceable market is $6.0 \times 0.40 \times 0.75 = \$1.8$ billion. If its factory can support only $0.5 billion of sales, its obtainable market is $0.5 billion.

Porter's generic competitive strategies (mature companies)

Michael Porter argued that a firm must pick one of three strategies. A firm with no clear advantage is **"stuck in the middle"**.

- **Cost leadership** (low-cost strategy): the lowest production costs in the industry, the lowest prices, and enough volume to earn a superior return. Common in mature or commoditized industries. It may serve a defensive purpose (holding on to market share) or an offensive one (e.g., aggressive price cuts to win share). Customers focus on cost; managerial incentives reward operating efficiency (scale, utilization, vertical integration).
- **Differentiation strategy**: products or services that are distinctive in type, quality or positioning and earn a price premium. It must create an **economic moat** (patents, brand, customer engagement) that rivals find hard to copy.
- **Focus strategy**: targeting a **niche**, such as a narrow group of customers, a region or a product stage. It is a hybrid that can combine elements of both cost leadership and differentiation.

	Cost leadership	Differentiation	Focus
Typical forms	Economies of scale and scope; supply-chain access; cost controls; aggressive pricing; low-cost distribution	Quality/features; brand and advertising; service and distribution; intellectual property; customer experience; premium pricing; bundling	Cost or differentiation elements aimed at one group or product
Five forces it counters	Threat of new entrants; customer bargaining power; industry rivalry	Threat of new entrants; threat of substitutes; customer bargaining power; industry rivalry	Threat of new entrants; threat of substitutes; customer bargaining power

	Cost leadership	Differentiation	Focus
Industry fit	Capital intensity; price-conscious customers; little product differentiation; limited industry innovation	Less price-conscious customers; customers who value product uniqueness; industry innovation with changing product features	Rivals find it hard to serve the specific customers, regions or products
Risks	Cost inflation, lack of discipline; technological change affecting costs or market share; customers wanting higher quality (premiumization)	Imitation; more sophisticated buyers; an excessive price premium; exclusivity that prevents market share growth	Large competitors outcompeting on price; narrowing differences among groups; more sophisticated buyers

COMMON EXAM TRAPS

- The business model is the broad description (customers, products, resources, suppliers). The competitive strategy is only the part about beating rivals.
- Keeping the status quo, following industry norms and managers pursuing their own incentives describe an **unintentional** strategy.
- Raising capital and forming partnerships belong to the start-up stage, defending market share to the mature stage, and cost management and restructuring to the decline stage.
- Cash flow may still be negative in the growth stage, but a growth firm has strong revenue growth; a start-up has little or no revenue.
- A focus strategy targets a narrow customer group and may mix cost and differentiation features. "Customers willing to pay more for perceived value" points to differentiation.
- Price-conscious customers plus slow innovation point to cost leadership; customers who value uniqueness point to differentiation.
- Threat of entry and power of buyers are industry-level forces; after industry analysis, the next company-level item is the competitive strategy.

EXAM SHORTCUTS

- Read the strategy from the customer and industry clues: price-conscious customers with little product differentiation and limited innovation point to cost leadership, customers who value uniqueness point to differentiation, and a narrow customer group, region or product points to focus.

BOTTOM LINE

- A business model explains how a company defines and creates value (its customers, products and services, key resources and protection from rivals), while the competitive strategy is only the part about how it wins against rivals.
- An intentional competitive strategy is proactive, with clear measurable goals and monitored performance, and is more likely to create long-term value than an unintentional one, which is reactive and may simply keep the status quo or copy industry norms.

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- Looking forward, a strategy is judged by whether it can respond to the threats found with Porter's five forces, whether it is neutral to or helped by the PESTLE influences, and whether the company can execute it.
- A start-up focuses on product and business-model development, partnerships and raising capital, a growth company on sales growth, a mature company on efficiency and defending market share, and a declining company on cost management, restructuring, reinvention or exit.
- The serviceable market is the part of the total addressable market the company can reach, $\text{TAM} \times \text{technical fit} \times \text{geographic reach}$, and the obtainable market is the share of it the company can realistically win given its scale and strategy.
- Porter's generic strategies are cost leadership (lowest costs and prices with enough volume), differentiation (distinctive products earning a price premium behind an economic moat) and focus (a niche, possibly mixing both), and a firm with no clear advantage is stuck in the middle.

QUICK CHECK

Question 1

Having finished her assessment of the competitive forces in the packaged-foods industry, an analyst now turns to one particular company in that industry. Which of the following is she *most likely* to examine next?

A The threat of substitute products.

B The bargaining power of suppliers.

C The company's competitive strategy.

Answer: C

The last step of industry and competitive analysis is to evaluate the individual company's competitive strategy: its competitive advantage, position within its industry, and resources and capabilities. Company analysis therefore adds the firm's own strategy (e.g., cost leadership or differentiation), along with its products and financial condition, to the industry picture.

Why the other options are wrong

- **A.** The threat of substitutes is one of Porter's five forces and is assessed during industry analysis, which the analyst has already completed.
- **B.** The bargaining power of suppliers is also an industry-level competitive force, already covered by the industry analysis.

Key takeaway The five forces belong to industry analysis; competitive strategy belongs to company analysis.

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MODULE 48.2

Revenue, Profitability, and Capital

LOS 48.b — Revenue drivers, operating profitability and pricing power

Revenue drivers

Linking performance to strategy starts with revenue on the income statement. Revenue can be broken into **revenue drivers** in two ways:

	Bottom-up analysis	Top-down analysis
Drivers	Company-specific: price and volume, business segments, geography	Macroeconomic and industry variables: economic growth (GDP), industry performance, market share
Starting point	The company's own lines of business	The economy and the industry

Best practice is to use both. Analysts track the level of drivers and how they change (recurring vs. episodic revenue), how they evolve over time (product mix, share of the obtainable market), and expected changes relative to other industry participants.

Operating profitability

$$\text{EBITDA} = \text{revenue} - \text{COGS} - \text{operating expenses (excluding D\&A)}$$

$$\text{EBIT} = \text{EBITDA} - \text{depreciation and amortization}$$

$$\text{Net income} = \text{EBIT} - \text{interest expense} - \text{taxes}$$

Dividing each by revenue gives the **EBITDA margin**, the **EBIT margin** (operating margin) and the **net margin**.

KEY CONCEPT

Metric	What it is	Strengths	Weaknesses
Net income	Total earnings after all expenses, interest and taxes	Most comprehensive (operating, financing, investing activities and taxes); useful in growth, mature and decline stages	Volatile; hard to compare across companies and to assess a company over time; distorted by changes in capital structure, expenditures or nonrecurring items; less useful for start-ups and highly cyclical industries
EBIT (operating income)	Earnings before interest, taxes and nonrecurring items	Removes financing and tax effects, so it improves comparability across firms in a period and for one firm over time	Distorted by depreciation/amortization (capex) policies: firms that invest heavily show lower operating margins than firms that underinvest

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Metric	What it is	Strengths	Weaknesses
EBITDA	EBIT plus depreciation and amortization	Compares firms with different investment policies, leverage and taxes	Not recognized by GAAP/IFRS; ignores the cost of replacing assets; adjustments may not be standard

Because EBITDA sits above interest, a jump in interest expense (e.g., after a shift from equity to debt financing) can push the net margin down even while the EBITDA margin rises. Analysts also compare the parts of the cost structure over time and against peers.

Pricing power

Pricing power is a company's ability to change the economic terms of its offer (mainly price) in its favor without hurting sales. It depends on price elasticity, market structure and competitive position.

- **Price elasticity** measures how much quantity demanded changes when price changes. It is the basic gauge of pricing power.
- **Inelastic demand** means high pricing power: buyers barely react to price (necessities such as health care, products with no substitutes).
- **Elastic demand** means little pricing power: buyers react strongly (luxuries and discretionary items such as leisure travel).
- Pricing power grows with product differentiation, switching costs, brand loyalty, barriers to entry and a lack of substitutes. Relative market share can matter more than absolute share.
- Less competitive structures (monopoly, oligopoly, monopolistic competition) allow more pricing power. In highly competitive markets with identical products, firms are **price takers**. In the long run price is driven to marginal cost and returns on capital to the cost of capital. A very low-cost producer can still keep its margins when price is set by the world market.

Operating leverage

KEY CONCEPT

$$\text{Operating profit} = Q \times (P - VC) - FC$$

where Q = units sold, P = price per unit, VC = variable cost per unit (materials, direct labor), FC = total fixed costs (rent, management salaries, insurance).

Operating leverage comes from fixed operating costs. The larger the share of fixed costs, the faster operating profit rises when volume rises, and the faster it falls when volume falls.

Example. A firm sells 20,000 units at \$15 with variable cost of \$9 per unit and fixed costs of \$80,000. Each unit sold adds $15 - 9 = \$6$ toward fixed costs and profit, so operating profit is $20{,}000 \times 6 - 80{,}000 = \$40{,}000$. The table shows the same firm after a 10% fall and after a 10% rise in volume. Its interest and net income rows are used for financial leverage under LOS 48.c.

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Operating profit and net income at three sales volumes ($)

Item	Volume down 10%	Base case	Volume up 10%
Units sold	18,000	20,000	22,000
Revenue ($15 per unit)	270,000	300,000	330,000
Variable costs ($9 per unit)	162,000	180,000	198,000
Fixed operating costs	80,000	80,000	80,000
Operating profit	28,000 (−30%)	40,000	52,000 (+30%)
Interest expense	10,000	10,000	10,000
Net income (no taxes)	18,000 (−40%)	30,000	42,000 (+40%)

A 2,000-unit change in volume moves operating profit by $2{,}000 \times 6 = \$12{,}000$, which is 30% of $40,000 and three times the 10% change in volume.

Regressing expenses on revenue helps separate fixed costs (low sensitivity) from variable costs (high sensitivity); this assumes the cost structure has not changed over the sample period.

COMMON EXAM TRAPS (48.B)

- Bottom-up analysis uses company-specific drivers (segments, geography, price × volume); top-down analysis uses GDP, industry performance and market share.
- Net income is less useful for start-ups and highly cyclical industries. It is a GAAP/IFRS measure; EBITDA is the non-GAAP one.
- A price rise followed by a large fall in volume signals elastic demand and low pricing power. A price rise with volume nearly unchanged signals inelastic demand.
- High operating leverage means the percentage change in operating profit is larger than the percentage change in revenue, in both directions.

LOS 48.c — Working capital, capital investments and capital structure

Free cash flow to equity

$$\text{FCFE} = \text{NI} + \text{D\&A} - \text{capital expenditures} - \Delta\text{working capital} + \text{net borrowing}$$

where:

- **Capital expenditures** are the gross cash investment in long-term (fixed) assets, before depreciation. D&A is added back because it is a noncash charge. When the data give the increase in net (book value) long-term assets instead, depreciation has already been netted out and D&A is not added back: $\text{FCFE} = \text{NI} - \text{net investment in long-term assets} - \Delta\text{working capital} + \text{net borrowing}$.
- $\Delta\text{working capital}$ is the increase in **operating (noncash) working capital**: current assets excluding cash and marketable securities, minus current liabilities excluding short-term debt and the current portion of long-term debt. Those debt items belong in net borrowing, and the change in cash is what FCFE measures.

- **Net borrowing** (**net debt issued**) = new borrowing − debt repaid.

Positive terms are sources of cash; negative terms are uses. Exam convention: the formula is written $\text{FCFE} = \text{NI} + \text{D\&A} - \Delta\text{working capital} - \Delta\text{LT assets} + \Delta\text{debt}$, with $\Delta\text{working capital} = \Delta\text{current assets} - \Delta\text{current liabilities}$. Current practice: the D&A add-back is correct only if $\Delta\text{LT assets}$ is the gross investment in long-term assets, and the working capital change leaves out cash and short-term debt, as defined above.

Example. NI is \$120 million, D&A \$35 million, the increase in noncash working capital \$15 million, capital expenditures \$70 million, new borrowing \$40 million and repayments \$25 million. FCFE is $120 + 35 - 15 - 70 + (40 - 25) = \$85$ million.

FCFE is often negative for start-ups and growth firms (heavy investment, no stable debt capacity) and typically positive for mature firms.

Working capital management

Working capital management is how a company uses current assets and current liabilities to run the business efficiently (working capital = current assets − current liabilities). The **operating cycle** (from receiving materials to collecting cash) is measured with efficiency ratios such as days of inventory on hand, days sales outstanding, days payables outstanding and receivables turnover. The **cash conversion cycle** narrows this to the cash gap (from paying for materials to collecting cash):

KEY CONCEPT

$$\text{CCC} = \text{DOH} + \text{DSO} - \text{DPO}$$

Example. With DOH of 45, DSO of 30 and DPO of 50, the CCC is $45 + 30 - 50 = 25$ days.

- A longer CCC means a greater need to finance working capital. A shorter CCC reduces financing needs and improves market position and returns.
- A negative CCC means operations fund the company, because suppliers are paid after customers pay.
- Industry examples: retailers use high turnover and bargaining power with suppliers; manufacturers are capital-intensive (production problems can swell inventory); service and software firms need little working capital; some brewers use long payables periods to run negative working capital, which works best while revenue grows and suppliers stay financially sound. Makers of long-lived products such as heavy equipment may measure inventory with **program accounting**, which looks at the profitability of the whole program rather than at unit costs.

Capital investments

Long-lived assets are capitalized and expensed through depreciation and amortization. **Net capital expenditure** shows whether a firm is keeping or expanding its productive capacity and future earnings growth:

$$\text{Net capital expenditure} = \text{total capital expenditure} - \text{D\&A}$$

Software and licensing businesses hold mostly intangible (amortized) assets; manufacturers expanding capacity buy tangible, depreciated assets.

Return on invested capital (ROIC) measures whether the company has created economic value from investors' capital:

$$\text{ROIC} = \frac{\text{NOPAT}}{\text{average invested capital}}, \qquad \text{invested capital} \approx \text{total long-term liabilities} + \text{total equity}$$

Average invested capital = (beginning + ending) / 2. The long-term liabilities plus equity measure is an approximation; a fuller definition is total debt plus total equity minus cash and equivalents. Value is created when ROIC exceeds WACC. *Example.* NOPAT is $15 million, and invested capital is $140 million at the start of the year and $160 million at the end, so ROIC is $15/150 = 10\%$. With a WACC of 8.5%, the firm created value. Judging a single project against the WACC is capital budgeting, a different use of the WACC.

Capital structure

A firm's capital structure reflects its business model, its stage in the life cycle and its asset conversion cycle.

- Firms in the early or growth stage whose cash flow is still negative rely mainly on **equity capital**.
- Mature firms with stable, predictable cash flows use more **debt** (e.g., public or private bonds).
- Capital-intensive industries (manufacturing, utilities) use more long-term debt; service and retail firms use more short-term debt.

The **degree of financial leverage (DFL)** shows how sensitive net income is to operating income:

$$\text{DFL} = \frac{\%\Delta \text{ net income}}{\%\Delta \text{ operating income}}$$

When judging debt, the analyst looks at loans outstanding, the type and maturity of the company's bonds, and its capacity to borrow more. Borrowing adds interest expense and raises DFL. Profits are magnified in good years and losses compounded in bad ones, and the risk of financial distress rises. When strategy changes (an acquisition, a restructuring, a large new investment), disaggregated financial models help confirm that cash will be there to service and repay debt. Other ratios: debt ratio $= \text{debt}/\text{assets}$ and interest coverage $= \text{EBITDA}/\text{interest expense}$.

Example. The bottom rows of the operating leverage table (LOS 48.b) add $10,000 of interest and no taxes. When volume rises 10%, net income goes from $30,000 to $42,000 (+40%) while operating profit rises 30%, so DFL $= 40\%/30\% = 1.33$. When volume falls 10%, net income drops by the same 40%, to $18,000.

COMMON EXAM TRAPS (48.C)

- DFL uses percentage changes, with net income on top. Dividing the dollar changes, or flipping the ratio, gives the wrong answer. In the example, dividing the $12,000 change in net income by the $12,000 change in operating profit gives 1.0, and flipping the ratio gives 0.75, instead of 1.33.
- Operating leverage links the change in operating income to the change in sales; the degree of financial leverage links the change in net income to the change in operating income.
- In FCFE, subtract an increase in noncash working capital, and use net borrowing (borrowing – repayments). In the example, using the $40 million of new borrowing instead of net borrowing gives $110 million, and adding the $15 million working capital increase gives $115 million, instead of $85 million.

- Net capital expenditure indicates productive capacity and growth, ROIC indicates economic value creation, and the CCC indicates working capital efficiency and financing needs.
- Issuing long-term debt to pay for fixed assets raises financial leverage. Once the proceeds are spent on those assets, net working capital is unchanged, and the extra interest reduces net income and so the addition to retained earnings.

EXAM SHORTCUTS

- With price, variable cost per unit and fixed costs unchanged, a change in units sold moves operating profit by the change in units times $P - VC$, so the whole profit figure need not be recomputed.

BOTTOM LINE

- Net income is the most comprehensive earnings measure but is volatile and less useful for start-ups and highly cyclical industries; EBIT removes financing and tax effects but is distorted by depreciation policies; EBITDA allows comparison across differing investment policies, leverage and taxes but is not a GAAP/IFRS measure and ignores the cost of replacing assets.
- Pricing power, the ability to change the terms of an offer in the company's favor without hurting sales, is high when demand is inelastic and grows with differentiation, switching costs, brand loyalty, barriers to entry and a lack of substitutes.
- Operating profit is $Q(P - VC) - FC$, and the larger the share of fixed costs, the more operating profit changes in percentage terms relative to a change in volume, in both directions.
- Exam convention: $\text{FCFE} = \text{NI} + \text{D\&A} - \Delta\text{working capital} - \Delta\text{LT assets} + \Delta\text{debt}$; current practice: the D&A add-back is correct only when the long-term asset change is gross investment, and working capital excludes cash and short-term debt.
- The cash conversion cycle is $\text{DOH} + \text{DSO} - \text{DPO}$; a longer CCC means a greater need to finance working capital, and a negative CCC means operations fund the company.
- Value is created when ROIC, NOPAT over average invested capital, exceeds the WACC, and the degree of financial leverage, $\%\Delta \text{ net income} / \%\Delta \text{ operating income}$, rises when borrowing adds interest expense.

QUICK CHECK

Question 2

An analyst is deciding whether to rely on net income when assessing a company's operating profitability. Which statement about net income is *most accurate*?

A It can be difficult to use for tracking a company's profitability over time.

B It is well suited to start-ups and to companies in highly cyclical industries.

C It is not recognized as a profitability measure under US GAAP or IFRS.

Answer: A

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Net income is often volatile because it is affected by nonrecurring items, changes in capital structure and changing expenditures or tax rates. That makes it hard to compare across companies and to assess one company's profitability over time.

Why the other options are wrong

- **B.** Net income is less useful for start-ups and highly cyclical industries, which are often loss-making or swing widely; it is most useful in the growth, mature and decline stages.
- **C.** Net income is a standard GAAP/IFRS measure. EBITDA is the metric not recognized by these accounting standards.

Key takeaway Net income: most comprehensive but volatile and distorted by capital structure; EBITDA: not a GAAP/IFRS metric.

Practice questions

Question 3

Fennwick Pumps makes solar-powered irrigation pumps. Across its home continent, annual demand for such pumps (the total addressable market) is estimated at $2.4 billion. Fennwick's pumps are certified only for the voltage standard used in 70% of that demand, and within the certified segment it holds sales licenses in countries representing 55% of that segment's demand. Its factory can supply at most $300 million of pumps a year. Fennwick's serviceable market is *closest* to:

A $924 million.

B $1,320 million.

C $1,680 million.

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Question 4

An analyst gathers the data in the exhibit for Pellworth Logistics.

Pellworth Logistics: selected data for the year ($ millions)

Item	Amount ($ millions)
Net income	54
Depreciation and amortization	18
Increase in noncash working capital	9
Capital expenditures (gross investment in long-term assets)	40
New long-term borrowing	25
Repayment of existing debt	10

Pellworth's free cash flow to equity (FCFE) for the year is *closest* to:

A $56 million.

B $48 million.

C $38 million.

Question 5

A long-established, capital-intensive electric utility with steady, predictable cash flows is *most likely* to finance its operations mainly by issuing:

A common stock.

B preferred stock.

C public bonds.

Answers: 3. A · 4. C · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/company-analysis-past-present-and-future/. The full bank has 40 questions on this reading.

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EQUITIES · READING 49

Equity Analyst Research Reports

What you'll learn

LOS 49.a Describe the sections of an initial company research report and the items in subsequent reports, including the rough buy/hold/sell guideline.

LOS 49.b Compare sell-side and buy-side research (producer, objective, format, audience, valuation methods, regulation), including screening and activist short selling.

LOS 49.c Explain why analysts using the same model reach different values: revenue and cost assumptions, capital investment, terminal value, financing.

MODULE 49.1

Equity Analyst Research Reports

This reading describes what an initiating-coverage report and a subsequent report contain and how a recommendation follows from the price target. It compares sell-side and buy-side research, and explains why analysts using the same model reach different values, including how to back an assumption out of a price target.

LOS 49.a — What a company research report contains

A **company research report** sets out the analyst's valuation and investment recommendation, based on the company's current and forecast performance and position. The first external report, **initiating coverage**, is thorough because it introduces the company to readers who may know little about it. Later **subsequent reports** are shorter updates, often triggered by expected events such as earnings releases or mergers.

Initial report: typical sections

KEY CONCEPT

Section	Content
Front page	Appears first but is **written last**: recommendation (buy/hold/sell), price target, **investment thesis**, company name, ticker, sector/industry, market cap and float, recent developments, summary financials
Business model	How the company creates value: revenue drivers (products, pricing, key customers) and expense drivers (suppliers, distribution)
Industry overview and competitive positioning	Introduce the industry (do not assume reader expertise); the analyst identifies and names peers even if the company does not; quantitative comparisons plus only the **relevant** parts of PESTLE and Porter's five forces
Financial analysis and valuation	Only the measures and ratios that support the **price target**, with no line-by-line commentary; forecasts draw on past results, company, industry and economic factors and any short- or long-term catalysts; several valuation techniques estimate intrinsic value, which sets the target; target is absolute or relative to a benchmark, a point or a range, usually over one to two years

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Section	Content
Investment risks	Detail on key company, industry and macro risks, including the **sensitivity** of the valuation to the analyst's assumptions
ESG factors	Content depends on industry and country; crucial where environmental issues matter (e.g., air travel, data storage)
Recommendation	A clear trading action: buy/hold/sell (or overweight/neutral/underweight)

A rough recommendation guide: buy if the stock is expected to rise by at least about 10%–15% (in absolute terms or relative to a benchmark), sell if it is expected to fall by at least about 10%–15%, and hold in between. It is a guideline, not a rule.

Subsequent report: typical items

A subsequent report has condensed front matter; the recommendation with the **rationale for any change**; analysis of new information, such as updated financial statements and **variance analysis** of expected versus actual results; changes to the valuation and the reasons for them; and changes in risks.

Every report should **distinguish facts from opinions**, disclose potential conflicts of interest, and explain the risks investors face.

COMMON EXAM TRAPS

- Business-model and industry sections belong to the initial report. Updates focus on what is new.
- The investment-risks section discusses sensitivity to assumptions. Reasons for a rating change go with the recommendation in an update.
- An expected gain (or outperformance) below the rough 10%–15% zone does not automatically mean "buy". It usually points to a hold.

LOS 49.b — Sell-side versus buy-side research

"Sell-side" and "buy-side" describe who produces the report. They say nothing about the recommendation it gives.

KEY CONCEPT

Feature	Sell-side research	Buy-side research
Producer	Investment banks/brokers, for their clients	Firms that invest (asset managers), for internal use
Objective	Provide information and recommendations to clients; support bank relationships (underwriting, trading volume, advisory)	Support internal investment decisions and monitoring
Detail and format	Wide range of industries and stocks for a diverse audience; **standardized format**	**Customized**, in-depth, focused

Feature	Sell-side research	Buy-side research
Audience	External investors and traders	Internal investment teams, portfolio managers
Valuation methods	DCF, multiples and other common models, with a focus on future earnings	Internal proprietary models plus traditional methods
Regulation	Constraints on **conflicts of interest** (e.g., underwriting relationships, no access to material nonpublic information held elsewhere in the bank, limits around IPOs/secondary offerings, research paid for separately from trade execution)	Mostly unregulated because it is not published

Each sell-side analyst usually specializes in one industry and follows selected companies in it. For **sell-side** firms, research is a cost unless it is sold, so coverage is limited and tilted toward stocks that generate trading volume. Coverage can add credibility and liquidity to a stock. Recommendations are mostly buy or hold and usually **in line with the consensus**, because a contrary view needs high conviction. Analysts may also avoid sell ratings to protect banking relationships.

Buy-side research is usually confidential and serves the firm's investment committee. Buy-side analysts read sell-side reports but reach their own conclusions. Because they do not need to generate commissions, they have more scope to cover smaller stocks. They narrow the universe with **stock screening**, increasingly with software and data platforms: they start from a universe (e.g., an index), apply quantitative criteria and refine them. Any arbitrary cutoff may exclude an attractive stock.

Activist short selling is a branch of buy-side research. Small teams look for stocks likely to fall sharply because of misinformation or undiscovered fraud. They often publish their research after sharing it with their own investors.

COMMON EXAM TRAPS

- Reading "sell-side" as "issues sell ratings": both sides give buy, hold and sell views.
- Placing activist short sellers on the sell side because they publish: they are a branch of buy-side research.
- Giving the sell side the small-stock coverage: its need for trading volume tilts coverage toward larger stocks, so the buy side has more room for small caps.

LOS 49.c — Why analysts using the same model reach different values

Valuation models need many forecasts, so two analysts using the same approach can reach opposite recommendations. This is not a flaw: differing views support **price discovery** and market efficiency. Stocks with widely dispersed estimates are **hard-to-value stocks**. Location can matter too; a foreign analyst, for example, may place a value on market access.

Two-stage FCFE valuation

An analyst's assumption can be backed out of a price target. The target is treated as the DCF value, and the equation is solved for the missing input, such as long-term growth.

With one year of explicit forecast and constant growth g from year 2, the value of equity is

$$V_0 = \frac{FCFE_1}{1+r} + \frac{FCFE_2/(r-g)}{1+r}$$

so, given the target value V_0, r and the two forecasts, the equation can be solved for g.

Example. The price target is \$20.00, there are 40 million shares, FCFE is \$40 million next year and \$42 million the year after, and the required return is 10%:

$$800 = \frac{40}{1.10} + \frac{42/(0.10-g)}{1.10} \Rightarrow 880 - 40 = \frac{42}{0.10-g} \Rightarrow 0.10 - g = 0.05 \Rightarrow g = 5\%$$

Main drivers of differences

- **Revenue and profitability.** Faster revenue growth usually goes with higher free cash flow and a higher value, but revenue growth alone can mislead, because what matters is how costs move. *Example.* Revenue is \$20 million and costs are \$14 million, a 30% margin, and revenue grows 5%. If costs also grow 5%, the margin stays at 30%. If costs grow 6%, it falls to 29.3%. If costs grow 3% because of economies of scale, it rises to 31.3%.
- **Capital investment.** Exam convention: if investment growth matches net-income growth, there is no overall change in free cash flow. Current practice: with other components held constant, equal dollar increases in investment and income leave free cash flow unchanged; equal percentage growth makes their difference grow at that rate. Free cash flow as a whole grows at that rate only if its other components are zero or grow at the same rate too.
  - When investment is smaller than income and grows more slowly, its dollar increase is also smaller, so free cash flow rises. The analyst should then ask whether that growth is sustainable without more investment.
  - When investment rises without income growth, free cash flow falls and the long-run return on capital declines.
  - Capital spending does not affect the operating margin directly.

  Example. Income of \$200 and investment of \$80 give free cash flow of \$120. If both rise by \$10, free cash flow is $210 - 90 = 120$, unchanged. If both rise 5%, it is $210 - 84 = 126$, also up 5%. If the firm also has other net cash items of \$20 that stay flat, free cash flow starts at \$140 and reaches only $210 - 84 + 20 = 146$ when income and investment grow 5%, a rise of 4.3%.
- **Terminal value.** In models such as FCFE, a large part of value comes from the **terminal value**, so the horizon and the long-term growth assumption matter a great deal.
- **Financing.** Financing choices matter less than operating and strategic decisions. A valuation driven mainly by financing deserves skepticism.

COMMON EXAM TRAPS

- Equating the same revenue forecast with the same margin: with revenue fixed, faster cost growth lowers the margin and slower cost growth raises it. In the example, keeping the 30% margin when costs grow 6% overstates the 29.3% that results.

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- Comparing growth rates alone: slower investment growth raises FCFE only if the dollar increase in investment is smaller than the dollar increase in income, as it is when investment starts below income.
- The terminal value is discounted from the period in which it is measured; a value at t = 1 is discounted one period. In the price-target example, discounting the terminal value of 840 two periods gives a total of 730.58, or $18.26 per share, instead of 800 and $20.00.

EXAM SHORTCUTS

- "Sell-side" and "buy-side" say who produces a report and nothing about its recommendation, so an answer that links sell-side research to sell ratings can be eliminated.

BOTTOM LINE

- An initiating-coverage report is thorough because readers may know little about the company, while subsequent reports are shorter updates often triggered by expected events such as earnings releases, and the front page is written last.
- The financial analysis and valuation section shows only the measures that support the price target, which is set by intrinsic value from several valuation techniques and can be absolute or relative, a point or a range, usually over one to two years.
- As a rough guideline, a stock expected to rise by at least about 10%–15% (in absolute terms or relative to a benchmark) is a buy, one expected to fall by at least that much is a sell, and one in between is a hold.
- A subsequent report gives the recommendation with the rationale for any change, variance analysis of expected versus actual results, and changes to the valuation and risks, and every report should distinguish facts from opinions and disclose potential conflicts of interest.
- Sell-side research is produced by investment banks and brokers for clients in a standardized format under conflict-of-interest constraints, whereas buy-side research is produced by investing firms for internal use, is customized and is mostly unregulated because it is not published.
- Sell-side coverage tilts toward stocks that generate trading volume and its recommendations are mostly buy or hold and in line with the consensus, while the buy side has more scope to cover smaller stocks and includes activist short selling.
- Analysts using the same model can reach different values through their assumptions on revenue and costs, capital investment, the terminal value and financing, and a valuation driven mainly by financing deserves skepticism.
- Exam convention: when investment grows at the same rate as net income, free cash flow shows no overall change; current practice: equal dollar increases leave free cash flow unchanged, while equal percentage growth makes their difference grow at that rate.

QUICK CHECK

Question 1

Which of the following statements about the front page of a report initiating coverage of a company is *most accurate*?

A It is written last, even though it is the first page investors read.

B It leaves out the recommendation so readers study the analysis first.

C It contains the full financial statement analysis behind the target.

Answer: A

Although it appears first, the front page is written last because it summarizes the whole report: the buy/hold/sell recommendation and price target, the investment thesis, the company's name, ticker, sector, market capitalization and float, recent developments, and summary financial data. Each of these is covered in more detail later in the report.

Why the other options are wrong

- **B.** The recommendation and price target are the main items the front page shows.
- **C.** The front page carries only a summary of recent and forecast financial data; the detailed analysis is in the financial analysis and valuation section.

Key takeaway The front page summarizes the recommendation, target, thesis and key data. It is the first page read and the last written.

Practice questions

Question 2

Compared with a report initiating coverage, a subsequent company research report is *most likely* to emphasize:

A variance analysis comparing forecast and actual results.

B a full introduction to the company's revenue and expense drivers.

C a detailed overview of the industry and competitive positioning.

Question 3

Which of the following is more characteristic of sell-side analysts than of buy-side analysts? Sell-side analysts tend to:

A keep their final investment conclusions confidential.

B specialize in activist short-selling strategies.

C issue recommendations in line with the market consensus.

Question 4

Selin Arslan covers a company with revenue of $48 million and operating costs of $30 million, an operating profit margin of 37.5%. She forecasts revenue growth of 4% for next year and assumes the company will benefit from economies of scale. The operating profit margin Arslan forecasts for next year will *most likely*:

A stay at 37.5%, because operating costs are expected to grow in line with revenue.

B decline, because new capital investment will be charged immediately against operating profit.

C increase, because operating costs are expected to grow more slowly than revenue.

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Answers: 2. A · 3. C · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/equity-analyst-research-reports/. The full bank has 24 questions on this reading.

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EQUITIES · READING 50

The Capital Asset Pricing Model, Market Model, and Other Factor-Based Equity Models

What you'll learn

LOS 50.a Explain how the CAPM, the market model, adjusted and comparables-based betas, market risk premium estimates and country risk premiums are used to estimate a company's required return on equity.

LOS 50.b Describe arbitrage pricing theory and multifactor models such as the four-factor model, and interpret factor betas.

MODULE 50.1

Estimating the Cost of Equity With the Market Model and CAPM

This reading shows how to estimate the required return on equity with the CAPM: estimating and adjusting beta, unlevering and relevering a peer beta, estimating the market risk premium and adding a country risk premium. It then covers arbitrage pricing theory and the four-factor model, including how to calculate an expected return from factor betas and premiums and how to read their signs.

LOS 50.a — Estimating the required return on equity with the CAPM and the market model

The **required rate of return** (cost of equity) is the discount rate in equity valuation models, so it must reflect the risk of the investment.

CAPM and the security market line

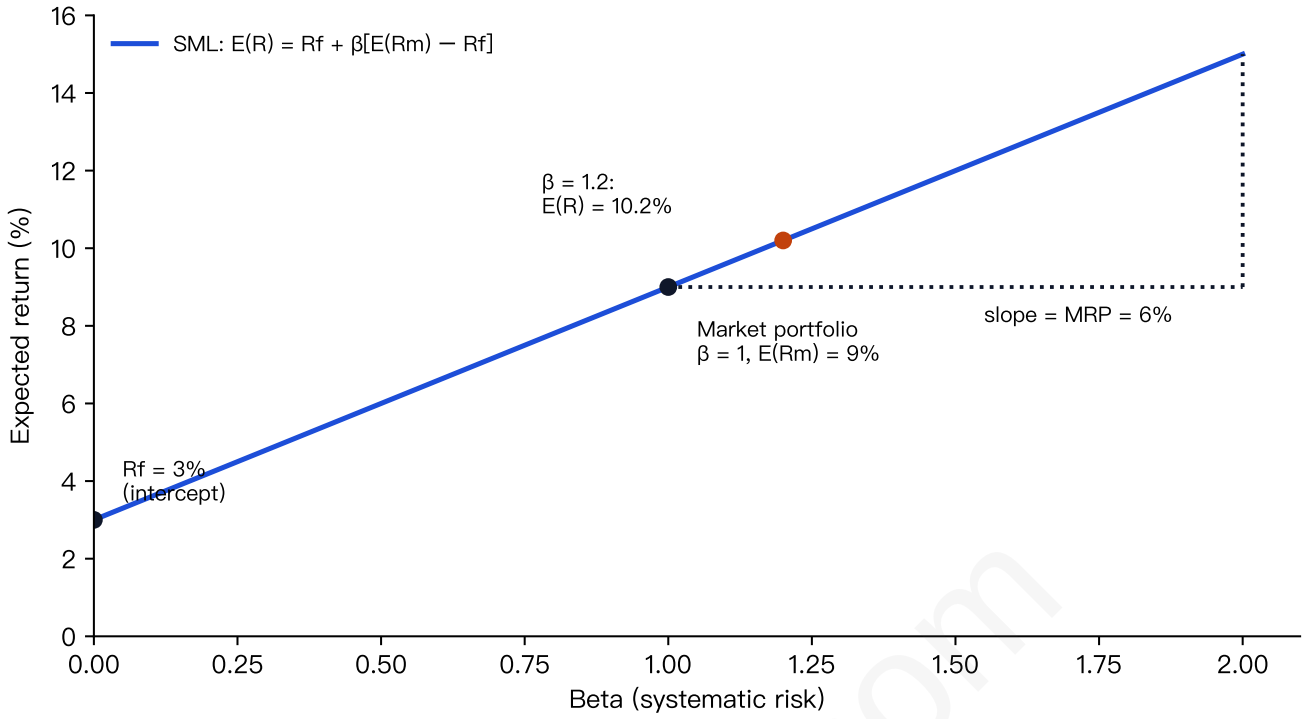
The **capital asset pricing model (CAPM)** is a single-factor model. Only **systematic risk (beta)** is priced:

KEY CONCEPT

$$E(R_i) = R_f + \beta_i [E(R_m) - R_f]$$

$E(R_m) - R_f$ is the **market risk premium (MRP)**. Plotted against beta, the CAPM is the **security market line (SML)**. Its intercept is the risk-free rate and its **slope is the market risk premium**. The SML shifts over time as the risk-free rate and return expectations change.

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The security market line with Rf = 3% and E(Rm) = 9%

Beta can be computed from correlation and volatilities: $\beta_i = \rho_{i,m}\,\sigma_i/\sigma_m$. *Example.* With $\sigma_i = 30\%$, $\rho = 0.60$ and $\sigma_m = 15\%$, beta is 1.20. If $R_f = 3\%$ and $E(R_m) = 9\%$, then $E(R_i) = 3\% + 1.2(6\%) = 10.2\%$.

Estimating beta: excess-return regression and market model

Beta is estimated by regressing the stock's returns on a market proxy (e.g., a broad index) over a **lookback period** (e.g., five years of monthly data):

- Excess-return form: $(R_{i,t} - R_{f,t}) = \alpha + \beta_i(R_{m,t} - R_{f,t}) + \varepsilon_t$
- **Market model** (actual returns): $R_{i,t} = \alpha + \beta_i R_{m,t} + \varepsilon_t$, which gives similar beta estimates.

Reading the output: the **standard error of beta** measures statistical noise, and a higher standard error means a less reliable estimate. R^2 measures explanatory power. Exam convention: $\sqrt{R^2}$ is described as the stock–index correlation. Current practice: $\sqrt{R^2}$ gives the correlation's magnitude; the sign follows beta (positive beta: positive correlation; negative beta: negative correlation).

The length of the lookback period involves a trade-off. A longer sample lowers the standard error. If it reaches back to a time when the company's business or capital structure was different, however, the beta describes a risk profile that no longer applies.

Adjusted beta (mean reversion)

Beta is estimated from past data, but the cost of equity it produces is used to discount future cash flows. Market model betas tend to revert toward 1.0. An **adjusted beta** is a weighted average of the raw beta and 1.0:

$$\text{adjusted beta} = \omega \times \text{raw beta} + (1 - \omega) \times 1.0, \qquad \omega \text{ commonly } 2/3$$

Example. A raw beta of 1.45 gives $(2/3)(1.45) + 1/3 = 1.30$. A raw beta above 1 is adjusted down, and a raw beta below 1 is adjusted up. With the common fixed weight of 2/3, the adjustment does not depend on the length of the lookback period.

Beta from comparables (pure-play approach)

For private companies (no price history) or firms whose capital structure has changed significantly, analysts use peer betas. Estimating beta for an equally weighted portfolio of peers also reduces statistical noise and captures the industry's business risk. A company's beta reflects both business risk and financial risk, so it is a **levered beta**. Remove the peers' leverage, then add back the subject company's leverage:

KEY CONCEPT

$$\beta_U = \frac{\beta_{L,\text{peers}}}{1 + (1 - t_{\text{peers}})\,\dfrac{D}{E}_{\text{peers}}} \qquad \beta_{L,\text{subject}} = \beta_U \left[1 + (1 - t_{\text{subject}})\,\frac{D}{E}_{\text{subject}}\right]$$

(D/E = debt to market value of equity.) The peers' average tax rate is used to unlever and the subject company's tax rate to relever. When all firms share one tax rate, the same t appears in both steps. *Example.* Peers have a beta of 1.25 and a D/E of 0.50, and t = 20%, so $\beta_U = 1.25/1.40 = 0.893$. With a subject D/E of 0.25, $\beta_L = 0.893 \times 1.20 = 1.07$. Lower leverage than the peers means a lower levered beta.

Estimating the market risk premium

Approach	How	Watch out for
Historical	average past excess return of an index over the risk-free rate (arithmetic or geometric mean)	assumes the past is an unbiased guide. A boom sample overstates the MRP, which leads to lower valuations. The geometric mean is below the arithmetic mean.
Forward-looking	implied by current prices, via the constant growth model	high current prices produce a lower estimate

Because of these biases, the historical and forward-looking estimates can conflict, and the analyst should revisit the assumptions behind each one.

Choice of risk-free rate: short-term rates are normally lower than long-term rates, so a short-term rate produces a higher MRP. Because equity valuation is long-term, a long-term government rate is more appropriate.

Forward-looking (implied) estimate:

$$E(R_m) = \frac{D_1}{P_0} + g, \qquad MRP = \frac{D_1}{P_0} + g - R_f$$

Example. A dividend yield of 1.8% and growth of 6.0% give $E(R_m) = 7.8\%$. With $R_f = 3.8\%$, the MRP is 4.0%.

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Emerging markets: country risk premium

For stocks in developing markets, add a **country risk premium (CRP)**, stated in a reserve currency (e.g., USD or EUR), to the market risk premium inside the beta term:

KEY CONCEPT

$$CRP = \text{sovereign bond yield spread} \times \frac{\sigma_{\text{equity index}}}{\sigma_{\text{sovereign debt}}} \qquad E(R_i) = R_f + \beta_i\,(MRP + CRP)$$

The sovereign yield spread is the country's reserve-currency bond yield minus the reserve-currency benchmark yield. The risk-free rate R_f is the benchmark (reserve-currency) yield. The emerging-market sovereign yield already contains the spread and is not used as R_f. *Example.* A spread of 3.0% with $\sigma_E = 21\%$ and $\sigma_D = 14\%$ gives a CRP of 4.5%. With $R_f = 3\%$, $\beta = 0.8$ and an MRP of 5%, $E(R) = 3\% + 0.8(9.5\%) = 10.6\%$.

COMMON EXAM TRAPS

- Adjusted beta: weight $2/3$ on the raw beta, $1/3$ on 1.0. Do not swap the weights or forget the $1/3$ term. For the raw beta of 1.45, swapping the weights gives 1.15 and leaving out the 1/3 term gives 0.97, instead of 1.30.
- Unlever with the peers' D/E and tax rate, then relever with the subject's D/E and tax rate. The $(1 - t)$ adjustment appears in both steps. In the example, leaving out $(1 - t)$ in both steps gives an unlevered beta of 0.833 and a levered beta of 1.04, instead of 0.893 and 1.07.
- CRP: multiply the spread by σ_E/σ_D (equity volatility in the numerator), and put CRP inside the bracket multiplied by beta. In the example, inverting the volatility ratio gives a CRP of 2.0% and $E(R) = 8.6\%$, and adding the CRP outside the beta term gives 11.5%, instead of 10.6%.
- Forward MRP: subtract R_f. Dividend yield plus growth alone is the expected market return. In the example, stopping at the dividend yield plus growth gives 7.8% instead of an MRP of 4.0%.

EXAM SHORTCUTS

- Adjusted beta is a weighted average of the raw beta and 1.0, so it lies between them: a raw beta above 1 is adjusted down and one below 1 is adjusted up, and any answer outside that range can be ruled out.
- When the subject and its peers share one tax rate, a subject with lower leverage than the peers has a lower levered beta than theirs, and one with higher leverage a higher beta.

BOTTOM LINE

- The CAPM, $E(R_i) = R_f + \beta_i[E(R_m) - R_f]$, prices only systematic risk, and its security market line has the risk-free rate as intercept and the market risk premium as slope.
- Beta equals $\rho_{i,m}\sigma_i/\sigma_m$ and is estimated by regressing the stock's returns on a market proxy over a lookback period, where a longer sample lowers the standard error but may describe a business or capital structure that no longer applies.
- Exam convention: the square root of R^2 is described as the stock–index correlation; current practice: it gives the correlation's magnitude, and the sign follows the sign of beta.
- Adjusted beta is $\omega \times \text{raw beta} + (1 - \omega) \times 1.0$, with ω commonly 2/3, because market model betas tend to revert toward 1.0.

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- In the pure-play approach the peers' levered beta is unlevered with the peers' D/E and tax rate and relevered with the subject company's D/E and tax rate, the $(1 - t)$ term appearing in both steps.
- The forward-looking market risk premium is $D_1/P_0 + g - R_f$, a long-term government rate is the more appropriate risk-free rate, and for emerging markets the country risk premium, the sovereign yield spread times $\sigma_{\text{equity index}}/\sigma_{\text{sovereign debt}}$, is added to the MRP inside the beta term.

QUICK CHECK

Question 1

Eighteen months ago, Galloway Industries sold its stable consumer division and financed a large acquisition mostly with new debt. An analyst estimating Galloway's beta with the market model decides to lengthen the lookback period from 2 years to 8 years. This change will *most likely*:

A make the beta more reliable, because a larger sample lowers statistical noise.

B increase the amount of mean reversion built into the adjusted beta.

C produce a beta that reflects a risk profile Galloway no longer has.

Answer: C

A longer lookback period gives a larger sample and a lower standard error of beta. However, most of the extra data now come from before Galloway's restructuring, when its business mix and leverage were different. The estimate will therefore describe past conditions rather than the company's future systematic risk.

Why the other options are wrong

- **A.** The standard error does fall with more observations, but a beta built largely on data from an obsolete business model and capital structure is a poorer guide to future risk.
- **B.** The mean-reversion adjustment is applied separately to the raw beta. With the usual fixed two-thirds weight on the raw beta, it does not change with the length of the lookback period.

Key takeaway A longer lookback period means less statistical noise but more stale data. After a major change in the business or capital structure, the stale data dominate, and a peer (pure-play) beta is usually the better choice.

MODULE 50.2

Arbitrage Pricing Theory

LOS 50.b — Arbitrage pricing theory and multifactor models

Why go beyond the CAPM?

Empirical studies show that a single market factor does not adequately explain why returns differ across stocks. This motivates models with several priced risk factors.

Arbitrage pricing theory (APT)

Arbitrage pricing theory (APT) is a framework for estimating expected equity returns with a **multifactor model**:

KEY CONCEPT

$$E(R_i) = R_f + \beta_{1i} f_1 + \beta_{2i} f_2 + \cdots + \beta_{ki} f_k$$

where β_{ki} is the beta of stock i to the kth risk factor and f_k is the risk premium for the kth risk factor.

APT is a **statistical factor model**: the theory leaves the choice of factors to the analyst, and researchers have tried both macroeconomic factors and company-specific (style or characteristic) factors. The table contrasts it with the CAPM.

KEY CONCEPT

	CAPM	APT
Number of factors	one	not specified
Identity of factors	the market portfolio (priced through the market risk premium)	not specified (macroeconomic, style or other company-specific factors)

The four-factor model

A widely used specification adds size, value and momentum factors to the market factor:

$$E(R_i) = R_f + \beta_{mkt}\,[E(R_m) - R_f] + \beta_{SMB}\,E(SMB) + \beta_{HML}\,E(HML) + \beta_{UMD}\,E(UMD)$$

Each nonmarket factor is the return on a **zero-cost, long-short portfolio** (equal amounts long and short):

KEY CONCEPT

Factor	Long	Short	Positive beta means...	Negative beta means...
Size (SMB, small minus big)	small-cap stocks	large-cap stocks	behaves like a small-cap stock	behaves like a large-cap stock
Value (HML, high-value minus low-value; high minus low book-to-market)	value stocks	growth stocks	value stock	growth stock
Momentum (UMD, up minus down)	recent winners	recent losers	benefits when recent winners keep outperforming	suffers when recent winners outperform

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The betas on these long-short factors are expected to average about zero across stocks, so individual betas are often negative. The market beta, by contrast, averages 1.0, and a market beta of 1.0 means average market risk. Betas are estimated with a multiple regression on historical data.

The sign of a factor beta shows how the stock moves with that factor's long-short return. A stock with a negative beta on a factor tends to underperform when that factor's long side beats its short side. If a factor beta is not given, nothing can be concluded about how the stock responds to that factor.

Example. $R_f = 3\%$. Betas and premiums are market 0.9 × 6%, size 0.5 × 2%, value −0.3 × 4%, momentum 0.2 × 5%.

$$E(R) = 3\% + 5.4\% + 1.0\% - 1.2\% + 1.0\% = 9.2\%$$

The stock behaves like a small-cap growth stock (size beta positive, value beta negative) with a mild momentum tilt.

Why multifactor models matter in equity investing

- *Estimating the cost of equity.* A multifactor expected return adds a premium for each factor exposure, so two stocks with the same market beta can have different required returns if one tilts toward small caps or value.
- *Describing a stock or portfolio.* The signs and sizes of the factor betas summarize its style (large or small, value or growth, momentum or contrarian) without looking at its holdings.
- *Choosing peers.* Firms with similar factor betas form a **statistical factor-based peer group** for multiples-based valuation (Reading 45). The caveat is that betas estimated from past data may not persist.

COMMON EXAM TRAPS

- Saying APT uses only macroeconomic factors, or that it specifies the factors. It specifies neither the number nor the identity.
- Expecting nonmarket factor betas to average 1 (that is the market beta), or confusing the risk-free rate with a beta.
- Mixing up the legs: size means long small caps, short large caps; value means long value stocks, short growth stocks; momentum means long recent winners, short recent losers.
- Dropping the sign of a negative factor beta when computing expected return. In the example, treating the value term as +1.2% gives 11.6% instead of 9.2%.

BOTTOM LINE

- A single market factor does not adequately explain why returns differ across stocks, which motivates multifactor models such as arbitrage pricing theory, $E(R_i) = R_f + \beta_{1i}f_1 + \cdots + \beta_{ki}f_k$.
- APT is a statistical factor model that specifies neither the number nor the identity of its factors, whereas the CAPM has one factor, the market portfolio.
- The four-factor model adds size (SMB), value (HML) and momentum (UMD) factors to the market factor, and each of these three is measured by a zero-cost portfolio holding equal amounts long and short.
- A positive SMB beta means the stock behaves like a small-cap stock and a positive HML beta like a value stock, while negative betas point to large-cap and growth behavior, and a negative factor beta must keep its sign in the expected return.

- Betas on the long-short factors average about zero across stocks, so individual betas are often negative, while the market beta averages 1.0.
- Multifactor models are used to estimate the cost of equity (two stocks with the same market beta can have different required returns), to describe a stock's or portfolio's style, and to form statistical factor-based peer groups, although betas estimated from past data may not persist.

QUICK CHECK

Question 2

An analyst estimates the following four-factor model betas for Tidewater Logistics:

Four-factor model betas for Tidewater Logistics

Factor	Beta
Market	0.90
Size (SMB)	−0.30
Value (HML)	0.42
Momentum (UMD)	−0.15

Based on these betas, Tidewater is *most appropriately* characterized as a:

A small-cap value stock.

B large-cap growth stock.

C large-cap value stock.

Answer: C

In the four-factor model the size factor (SMB) is the return on a zero-cost portfolio that buys small-cap stocks and shorts large-cap stocks. Tidewater's negative size beta (−0.30) means it tends to move inversely with small caps, which marks it as a large-cap stock. The value factor is long high book-to-market (value) stocks and short low book-to-market (growth) stocks; Tidewater's positive value beta (0.42) means it moves with value stocks. Together the two signs describe a large-cap value stock. The market beta (0.90) and momentum beta (−0.15) do not bear on the size or style classification.

Why the other options are wrong

- **A.** A small-cap stock would show a positive beta to the size factor, because it would move with the small-cap long side of that portfolio. Tidewater's size beta is negative (−0.30), so it behaves like a large-cap stock, even though its positive value beta does point to value.
- **B.** A growth stock would show a negative beta to the value factor, because it would move with the short (low book-to-market) side of that portfolio. Tidewater's value beta is positive (0.42), so the growth label is wrong, even though the large-cap part is right.

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Key takeaway Size beta sign: + small, − large. Value beta sign: + value, − growth.

Practice questions

Question 3

A broad equity index currently offers a dividend yield of 2.2%, and its earnings and dividends are expected to grow at 5.5% a year over the long term. The risk-free rate is 3.1%. A forward-looking estimate of the market risk premium is *closest* to:

A 4.6%.

B 7.7%.

C 10.8%.

Answers: 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/equities/the-capital-asset-pricing-model-market-model-and-other-factor-based-equity-models/. The full bank has 14 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Fixed Income

Exam weight 11–14% · 19 readings

Bond features, cash flows and markets; prices, yields and spreads; the term structure of interest rates; duration and convexity as measures of interest rate risk; credit analysis; and securitized debt such as asset-backed and mortgage-backed securities.

Reading 51	Bond Indenture
Reading 52	Zero Coupon Bond
Reading 53	Fixed-Income Issuance and Trading
Reading 54	Commercial Paper
Reading 55	Treasury Bonds
Reading 56	Yield to Maturity
Reading 57	Credit Spread
Reading 58	Yield and Yield Spread Measures for Floating-Rate Instruments
Reading 59	The Term Structure of Interest Rates: Spot, Par, and Forward Curves
Reading 60	Interest Rate Risk and Return
Reading 61	Macaulay Duration
Reading 62	Bond Convexity
Reading 63	Curve-Based and Empirical Fixed-Income Risk Measures
Reading 64	Credit Risk
Reading 65	Credit Analysis for Government Issuers
Reading 66	Credit Analysis for Corporate Issuers
Reading 67	Fixed-Income Securitization
Reading 68	Asset-Backed Security (ABS) Instrument and Market Features
Reading 69	Mortgage-Backed Security (MBS) Instrument and Market Features

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FIXED INCOME · READING 51

Bond Indenture

What you'll learn

LOS 51.a Describe the basic features of a fixed-income security: issuer, maturity, par value, coupon rate and frequency, seniority, contingency provisions, and the price-yield relationship.

LOS 51.b Describe what a bond indenture contains, the sources of repayment, and contrast affirmative and negative covenants.

MODULE 51.1

Fixed-Income Instrument Features

This reading introduces the basic features of a fixed-income security and the contents of a bond indenture. A candidate must be able to compute a periodic coupon, classify bonds by issuer, maturity and seniority, and tell affirmative covenants from negative covenants.

LOS 51.a — Features of a fixed-income security

Fixed-income instruments come in two broad forms. A **loan** is a private, nontradable agreement between a borrower and a lender. A **bond** (a **fixed-income security**) is a standardized, tradable security that represents a debt investment. Bond investors lend capital, the **principal**, also called **par value** or **face value**. The issuer promises to repay it together with interest, usually as a periodic **coupon** stated as a percentage of par. Issuers typically use the money raised to fund long-term investments. For a corporate issuer, bonds and long-term loans are classified as **long-term liabilities** on the balance sheet.

Key bond features

Feature	What it specifies
Issuer	Who borrows: **sovereign** (national) governments, corporations, local governments, **supranational entities** (e.g., the IMF), **quasi-government entities** (e.g., a state railway), and **special purpose entities** (SPEs) that buy financial assets and issue **asset-backed securities** backed by the cash flows of those assets
Maturity	The date the final cash flow is paid. The time left until then is the **tenor**. Original maturity of one year or less = **money market securities**; more than one year = **capital market securities**; no stated maturity = **perpetual bonds**
Par value	The principal to be repaid — usually in one amount at maturity, but some instruments (e.g., a mortgage) repay it gradually
Coupon rate and frequency	The **coupon rate** is the *annual* percentage of par paid as interest; payments may be annual, semiannual, quarterly or monthly
Seniority	Debt ranks ahead of equity in bankruptcy or liquidation; **senior debt** ranks ahead of **junior debt** (**subordinated debt**), so junior debt carries more credit risk and must offer a higher yield

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Feature	What it specifies
Contingency provisions	Some bonds include embedded options, such as a **call option**, a **put option** or a conversion right (covered in the next reading)

KEY CONCEPT

Each periodic coupon $= \dfrac{\text{coupon rate} \times \text{par value}}{\text{payments per year}}$. A bond described as a "6% semiannual coupon bond" pays 3% of par every six months.

Example. A $5,000 par bond with a 4% coupon paid semiannually pays $0.04 \times 5{,}000/2 = \$100$ every six months, or $200 a year. If it paid quarterly, each coupon would be $50.

Floating-rate notes (FRNs), or **floaters**, pay a coupon equal to a variable **market reference rate (MRR)** plus a fixed margin. The margin is quoted in **basis points** (1 bp = 0.01%).

Zero-coupon bonds (**pure discount bonds**) pay no interest before maturity. They are sold at a discount to par, and the investor's whole return is the difference between the purchase price and the par value received at maturity. For a given maturity, the lower the price, the higher the return.

Price, yield and the yield curve

A bond's **yield** is the return an investor expects from buying it at today's price and receiving its promised cash flows. For a fixed-coupon bond, price and yield move in opposite directions. Fixed cash flows can deliver a higher return only if the investor pays less for them.

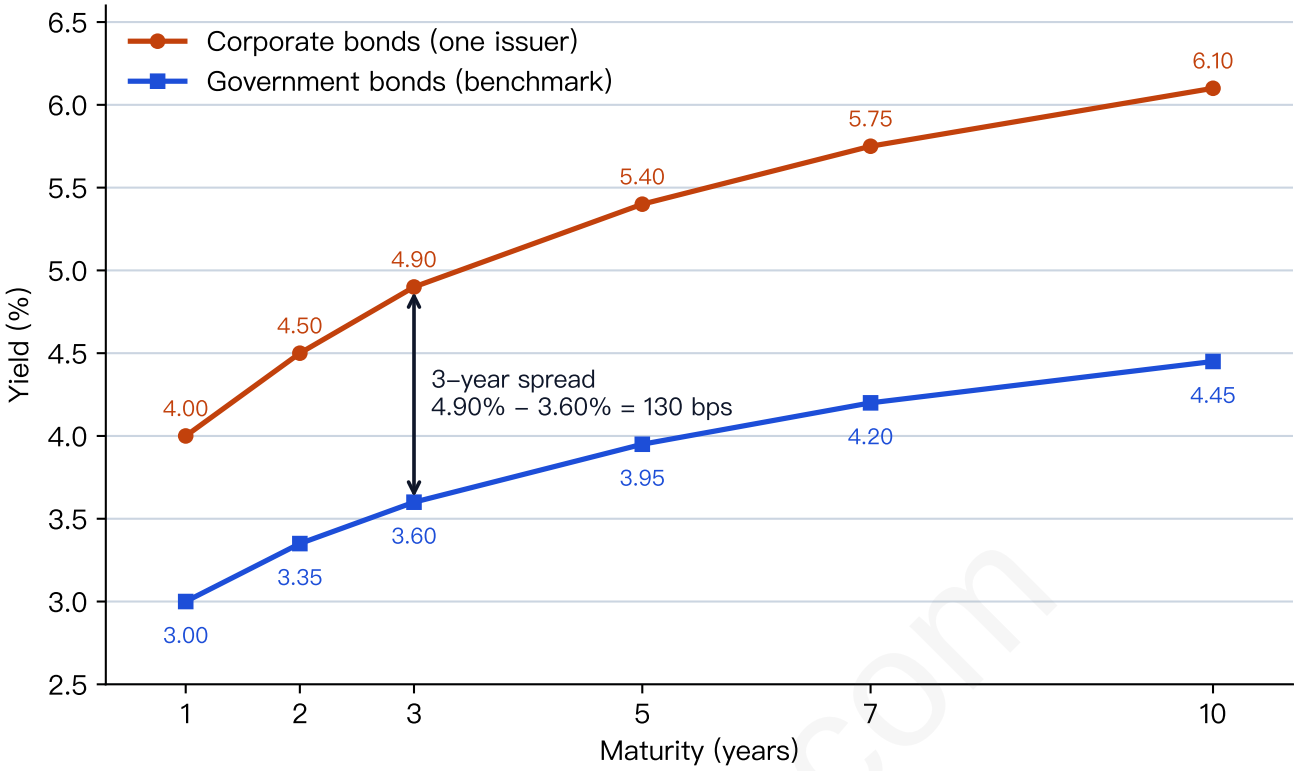
KEY CONCEPT

- **Premium bond**: price above par (its yield is below its coupon rate).
- **Discount bond**: price below par (its yield is above its coupon rate).

A plot of yields against maturity for one issuer or class of bonds is a **yield curve**. An upward-sloping curve is a **normal yield curve**. It is the most common shape, because investors usually demand a higher yield for longer, more uncertain horizons. A downward-sloping curve is an **inverted yield curve**, which is less common. The US Treasury curve, for instance, was upward sloping in mid-2018 and inverted in mid-2023. Government bonds are usually the lowest-credit-risk bonds in a market, so the government curve is the benchmark, and the extra yield on a riskier bond over a government bond of the same maturity is its **spread**.

Example. A 3-year corporate bond yields 4.9% and the 3-year government bond yields 3.6%, so the spread is $4.9\% - 3.6\% = 1.3\% = 130$ bps.

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Government and corporate yield curves; the spread is the vertical gap at one maturity

Common exam traps

- Confusing the per-period coupon with the coupon rate: 2% paid quarterly is an 8% coupon rate. For the $5,000 par, 4% semiannual bond in the example, applying the full 4% to each payment gives a coupon of $200 instead of $100.
- Classifying money market and capital market securities by remaining maturity. The original maturity is what counts.
- Thinking a zero-coupon bond can trade above par when rates fall. As long as its yield is positive, it stays below par until maturity.
- Measuring a spread against a government bond of a different maturity.
- Assuming senior and junior bonds of the same issuer and maturity offer the same yield. The junior bonds are riskier and yield more.

LOS 51.b — The bond indenture and covenants

The **bond indenture** (also called the **trust deed**) is the legal contract between the issuer (borrower) and the bondholders (lenders). It sets out the bondholders' rights and the issuer's obligations and restrictions, and it governs all later dealings between the two sides. It specifies the issuer, maturity, par value, coupon rate and frequency, currency, the **sources of repayment**, any **collateral** pledged, **credit enhancements**, and the **covenants** the issuer must follow.

The indenture does not name the individual bondholders. Bonds are tradable securities, so the holders change over the bond's life.

Sources of repayment

Issuer / bond type	Repaid from
Sovereign bonds	Taxes on economic activity and, in some cases, the power to create currency — hence usually the lowest credit risk in a region
Local government bonds	Local taxes, or revenue from projects such as toll roads
Secured bond	The company's operating cash flow plus a legal claim (**lien** or **pledge**) on specific assets (**collateral**) if the issuer defaults
Unsecured bond	Operating cash flow only
Asset-backed security	Cash flows from the financial assets held by the SPE that issued it

Affirmative versus negative covenants

Bondholders have no votes, but **covenants** written into the indenture protect them.

KEY CONCEPT

	Affirmative covenants	Negative covenants
Nature	Things the issuer must do (mostly administrative)	Things the issuer must not do (restrictions)
Examples	Deliver timely financial reports; use proceeds as stated; pay taxes and comply with laws; insure and maintain assets pledged as collateral; redeem at a premium if acquired; **cross-default clause** (a default on any other debt is a default on this bond); **pari passu clause** (this bond ranks equally with the issuer's other senior debt)	Limits on asset sales and **sale-and-leaseback** deals; no pledging the same collateral twice; no new debt senior to existing debt (**negative pledge clause**); limits on additional borrowing, share repurchases or dividends, often subject to an **incurrence test** (e.g., allowed only while debt/EBITDA is below a stated level)

Negative covenants protect bondholders by stopping actions that would raise default risk, but they must not be so tight that the firm cannot respond to opportunities or changing conditions.

Common exam traps

- Keeping collateral insured is affirmative (a required action). Not selling or re-pledging that collateral is negative (a restriction).
- Covenants are provisions inside the indenture. The indenture is the whole contract.
- Wording can mislead. Cross-default and pari passu clauses sound restrictive, yet they are affirmative covenants. A clause such as "dividends may be paid only while debt/EBITDA is below 2.5×" reads like a duty, but it limits what the issuer may do, so it is a negative covenant (an incurrence test).

EXAM SHORTCUTS

- A bond priced above par has a yield below its coupon rate, and a bond priced below par has a yield above it, so comparing the price with par answers a yield-versus-coupon question without any calculation.

BOTTOM LINE

- The coupon rate is the annual percentage of par paid as interest, so each periodic coupon equals the coupon rate times par value divided by the number of payments per year.
- A bond with an original maturity of one year or less is a money market security, one with an original maturity of more than one year is a capital market security, and a bond with no stated maturity is a perpetual bond.
- A zero-coupon (pure discount) bond pays no interest before maturity and is sold at a discount to par, so the investor's whole return is the gap between the price paid and the par value received.
- For a fixed-coupon bond, price and yield move in opposite directions; a premium bond yields less than its coupon rate and a discount bond yields more.
- Senior debt ranks ahead of junior (subordinated) debt in bankruptcy or liquidation, so junior debt carries more credit risk and must offer a higher yield.
- A bond's spread is its extra yield over a government bond of the same maturity, with the government yield curve used as the benchmark.
- The bond indenture (trust deed) is the legal contract between issuer and bondholders; it sets out the sources of repayment, any collateral, credit enhancements and the covenants.
- Affirmative covenants state what the issuer must do, including cross-default and pari passu clauses; negative covenants restrict what the issuer may do, including the negative pledge clause and limits subject to an incurrence test.

QUICK CHECK

Question 1

A dealer's inventory includes the three fixed-income securities described below. Which one is classified as a money market security?

A A note issued three years ago that matures in four months.

B A bill issued today that matures in nine months.

C A bond issued ten months ago that matures in 14 months.

Answer: B

Money market securities are those with an original maturity of one year or less. The bill issued today with nine months to maturity has an original maturity of nine months, so it is a money market security.

Why the other options are wrong

- **A.** The note had an original maturity of about three years and four months. Its short remaining maturity does not change its classification: it is a capital market security.

- **C.** The bond's original maturity is 10 + 14 = 24 months, which is longer than one year, so it is a capital market security.

Key takeaway Money market and capital market securities are classified by original maturity at issuance. The time remaining does not change the classification.

Practice questions

Question 2

Which of the following describes a bond that is trading at a premium?

A Its coupon rate is below its market yield.

B Its market price is above its par value.

C Its face value is below its redemption value.

Question 3

Government bond yields are 4.60% for 2 years, 4.10% for 7 years and 3.90% for 10 years. Ardent Rail's 7-year bond yields 5.85%. The spread on Ardent Rail's bond and the shape of the government yield curve are best described as:

A 1.25%; inverted yield curve.

B 1.75%; normal yield curve.

C 1.75%; inverted yield curve.

Answers: 2. B · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/bond-indenture/. The full bank has 17 questions on this reading.

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FIXED INCOME · READING 52

Zero Coupon Bond

What you'll learn

LOS 52.a Describe bullet, amortizing and sinking fund cash flow structures and coupon structures, and contrast embedded options that benefit issuers (call) with those that benefit investors (put, conversion).

LOS 52.b Describe how legal, regulatory and tax considerations (domestic, foreign, Eurobond and global bonds; securitization; taxation of interest, capital gains and OID) affect the issuance and trading of fixed-income securities.

MODULE 52.1

Fixed-Income Cash Flows and Types

This reading covers how bonds repay principal and pay coupons, the embedded options that can change those cash flows, and the legal, regulatory and tax factors that shape where and how bonds are issued. A candidate must be able to compute amortizing and balloon payments, floating-rate and inflation-linked coupons and conversion values, and say which party an embedded option benefits.

LOS 52.a — Cash flow structures and contingency provisions

Principal repayment structures

KEY CONCEPT

Structure	Periodic payments	At maturity
Bullet structure (non-amortizing)	Interest only (the coupons)	Final coupon plus the entire principal
Fully amortizing	Equal payments, each part interest and part principal	Nothing left to repay; the last equal payment retires the debt
Partially amortizing	Level payments that repay only part of the principal	Last payment plus a **balloon payment** of the remaining principal

Example. A $10,000, 4-year loan at 6% that is fully amortizing needs a level payment of

$$PMT = \frac{P \times i}{1-(1+i)^{-n}} = \frac{10{,}000 \times 0.06}{1-1.06^{-4}} = \$2{,}885.91$$

Calculator (END mode, P/Y = C/Y = 1): N = 4; I/Y = 6; PV = −10,000; FV = 0; CPT PMT = 2,885.91. The amortization schedule is then built one year at a time:

1. Interest for the year = opening balance × 6%.
2. Principal repaid = payment − interest.
3. Closing balance = opening balance − principal repaid. It becomes the next year's opening balance.

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Year	Opening balance	Payment	Interest	Principal repaid	Closing balance
1	10,000.00	2,885.91	600.00	2,285.91	7,714.09
2	7,714.09	2,885.91	462.85	2,423.06	5,291.03
3	5,291.03	2,885.91	317.46	2,568.45	2,722.58
4	2,722.58	2,885.93	163.35	2,722.58	0.00

The payment stays level while the balance falls, so the interest part shrinks and a larger part of each payment retires principal. The last payment is 2 cents higher because of rounding.

A partially amortizing version of the same loan that leaves a $2,000 balloon keeps the same inputs except FV = 2,000: N = 4; I/Y = 6; PV = −10,000; FV = 2,000; CPT PMT = 2,428.73. The final payment is $2{,}428.73 + 2{,}000 = \$4{,}428.73$. The three structures compare as follows:

Year	Bullet	Fully amortizing	Partially amortizing ($2,000 balloon)
1 to 3 (each year)	600.00	2,885.91	2,428.73
4	10,600.00	2,885.93	4,428.73

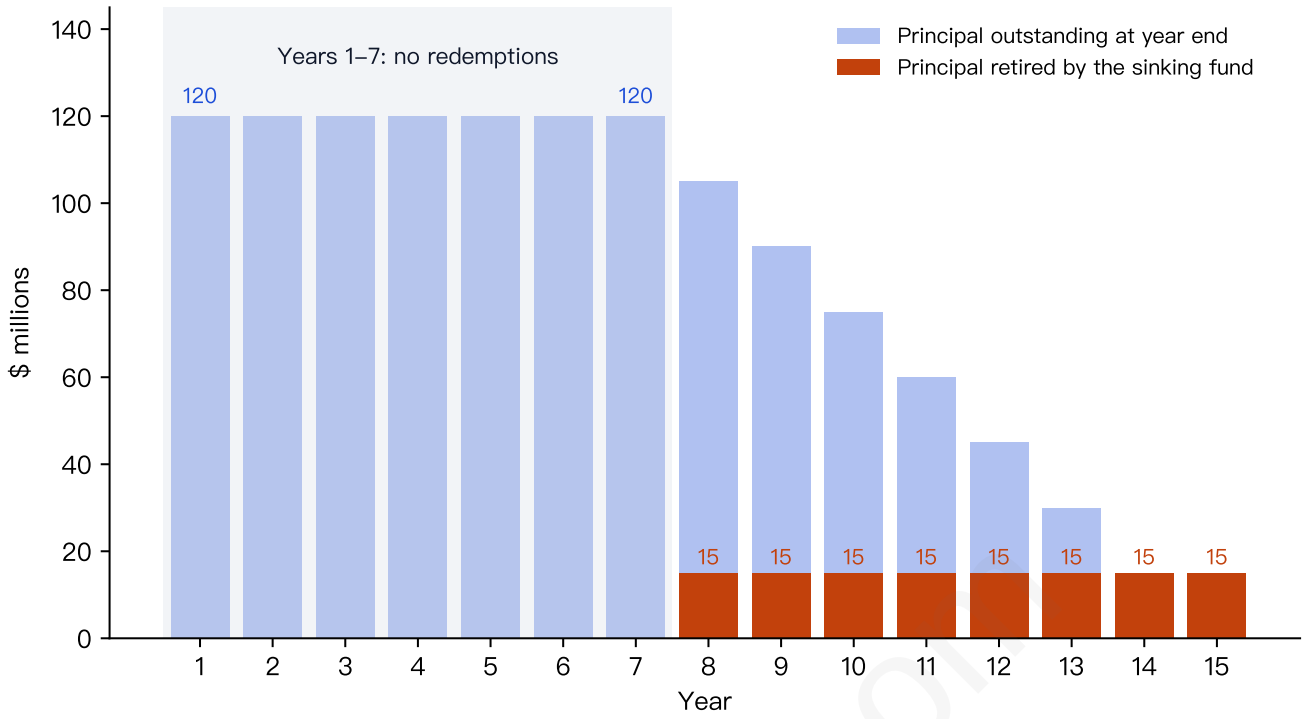
Compared with a fully amortizing loan, a partially amortizing one has lower regular payments and a higher final payment. It repays less principal along the way, so more principal stays outstanding, and its interest in every year after the first is higher (year-1 interest is the same, because no principal has been repaid yet).

A **sinking fund provision** obliges the issuer to pay off part of the issue according to a set schedule over the bond's life. Typically the bond trustee redeems a fixed amount of principal each year from holders chosen at random. For bondholders, a sinking fund has two effects:

- It lowers **credit risk**, because less principal is left to be repaid at maturity.
- It adds **reinvestment risk**, the risk of receiving cash early and having to reinvest it at lower yields. If rates have fallen, the bonds trade above par, yet holders whose bonds are drawn receive only par. Holders of premium bonds therefore prefer that their bonds are not drawn.

> *Example.* A $120 million, 15-year issue carries a sinking fund that retires $15 million of principal each year from year 8 through year 15. Nothing is redeemed in years 1 to 7, and the last $15 million is retired at maturity.

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Sinking fund schedule of an illustrative $120 million, 15-year issue

Amortizing structures carry the same trade-off, since a sinking fund is one form of amortization. The more principal a structure repays before maturity (bullet, then partially amortizing, then fully amortizing), the lower its credit risk and the higher its reinvestment risk.

A sinking fund is an obligation of the issuer and is not an embedded option. **Waterfall structures**, another form of amortization schedule, are used for ABS and MBS. They pay principal to senior **tranches** first. Junior tranches receive principal only after senior tranches are repaid, although all tranches still receive interest.

Coupon structures

- **Floating-rate notes (FRNs)** pay the **market reference rate (MRR)** plus a fixed margin (the **credit spread**) stated in basis points. Most reset and pay quarterly: the MRR observed at the start of a quarter sets the coupon paid at the end of that quarter. The per-period coupon is $(\text{MRR} + \text{margin})/\text{payments per year}$. *Example.* With an MRR of 3.10%, a margin of 60 bps and quarterly payments, the coupon is $(3.10\% + 0.60\%)/4 = 0.925\%$ of par.
- **Step-up coupon bonds**: the coupon rises on a predetermined schedule, which gives investors some protection against rising interest rates.
- **Credit-linked coupon bonds** and **credit-linked notes**: the coupon rises if the issuer's credit rating falls and falls if it improves. Leveraged loans may step up if debt/EBITDA rises. Some **green bonds** pay more if environmental targets are missed.
- **Payment-in-kind (PIK) bonds**: the issuer can pay interest by increasing the principal (paying with more bonds). They are used by highly leveraged issuers, so yields are higher.
- **Index-linked bonds** tie coupons or principal to a published index. **Inflation-linked bonds** (**linkers**) are the most common.
  - **Interest-indexed bonds**: the coupon rate is adjusted for inflation, and the principal is unchanged (not protected).

- **Capital-indexed bonds** (the most common type, e.g., US **Treasury Inflation-Protected Securities (TIPS)**): the coupon rate stays fixed, and the principal is adjusted up for inflation (down for deflation). The fixed coupon rate is effectively a **real rate**. Because the coupons and the principal repaid both rise with the index, the investor's purchasing power is fully protected over the bond's life. Adjusted principal can fall below par during the bond's life, but at maturity TIPS repay the greater of the inflation-adjusted principal and par.
- *Example.* A bond has €5,000 par and a 2% annual coupon paid semiannually. Inflation over the first six months is 1.5%, so the principal becomes $5{,}000 \times 1.015 = €5{,}075$ and the coupon is $5{,}075 \times 0.02/2 = €50.75$.

- **Zero-coupon bonds** make one payment of par at maturity, so they must trade below par to give investors a positive return. They appeal to investors who want to minimize reinvestment risk. A zero-coupon bond is the extreme case of a deferred coupon bond.
- **Deferred coupon bonds** pay no coupons for an initial period after issuance and then pay regular coupons to maturity. In some issues the skipped coupons accrue and are paid as one lump sum when the deferral period ends; otherwise the bond is simply sold at a deeper discount. Like zero-coupon bonds, they often trade below par. The structure suits low-rated issuers or projects that generate cash only later.

Periodic coupons also have to be reinvested, so reinvestment risk rises with the coupon rate, and a call feature adds to it. Holding a coupon bond to maturity does not remove it; only a zero-coupon bond held to maturity has none.

Contingency provisions (embedded options)

A **contingency provision** allows an action to be taken if a specified event occurs. In a bond indenture such provisions are **embedded options**: they are part of the bond contract and are not separate securities. An embedded option benefits whoever holds the right to exercise it. Bonds without them are **straight bonds** (**option-free bonds**).

KEY CONCEPT

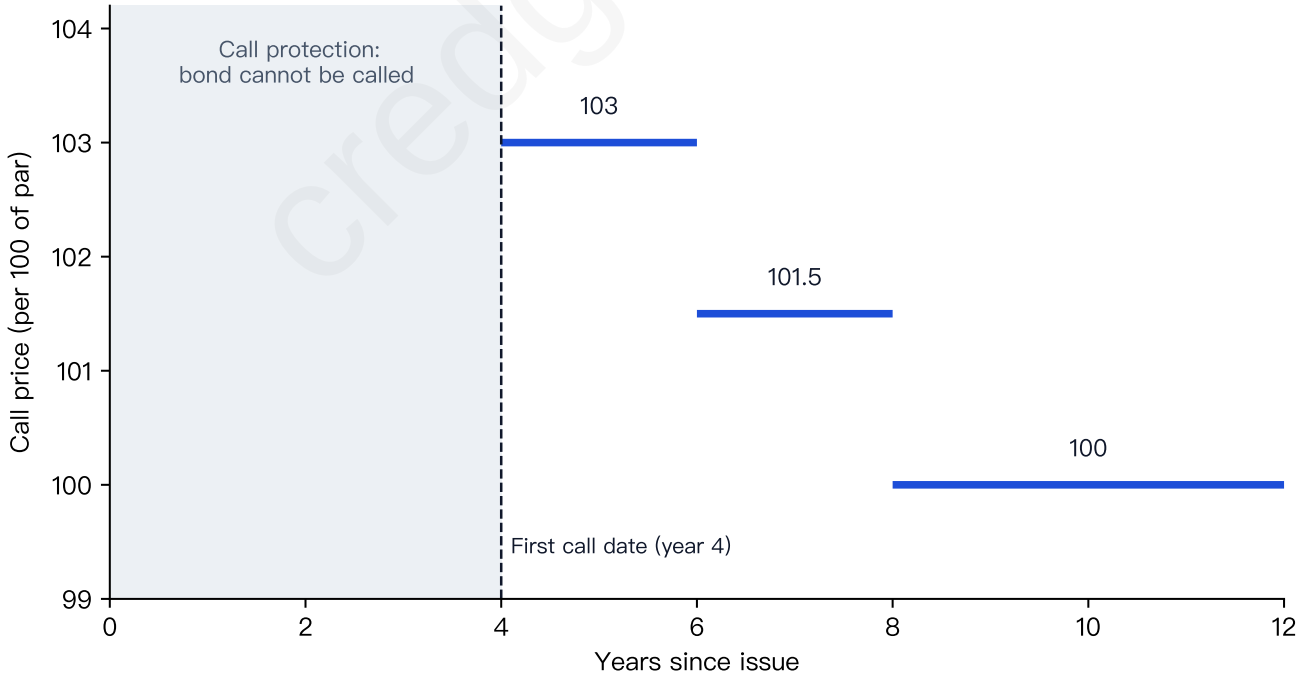
Bond	Who holds the option	Likely exercised when	Price vs straight bond	Yield vs straight bond
Callable bond (issuer may redeem early at the **call price**)	Issuer	The bond's market yield has fallen (rates dropped or the issuer's credit improved), so the issuer can refinance more cheaply	Lower: callable = straight − call	Higher
Putable bond (holder may sell back, usually at par)	Bondholder	The bond's price is below the put price (rates rose or the issuer's credit weakened)	Higher: putable = straight + put	Lower

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Bond	Who holds the option	Likely exercised when	Price vs straight bond	Yield vs straight bond
Convertible bond (holder may exchange for common shares)	Bondholder	The share price has risen, lifting the conversion value	Higher	Lower

- **Callable bonds**: the period before the first call date is **call protection**. The call price often steps down over time (e.g., 103, then 101.5, then par). Holders face **call risk**, and during the call period the call price caps the bond's market value. *Example.* A 5.5% bond with $1,000 par reaches its first call date, callable at 101, just as its market yield falls to 3.5%. The issuer pays $1,010 per bond and borrows that amount at 3.5%, which cuts annual interest from $55 to $1{,}010 \times 0.035 = \$35.35$ per bond. The holder receives $1,010 early and can reinvest it only at the lower yield.
- **Convertible bonds**: the **conversion price** is the par amount per share received, the **conversion ratio** is par divided by the conversion price, and the **conversion value** is the conversion ratio times the current share price. *Example.* A $1,000 par bond with a conversion price of $25 converts into 40 shares. At a share price of $28, the conversion value is $40 \times 28 = \$1{,}120$.
- **Warrants** attached to straight bonds let holders buy common shares at a fixed price. They can be detached and traded separately.
- **Contingent convertible bonds (CoCos)** convert automatically into equity if a trigger occurs (e.g., a bank's equity capital falls below a required level).

A typical call schedule steps down toward par after a period of call protection:



Call price schedule of an illustrative 12-year callable bond

Options are worth more when the underlying is more volatile. Higher interest rate volatility therefore raises the value of the call held by the issuer, so the callable bond's value falls. It also raises the value of the put held by the investor, so the putable bond's value rises.

COMMON EXAM TRAPS

- Level payments that leave nothing owing at maturity mean the bond is fully amortizing. Level payments followed by a balloon mean it is partially amortizing, and "bullet" means all principal is repaid at maturity.
- For a partially amortizing loan, enter the balloon as FV. Leaving FV = 0 gives the fully amortizing payment instead. For the $10,000, 4-year, 6% loan with a $2,000 balloon, FV = 0 gives a payment of $2,885.91 instead of $2,428.73.
- A sinking fund is an obligation, and it can hurt holders of premium bonds.
- TIPS have a fixed coupon rate and an adjusted principal. The coupon amount changes because the rate is applied to the adjusted principal. In the €5,000 example, applying the 2% rate to par gives a coupon of €50.00 instead of €50.75.

LOS 52.b — Legal, regulatory and tax considerations

Where bonds are issued and traded

KEY CONCEPT

Type	Definition
Domestic bonds	Issuer domiciled in the same country as the market where the bonds are issued and traded
Foreign bonds	Issued by an issuer from another country and traded in that country's domestic market (e.g., a UK firm selling US dollar bonds that trade in the US)
Eurobonds	Issued outside the jurisdiction of any one country, in any currency; less regulated than domestic bonds in most jurisdictions (they began as a way around US regulation). Named by currency: **Eurodollar bonds** (USD), **Euroyen bonds** (yen)
Global bonds	Trade in the Eurobond market and in at least one domestic market

The "Euro" in Eurobond does not mean the bonds are denominated in euros, sold in Europe or issued by a European firm; the name comes from the market's origins in Europe, where most Eurobonds are still traded. Foreign bonds, global bonds and Eurobonds are collectively **international bonds**. **Bearer bonds** (ownership shown simply by possession, with no record kept by the issuer) were once the norm for Eurobonds. Today most bonds are **registered bonds**. Markets also differ in laws, regulation and conventions such as coupon frequency, but the currency of a bond is the factor that best explains yield differences across markets, because a bond's yield is driven by interest rates in the country of its currency. **Sukuk** are Sharia-compliant bonds whose payments are treated as rent on underlying assets.

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Taxation of bond income

- Interest income is usually taxed as **ordinary income**. In the US, interest on **municipal bonds** is usually not subject to national income tax, and often not to state tax in the issuing state either.
- Gains or losses on selling a bond before maturity are **capital gains** or losses. They are often taxed at lower rates, and lower still if the gain is long-term.
- **Original issue discount (OID)** bonds (e.g., zero-coupon bonds): the rise in value toward par is really interest, so in many jurisdictions part of the discount is taxed each year as interest income even though no cash is received.
- Some jurisdictions give bonds issued at a **premium** the symmetric treatment: part of the premium is used to reduce the taxable portion of the coupon interest.

COMMON EXAM TRAPS

- A euro-denominated bond sold in the euro area by a local issuer is a domestic bond.
- A bond sold in a country's own currency and market by a foreign issuer is a foreign bond, not a Eurobond.
- Currency alone does not decide the category. A Eurobond is defined by issuance outside the jurisdiction of any one country, and it can be in any currency.
- The accretion on a zero-coupon bond is interest income and is not treated as a capital gain.

EXAM SHORTCUTS

- A partially amortizing payment uses the same calculator entries as the fully amortizing payment, with the balloon entered as FV instead of zero.
- Only a zero-coupon bond held to maturity has no reinvestment risk, so any coupon bond can be eliminated from that answer even if it is held to maturity.
- A callable bond is worth less and a putable bond more than an otherwise identical straight bond, so the three can be ranked by price, and in reverse by yield, without valuing any of them.

BOTTOM LINE

- A bullet bond pays only interest before maturity and repays all principal at maturity; a fully amortizing bond's level payments retire the debt by the last payment; a partially amortizing bond's level payments leave a balloon payment at maturity.
- A sinking fund provision obliges the issuer to retire part of the issue on a set schedule, which lowers bondholders' credit risk but adds reinvestment risk.
- A floating-rate note pays the market reference rate plus a fixed margin, so its per-period coupon is (MRR + margin) divided by the number of payments per year.
- A capital-indexed bond such as TIPS keeps a fixed coupon rate, effectively a real rate, and adjusts its principal for inflation, so each coupon is that rate applied to the adjusted principal; an interest-indexed bond adjusts the coupon rate and leaves the principal unprotected.
- An embedded option benefits whoever holds the right to exercise it: callable bond = straight bond − call, with a higher yield, and putable bond = straight bond + put, with a lower yield.
- For a convertible bond, the conversion ratio is par divided by the conversion price, and the conversion value is the conversion ratio times the current share price.

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- A foreign bond is issued by a foreign issuer and traded in another country's domestic market, while a Eurobond is issued beyond the jurisdiction of any single country and may be denominated in any currency.
- On an original issue discount bond such as a zero-coupon bond, the rise in value toward par is interest, and many jurisdictions tax part of the discount each year as interest income even though no cash is received.

QUICK CHECK

Question 1

A bond with a par value of $2,000 carries a 6.8% coupon rate, paid semiannually, and is quoted at a price of 104.35. The dollar amount of each semiannual coupon is:

A $136.00.

B $70.96.

C $68.00.

Answer: C

Coupons are based on par value, and the market price plays no part. Half of the annual coupon is paid every six months.

$$\text{Semiannual coupon} = \frac{\text{coupon rate} \times \text{par}}{2} = \frac{0.068 \times \$2{,}000}{2} = \$68.00$$

The quoted price of 104.35 (104.35% of par) does not enter the calculation.

Why the other options are wrong

- **A.** $136.00 is the full annual coupon; it is not divided by two for semiannual payments.
- **B.** $70.96 applies the coupon rate to the market price ($2{,}000 \times 1.0435$) instead of to the par value.

Key takeaway The price quote is irrelevant to the coupon amount. Each coupon equals the coupon rate times par, divided by the number of payments per year.

Practice questions

Question 2

Which statement about a sinking fund provision is *most accurate*?

A It obliges the issuer to deposit cash in a reserve that is used to repay the whole issue at maturity.

B It gives bondholders the right to sell their bonds back to the issuer at par each year.

C It obliges the issuer to retire part of the issue at scheduled dates before the final maturity.

Question 3

Priya Nair's main concern in her bond portfolio is reinvestment risk. Which of the following is she *least likely* to do?

A Choose a noncallable bond over an otherwise identical callable bond.

B Choose a low-coupon bond over a high-coupon bond of the same maturity.

C Rely on holding a coupon-paying bond to maturity to remove the risk.

Question 4

Which of the following bond issues best fits the definition of a Eurobond?

A A Mexican firm's Swiss franc bonds sold to investors in Switzerland.

B An Indian firm's Australian dollar bonds sold to investors in Singapore.

C A German firm's euro-denominated bonds sold to investors in Germany.

Question 5

Maren Holt buys a newly issued 8-year corporate bond at 106.40 per 100 of par. It pays a 6.5% annual coupon, and she plans to hold it to maturity. Her country taxes coupon interest as ordinary income and gives bonds issued at a premium the treatment that mirrors its original issue discount (OID) provision. Each year, the tax treatment of her coupons is *most likely* that:

A part of the 6.40 premium reduces the taxable portion of her coupon interest.

B the full 6.50 coupon is taxed as ordinary income, with no adjustment for the premium.

C part of the 6.40 premium is added to her taxable interest income.

Answers: 2. C · 3. C · 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/zero-coupon-bond/. The full bank has 42 questions on this reading.

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FIXED INCOME · READING 53

Fixed-Income Issuance and Trading

What you'll learn

LOS 53.a Describe how bond markets are segmented (issuer type, credit quality, original maturity and other features) and where issuers and investors sit on the credit/maturity spectrum.

LOS 53.b Describe aggregate and narrower fixed-income indexes and how bond indexes differ from equity indexes (constituents, turnover, weights, benchmark fit).

LOS 53.c Compare primary bond issuance (public offerings, private placements, underwritten, best-efforts, shelf, auctions) and OTC secondary trading with equity markets.

MODULE 53.1

Fixed-Income Issuance and Trading

This reading maps the bond market by issuer type, credit quality and original maturity, shows where different issuers and investors sit on that map, and compares bond indexes and bond trading with their equity counterparts. A candidate must be able to classify a bond by segment, match a funding need or an investor to a part of the spectrum, and explain how primary offerings and dealer markets work.

LOS 53.a — Market segments, issuers and investors

Global bond markets are split mainly along three lines: the **type of issuer** (also called the sector), **credit quality** and **original maturity**. Other ways to classify bonds include currency, the issuer's geography (for example **developed market** versus **emerging market** bonds) and environmental, social and governance (**ESG**) features.

KEY CONCEPT

Dimension	Main categories
Type of issuer	Governments (sovereign and non-sovereign), corporates, and special purpose entities issuing **asset-backed securities (ABSs)**
Credit quality	**Investment-grade bonds**: AAA down to BBB− (S&P) or Aaa down to Baa3 (Moody's). **High-yield bonds** (speculative or "junk"): BB+ or lower (S&P), Ba1 or lower (Moody's)
Original maturity	Short-term, 1 year or less (**money market securities**); intermediate-term, 1 to 10 years; long-term, over 10 years

Credit ratings come from **credit rating agencies** such as S&P and Moody's.

Exam convention: money market securities have an original maturity of one year or less; the same segment is also described as maturities of less than one year. Current practice: the money market label is used for instruments issued with maturities of up to about one year, such as Treasury bills, commercial paper and repos, so the two descriptions differ only for an original maturity of exactly one year.

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The issuer credit/maturity spectrum

Credit segment	Short-term (≤ 1 year)	Intermediate-term (1–10 years)	Long-term (> 10 years)
"Default risk free"	Treasury bills	Treasury notes	Treasury bonds
Investment grade	Repos, **commercial paper**, **asset-backed commercial paper (ABCP)**	Unsecured corporate bonds, ABS	Unsecured corporate bonds, MBS
High yield	—	Secured corporate bonds, **leveraged loans**	—

- Secured new corporate issues sit in the high-yield row. An issuer with less dependable operating cash flows has to pledge collateral to attract lenders.
- ABCP is commercial paper backed by financial assets, so it counts as a short-term asset-backed security. The spectrum places it in the investment-grade row, not in the default-risk-free row.
- **Fallen angels** are bonds of issuers that were investment grade but have since been downgraded to high yield because their credit quality deteriorated. They are part of the high-yield sector.

Issuer choices

What a corporation issues depends on its access to capital markets and on what the money is for. A well-established investment-grade company can match funding to use:

- **Commercial paper** funds short-term (including seasonal) working capital needs.
- **Intermediate-term debt** funds medium-term investments and permanent working capital.
- **Long-term debt** funds capital investment in fixed assets.

Short- and medium-term borrowing is often arranged, for a fee, with a **syndicate** of banks that provides **credit facilities**, so the company can draw funds when it needs them. For a riskier company whose operating cash flows are less stable, access to secured short-term financing and leveraged loans is likely to be limited.

Investor positioning

Investors choose where to sit on the spectrum according to the interest rate and credit risk they want and the timing of the obligations they must meet.

Investor	Typical position	Why
Pension funds, insurance companies	Long-term, investment grade	Match long-dated liabilities (pensions, claims); regulation often bars high-yield holdings
Money market funds	Short-term, at the boundary between default-risk-free and investment-grade instruments	Hold short, high-quality paper for liquidity
Corporations	Short-term investment grade (commercial paper, repos, ABCP)	Earn a return on excess liquidity

Investor	Typical position	Why
Central banks	Intermediate-term Treasury notes	Monetary policy: buying adds to banks' reserves, selling drains them
Bond funds and ETFs	Intermediate investment grade (excluding Treasuries), per mandate	Stated fund objective
Asset managers seeking higher returns, hedge funds, **distressed debt** funds	Intermediate high yield; distressed debt at the lowest credit quality	Higher expected return
Financial intermediaries (banks)	Treasuries across all maturities	Manage interest rate and liquidity risk

Example. Orchard Valley Foods, rated A, has three funding needs: (1) a build-up of inventory ahead of its peak selling season, repaid within five months; (2) a four-year warehouse-automation project; (3) a processing plant with a 30-year useful life. The natural matches are commercial paper for (1), an intermediate-term note for (2), and a long-term bond for (3).

COMMON EXAM TRAPS

- BBB− (S&P) and Baa3 (Moody's) are the lowest investment-grade ratings. BB+ and Ba1 are the highest high-yield ratings.
- Classifying a bond as a money market security because little time is left to maturity. Only the original maturity (one year or less) counts.
- A fallen angel has been downgraded to high yield. That alone does not make it distressed debt, which requires the issuer to be in, or expected to file for, bankruptcy.
- Placing central banks in the short-term segment. On the investor map they sit in intermediate-term Treasury notes.

LOS 53.b — Fixed-income indexes

Bond indexes differ from equity indexes in three ways:

1. *More constituents.* One corporate issuer may have dozens of bonds outstanding but only two or three share classes. Because a full replica would be impractical, bond index tracker funds use **sampling** to keep transaction complexity reasonable.
2. *Higher turnover.* Bonds mature, get called or retired by sinking funds, and new bonds are issued far more often than new shares appear. Constituents are therefore removed and replaced (**turnover**) more often than in equity indexes, and the bond universe is less stable than the stock universe.
3. *Weights driven by issuance.* Governments are usually the largest borrowers, so broad bond indexes carry large weights in **sovereign bonds**. As issuance trends in maturity and credit quality shift, index weights shift with them.

KEY CONCEPT

Feature	Equity index	Bond index
Number of constituents	Fewer	Many more
Turnover	Lower	Higher
Tracking method	Full replication common	Sampling
Largest weights	Largest listed companies	Usually sovereign issuers

Aggregate indexes hold a broad selection of bonds across many sectors and currencies, but they still apply entry rules. Narrower indexes concentrate on a particular geography, sector, credit quality or maturity range, and some also apply **ESG** screens. Three well-known indexes show the range:

Index	Type	What it holds	Credit rule	Other rules
Bloomberg Barclays Aggregate Index	Aggregate	Bonds from all sectors in 28 currencies	High-yield and unrated issues excluded	Minimum issue size
JP Morgan Emerging Markets Bond Index Plus	Narrower: geography and credit quality	US dollar-denominated emerging market sovereign debt	Baa1/BBB+ or lower	Minimum size and maturity limits
Bloomberg Barclays MSCI Euro Corporate Sustainable SRI Index	Narrower: ESG	Euro corporate bonds	Baa3/BBB– or higher	Minimum ESG rating of BBB; sectors such as alcohol and thermal coal generation screened out

A bond fund's benchmark should have the same sector focus, credit quality and maturity profile as the fund.

COMMON EXAM TRAPS

- Reversing the index comparison. Bond indexes have more constituents and higher turnover than equity indexes.
- An aggregate index is broad but does not hold everything. High-yield, unrated and small issues are left out.
- The benchmark-matching dimensions are sector, credit quality and maturity. Coupon level and return volatility are not on the list.

LOS 53.c — Primary and secondary bond markets

Primary market

In a primary market transaction the issuer sells newly created bonds and receives the cash. Two routes are available:

- **Public offering**: the issue is registered with securities regulators and sold to the public.

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- **Private placement**: the issue is sold only to one investor or a small group of selected investors.

Both routes normally use investment banks as intermediaries. A company selling its first-ever bond is a **debut issuer**. It is typically a growing firm replacing bank loans with bonds. A debut issue needs weeks of investor **roadshows** and is usually slower to complete than later issues. **Repeat issues** are much quicker. When an investment-grade issuer that borrows frequently has a fully underwritten shelf registration, a deal can be agreed, completed and allocated within hours, because investors already know the issuer and the regulatory registrations are already in place. A small high-yield issue with detailed covenants and collateral, sold on a best-efforts basis, takes far longer.

KEY CONCEPT

Method	Price risk	Key feature
Underwritten offering	Intermediaries guarantee the issue price	Bank bears the risk of unsold bonds
Best-efforts offering	No price guarantee	Bank earns a commission for placing the bonds at the best price it can
Shelf registration	Either	Aggregate amount registered once with a master prospectus; bonds issued over time as funds are needed
Public auction	Set by bids	Used for some bonds, especially government bonds

Secondary market

In the secondary market investors trade bonds that are already outstanding. Some trading uses electronic platforms or exchanges, but most bond trading is in **over-the-counter (OTC)** dealer markets. Most equity trading, by contrast, happens on exchanges. Dealers quote a **bid** price (at which they buy) and an **ask** or **offer** price (at which they sell). The ask is always above the bid, and the difference is the **dealer's spread**. It depends on **liquidity**:

- A fraction of a basis point applies to liquid, recently issued (**on-the-run**) developed-market sovereign bonds and to bonds of high-quality frequent corporate issuers.
- 10–20 basis points or more applies to less liquid, smaller or older (**seasoned**) corporate issues.

The dealer's spread therefore signals how liquid an issue is.

Example. A dealer quotes a bond at 97.20 bid and 97.32 ask per 100 of par. The spread is 0.12 per 100 of par, or 12 basis points. That is typical of a seasoned, thinly traded corporate issue. An on-the-run government bond would be quoted far more tightly.

Distressed debt is the debt of issuers that are in bankruptcy or expected to file for it. It has a low credit rating, high risk and a high promised yield. Distressed debt investors often buy from institutions that are not allowed to hold low-rated bonds. They hope to profit from a turnaround, a better-than-expected recovery, or a value-creating restructuring. Entering distress can temporarily lift trading in a normally quiet issue.

COMMON EXAM TRAPS

- In an underwritten offering the price is guaranteed. In a best-efforts offering the bank earns a commission and gives no guarantee. Shelf registration concerns timing (issuing over time) and says nothing about price risk.
- Selling an entire issue to a single investor is a private placement.
- National governments are the typical auction issuers. Corporations and municipalities usually sell through intermediaries.
- A wide bid-ask spread signals illiquidity. On its own it says nothing definite about credit rating or issuer type.

BOTTOM LINE

- Bond markets are segmented mainly by type of issuer (sector), credit quality and original maturity.
- Investment grade means a rating from AAA down to BBB– at S&P, or from Aaa down to Baa3 at Moody's; a rating of BB+ or below (Ba1 or below at Moody's) is high yield.
- Exam convention: money market securities have an original maturity of one year or less, intermediate-term ones between 1 and 10 years, and long-term ones more than 10 years; current practice applies the money market label to instruments issued with maturities of up to about one year, such as Treasury bills, commercial paper and repos.
- Fallen angels are bonds of issuers downgraded from investment grade to high yield because their credit quality deteriorated, while distressed debt is the debt of issuers that are in bankruptcy or expected to file for it.
- A well-established investment-grade company can fund short-term (including seasonal) working capital with commercial paper, medium-term investments and permanent working capital with intermediate-term debt, and fixed-asset investment with long-term debt.
- Compared with equity indexes, bond indexes have many more constituents and higher turnover, tracker funds use sampling, and broad bond indexes usually carry large weights in sovereign bonds.
- In an underwritten offering the intermediaries guarantee the issue price; in a best-efforts offering they earn a commission with no price guarantee; a shelf registration registers the aggregate amount once and lets the bonds be issued over time.
- Most secondary bond trading takes place in over-the-counter dealer markets, and the dealer's spread between ask and bid widens as liquidity falls: a fraction of 1 bp for on-the-run sovereign debt of developed markets, and 10–20 bp or more for seasoned, smaller or less liquid corporate issues.

QUICK CHECK

Question 1

A bond database lets users group bonds along several dimensions: type of issuer (sector), credit quality, original maturity and currency. Which of the following groups results from sorting bonds by type of issuer?

A Bonds denominated in euros.

B Short-term money market securities.

C Bonds issued by special purpose entities.

Answer: C

Classification by type of issuer (sector) separates governments (sovereign and non-sovereign), corporates, and special purpose entities issuing asset-backed securities. Bonds issued by special purpose entities are grouped by who the issuer is, so this group results from sorting by issuer. The other two groups come from the currency and original-maturity dimensions.

Why the other options are wrong

- **A.** Bonds denominated in euros are grouped by currency, which is one of the other ways to classify bonds. Any type of issuer can sell bonds in euros.
- **B.** Money market securities are defined by original maturity (one year or less). Any type of issuer can sell them.

Key takeaway Sort the labels: type of issuer (governments, corporates, special purpose entities issuing ABSs), credit quality, original maturity (money market, intermediate, long-term), and other dimensions such as currency, issuer geography and ESG features.

Practice questions

Question 2

An analyst compares bond market indexes with stock market indexes. Which of the following statements is *least accurate*?

A A bond index typically contains far more constituents than a stock index.

B A single company often has many bonds outstanding but only two or three share classes.

C The set of bonds in an index changes less often than the set of stocks in a stock index.

Question 3

Which of the following issuers *most likely* sells its new bonds through a public auction?

A A national government.

B A corporation.

C A municipality.

Question 4

Tamsin Hale wants to buy $2 million face value of a mid-sized corporate bond that was issued six years ago and now trades only infrequently. She is *most likely* to:

A submit a competitive bid in the issuer's next bond auction.

B buy from a dealer at its ask price, paying a relatively wide spread.

C buy on an exchange order book, paying a spread similar to an on-the-run sovereign bond.

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Answers: 2. C · 3. A · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/fixed-income-issuance-and-trading/. The full bank has 26 questions on this reading.

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FIXED INCOME · READING 54

Commercial Paper

What you'll learn

LOS 54.a Compare short-term funding sources for nonfinancial corporations (lines of credit, secured loans, factoring, commercial paper) and for financial institutions (deposits, CDs, interbank and central bank funds, CP/ABCP).

LOS 54.b Describe repurchase agreements: mechanics (purchase price, repo rate, initial and variation margin, haircut), uses, repo-rate drivers and risks.

LOS 54.c Contrast long-term funding of investment-grade and high-yield corporate issuers (yields and spreads, covenants, collateral, maturity, standardization, prepayment features).

MODULE 54.1

Fixed-Income Markets for Corporate Issuers

This reading compares the short-term funding sources open to nonfinancial companies and to financial institutions, explains how repurchase agreements work, and contrasts long-term funding for investment-grade and high-yield issuers. A candidate must be able to rank lines of credit by reliability, compute a repo's purchase price, haircut, repurchase price and variation margin, and state how high-yield issues differ from investment-grade issues.

LOS 54.a — Short-term funding alternatives

Nonfinancial corporations

A nonfinancial company funds short-term assets (cash, short-term investments, receivables and inventory) through intermediaries. The intermediaries supply this funding in two forms: loan financing and security-based financing.

Bank lines of credit let a borrower draw funds as needed, usually at a floating **market reference rate (MRR)** plus a fixed credit spread.

KEY CONCEPT

Type	Bank's commitment	Cost	Reliability
Uncommitted line of credit	Bank may refuse to lend if circumstances change	Usually interest only, no fees; flexible, and may be unsecured if the borrower keeps stable cash balances at the bank	Least reliable
Committed (regular) line of credit	Bank commits for a set period	Interest plus a **commitment fee** (about 50 bp on the full or unused amount)	More reliable; **renewal risk** at maturity
Revolving (operating) line of credit	Longer commitment, sometimes years	Similar to a committed line; **restrictive covenants** usually imposed	Most reliable

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Banks hold more regulatory reserves against committed lines. They manage that risk by keeping commitments under one year or by lending in a **syndicate**.

- **Secured (asset-backed) loans**: weaker borrowers pledge fixed assets, receivables or inventory as collateral. Receivables can also be assigned to a lender as collateral.
- **Factoring**: the company actually transfers its receivables, together with the credit-granting and collection functions, to a **factor**, which pays less than face value. The discount is effectively the interest rate and depends on the customers' creditworthiness and on collection costs.
- **Commercial paper (CP)**: short-term unsecured debt issued by large, highly rated firms, usually maturing in under three months. For these issuers it is cheaper than a bank loan. CP funds working capital and serves as **bridge financing** until permanent (long-term) financing is arranged. CP is normally rolled over at maturity. The danger of being unable to sell new paper to repay maturing paper is **rollover risk**, which issuers manage with **backup lines of credit** (liquidity enhancement). US CP is a pure discount security, like a T-bill. A smaller, less liquid international market is **Eurocommercial paper (ECP)**.

Financial institutions

- Deposits: **demand deposits** (checking accounts, immediate access, usually no interest), **operational deposits** (large customers needing cash management, custody and clearing) and **savings deposits**, which carry a stated term and interest rate. A **certificate of deposit (CD)** has a stated rate and maturity of less than a year. A **nonnegotiable CD** carries a penalty for early withdrawal. A **negotiable CD** can be sold in the open market, and wholesale negotiable CDs, traded in domestic bond markets and in the Eurobond market, are an important bank funding source.
- **Interbank funds**: loans from one bank to another for one day to one year, secured or unsecured, priced off an MRR such as SOFR. The most common secured form is the repo.
- **Central bank funds market**: banks with **excess reserves** at the central bank lend them to banks that are short. The rate is the **central bank funds rate**, which the central bank's open market operations strongly influence. Borrowing directly from the central bank as **lender of last resort** ("discount window lending") costs more and brings extra scrutiny.
- CP and ABCP: financial institutions issue more CP than nonfinancial companies. They also sponsor **asset-backed commercial paper (ABCP)**, which is created in two steps. (1) The bank transfers short-term loans it has made to an off-balance-sheet **special purpose entity (SPE)** in exchange for cash. (2) The SPE sells ABCP to investors, who take on the risk and return of those loans, and the bank provides a backup liquidity line.

COMMON EXAM TRAPS

- Reliability rises from uncommitted to committed to revolving lines of credit, and revolving lines usually add restrictive covenants. Committed and revolving lines carry similar fees, including a commitment fee; an uncommitted line usually costs only the interest on amounts drawn.
- Assigning receivables as collateral is not factoring. Factoring transfers the receivables and their collection to the factor.
- Excess reserves lent bank-to-bank earn the central bank funds rate. Borrowing from the central bank itself is discount window (lender of last resort) lending.

LOS 54.b — Repurchase agreements

In a **repurchase agreement (repo)**, one party sells a security and commits to repurchase it at a later date for a higher price fixed in advance. Economically, the security buyer lends cash to the security seller and holds the security as collateral.

Leg	Security buyer (cash lender)	Security seller (cash borrower)
Opening (today)	Pays the purchase price (loan amount)	Delivers the collateral securities
Closing (the **repurchase date**)	Returns the collateral	Pays the repurchase price (loan plus interest)

The **repo rate** is the annualized interest rate implied by the gap between the repurchase price and the purchase price. The borrower keeps the economic benefits of the bond, such as coupons, during the term.

Initial margin protects the lender. Collateral worth more than the loan must be posted, so the loan is a discount to the market value:

KEY CONCEPT

$$\text{purchase price} = \frac{\text{market value of collateral}}{\text{initial margin}}$$

$$\text{repurchase price} = \text{purchase price} \times \left(1 + \text{repo rate} \times \frac{\text{days}}{360}\right)$$

$$\text{haircut} = \frac{\text{market value} - \text{purchase price}}{\text{market value}} = 1 - \frac{1}{\text{initial margin}}$$

The loan value grows at the repo rate. If the collateral's value drops below (accrued loan value × initial margin), the lender calls for **variation margin**:

$$\text{variation margin} = \text{initial margin} \times \text{adjusted purchase price} - \text{market value of collateral}$$

A negative result signals overcollateralization, so the borrower may request the return of some collateral.

Example. Collateral is worth \$40 million, the initial margin is 105%, the repo rate is 3% and the term is 30 days (360-day year).

1. Loan amount: the purchase price is $40{,}000{,}000/1.05 = \$38{,}095{,}238$.
2. Haircut: $1 - 1/1.05 = 4.76\%$.
3. Amount due on the repurchase date: $38{,}095{,}238 \times (1 + 0.03 \times 30/360) = \$38{,}190{,}476$.
4. Margin check during the term. After 10 days the loan has grown to $38{,}095{,}238 \times (1 + 0.03 \times 10/360) = \$38{,}126{,}984$, so the required collateral is $38{,}126{,}984 \times 1.05 = \$40{,}033{,}333$. If the collateral is then worth \$39.4 million, the borrower must post variation margin of about \$633,333. If it is worth \$40.5 million instead, variation margin is $40{,}033{,}333 - 40{,}500{,}000 = -\$466{,}667$, so the borrower can ask for about \$466,667 of collateral to be released.

Term	Meaning
Overnight repo	Lasts one day
Term repo	Lasts longer than one day
General collateral repo	Any security of a given type is acceptable, such as Treasuries in a maturity range
Master repurchase agreement	The document that holds the contract terms
Tri-party repo	A custodian bank or clearinghouse acts as agent
Bilateral repo	Struck directly between the two parties

Uses

- Dealers and other financial institutions borrow through repos (as security sellers) to finance their securities positions.
- Banks, mutual funds and pension funds lend (as security buyers) to earn the repo rate on excess cash.
- Central banks lend (buy) to add money and borrow (sell) to drain it.
- Short sellers such as hedge funds buy the security and lend cash in a repo, sell the security short, buy it back later and return it at maturity. A trade entered for the purpose of borrowing a security in this way is a **reverse repo** from that participant's side. Asking for a specific security is a **special trade**. For a hard-to-borrow security the lender of cash accepts a lower, sometimes negative, repo rate.

What drives the repo rate?

Factor	Effect on repo rate
Higher rates on alternative money market funding	Higher
Higher credit quality of collateral	Lower
Longer repo term (when longer rates exceed short rates)	Higher
Collateral in high demand or low supply	Lower
Undercollateralized, or collateral specified but not delivered	Higher

Because repos are short-term and collateralized, usually by high-quality sovereign bonds, repo rates are normally below bank loan rates.

A repo is safer than most short-term lending, but it is still debt for the borrower, and heavy reliance on it can lead to financial distress. Repos carry **default risk** (the cash borrower fails to repurchase), **collateral risk**, **margining risk**, **legal risk**, and **netting and settlement risk**. A tri-party repo helps mitigate many of these risks and improves cost efficiency in accessing, valuing and safekeeping collateral, but it does not reduce credit risk.

COMMON EXAM TRAPS

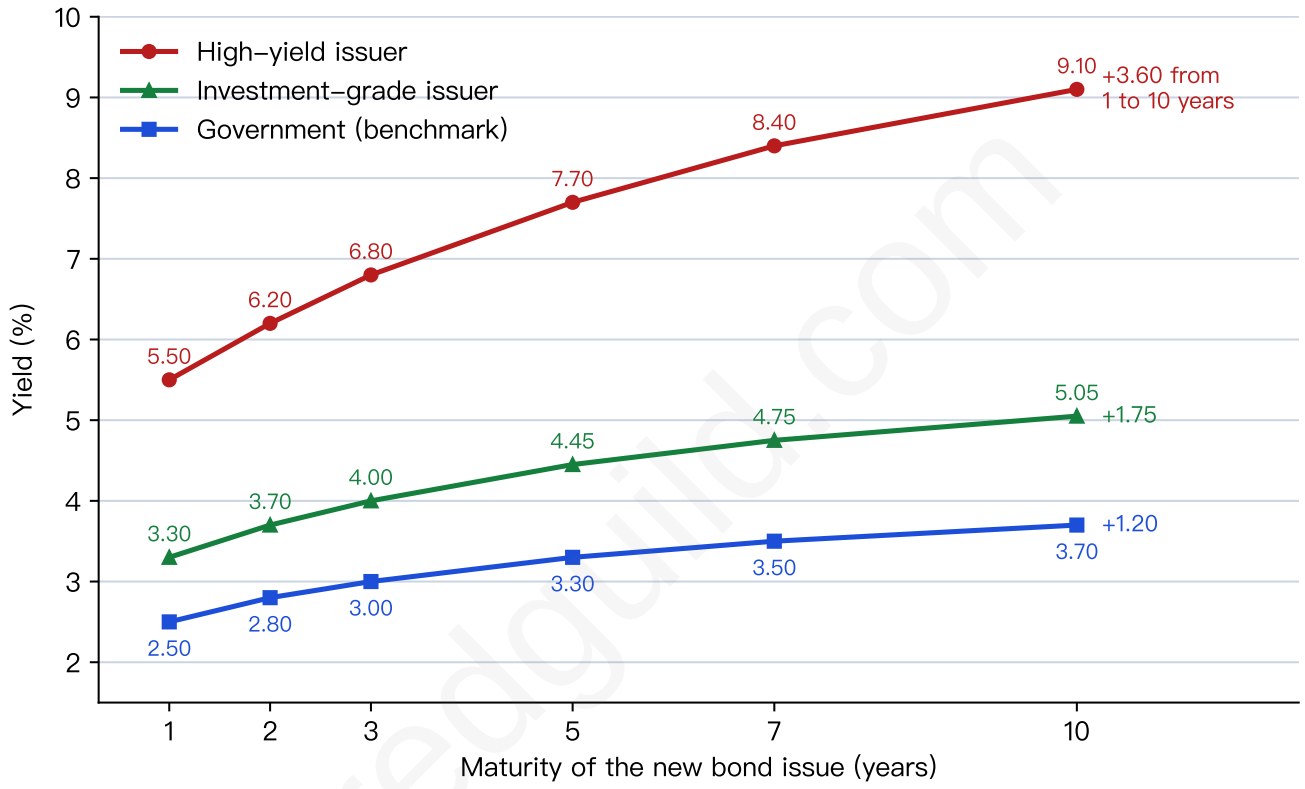
- Divide by the initial margin to get the loan amount. The haircut is not the margin in excess of 100%: a 105% initial margin means a haircut of 4.76%, not 5%. Applying a 5% haircut to the $40 million of collateral in the example gives a loan of $38,000,000 instead of $38,095,238.

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- The borrower in a repo is the security seller. A participant who buys the security to obtain it is in a reverse repo.
- Scarce ("special") collateral lowers the repo rate the cash lender accepts.

LOS 54.c — Investment-grade vs. high-yield long-term funding

With a normal (upward-sloping) yield curve, both investment-grade and high-yield issuers must pay higher yields at longer maturities. The increase is bigger for high-yield issuers because their credit spreads are larger. Issuing shorter bonds to save on yield adds rollover risk.



Yields on new issues by maturity for government, investment-grade and high-yield borrowers

In the figure, moving from a 1-year to a 10-year issue adds 1.75 percentage points to the investment-grade yield and 3.60 points to the high-yield yield. The government curve rises by only 1.20 points, so most of the extra cost for the high-yield issuer comes from a credit spread that widens with maturity.

KEY CONCEPT

Feature	Investment grade	High yield
Main credit concern of investors	Downgrade risk and a rising probability of future default	Default risk and **loss given default**
Credit spread as a share of yield	Small; yield tied mostly to benchmark (sovereign) rates	Large

Feature	Investment grade	High yield
Covenants	Few (e.g., limits on liens and sale-and-leaseback of core assets)	Many: debt ratios, limits on additional debt and on distributions such as dividends
Collateral	Usually unsecured	Often secured
Standardization and maturities	Fairly standardized; issued across many maturities, which lowers **rollover risk** and lets issuers exploit market conditions	Less standardized; usually 10 years or less; less able to refinance when rates fall
Early repayment features	Less common	**Leveraged loans** with prepayment options or **callable** bonds, so debt can be refinanced if credit improves
Return profile	Bond-like	More **equity-like**

COMMON EXAM TRAPS

- Covenants protect lenders, so they set maximums on leverage, debt and dividends.
- An issuer hoping its credit will improve wants the right to repay early (call or prepay). A put option benefits the investor.

EXAM SHORTCUTS

- The haircut depends only on the initial margin, $1 - 1/\text{initial margin}$, so it can be found without the collateral value, the repo rate or the term.

BOTTOM LINE

- Lines of credit become more reliable from uncommitted to committed to revolving; committed and revolving lines carry a commitment fee and revolving lines usually add restrictive covenants, while an uncommitted line usually costs only interest on the amount drawn.
- Factoring transfers receivables, with the credit-granting and collection functions, to a factor at a discount to face value, whereas assigning receivables as collateral keeps them with the borrower as security for a loan.
- Commercial paper is short-term unsecured debt issued by large, highly rated firms, usually maturing in under three months, and issuers manage its rollover risk with backup lines of credit.
- In a repo the security seller borrows cash and the security buyer lends it against the security; $\text{purchase price} = \text{market value of collateral}/\text{initial margin}$ and $\text{repurchase price} = \text{purchase price} \times (1 + \text{repo rate} \times \text{days}/360)$.
- $\text{Variation margin} = \text{initial margin} \times \text{adjusted purchase price} - \text{market value of collateral}$; a negative figure means the loan is overcollateralized and the borrower may ask for collateral back.
- The repo rate is higher when alternative money market rates are higher, when the term is longer and longer rates exceed short rates, and when the repo is undercollateralized or collateral is not delivered; it is lower for higher-quality collateral and for collateral in high demand or low supply.

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- With a normal yield curve, both investment-grade and high-yield issuers pay higher yields at longer maturities, and the increase is bigger for high-yield issuers because their credit spreads are larger.
- Compared with investment-grade issues, high-yield issues carry more covenants, are often secured, usually mature in 10 years or less, more often allow early repayment through callable bonds or prepayable leveraged loans, and have more equity-like returns.

QUICK CHECK

Question 1

A corporate treasurer is reviewing the bank credit lines available to her company. Which of the following statements about these lines is *most accurate*?

A An uncommitted line typically carries a commitment fee on its unused portion.

B A committed line is the least reliable source of bank funding for a company.

C A revolving line often runs for years and usually comes with restrictive covenants.

Answer: C

A revolving (operating) line of credit is the most reliable source of bank funding. It is typically arranged for a longer term than a committed line, sometimes several years, and because the bank is committed for that long it usually imposes restrictive covenants that oblige the borrower to take, or to avoid, specified actions, much as a bond indenture does. Its fees and rates are similar to those of a committed line.

Why the other options are wrong

- **A.** An uncommitted line is flexible and usually has no fees beyond interest on the amounts drawn, because the bank can decline to lend if conditions change. Commitment fees (about 50 bp on the full or unused amount) are charged on committed lines, and revolvers carry similar fees.
- **B.** The least reliable form is the uncommitted line, which the bank can decline to honor. A committed line binds the bank for the stated period, although the bank may decline to renew it at maturity (renewal risk).

Key takeaway Reliability rises from uncommitted to committed to revolving lines. Commitment fees apply to committed lines and revolvers, and revolvers usually add restrictive covenants over a multiyear term.

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Practice questions

Question 2

A bond dealer finances a position by entering a 60-day repurchase agreement. It delivers government bonds with a market value of $25,000,000. The lender requires an initial margin of 102% and a repo rate of 3.6%, using a 360-day year. The repurchase price the dealer will pay at the end of the term is *closest* to:

A $24,656,863.

B $24,647,000.

C $24,509,804.

Question 3

An analyst splits the yield of each of three bonds into a benchmark rate and a credit spread. The spread accounts for 4% of Bond P's yield, 12% of Bond Q's yield and 45% of Bond R's yield. Which type of bond is Bond R *most likely* to be?

A A government bond.

B An investment-grade corporate bond.

C A high-yield corporate bond.

Answers: 2. A · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/commercial-paper/. The full bank has 16 questions on this reading.

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FIXED INCOME · READING 55

Treasury Bonds

What you'll learn

LOS 55.a Describe how sovereign, non-sovereign (regional/local), quasi-government (agency) and supranational issuers fund themselves, including developed vs. emerging market and domestic vs. external sovereign debt.

LOS 55.b Contrast government and corporate bond issuance and trading: auctions (competitive and noncompetitive bids, cut-off yield, single- vs. multiple-price), primary dealers, on-the-run bonds and noneconomic investors.

MODULE 55.1

Fixed-Income Markets for Government Issuers

This reading describes how sovereign, non-sovereign, agency and supranational issuers fund themselves, and how government bonds are issued at auction and traded. A candidate must be able to classify a government-related issuer, explain the currency risk in emerging market debt and the maturity choices of a government, and work through an auction to find the cut-off yield and each bidder's allocation.

LOS 55.a — Government issuers and their funding choices

Sovereign debt

National governments borrow to pay for public goods and services and for infrastructure. **Sovereign bonds** are backed by the power to tax, so they usually carry the highest credit rating in their home market, and the sovereign is usually the largest issuer in that market.

Public-sector accounting standards differ from private-sector ones: they lean on cash transactions more than on accruals such as depreciation and unfunded liabilities. Analysts should therefore think of an **economic balance sheet** that adds implied assets (expected future tax revenue) and implied liabilities (promised future spending) to the reported financial assets and liabilities.

A government's financial balance sheet and economic balance sheet

	Assets side	Liabilities side
Financial balance sheet (reported)	Financial assets, such as short-term claims, holdings in state-owned firms and foreign exchange reserves	Debt outstanding (short- and long-term, in domestic or foreign currency) and other recorded obligations
Added in the economic balance sheet	Implied asset: expected future tax revenues	Implied liability: promised future expenditures, such as pensions and public services

	Developed market sovereign	Emerging market sovereign
Economy	Stable, diversified	Faster-growing, less stable, more concentrated
Public finances	Consistent, transparent fiscal policy	Less stable tax revenue, sometimes tied to one dominant industry or commodity
Currency of debt	A **reserve currency**: a major currency held by central banks worldwide and widely used in trade (e.g., US dollar, Chinese renminbi)	Home currency or a foreign reserve currency

Emerging market debt often funds investment in economic growth. It can be:

- **Domestic debt**: issued in the home currency and held by domestic investors. The currency may not be freely convertible, because of illiquidity or capital controls.
- **External debt**: owed to foreign creditors, in the home currency or in a reserve currency. If it is in a reserve currency, foreign investors avoid direct currency risk. They still bear indirect currency risk, because the government must earn enough of the reserve currency to repay.

A government's **debt management policy** sets how much debt it will issue and in what form. To forecast issuance, focus on **fiscal policy**, such as tax cuts and spending increases when the economy is below full employment, and on how cyclical and inflation-sensitive revenues and spending are. Floating-rate or inflation-indexed features and any government guarantees of non-sovereign debt also matter.

Ricardian equivalence says taxpayers expect today's debt to be repaid with future taxes. Under that view a government should be indifferent to the maturity of its debt, but only if taxpayers save more when they expect higher taxes, have rational expectations, can borrow and lend without transaction costs, and pass tax savings on to future generations. These conditions do not hold in practice. Governments therefore issue across the maturity spectrum and keep a fairly stable mix of short- and long-term debt.

- Short-term sovereign debt is seen as safe and highly liquid and can serve as an alternative to bank deposits. That liquidity benefit likely keeps its yields below where they would otherwise be. Relying on it too heavily creates **rollover risk**.
- Longer-term sovereign debt provides **benchmark** yields for pricing the credit risk of other borrowers. It is used for interest rate risk management and as repo collateral, and central banks use it to conduct monetary policy.

Other government-related issuers

KEY CONCEPT

Issuer type	Who issues	Repayment / key feature
Non-sovereign government bonds	States, provinces, counties, cities and entities providing public services (hospitals, airports)	See GO vs. revenue bonds below
General obligation (GO) bonds	Local or regional governments	General public spending, backed by local taxing power

Issuer type	Who issues	Repayment / key feature
Revenue bonds	Local or regional governments	Fund one project; repaid from fees generated by it (e.g., a toll road or bridge)
Agency (quasi-government) bonds	Entities created by a national government for a specific purpose (infrastructure, mortgage finance; e.g., Ginnie Mae)	Repaid from the agency's own revenue (Ginnie Mae: guarantee fees) plus any government backing; if backed by the sovereign, yields and ratings sit close to the government's
Supranational bonds	International institutions set up by several sovereign governments (World Bank, IMF, Asian Development Bank) to promote cooperation, trade or growth	Typically high credit quality; some issues very liquid

COMMON EXAM TRAPS

- Calling the bonds of an institution set up by several governments quasi-government bonds. They are supranational bonds; an agency is created by one national government.
- Mixing classification by geography (developed vs. emerging market) with classification by issuer type (sovereign, non-sovereign, agency, supranational).
- Treating reserve-currency external debt as free of currency risk. Only the direct risk is removed.

LOS 55.b — Issuing and trading government vs. corporate bonds

Corporations raise debt financing as they need it. Sovereign issuers use **regular public auctions**.

How a government bond auction works

KEY CONCEPT

1. **Noncompetitive bids** are filled first. These bidders are guaranteed their allocation at the price the auction sets, but they do not help set it.
2. **Competitive bids** state a yield (price). They are ranked from the highest price (lowest yield) down.
3. Bonds go to competitive bidders from the highest price downward until the offering is filled.
4. The yield on the lowest-priced accepted competitive bid is the **cut-off yield**.

KEY CONCEPT

Auction type	Price paid by successful bidders	Consequences
Single-price auction	Every winner pays the price at the cut-off yield	All bonds issued at one yield, so lower yield volatility. Tends to widen distribution, raise the chance of a successful auction and lower the issuer's funding cost

Auction type	Price paid by successful bidders	Consequences
Multiple-price auction	Each winner pays its own bid	Bids tend to cluster close together and be large in size

Example. A treasury offers 500 million of 7-year notes. It receives 80 million of noncompetitive bids and the competitive bids shown below. Allocation runs down the table, adding up the amounts until the 500 million is used:

Bid	Yield bid	Amount bid	Cumulative amount	Allocated
Noncompetitive	—	80	80	80
Competitive	3.10%	150	230	150
Competitive	3.12%	120	350	120
Competitive	3.15%	200	550	150 (partial)
Competitive	3.18%	100	650	0

The offering runs out within the 3.15% bids, so the cut-off yield is 3.15%. Those bidders receive 150 million of the 200 million they asked for, and the 3.18% bid gets nothing. In a single-price auction every successful bidder pays the price that corresponds to 3.15%. In a multiple-price auction each pays the price of its own bid.

Primary dealers

A sovereign designates certain financial institutions as **primary dealers**. They must bid competitively in auctions, submit bids for third parties, and act as counterparties when the central bank buys (expansionary) or sells (contractionary) government securities to conduct monetary policy. Fiscal policy, meaning spending and taxation, is run by the government.

Secondary trading and yields

Like corporate bonds, government bonds trade mainly in quote-driven **OTC dealer markets**. Trading is heaviest and prices most informative for **on-the-run** bonds, the most recently issued bonds of each maturity. Their yields serve as the default-risk-free **benchmark** yields used to build yield curves.

Some holders of government bonds have **noneconomic objectives**. Central banks hold them for monetary policy, foreign governments hold them as reserves, and regulations require some financial institutions to hold them. This extra demand pushes sovereign yields below those of non-sovereign issuers. Lower credit risk also contributes to the lower yields.

COMMON EXAM TRAPS

- Filling competitive bids before noncompetitive ones, or starting the competitive bids at the highest yield. In the 500 million example, starting from the 3.18% bid gives a cut-off yield of 3.12%, and filling the competitive bids before the noncompetitive ones gives 3.18%, instead of 3.15%.
- Taking the cut-off yield as the lowest accepted yield. It is the highest accepted yield, which matches the lowest accepted price. In the example that error gives 3.10% instead of 3.15%.

- Assuming a competitive bidder is sure of an allocation. Only noncompetitive bidders are, in either auction format.

BOTTOM LINE

- Sovereign bonds are backed by the power to tax, so they usually carry the highest credit rating in their home market, and the sovereign is usually the largest issuer there.
- External debt of an emerging market government in a reserve currency spares foreign investors the direct currency risk but leaves indirect currency risk, because the government must earn enough of that currency to repay.
- Ricardian equivalence would make a government indifferent to the maturity of its debt only if taxpayers save more when they expect higher taxes, have rational expectations, borrow and lend without transaction costs and pass tax savings on to future generations; these conditions do not hold, so governments keep a fairly stable mix of short- and long-term debt.
- Short-term sovereign debt is seen as safe and highly liquid, which likely holds its yields down, but relying on it too heavily creates rollover risk; longer-term sovereign debt supplies the benchmark yields used to price other borrowers' credit risk.
- General obligation bonds are backed by local taxing power, revenue bonds are repaid from the fees of the one project they fund, agency (quasi-government) bonds are issued by entities a national government creates, and supranational bonds are issued by institutions set up by several sovereign governments.
- In a government bond auction, noncompetitive bids are filled first, then competitive bids from the highest price (lowest yield) down until the offering is used up; the cut-off yield is the highest accepted yield, which matches the lowest accepted price.
- In a single-price auction every successful bidder pays the price at the cut-off yield, while in a multiple-price auction each successful competitive bidder pays its own bid.
- On-the-run government bonds, the most recently issued of each maturity, trade most actively and supply the default-risk-free benchmark yields, and demand from holders with noneconomic objectives pushes sovereign yields below non-sovereign yields.

QUICK CHECK

Question 1

A bond analyst wants to group bonds according to the geography of their issuers. Which of the following labels fits that approach?

A Supranational bonds.

B Municipal bonds.

C Emerging market bonds.

Answer: C

Grouping bonds as developed market or emerging market bonds classifies them by the issuer's geography. The label emerging market bonds belongs to this approach.

Why the other options are wrong

- **A.** Supranational bonds are a category of issuer: international institutions owned by several governments. The label is not geographic.
- **B.** Municipal bonds are grouped by type of issuer (local governments), and sometimes by tax status.

Key takeaway Geographic labels are developed market and emerging market. Issuer-type labels are sovereign, municipal, agency, supranational and corporate.

Practice questions

Question 2

The central bank of Valdoria conducts much of its policy through a group of designated primary dealers. Which transaction is it *most likely* to carry out with them?

A Expansionary monetary policy by selling government bonds.

B Expansionary monetary policy by buying government bonds.

C Contractionary fiscal policy by cutting government spending.

Question 3

An analyst is building a government yield curve to use as the default-risk-free benchmark. The yields she uses are *most likely* taken from:

A older government bonds that have been outstanding for many years.

B the most recently issued government bonds of each maturity.

C highly rated supranational bonds.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/treasury-bonds/. The full bank has 11 questions on this reading.

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FIXED INCOME · READING 56

Yield to Maturity

What you'll learn

LOS 56.a Calculate a bond's price from its yield to maturity on a coupon date or between coupon dates (accrued interest, flat and full price).

LOS 56.b Identify how price relates to coupon rate, maturity and yield to maturity, including convexity and pull to par.

LOS 56.c Describe matrix pricing for bonds that are not traded or trade infrequently.

MODULE 56.1

Fixed-Income Bond Valuation: Prices and Yields

This reading values a fixed-rate bond as the present value of its promised cash flows, on a coupon date and between coupon dates, and links its price to its coupon rate, maturity and yield. A candidate must be able to compute a bond's price or YTM for any coupon frequency, split a full price into flat price and accrued interest, explain why prices move toward par, and estimate an illiquid bond's yield by matrix pricing.

LOS 56.a — Pricing a bond on and between coupon dates

Price = present value of the promised cash flows

A bond's value is the sum of the present values of every promised coupon and the principal. The single discount rate that makes this sum equal to the market price is the **yield to maturity (YTM)**, also called the **market discount rate**. Given the YTM, the price can be computed; given the price, the YTM can be solved for.

For an **annual-pay bond** with N years left, coupon C and par FV:

$$P = \frac{C}{(1+YTM)} + \frac{C}{(1+YTM)^2} + \cdots + \frac{C+FV}{(1+YTM)^N}$$

For a **semiannual-pay bond**, use the semiannual coupon $C/2$, the periodic rate $YTM/2$ and $2N$ periods:

KEY CONCEPT

$$P = \sum_{t=1}^{2N} \frac{C/2}{(1+YTM/2)^t} + \frac{FV}{(1+YTM/2)^{2N}}$$

The same logic works for any **periodicity**: a quarterly-pay bond uses $C/4$, $YTM/4$ and $4N$ periods. A quoted YTM is a **stated annual rate**: the periodic yield times the number of periods per year. It is an effective annual rate only if the bond pays annually. For a semiannual bond the periodic yield is therefore the quoted YTM divided by 2, and no square root is taken.

A **zero-coupon bond** has only one cash flow, so $P = FV/(1+YTM/m)^{mN}$, where m is the number of periods per year.

Example (annual vs. semiannual). A 6-year, 4.5% bond is valued at a YTM of 5.5% per 100 of par.

- Annual pay: `N = 6, I/Y = 5.5, PMT = 4.5, FV = 100, CPT PV = −95.00`, so the price is 95.00.
- Semiannual pay: `N = 12, I/Y = 2.75, PMT = 2.25, FV = 100, CPT PV = −94.95`, so the price is 94.95.

On the **TI BA II Plus**, keep P/Y = C/Y = 1 and END mode, enter the periodic values, and clear the worksheet first (`[2ND] [CLR TVM]`). The PV comes out negative: it is the cash the buyer pays, while PMT and FV are cash received. Enter the rate as a percentage figure (5.5 for 5.5%).

Example (solving for YTM). An 8-year, 3% semiannual bond trades at 94.00: `N = 16, PV = −94, PMT = 1.5, FV = 100, CPT I/Y = 1.94`. That is the semiannual yield, so the stated YTM $= 1.94\% \times 2 = 3.88\%$. If the PV is entered with the same sign as PMT and FV, the calculator returns an error.

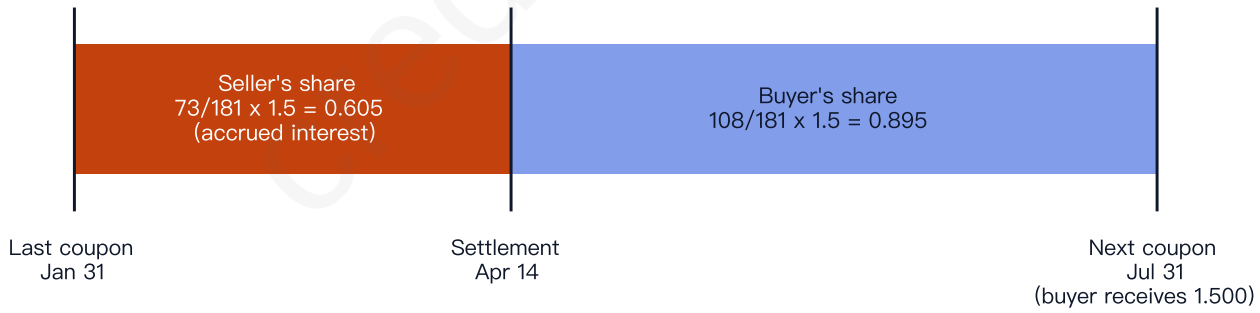
An investor actually earns the YTM only if (1) the bond is held to maturity, (2) the issuer makes every promised payment, and (3) each coupon is reinvested at that same YTM.

Accrued interest, flat price and full price

Trades usually settle between coupon dates. The buyer receives the entire next coupon, but part of it was earned by the seller. That part is the **accrued interest**:

$$AI = \frac{t}{T} \times PMT$$

Here t is the number of days from the last coupon date to the **settlement date**, and T is the number of days in the coupon period. The figure uses the example below.



Splitting a coupon between seller and buyer when a trade settles between coupon dates

Day-count convention	How days are counted	Typical use
Actual/actual	actual calendar days in the accrual period and in the coupon period	government bonds

Day-count convention	How days are counted	Typical use
30/360	every month has 30 days, the year has 360 days	corporate bonds

Example. An annual 3.6% bond paid its coupon on October 12, 2025, and a trade settles on January 3, 2026. Under 30/360 the accrued days are $18 + 30 + 30 + 3 = 81$, so $AI = 3.6 \times 81/360 = 0.810$ per 100. Under actual/actual they are $19 + 30 + 31 + 3 = 83$, so $AI = 3.6 \times 83/365 = 0.819$.

- The **flat price** (also called the **clean price** or **quoted price**) excludes accrued interest. Quoting flat prices stops the price from rising every day and dropping on each payment date.
- The **full price** (also called the **invoice price** or **dirty price**) is what the buyer actually pays: full price = flat price + accrued interest, so flat price = full price − accrued interest.

KEY CONCEPT

To value a bond between coupon dates, compute the full price first and then subtract AI:

1. Price the bond on the last coupon date (PV_0), using the remaining coupons.
2. Compound that value forward at the periodic YTM r for the fraction of the period elapsed: $PV^{full} = PV_0 \times (1 + r)^{t/T}$.
3. Flat price $= PV^{full} - AI$.

Example. A 3% semiannual government bond has coupons on January 31 and July 31 and a YTM of 3.4%. A trade settles on April 14, 2026, with 8 coupons remaining. Actual days are $t = 73$ and $T = 181$. PV_0: `N = 8, I/Y = 1.7, PMT = 1.5, FV = 100, CPT PV = −98.516`. Full price $= 98.516 \times 1.017^{73/181} = 99.188$. $AI = 1.5 \times 73/181 = 0.605$. Flat price $= 99.188 - 0.605 = 98.583$. The flat price differs from PV_0.

WORKED EXAMPLE: FULL AND FLAT PRICE OF A CORPORATE BOND

A corporate bond pays a 5.4% coupon semiannually on March 15 and September 15, uses the 30/360 day count and matures on September 15, 2030. A trade settles on June 27, 2026, at a YTM of 4.6%. Compute the accrued interest, the full price and the flat price per 100 of par.

Step 1. The last coupon was paid on March 15, 2026, so 9 semiannual coupons of 2.7 remain. Under 30/360 the accrued days are $30 + 30 + 30 + 12 = 102$ out of 180.

Step 2. Value on the last coupon date, at a periodic yield of $4.6\%/2 = 2.3\%$: N = 9; I/Y = 2.3; PMT = 2.7; FV = 100; CPT PV = −103.22.

Step 3. Full price $= 103.22 \times 1.023^{102/180} = 104.56$.

Step 4. Accrued interest $= 2.7 \times 102/180 = 1.53$.

Step 5. Flat price $= 104.56 - 1.53 = 103.03$.

Result. The buyer pays the full price of 104.56 per 100 of par, and the bond is quoted at a flat price of 103.03. Both are above par, as expected for a coupon (5.4%) above the YTM (4.6%).

Common exam traps (56.a)

- Using annual N, I/Y and PMT for a semiannual or quarterly bond: all three must be converted to per-period values. For the 6-year, 4.5% semiannual bond at 5.5% in the example, annual inputs give 95.00 instead of 94.95.
- Counting years from issuance instead of the remaining maturity. When a question asks for a future price, price the bond with the periods left at that future date; the original purchase price is irrelevant.
- Treating the quoted price as the amount paid. The seller receives the full price, which is the quote plus accrued interest.
- Adding accrued interest to the value on the last coupon date, or discounting accrued interest along with the cash flows. Neither is correct. In the April 14 example, adding AI to PV_0 gives a full price of 99.121 instead of 99.188.
- Valuing a **deferred coupon bond** as if its coupons started in Year 1. Discount each coupon from the year in which it is actually paid; the deferral pushes the price well below that of an otherwise identical bond whose coupons start at once.

LOS 56.b — How price, coupon, maturity and yield are related

KEY CONCEPT

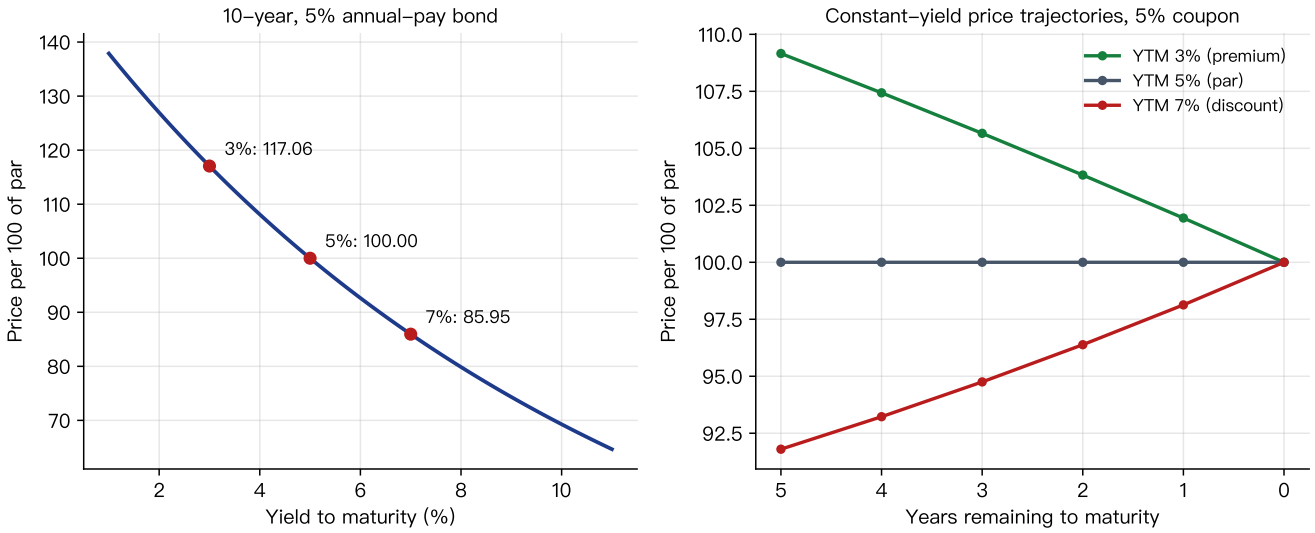
Coupon rate vs. YTM	Price vs. par	Label
coupon rate > YTM	price > par	**premium** bond
coupon rate = YTM	price = par	par bond
coupon rate < YTM	price < par	**discount** bond

A coupon below the YTM is sometimes called a **deficient coupon**, and a coupon above the YTM an **excessive coupon**.

The key relationships for **option-free bonds**, other things equal:

1. *Inverse relationship.* A higher YTM means a lower price, and a lower YTM means a higher price.
2. *Coupon effect.* A lower-coupon bond's price is more sensitive to a given yield change than a higher-coupon bond's price.
3. *Maturity effect.* A longer-maturity bond's price is more sensitive to a given yield change than a shorter-maturity bond's price.
4. **Convexity**. The price-yield curve is convex. For equal yield changes, the price rise when yields fall is larger than the price drop when yields rise. As yield increases, price decreases at a decreasing rate.

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Price-yield convexity and constant-yield price trajectories

In the left panel, the 10-year, 5% annual bond is worth 117.06 at a 3% YTM, 100.00 at 5% and 85.95 at 7%. A 2-point fall in yield adds 17.06, while a 2-point rise takes away only 14.05.

Pull to par: whatever its yield, a bond's price converges to par as maturity approaches. If the YTM stays constant, the path the price follows is the **constant-yield price trajectory**:

- A **premium bond**'s price falls toward par over time. In the right panel (YTM 3%) it goes from 109.16 to 107.43 and on down to 100.
- A **discount bond**'s price rises toward par. At a YTM of 7% it goes from 91.80 to 93.23 and on up to 100.
- A **par bond** stays at par while its YTM equals its coupon rate.
- At a positive yield a zero-coupon bond is a discount bond, so with an unchanged yield its price rises steadily toward par.

The same logic runs in reverse. If a discount bond's price is unchanged after some time has passed, its YTM must have risen. The constant-yield price would have moved up, so an unchanged price implies a higher yield. For a premium bond with an unchanged price, the YTM must have fallen.

Common exam traps (56.b)

- Mixing up the sensitivity directions. A low coupon and a long maturity give higher price sensitivity.
- Thinking prices are more sensitive to yield rises than to yield falls. Convexity makes the opposite true.
- Assuming a bond keeps its issue-date premium or discount status. What matters is today's YTM compared with the coupon rate.
- Forgetting that time alone changes the price of a non-par bond even if the yield is unchanged.

LOS 56.c — Matrix pricing

Matrix pricing estimates the YTM (and therefore the price) of a bond that is not traded or trades infrequently, i.e., a bond with low liquidity. It uses the observed YTMs of actively traded bonds with the same or very similar credit quality and similar coupon and maturity. When no comparable bond has exactly the same maturity, the analyst uses **linear interpolation**:

$$YTM_{target} = YTM_{short} + \frac{T_{target} - T_{short}}{T_{long} - T_{short}}\left(YTM_{long} - YTM_{short}\right)$$

The procedure has four steps:

1. Select the comparable traded bonds and record their YTMs.
2. If several comparable bonds share a maturity, average their yields.
3. Interpolate between the two maturities that bracket the target bond's maturity.
4. Price the target bond by discounting its cash flows at the estimated YTM.

Example. A thinly traded 4-year, 5% annual bond is to be priced. Comparable bonds with the same rating yield 3.9% (a 3-year bond) and 5.0% and 5.2% (two 7-year bonds). The 7-year average is 5.1%, the interpolated yield is $3.9\% + \frac{1}{4}(5.1\% - 3.9\%) = 4.2\%$, and the price is `N = 4, I/Y = 4.2, PMT = 5, FV = 100, CPT PV = -102.89`.

A spread-based variant is used to set the yield on a new bond:

1. Find an existing bond of similar credit quality from the same issuer or sector.
2. Measure its spread over a benchmark government yield of the same maturity, interpolating the benchmark if needed.
3. Add that spread to the benchmark yield at the new bond's maturity.

Example. Two-year and 4-year government bonds yield 1.60% and 2.20%, and an existing 3-year AA corporate bond yields 3.05%. The interpolated 3-year government yield is 1.90%, so the spread is 1.15%. A new 4-year AA issue should yield about $2.20\% + 1.15\% = 3.35\%$.

Common exam traps (56.c)

- Basing a matrix price on the issuer's cost of capital or on a default-probability model instead of the yields of comparable traded bonds.
- Interpolating from the wrong end, or simply averaging two yields when the target maturity is not the midpoint. In the 4-year example, averaging 3.9% and 5.1% gives 4.5% instead of 4.2%.
- In the spread method, measuring the spread against a benchmark of the wrong maturity. Measuring the 3-year AA bond's spread against the 4-year government yield gives 0.85% and a new-issue yield of 3.05%, instead of 1.15% and 3.35%.

EXAM SHORTCUTS

- Comparing the coupon rate with the YTM shows whether the price is above, at or below par before any calculation, so answer choices on the wrong side of par can be ruled out at once.
- If a discount bond's price is unchanged after time has passed, its YTM must have risen, and if a premium bond's price is unchanged, its YTM must have fallen; no repricing is needed to answer the direction question.

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BOTTOM LINE

- A bond's price is the present value of its coupons and principal discounted at the YTM; a semiannual bond uses the coupon $C/2$, the periodic rate $YTM/2$ and $2N$ periods.
- A quoted YTM is a stated annual rate, the periodic yield times the number of periods per year, and it is an effective annual rate only for an annual-pay bond.
- An investor earns the YTM only if the bond is held to maturity, the issuer makes every promised payment and each coupon is reinvested at that same YTM.
- Between coupon dates, the full price is the value on the last coupon date compounded at the periodic YTM, $PV^{full} = PV_0 \times (1+r)^{t/T}$; accrued interest is $t/T \times PMT$, and subtracting accrued interest from the full price gives the flat (quoted) price.
- Accrued interest is usually counted on an actual/actual basis for government bonds and on a 30/360 basis for corporate bonds.
- A bond whose coupon rate is above its YTM trades at a premium, one whose coupon equals its YTM trades at par, and one whose coupon is below its YTM trades at a discount.
- For option-free bonds, other things equal, a lower coupon and a longer maturity make the price more sensitive to a yield change, and convexity makes the price rise from a yield fall larger than the price drop from an equal yield rise.
- Matrix pricing estimates the YTM of a bond that is not traded or trades infrequently from the yields of traded bonds with very similar credit quality, coupon and maturity, interpolating linearly between the maturities that bracket the target.

QUICK CHECK

Question 1

Five years ago, Kessler Freight Lines sold 25-year bonds with a 5.5% annual coupon, paid semiannually, to finance new rail terminals. The bonds now yield 5.2%. The price of the bonds per 100 of par is *closest* to:

A 103.70.

B 102.32.

C 96.39.

Answer: A

Only the remaining 20 years (40 semiannual periods) of cash flows matter. Because the 5.5% coupon exceeds the 5.2% yield, the bond trades at a premium.

Remaining maturity $= 25 - 5 = 20$ years, so $N = 40$; $PMT = 5.5/2 = 2.75$; periodic yield $= 5.2\%/2 = 2.6\%$.

$$P = \sum_{t=1}^{40} \frac{2.75}{1.026^{t}} + \frac{100}{1.026^{40}} = 103.70$$

BA II Plus (P/Y = C/Y = 1, END mode; clear with [2ND] [CLR TVM]): `N = 40; I/Y = 2.6; PMT = 2.75; FV = 100; CPT PV = −103.70`.

Why the other options are wrong

- **B.** 102.32 uses $N = 20$, the number of remaining years, as the number of semiannual periods. It should be 40.
- **C.** 96.39 swaps the coupon and the yield (a 5.2% coupon discounted at 2.75% per half-year). This produces a discount price for what is actually a premium bond.

Key takeaway Count the remaining maturity from today and convert it to periods. The issue date does not matter.

Practice questions

Question 2

With respect to a bond traded between coupon dates, accrued interest:

A is the interest earned from the last coupon date up to the settlement date.

B is the part of the next coupon that the seller has not earned.

C is discounted together with the other cash flows to obtain the full price.

Question 3

Holding all other features constant, which of the following statements about option-free bonds is *most accurate*?

A A long-maturity bond's price is more sensitive to yield changes than a short-maturity bond's price.

B A bond's price is more sensitive to increases in yield than to decreases in yield of the same size.

C A high-coupon bond's price is more sensitive to yield changes than a low-coupon bond's price.

Question 4

Marguerite Olsen wants to estimate the yield to maturity of a non-traded 7-year, annual-pay bond rated A. Actively traded A-rated bonds yield 4.10% at a 5-year maturity and 5.60% at a 10-year maturity. Using matrix pricing, the estimated YTM for the non-traded bond is *closest* to:

A 5.00%.

B 4.85%.

C 4.70%.

Answers: 2. A · 3. A · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/yield-to-maturity/. The full bank has 53 questions on this reading.

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FIXED INCOME · READING 57

Credit Spread

What you'll learn

LOS 57.a Calculate a bond's annual yield for different compounding periods (periodicity, street vs. true yield, current, simple, YTM, yield to call and yield to worst, option-adjusted yield).

LOS 57.b Compare, calculate and interpret yield spread measures: G-spread, I-spread, Z-spread and option-adjusted spread.

MODULE 57.1

Yield and Yield Spread Measures for Fixed-Rate Bonds

This reading covers the yield measures quoted for fixed-rate bonds and the spreads that compare a bond's yield with a benchmark. A candidate must be able to restate a yield for a different periodicity, compute current yield, simple yield, yield to call and yield to worst, and calculate or interpret G-spreads, I-spreads, Z-spreads and option-adjusted spreads.

LOS 57.a — Annual yields for different compounding periods

Yield to maturity and the semiannual bond basis

The **yield to maturity (YTM)** is the single discount rate that makes the present value of all of a bond's promised cash flows equal to its market price. It is an internal rate of return, so on the calculator it is found by solving for I/Y given the price, the coupon, the number of periods and the redemption value.

For a bond that pays coupons more than once a year, the calculator gives a periodic rate. By convention the quoted YTM is the periodic rate multiplied by the number of periods per year. For a semiannual-pay bond, the quoted YTM is $2\times$ the semiannual rate. Such a yield is quoted on a **semiannual bond basis**.

Calculator (TI BA II Plus), semiannual bond. Set P/Y = C/Y = 1 and END mode, then N = years $\times 2$, PMT = annual coupon / 2, FV = par, PV = −price (cash paid out is negative), CPT I/Y, and multiply the result by 2.

Periodicity and the effective annual yield

The number of coupon payments per year is the bond's **periodicity**. For a given stated yield, more compounding periods per year produce a higher **effective annual yield** (EAY):

KEY CONCEPT

$$EAY = \left(1 + \frac{\text{stated annual yield}}{n}\right)^{n} - 1$$

where n is the periodicity. The same relationship links any **stated annual rate** (also called the nominal rate or APR) with its **effective annual rate (EAR)**: the stated rate divided by n is the **periodic rate**, and compounding the periodic rate n times gives the EAR. The stated rate does not adjust for compounding; the EAR does.

To restate a yield for a different periodicity, keep the EAY fixed:

$$\text{stated yield}_m = m\left[(1 + EAY)^{1/m} - 1\right]$$

Example. A bond yields 6% on a semiannual bond basis, which is 3% per half-year.

Basis	Calculation	Yield
Annual-pay equivalent (EAY)	$1.03^2 - 1$	6.09%
Semiannual bond basis	given	6.00%
Quarterly basis	$4\left(1.03^{0.5} - 1\right)$	5.96%

All three describe the same return. Holding the EAY constant, higher periodicity means a lower stated yield. Holding the stated yield constant, higher periodicity means a higher EAY. An annual-pay bond's YTM already is an effective annual yield.

Street convention and true yield

A yield computed with the scheduled coupon dates follows the **street convention**. When a scheduled date falls on a weekend or holiday the cash actually arrives on the next business day; a yield computed with these actual payment dates is the **true yield**. Because cash arrives slightly later, the true yield is usually slightly lower than the street convention yield, by a few basis points, and equal to it when no payment date moves.

Current yield and simple yield

Current yield (also **income yield** or **running yield**) considers only coupon income:

$$\text{current yield} = \frac{\text{annual cash coupon}}{\text{flat price}}$$

It ignores capital gains or losses and reinvestment income, and it is the same for annual-pay and semiannual-pay bonds with the same coupon rate and price. *Example.* A 5% bond priced at 96 has a current yield of $5/96 = 5.21\%$.

Simple yield adds (subtracts) the straight-line amortization of a discount (premium) to the coupon:

$$\text{simple yield} = \frac{\text{annual coupon} \pm \text{straight-line amortization}}{\text{flat price}}$$

Example. A 3-year 5% bond at 97 amortizes $3/3 = 1$ per year, so the simple yield is $(5 + 1)/97 = 6.19\%$.

Ranking for option-free bonds:

KEY CONCEPT

Bond priced at	Ranking
Discount (price < par)	coupon rate < current yield < YTM
Par	coupon rate = current yield = YTM

Bond priced at	Ranking
Premium (price > par)	coupon rate > current yield > YTM

Yield to call and yield to worst

For a **callable bond**, the **yield to call** is found like a YTM but with N = periods to the call date and FV = the **call price** instead of par. A yield to call can be computed for each call date. The lowest of the YTM and all the yields to call is the **yield to worst**, the conservative yield to rely on. For a premium bond callable at or near par, a call cuts short the period over which the premium is amortized, so a yield to call is usually the lowest.

Example. A 4-year, 5.5% semiannual bond at 103 is callable at 101 in 2 years. YTM: N = 8, PMT = 2.75, FV = 100, PV = −103, CPT I/Y = 2.335, so the YTM is 4.67%. Yield to call: N = 4, FV = 101, same PMT and PV, CPT I/Y = 2.200, so the yield to call is 4.40%. The yield to worst is 4.40%.

Option-adjusted price and option-adjusted yield

A callable bond equals a straight bond minus the call option the investor has sold to the issuer:

$$\text{callable bond value} = \text{straight bond value} - \text{call option value}$$

Adding the call value back gives the **option-adjusted price** (the value of the equivalent option-free bond); the yield computed from it is the **option-adjusted yield**. For a callable bond the option-adjusted yield is lower than the bond's YTM, because investors demand extra yield for bearing the call. "Option-adjusted" means the option has been removed. Because the option's effect is stripped out, option-adjusted yields put bonds with different embedded options, and comparable option-free bonds, on a like-for-like footing.

COMMON EXAM TRAPS

- Forgetting to double the semiannual I/Y, or doubling it for an annual-pay bond. In the callable-bond example, the undoubled figure is 2.335% instead of a YTM of 4.67%.
- Using par instead of the call price as FV in a yield to call. In the callable-bond example, FV = 100 gives a yield to call of 3.93% instead of 4.40%.
- Using years instead of periods for N, or the time to maturity instead of the time to the call date. In the callable-bond example, N = 2 for the call date gives a yield to call of 3.41% instead of 4.40%.
- Converting in the wrong direction: the annual-pay equivalent of a semiannual-basis yield is higher. For the 6% semiannual-basis yield in the example, converting the wrong way gives 5.91% instead of 6.09%.
- Expecting a negative yield to move toward zero at a higher periodicity. Holding the EAY constant, the stated yield still falls, so a negative yield becomes more negative.
- Treating the current yield as a total return: it ignores the pull to par.

LOS 57.b — Yield spreads for fixed-rate bonds

Benchmark spreads: G-spread and I-spread

A **yield spread** (or **benchmark spread**) is the bond's yield minus a benchmark yield, usually in basis points. When the benchmark is a government bond, typically the **on-the-run** issue of the same or nearest maturity, the spread is a **G-spread**. If no government bond has exactly the same maturity, the benchmark yield is

interpolated.

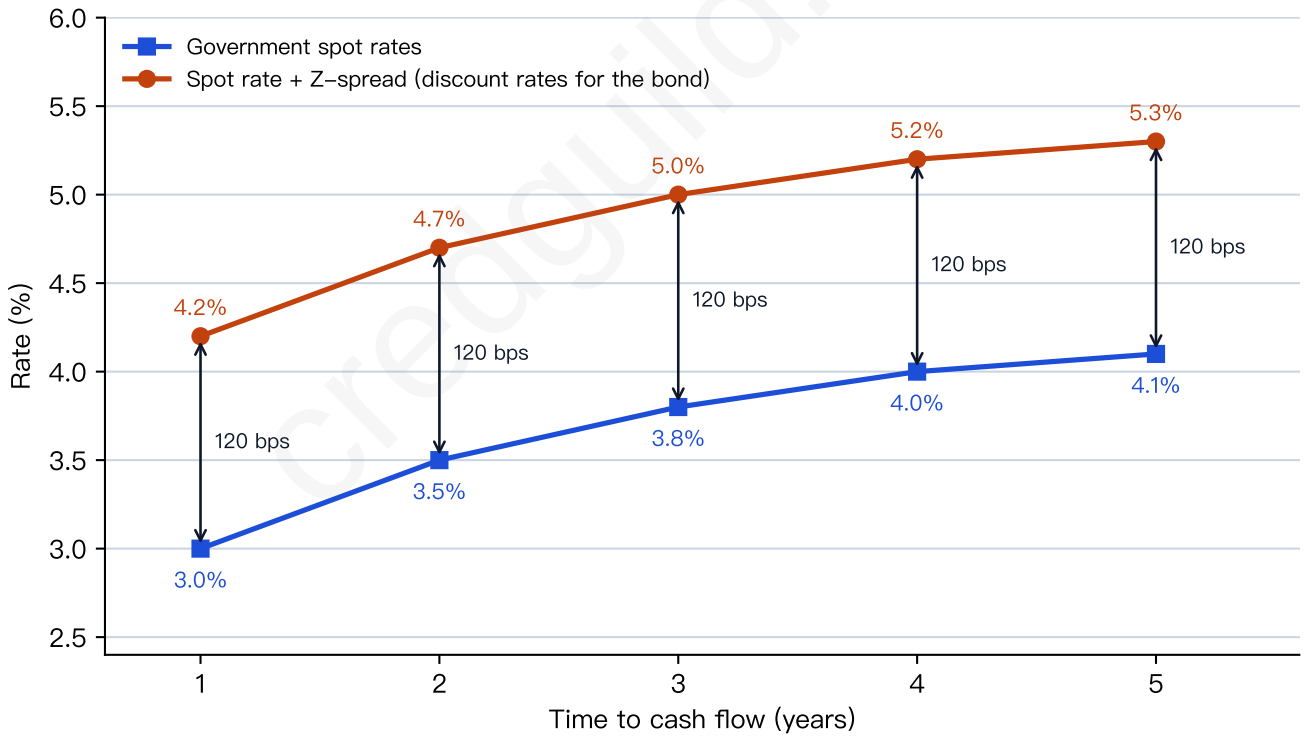
> *Example.* Two-year and 5-year government yields are 3.2% and 3.8%. A 4-year corporate bond yields 5.1%. Interpolated 4-year yield $= 3.2\% + (3.8\% - 3.2\%) \times 2/3 = 3.6\%$, so the G-spread is 150 bp.

A spread measured against **swap rates** of the same currency and tenor is an **interpolated spread (I-spread)**. It measures the bond's extra return over the interbank **market reference rates (MRRs)** used in swaps. I-spreads are common for euro-denominated bonds.

If a bond's yield rises and its spread is unchanged, the benchmark yield rose too. **Macroeconomic** factors moved yields in general; the benchmark yield is made up of the real risk-free rate plus expected inflation. If the spread widens, **microeconomic** factors are at work, such as higher issuer **credit risk** or poorer **liquidity**. Differences in taxation can also affect spreads.

Z-spread

G- and I-spreads compare the bond's YTM with a single benchmark yield, so they ignore the shape of the curve. Individual cash flows are discounted at different **spot rates**, and a YTM is only a kind of weighted average of them. The **zero-volatility spread (Z-spread)** is the constant spread that, added to each benchmark spot rate, discounts the bond's cash flows to a present value equal to its market price. The figure shows the idea: one spread is added to every point of the spot curve.



The Z-spread: one constant spread added to every government spot rate

It is found by trial and error:

$$\text{price} = \sum_{t=1}^{N} \frac{CF_t}{(1 + S_t + Z)^t}$$

The search runs in four steps:

1. Guess a spread Z.
2. Add it to every spot rate and discount each cash flow at its own $S_t + Z$.
3. Compare the total with the market price. A total above the price means the guess is too small; a total below the price means it is too large.
4. Adjust Z and repeat until the present value equals the price.

Example. Spot rates are 3.0% (1 year) and 3.5% (2 years), and a 2-year 5% annual-pay bond trades at 100.58. A first guess of 100 bp gives $5/1.04 + 105/1.045^2 = 100.96$, which is above the price, so the spread must be larger. At 120 bp the value is $5/1.042 + 105/1.047^2 = 100.58$, equal to the price, so the Z-spread is 120 bp.

When the spot curve is flat, every spot rate equals the benchmark YTM, so the G-spread and the Z-spread are the same. YTM-based spreads are theoretically correct only for a flat spot curve.

Option-adjusted spread (OAS)

The **option-adjusted spread (OAS)** is the spread over the benchmark spot curve that remains once the effect of the embedded option is removed. It equals the Z-spread minus the **option value** (also called the option cost), the part of the spread that reflects the embedded option:

$$\text{OAS} = \text{Z-spread} - \text{option value (bp)} \qquad \text{option value} = \text{Z-spread} - \text{OAS}$$

For a **callable** bond the option works against the investor, so the investor demands extra spread and the OAS is below the Z-spread. The difference, the option value, is the extra yield that compensates bondholders for the call. *Example.* A Z-spread of 145 bp and an option value of 40 bp give an OAS of 105 bp. The Z-spread compensates for credit, liquidity, taxation and optionality risk, while the OAS compensates for credit, liquidity and taxation risk only. For an option-free bond, $\text{OAS} = \text{Z-spread}$. An option that benefits the bondholder (a conversion or put feature) makes the bond trade at a lower spread than an otherwise identical straight bond.

KEY CONCEPT

Measure	Benchmark	Accounts for curve shape?	Removes option?
G-spread	Government bond YTM (interpolated if needed)	No	No
I-spread	Swap rate of same tenor	No	No
Z-spread	Each government spot rate	Yes	No
OAS	Each government spot rate	Yes	Yes

COMMON EXAM TRAPS

- Measuring the option value as the G-spread minus the OAS. The G-spread stands in for the Z-spread only when the curve is flat.

- Expecting a callable bond's OAS to exceed its Z-spread. That ordering fits a bond whose option helps the investor, such as a putable bond.
- Comparing a bond with a benchmark of a different maturity. Interpolate instead.
- Subtracting two yields quoted at different periodicities. Restate them on the same periodicity first; for example, a quarterly-basis yield must be converted to a semiannual basis before it is compared with a semiannual-basis yield.
- Blaming the issuer for a higher yield when the spread is unchanged. The benchmark moved, so macro factors caused the rise.

EXAM SHORTCUTS

- For an option-free bond, the price relative to par fixes the order of coupon rate, current yield and YTM, so a ranking question needs no yield calculation.
- A yield to call reuses the YTM inputs; only N, set to the periods to the call date, and FV, set to the call price, change.
- When the spot curve is flat, the Z-spread equals the G-spread, so no trial-and-error search is needed.

BOTTOM LINE

- For a bond that pays more than once a year, the quoted YTM is the periodic yield times the number of periods per year; for a semiannual-pay bond this is the semiannual bond basis.
- $EAY = (1 + \text{stated annual yield}/n)^n - 1$; holding the EAY constant, a higher periodicity means a lower stated yield, and holding the stated yield constant, a higher periodicity means a higher EAY.
- Current yield is the annual cash coupon divided by the flat price and ignores capital gains, losses and reinvestment income; simple yield adds the straight-line amortization of a discount, or subtracts that of a premium, before dividing by the flat price.
- For an option-free bond priced at a discount, coupon rate < current yield < YTM; at a premium the order reverses, and at par all three are equal.
- A yield to call uses the periods to the call date and the call price in place of maturity and par, and the yield to worst is the lowest of the YTM and all the yields to call.
- A G-spread is the bond's yield minus a government benchmark yield of the same maturity, interpolated if needed, while an I-spread is measured against the swap rate of the same currency and tenor.
- The Z-spread is the constant spread that, added to every benchmark spot rate, discounts the bond's cash flows to its market price, and it equals the G-spread when the spot curve is flat.
- $\text{OAS} = \text{Z-spread} - \text{option value}$; for a callable bond the OAS is below the Z-spread, and for an option-free bond the two are equal.

QUICK CHECK

Question 1

An analyst discounts every cash flow of a bond at one uniform rate, choosing the rate so that the total present value equals the bond's market price. This rate is the bond's:

A Yield to maturity.

B Current yield.

C Simple yield.

Answer: A

The yield to maturity is the single discount rate that sets the present value of all of a bond's cash flows equal to its price. It is the bond's internal rate of return.

Why the other options are wrong

- **B.** Current yield is the annual coupon divided by the price; it does not discount any cash flows.
- **C.** Simple yield adds straight-line amortization of a discount (or subtracts that of a premium) to the coupon and divides by the price; it is not a discount rate.

Key takeaway YTM = IRR of the bond's cash flows at the market price.

Practice questions

Question 2

Delmont Paper's $1,000 par value bonds carry a 9% coupon rate and are quoted at a price of $962. The bonds' current yield is *closest* to:

A 8.66%.

B 9.00%.

C 9.36%.

Question 3

An analyst calculates a bond's yield to maturity using its scheduled coupon dates. Two of those dates fall on a Sunday, so the issuer will pay on the following Monday. The analyst then recalculates the yield using the dates on which the cash will actually be received. The recalculated figure is best described as:

A the true yield, slightly higher than the first figure.

B the street convention yield, slightly lower than the first figure.

C the true yield, slightly lower than the first figure.

Question 4

Which description of a callable bond's yield to worst is *most accurate*? It is the:

A yield the investor earns if the issuer defaults on its payments.

B lowest of the bond's yield to maturity and its yields to each call date.

C yield implied by the lowest price at which the bond has traded.

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Question 5

A callable corporate bond has a zero-volatility spread of 160 basis points, and its embedded call option has a positive value. The bond's option-adjusted spread (OAS) is *most likely*:

A greater than 160 basis points.

B equal to 160 basis points.

C less than 160 basis points.

Answers: 2. C · 3. C · 4. B · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-spread/. The full bank has 61 questions on this reading.

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FIXED INCOME · READING 58

Yield and Yield Spread Measures for Floating-Rate Instruments

What you'll learn

LOS 58.a Calculate and interpret the quoted margin and discount (required) margin of a floating-rate note and estimate its value on a reset date.

LOS 58.b Calculate and interpret money market yields on discount and add-on bases, 360- and 365-day years, and the bond equivalent yield.

MODULE 58.1

Yield and Yield Spread Measures for Floating-Rate Instruments

This reading covers the spread measures used for floating-rate notes and the yield conventions used for money market instruments. A candidate must be able to value an FRN on a reset date from its quoted and discount margins, solve for the discount margin from a price, and convert money market quotes among discount yields, add-on yields, the bond equivalent yield and a semiannual bond basis.

LOS 58.a — Quoted margin, discount margin and FRN value

How a floating-rate note pays

A **floating-rate note (FRN)** pays a coupon that is reset each period to the current **market reference rate (MRR)** plus a fixed spread. The MRR (typically an interbank or overnight-based rate) is set at the start of the period and paid at the end, so interest is **paid in arrears**. Because the coupon keeps catching up with market rates, an FRN's price is more stable than the price of a fixed-rate bond with the same maturity.

- The fixed spread written into the indenture is the **quoted margin (QM)**. It reflects the issuer's credit risk *at issuance* compared with that of the banks behind the MRR (liquidity and tax treatment can also affect it).
- An issuer riskier than those banks pays MRR plus a margin; an issuer with less credit risk than those banks pays MRR minus a margin, so its coupon rate is below the MRR.
- The spread investors *currently* require over the MRR is the **required margin**, also called the **discount margin (DM)**. It is the margin that would make the FRN price equal to par on a reset date.

QM versus DM: the price rule

KEY CONCEPT

Situation since issuance	Relation	Price on a reset date
Credit quality unchanged	QM = DM	At par
Credit quality worsened	DM > QM	Below par (discount)
Credit quality improved	DM < QM	Above par (premium)

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FRNs are normally issued at par, with QM = DM. As long as the issuer's credit quality is unchanged, QM stays equal to DM throughout the note's life. The passage of time does not open a gap between them. A change in the MRR itself also does not create a gap, because both the coupon (MRR + QM) and the required return (MRR + DM) move with it. The analogy with fixed-rate bonds: coupon rate < required yield means a discount price; for an FRN, MRR + QM < MRR + DM means the QM is **deficient** and the note trades below par.

Simplified valuation on a reset date

Project every future coupon using today's MRR plus the QM, and discount at today's MRR plus the DM, with the rates divided by the number of periods per year:

$$V = \sum_{t=1}^{N} \frac{(\text{MRR} + \text{QM}) \times FV/m}{\left(1 + \frac{\text{MRR}+\text{DM}}{m}\right)^{t}} + \frac{FV}{\left(1 + \frac{\text{MRR}+\text{DM}}{m}\right)^{N}}$$

Example. A $1 million FRN pays annual coupons of MRR + 60 bp and has three years left. On a reset date the MRR is 2.80% and investors now require a DM of 90 bp.
Coupon $= (2.80\% + 0.60\%) \times 1{,}000{,}000 = \$34{,}000$; discount rate $= 2.80\% + 0.90\% = 3.70\%$.
TI BA II Plus (P/Y = C/Y = 1, END mode): N = 3, I/Y = 3.70, PMT = 34,000, FV = 1,000,000, CPT PV = −991,627. The value is about $991,627, below par because DM > QM. This shortcut ignores expected changes in the MRR; more complex models give better estimates of value.

Solving for the DM. When the price is known, the same formula is run backward:

1. Coupon per period $= (\text{MRR} + \text{QM}) \times FV/m$.
2. Solve for the periodic discount rate (I/Y) with N, PMT, FV and PV = −price.
3. Multiply the periodic rate by m to annualize it.
4. Subtract the MRR. What remains is the DM.

Example. Suppose the same note were priced at $988,000. With $m = 1$: N = 3, PMT = 34,000, FV = 1,000,000, PV = −988,000, CPT I/Y = 3.831, so $DM = 3.831\% - 2.80\% = 1.03\%$, about 103 bp. The DM is above the 60 bp QM, as it must be for a note priced below par on a reset date.

Common exam traps

- Swapping the roles. The coupon uses the QM; the discount rate uses the DM. In the $1 million example, swapping them values the note at about $1,008,421 instead of $991,627.
- Subtracting the MRR from a quarterly periodic rate. Multiply the periodic rate by 4 first, then subtract the MRR.
- Assuming an FRN always trades at par on a reset date. This holds only when QM = DM.
- Expecting the approach of maturity or a move in the MRR to open a gap between the QM and the DM.
- Treating the MRR as a floor on the coupon rate. A lower-risk issuer pays less than the MRR.

LOS 58.b — Money market yield conventions

Money market securities mature in one year or less. Their yields are quoted under two sets of choices:

Choice	Option 1	Option 2
What is annualized	**Add-on yield**: interest earned ÷ amount invested today	**Discount yield**: discount from face value ÷ face value
Day-count year	360 days	365 days

KEY CONCEPT

- Annualized add-on rates are the quoting convention for bank CDs, repos and market reference rates.
- Annualized discount yields on a 360-day year are the convention for US Treasury bills and commercial paper.
- The **bond equivalent yield (BEY)** of a money market instrument is an **add-on yield based on a 365-day year**.

Key formulas: money market yields

Holding period yield: $HPY = \dfrac{\text{amount received at maturity}}{\text{price paid}} - 1$

KEY CONCEPT

$$\text{Add-on yield (365-day)} = HPY \times \frac{365}{\text{days}} \qquad \text{Discount yield} = \frac{FV - P}{FV} \times \frac{360}{\text{days}}$$

The 360 in the discount-yield formula becomes 365 when an instrument is quoted as a discount yield on a 365-day year.

Rearranged for pricing, the two conventions run in opposite directions. A discount yield starts from the face value and works back to today's price. An add-on yield starts from the amount invested today and works forward to the amount received at maturity:

$$P = FV \times \left(1 - \text{discount yield} \times \frac{\text{days}}{\text{year}}\right) \qquad FV = PV \times \left(1 + \text{add-on yield} \times \frac{\text{days}}{\text{year}}\right)$$

Here "year" is 360 or 365 days, whichever basis the quote uses. Converting an add-on yield from a 360-day to a 365-day basis: multiply by $365/360$.

Because a discount yield divides by face value (larger than the price) and uses a 360-day year, the BEY of a discount instrument priced below face value is higher than its quoted discount yield.

Example 1. A 60-day T-bill is quoted at a 3.00% discount yield.
Discount $= 3.00\% \times 60/360 = 0.50\%$, so the price is 99.50 per 100 of face value.
$HPY = 100/99.50 - 1 = 0.5025\%$; $BEY = 0.5025\% \times 365/60 = 3.057\%$, above the 3.00% quote.

Example 2. A 45-day interbank deposit quoted at a 2.00% add-on yield on a 360-day basis has a BEY of $2.00\% \times 365/360 = 2.028\%$.

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Comparing with a semiannual-pay bond

To compare a money market yield with the YTM of a semiannual-pay bond (a **semiannual bond basis** yield): find the HPY, compound it to an effective annual yield, convert to an effective semiannual rate and double it:

$$EAY = (1 + HPY)^{365/\text{days}} - 1, \qquad \text{semiannual bond basis} = 2\left[(1 + EAY)^{1/2} - 1\right]$$

Example 3. A 200-day deposit quoted at 3.60% add-on (365-day basis): $HPY = 3.60\% \times 200/365 = 1.9726\%$; $EAY = 1.019726^{365/200} - 1 = 3.6293\%$; semiannual bond basis $= 2(1.036293^{0.5} - 1) = 3.597\%$. With a holding period longer than half a year (periodicity below 2), the semiannual bond basis yield is slightly *below* a simple 365-day add-on quote; with a holding period shorter than half a year it is slightly *above*.

Common exam traps

- Annualizing an HPY by $\text{days}/365$ instead of $365/\text{days}$. For the 60-day T-bill in Example 1, that gives 0.083% instead of a BEY of 3.057%.
- Using 360 instead of 365 when the question asks for a BEY. For the T-bill in Example 1, this gives 3.015% instead of 3.057%.
- Treating a discount yield as a return on the price: a discount yield is computed on face value.
- Compounding when the BEY is required. The BEY is a simple (non-compounded) annualization. For the T-bill in Example 1, compounding gives 3.096% instead of 3.057%.

EXAM SHORTCUTS

- An FRN priced below par on a reset date must have DM > QM, and one priced above par must have DM < QM, so answer choices on the wrong side of the quoted margin can be eliminated without solving for I/Y.
- An add-on yield quoted on a 360-day basis converts to a 365-day basis, the BEY, by multiplying by 365/360; the holding period yield is not needed.

BOTTOM LINE

- An FRN's coupon rate is reset each period to the market reference rate set at the start of the period plus the quoted margin, and the coupon is paid at the end of the period, in arrears.
- The quoted margin is fixed in the indenture and reflects the issuer's credit risk at issuance, while the discount (required) margin is the spread investors currently require, the margin that would make the FRN worth par on a reset date.
- On a reset date an FRN trades at par when QM = DM, below par when DM > QM because credit quality has worsened, and above par when DM < QM because credit quality has improved.
- While the issuer's credit quality is unchanged, the QM stays equal to the DM; neither the passage of time nor a change in the MRR opens a gap between them.
- The simplified FRN valuation projects each coupon at today's MRR plus the QM and discounts at today's MRR plus the DM, both divided by the periods per year, and it ignores expected changes in the MRR.

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- US Treasury bills and commercial paper are quoted as discount yields on a 360-day year, computed on face value, while the convention for bank CDs, repos and market reference rates is an annualized add-on rate.
- A money market instrument's BEY annualizes the add-on return over a 365-day year, $HPY \times 365/\text{days}$, and for a discount instrument priced below face value it is higher than the quoted discount yield.
- To compare a money market yield with a semiannual-pay bond's YTM, compound the HPY to an effective annual yield, $(1+HPY)^{365/\text{days}}-1$, and restate it as $2[(1+EAY)^{1/2}-1]$.

QUICK CHECK

Question 1

Carrow Logistics has a $5 million floating-rate note outstanding that pays an annual coupon equal to the market reference rate (MRR) plus a quoted margin of 110 basis points. Today is a reset date with exactly four years remaining to maturity. The MRR is 4.20%, and the discount margin the market currently requires on Carrow's credit is 70 basis points. Using the simplified approach that holds the MRR constant, the estimated value of the note is *closest* to:

A $5,071,084.

B $5,000,000.

C $4,929,572.

Answer: A

Coupons are projected at MRR + QM and discounted at MRR + DM. The QM (110 bp) exceeds the DM (70 bp), so the note is worth more than par.

Coupon rate $= 4.20\% + 1.10\% = 5.30\%$, so the annual coupon $= 0.053 \times \$5{,}000{,}000 = \$265{,}000$.

Discount rate $= 4.20\% + 0.70\% = 4.90\%$.

$$V = \sum_{t=1}^{4} \frac{265{,}000}{1.049^{t}} + \frac{5{,}000{,}000}{1.049^{4}} = \$5{,}071{,}084$$

TI BA II Plus (P/Y = C/Y = 1, END mode): N = 4, I/Y = 4.90, PMT = 265,000, FV = 5,000,000, CPT PV = −5,071,084.

Why the other options are wrong

- **B.** $5,000,000 assumes the FRN must trade at par on a reset date. That holds only when the quoted margin equals the discount margin; here QM > DM.
- **C.** $4,929,572 swaps the two margins: coupons at 4.20% + 0.70% = 4.90% discounted at 4.20% + 1.10% = 5.30%, which produces a discount price.

Key takeaway Coupons come from the QM and the discount rate from the DM. If QM > DM the note trades at a premium; if QM < DM, at a discount.

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Practice questions

Question 2

Tenby Savings Bank issues a 91-day negotiable certificate of deposit. An investor who deposits $1,000 today receives $1,006.50 at maturity. The certificate's bond equivalent yield is *closest* to:

A 2.57%.

B 2.61%.

C 2.63%.

Question 3

A floating-rate note issued by Penhale Chemicals pays the market reference rate plus 85 basis points. Dealers now price the note at a discount margin of 55 basis points. Which of the following is the *most likely* explanation for the difference?

A The market reference rate has fallen since the note was issued.

B The issuer's credit quality has improved since the note was issued.

C The note is currently priced below its par value.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/yield-and-yield-spread-measures-for-floating-rate-instruments/. The full bank has 15 questions on this reading.

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FIXED INCOME · READING 59

The Term Structure of Interest Rates: Spot, Par, and Forward Curves

What you'll learn

LOS 59.a Define spot rates and the spot curve, and price a bond by discounting each cash flow at its own spot rate.

LOS 59.b Define par and forward rates; compute par rates, forward rates from spot rates, spot rates from forward rates, and bond prices using forward rates.

LOS 59.c Compare the spot curve, par curve and forward curve and how they are ordered in normal, inverted and flat environments.

MODULE 59.1

The Term Structure of Interest Rates: Spot, Par, and Forward Curves

This reading prices bonds with a separate spot rate for each cash flow and links spot rates, par rates and forward rates. A candidate must be able to price a bond from spot rates, compute par rates, derive forward rates from spot rates and spot rates from forward rates, price a bond with forward rates, and describe how the spot, par and forward curves relate.

LOS 59.a — Spot rates, the spot curve and pricing a bond with spot rates

A bond's **yield to maturity (YTM)** discounts every cash flow at one rate. In reality the market applies a different rate to each date. The market discount rate for a single payment at a future date is the **spot rate** for that maturity. Because a **zero-coupon bond** makes exactly one payment, its YTM is the spot rate for its maturity. For this reason spot rates are also called **zero-coupon rates** or **zero rates**. At a positive yield, all of a zero-coupon bond's return comes from price appreciation toward par; there are no coupons to reinvest.

The **spot curve** plots spot rates against time to maturity for one type of issuer (e.g., government spot rates). It usually slopes upward, a **normal yield curve**, because investors want higher returns for committing money over longer horizons. When longer-maturity spot rates are lower than shorter ones, the curve slopes downward and is called **inverted**.

Pricing with spot rates

Discount each cash flow at the spot rate that matches its own date and add the present values:

$$P = \frac{PMT}{(1+S_1)} + \frac{PMT}{(1+S_2)^2} + \cdots + \frac{PMT+FV}{(1+S_N)^N}$$

This is the **no-arbitrage price** (the **arbitrage-free valuation** approach). If a bond traded at any other price, a dealer could strip it into zero-coupon pieces (or rebuild it from zeros) and lock in a riskless profit. Absent arbitrage opportunities, the market price therefore equals this value whatever the shape of the curve. The difference between market price and no-arbitrage value is the potential arbitrage profit. A government bond is valued with the **government spot rate** for each cash flow's maturity; the same spot rate applies to a given date whichever bond the cash flow belongs to (the 6-year spot rate discounts the year-6 coupon of a 20-year bond too).

Example. Spot rates: $S_1 = 2.0\%$, $S_2 = 2.8\%$, $S_3 = 3.3\%$. A 3-year, 4% annual-pay bond:

$$P = \frac{4}{1.02} + \frac{4}{1.028^2} + \frac{104}{1.033^3} = 3.922 + 3.785 + 94.348 = 102.055$$

Its YTM (N = 3, PMT = 4, FV = 100, PV = −102.055, CPT I/Y) is 3.27%. It is a cash-flow-weighted blend of the spot rates, dominated by S_3 because most of the cash arrives at year 3. The YTM is found as the bond's internal rate of return; a simple or geometric average of spot rates does not give it.

Common exam traps

- Discounting every cash flow at the longest spot rate, or at the first one. For the 3-year, 4% bond in the example, discounting everything at 3.3% gives 101.969 and at 2.0% gives 105.768, instead of 102.055.
- Forgetting the final coupon (discounting only the principal at maturity). For the same bond, discounting only the 100 of principal at year 3 gives 98.426 instead of 102.055.
- Using annual spot rates with semiannual periods: for semiannual cash flows, use $S_t/2$ and count half-years.

LOS 59.b — Par rates, forward rates, and moving between spot and forward rates

Par rates

A **par rate** is the coupon rate that makes a bond of a given maturity price at exactly par, given the spot curve. Equivalently, it is the YTM of a hypothetical **par bond**. For maturity N with coupon c per 100:

$$100 = \frac{c}{1+S_1} + \frac{c}{(1+S_2)^2} + \cdots + \frac{c+100}{(1+S_N)^N} \;\Rightarrow\; c = \frac{100\left[1 - \frac{1}{(1+S_N)^N}\right]}{\sum_{t=1}^{N} \frac{1}{(1+S_t)^t}}$$

With the spot rates above (2.0%, 2.8%, 3.3%), the 3-year par rate is 3.27%, slightly below the 3-year spot rate. On the exam it is often quicker to plug in the middle answer choice: a price below 100 means the par rate is higher; above 100 means it is lower.

Forward rates and notation

A **forward rate** is a rate agreed today for a loan that starts at a future date. The notation **AyBy** means a B-year loan starting A years from now: **1y1y** is a 1-year rate one year from now, **2y1y** a 1-year rate two years from now, **2y2y** a 2-year rate two years from now.

No arbitrage requires that investing for N years at the N-year spot rate grows to the same amount as rolling over at the forward rates:

KEY CONCEPT

$$(1+S_3)^3 = (1+S_1)(1+{}_{1y1y})(1+{}_{2y1y}) = (1+S_2)^2(1+{}_{2y1y})$$

The two-year case shows the logic. An investor who buys a 1-year zero at S_1 instead of a 2-year zero at a higher S_2 accepts less in year 1. That choice makes sense only if the rate available for year 2 is high enough to catch up. The forward rate 1y1y is the year-2 rate at which both plans end with the same amount.

Spot from forwards. The spot rate is the **geometric mean** of the one-period rates:

$$S_N = \left[(1+S_1)(1+{}_{1y1y})\cdots(1+{}_{(N-1)y1y})\right]^{1/N} - 1$$

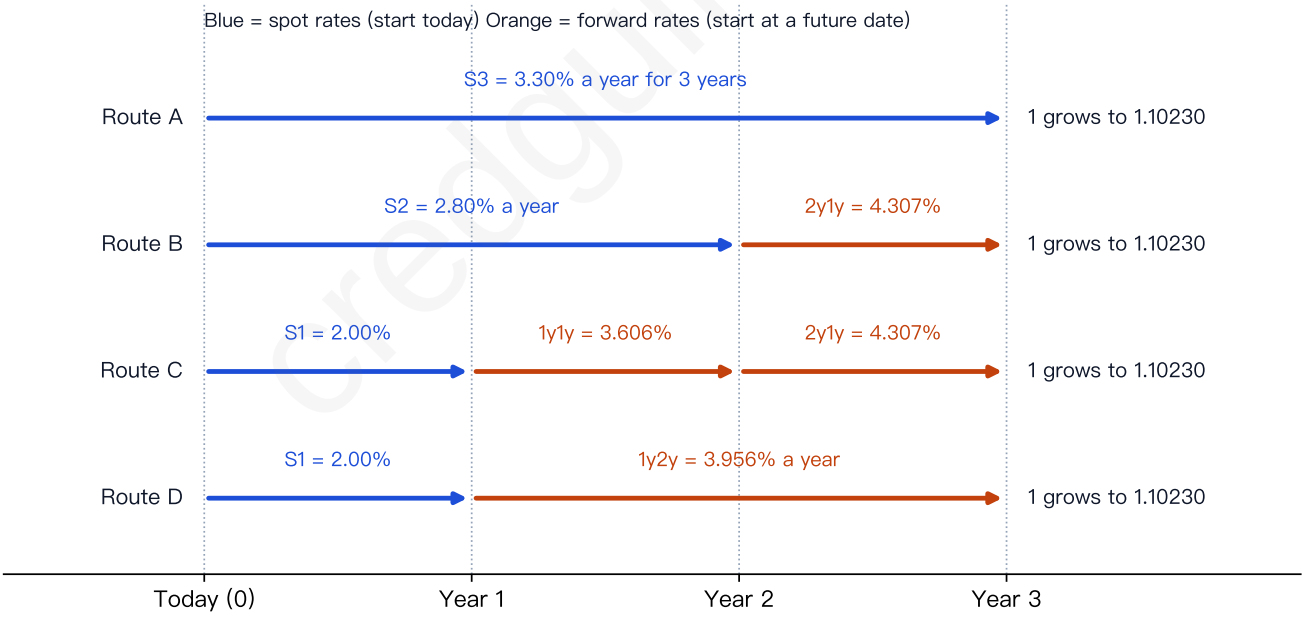
Forward from spots:

KEY CONCEPT

$$_{AyBy} = \left[\frac{(1+S_{A+B})^{A+B}}{(1+S_A)^A}\right]^{1/B} - 1$$

Example. With $S_1 = 2.0\%$, $S_2 = 2.8\%$, $S_3 = 3.3\%$:

- $1y1y = 1.028^2/1.02 - 1 = 3.606\%$
- $2y1y = 1.033^3/1.028^2 - 1 = 4.307\%$
- $1y2y = (1.033^3/1.02)^{1/2} - 1 = 3.956\%$. The square root annualizes the 2-year forward rate.
- Going back: $S_3 = [(1.02)(1.03606)(1.04307)]^{1/3} - 1 = 3.30\%$.



Four ways to invest from today to year 3 that must grow to the same amount

Each route in the figure turns 1 into 1.10230 by year 3, so any one rate on the timeline can be backed out from the others. A 1-year spot rate, for example, is $1 + S_1 = (1+S_2)^2/(1+{}_{1y1y})$.

Approximation check: forward ≈ difference of "total" rates, e.g. $2y1y \approx 3 \times 3.3 - 2 \times 2.8 = 4.3\%$. Useful to screen answers; the exact figure uses compounding. The same shortcut works for a multi-year forward: $2y2y \approx (4S_4 - 2S_2)/2$.

Semiannual rates: with rates stated on a **semiannual bond basis**, work with half-year rates and periods, then double the result (e.g., the 6-month rate six months forward, $6m6m$).

WORKED EXAMPLE: A 6-MONTH FORWARD RATE AND A SEMIANNUAL BOND PRICE

Spot rates on a semiannual bond basis are 2.3% for 6 months and 3.1% for 1 year. Compute the 6-month rate six months forward (6m6m) on a semiannual bond basis, and price a 1-year bond with a 4.4% coupon paid semiannually, per 100 of par.

Step 1. Half-year spot rates: $2.3\%/2 = 1.15\%$ and $3.1\%/2 = 1.55\%$.

Step 2. Half-year forward rate: $1.0155^2/1.0115 - 1 = 1.9516\%$. On a semiannual bond basis, $6m6m = 2 \times 1.9516\% = 3.903\%$.

Step 3. The bond pays 2.2 after 6 months and 102.2 after 1 year. Price $= 2.2/1.0115 + 102.2/1.0155^2 = 2.175 + 99.104 = 101.279$.

Result. The 6m6m forward rate is 3.903%, and the bond is worth 101.279 per 100 of par.

Valuing a bond with forward rates

Each cash flow is discounted by the product of one plus each one-period rate up to its date:

$$P = \frac{PMT}{1+S_1} + \frac{PMT}{(1+S_1)(1+{}_{1y1y})} + \frac{PMT+FV}{(1+S_1)(1+{}_{1y1y})(1+{}_{2y1y})}$$

These products are the spot-rate discount factors, so the price is the same no-arbitrage price. Example: with rates 2%, 3% and 3.5% for years 1–3, a 3-year 4% bond is worth $4/1.02 + 4/(1.02 \times 1.03) + 104/(1.02 \times 1.03 \times 1.035) = 103.37$. A zero-coupon bond is priced the same way with only the final term.

Common exam traps

- Using the arithmetic average of forward rates instead of the geometric mean. The gap widens as the rates spread apart.
- Forgetting to annualize a multi-year forward rate (take the $1/B$ root), or dividing by B instead of taking the root. For 1y2y in the example, leaving out the root gives 8.069%, and dividing by 2 gives 4.034%, instead of 3.956%.
- Off-by-one exponents: the year-4 to year-5 forward uses $(1+S_5)^5/(1+S_4)^4$.
- Discounting a later cash flow at only one forward rate instead of the compounded chain. With the 2%, 3% and 3.5% rates in the example, discounting each cash flow at its own year's rate alone gives 108.29 instead of 103.37.

LOS 59.c — Comparing the spot, par and forward curves

Curve	What it plots	Observed or derived?
Spot curve (**zero curve**, **strip curve**)	Spot rates (YTMs of zero-coupon bonds) vs maturity	From zero-coupon or stripped Treasury prices
Yield curve for coupon bonds	YTMs of actively traded coupon bonds of the same or similar issuer vs maturity	Observed; gaps filled by linear interpolation

Curve	What it plots	Observed or derived?
Par curve	Coupon rates at which bonds of each maturity would price at par	Derived (hypothetical) from the spot curve
Forward curve	Forward rates (e.g., 1-year rates) for successive future periods	Derived from spot rates

A coupon-bond yield curve built from market prices suffers from illiquidity and from tax distortions for bonds trading away from par; using **on-the-run** issues helps with liquidity, but there may be too few of them. A **par curve** derived from the spot curve avoids these problems. Rates on these curves are usually quoted on a **semiannual bond basis**.

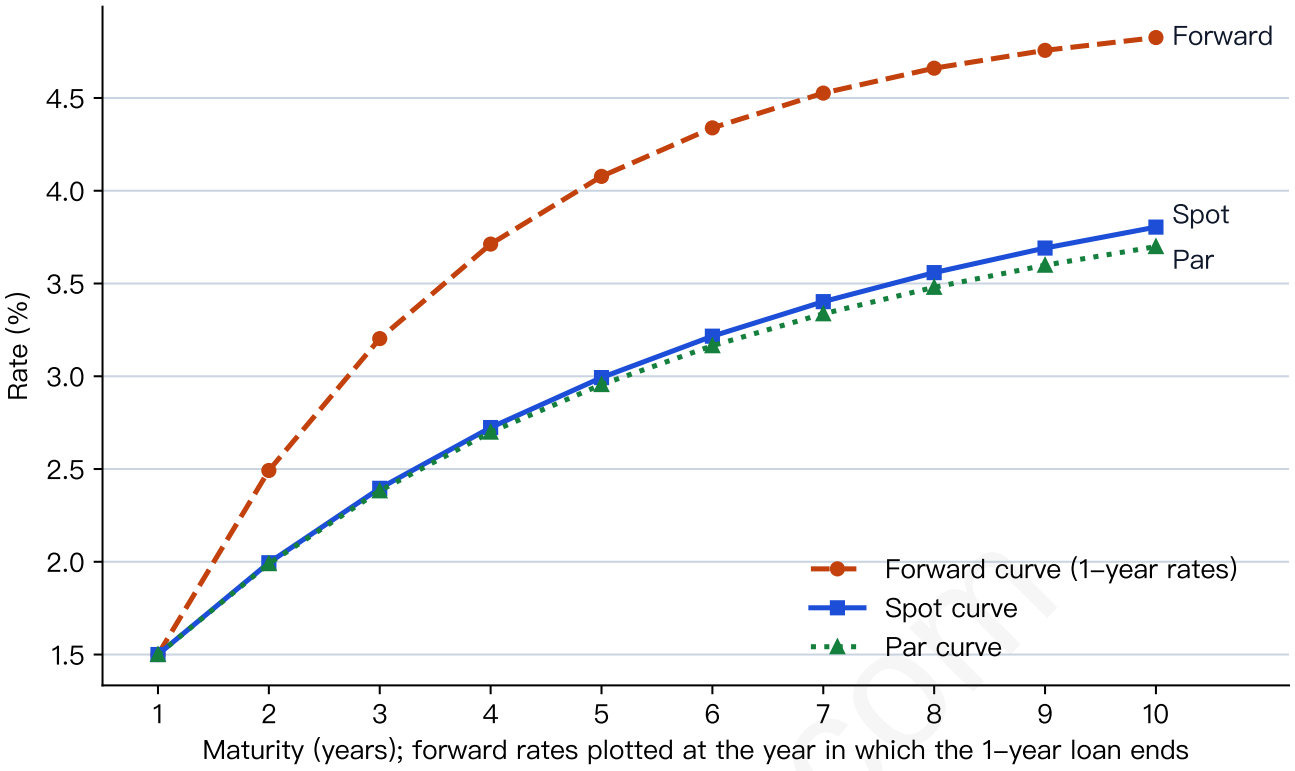
How the curves relate

Forward rates drive spot rates (a spot rate is a geometric mean of forwards), and spot rates drive par rates (a par rate is a weighted average of spot rates). The par rate is weighted most heavily toward the final cash flow.

KEY CONCEPT

Environment	Ordering
Normal (upward-sloping)	forward > spot > par beyond the first year; spot rises more slowly than forward; par just below spot
Inverted (downward-sloping)	forward < spot < par beyond the first year; par just above spot
Flat	forward = spot = par at all maturities

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Forward, spot and par curves when forward rates rise with maturity

An inverted curve is consistent with the market expecting short-term rates to fall.

Common exam traps

- Saying the par curve is observed in the market. It is theoretical, derived from spot rates.
- Reversing the ordering in an inverted environment: there, par yields sit above spot rates and forwards are lowest.
- Confusing "the yield curve" of coupon bonds (same issuer, different maturities) with a plot across issuers.

EXAM SHORTCUTS

- A forward rate is roughly the difference of the total rates, for example $2y1y \approx 3S_3 - 2S_2$; the exact figure needs compounding, but the approximation screens out wrong answer choices quickly.
- To find a par rate from answer choices, plug in the middle choice: a price below 100 means the par rate is higher, and a price above 100 means it is lower.

BOTTOM LINE

- A spot rate is the market discount rate for a single payment at a future date, which is why it equals the YTM of a zero-coupon bond of that maturity.
- The no-arbitrage price of a bond discounts each cash flow at the spot rate for its own date, and the bond's YTM is the internal rate of return at that price, not a simple or geometric average of the spot rates.

- A par rate is the coupon rate at which a bond of a given maturity would be priced at par, given the spot curve.
- ${}_{A}y_{B} = \left[(1+S_{A+B})^{A+B}/(1+S_A)^{A}\right]^{1/B} - 1$, where the $1/B$ root annualizes a multi-year forward rate.
- A spot rate is the geometric mean of the one-period rates up to its maturity: $S_N = \left[(1+S_1)(1+{}_{1}y_{1})\cdots(1+{}_{(N-1)}y_{1})\right]^{1/N} - 1$.
- Discounting each cash flow by the compounded chain of one-period forward rates up to its date gives the same no-arbitrage price as discounting at spot rates.
- Beyond the first year, forward > spot > par when the curve is normal (upward-sloping), forward < spot < par when it is inverted, and all three are equal when it is flat.
- The par curve and the forward curve are derived from the spot curve, while the yield curve for coupon bonds is observed from the YTMs of actively traded bonds, with gaps filled by linear interpolation.

QUICK CHECK

Question 1

Which of the following statements about zero-coupon bonds and spot rates is *most accurate*?

A If a 4-year zero-coupon bond yields 5.2%, the 4-year spot rate is 1.3%.

B Spot rates are the same for every maturity on the term structure.

C At a positive yield, a zero-coupon bond's whole return is price appreciation.

Answer: C

A zero-coupon bond pays no coupons: at a positive yield it is bought below par and pays par at maturity, so its entire return is price appreciation and there is no uncertainty about reinvesting coupons. Its yield to maturity is the spot rate for its maturity.

Why the other options are wrong

- **A.** The YTM of a zero-coupon bond *is* the spot rate for that maturity, so the 4-year spot rate is 5.2%; it is not divided by the number of years.
- **B.** Spot rates generally differ across maturities, and that variation is what the spot curve shows. Only in a flat environment are they all equal.

Key takeaway Zero-coupon bond: YTM = spot rate; at a positive yield the whole return is the price rising toward par; no reinvestment risk.

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Practice questions

Question 2

Which of the following *best* describes how the arbitrage-free approach values an option-free bond?

A It discounts the cash flows at a set of spot rates that reflects the current term structure.

B It discounts the cash flows at the geometric average of the current spot rates.

C It discounts all the cash flows at a single discount rate.

Question 3

Government spot rates are 2.1% for one year, 2.9% for two years and 3.6% for three years (annual compounding). The coupon rate at which a 3-year annual-pay government bond would be priced at par (the 3-year par rate) is *closest* to:

A 2.87%.

B 3.57%.

C 3.60%.

Question 4

Current market rates (annual compounding) are as follows: the 1-year spot rate is 2.8%, the 1-year forward rate one year from now is 4.6%, and the 1-year forward rate two years from now is 6.2%. The price per 100 of par of a 3-year zero-coupon bond is *closest* to:

A 92.05.

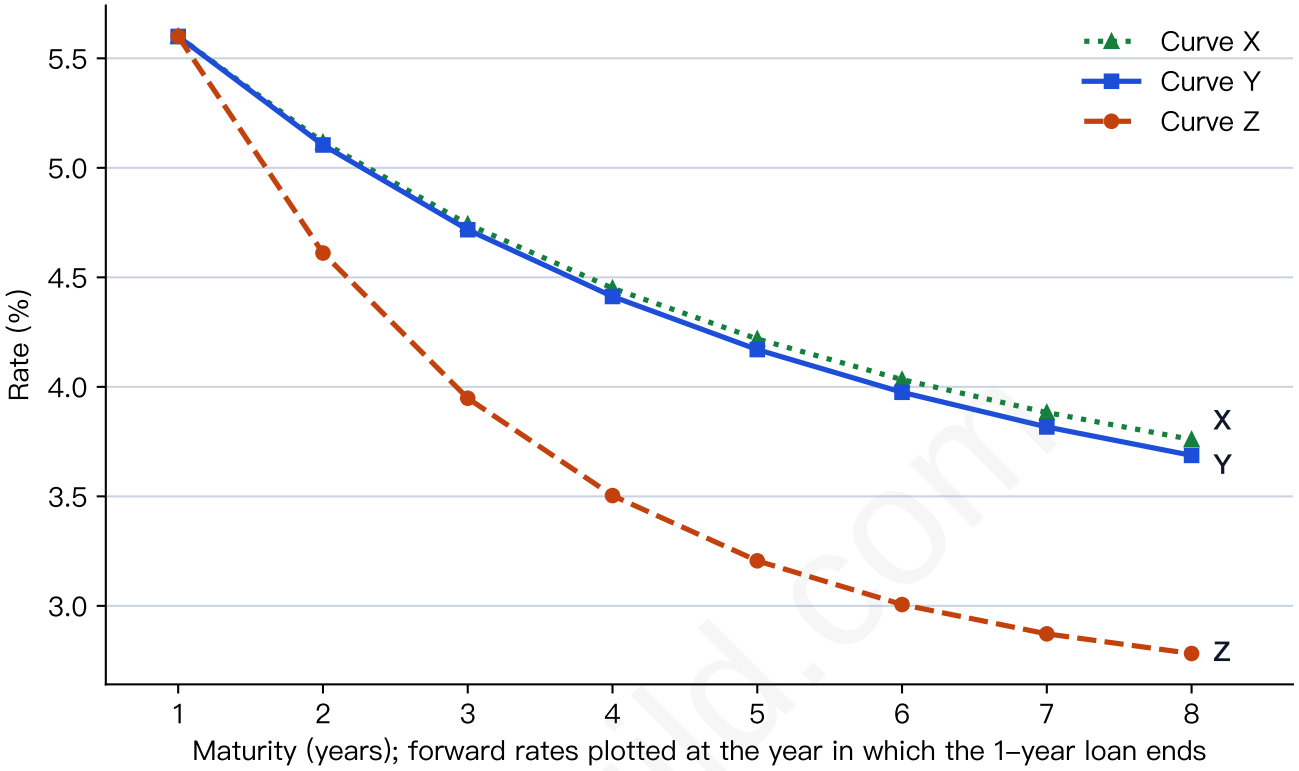
B 87.57.

C 83.49.

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Question 5

The chart below shows the spot curve, the par curve and the forward curve of 1-year rates, all derived from the same set of government bond prices (annual compounding). The curves are labeled only X, Y and Z.



Three curves derived from the same government bond prices

Curve X is *most likely* the:

A par curve.

B spot curve.

C forward curve.

Answers: 2. A · 3. B · 4. B · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/the-term-structure-of-interest-rates-spot-par-and-forward-curves/. The full bank has 41 questions on this reading.

FIXED INCOME · READING 60

Interest Rate Risk and Return

What you'll learn

LOS 60.a Calculate and interpret the sources of return from a fixed-rate bond: coupons and principal, reinvestment income, and capital gains or losses relative to carrying value.

LOS 60.b Describe how a bond's holding period return depends on its Macaulay duration relative to the investment horizon (duration gap).

LOS 60.c Define, calculate, and interpret Macaulay duration as the present-value-weighted average time to receipt of a bond's cash flows.

MODULE 60.1

Interest Rate Risk and Return

This reading traces the sources of return on a fixed-rate bond and shows how market price risk and reinvestment risk trade off over the investment horizon. A candidate must be able to compute a horizon yield and a capital gain or loss measured against carrying value, explain the duration gap, and calculate and interpret Macaulay duration.

LOS 60.a — Where a fixed-rate bond's return comes from

An investor in a fixed-rate bond can earn return from three sources:

1. **Coupon and principal payments** promised by the issuer.
2. **Reinvestment income**: the interest earned by reinvesting coupons over the investor's holding period.
3. A **capital gain or loss** if the bond is sold before it matures.

Throughout this reading we assume the issuer makes every payment on time (no credit risk) and that coupons are reinvested at the bond's prevailing **yield to maturity (YTM)**.

Horizon yield

The **investment horizon** is how long the investor plans to hold the bond; it can be shorter than the bond's maturity. The **horizon yield** (annualized horizon return) is the compound annual rate that turns the purchase price into the total value at the horizon:

$$\text{Horizon yield} = \left(\frac{\text{sale price (or principal)} + \text{coupons} + \text{reinvestment income}}{\text{purchase price}} \right)^{1/H} - 1$$

where H is the horizon in years. Coupons plus reinvestment income equal the future value of the coupon annuity at the reinvestment rate. Working backward, the reinvestment income needed to reach a target yield is the purchase price compounded at that yield over the horizon, minus the coupons and the principal (or sale price) received.

> *Example.* A 4-year, 3% annual-coupon bond is bought at a YTM of 4%, so its price is 96.370 (N = 4, I/Y = 4, PMT = 3, FV = 100, CPT PV). Held to maturity with coupons reinvested at 4%, the coupons grow to $3(1.04)^3 + 3(1.04)^2 + 3(1.04) + 3 = 12.739$, of which $12.739 - 12 = 0.739$ is reinvestment income.

Ending value $= 112.739$, and $(112.739/96.370)^{1/4} - 1 = 4.00\%$, the YTM at purchase.

Five key results

KEY CONCEPT

Situation (any change in YTM occurs before the first coupon date)	Realized return vs. YTM at purchase
YTM unchanged, bond held to maturity	Equal to the YTM
YTM unchanged, bond sold before maturity	Equal to the YTM (sale at the carrying value)
YTM rises (falls), bond held to maturity	Higher (lower); only reinvestment income changes
YTM rises, short horizon (below the bond's Macaulay duration, LOS 60.b)	Lower; the lower sale price dominates
YTM falls, long horizon (beyond the bond's Macaulay duration, LOS 60.b)	Lower; lost reinvestment income dominates

Continuing the example: if the YTM moves to 4.75% immediately after purchase and the bond is held to maturity, coupons grow to 12.882 and the realized return is 4.03% (> 4%). If instead the bond is sold after one year, its price is 95.212 and the one-year return is $(95.212 + 3)/96.370 - 1 = 1.91\%$ (< 4%). With a move to 3.25%, the hold-to-maturity return is 3.97% but the one-year return is $(99.296 + 3)/96.370 - 1 = 6.15\%$. In every hold-to-maturity case the realized return lies between the original YTM and the new reinvestment rate.

Two sources of interest rate risk

- **Market price risk** (price risk): uncertainty about the price at which the bond can be sold at the horizon. Higher yields mean lower prices.
- **Reinvestment risk**: uncertainty about the income from reinvesting coupons. Higher yields mean more reinvestment income.

A rise in yields therefore *hurts* the sale price but *helps* reinvestment income, and a fall does the opposite. The investment horizon decides which effect is larger. An investor who holds to maturity has no price risk (the bond pays par), only reinvestment risk. An investor who sells on the first coupon date has not yet earned any interest on reinvested coupons, so that investor faces price risk only.

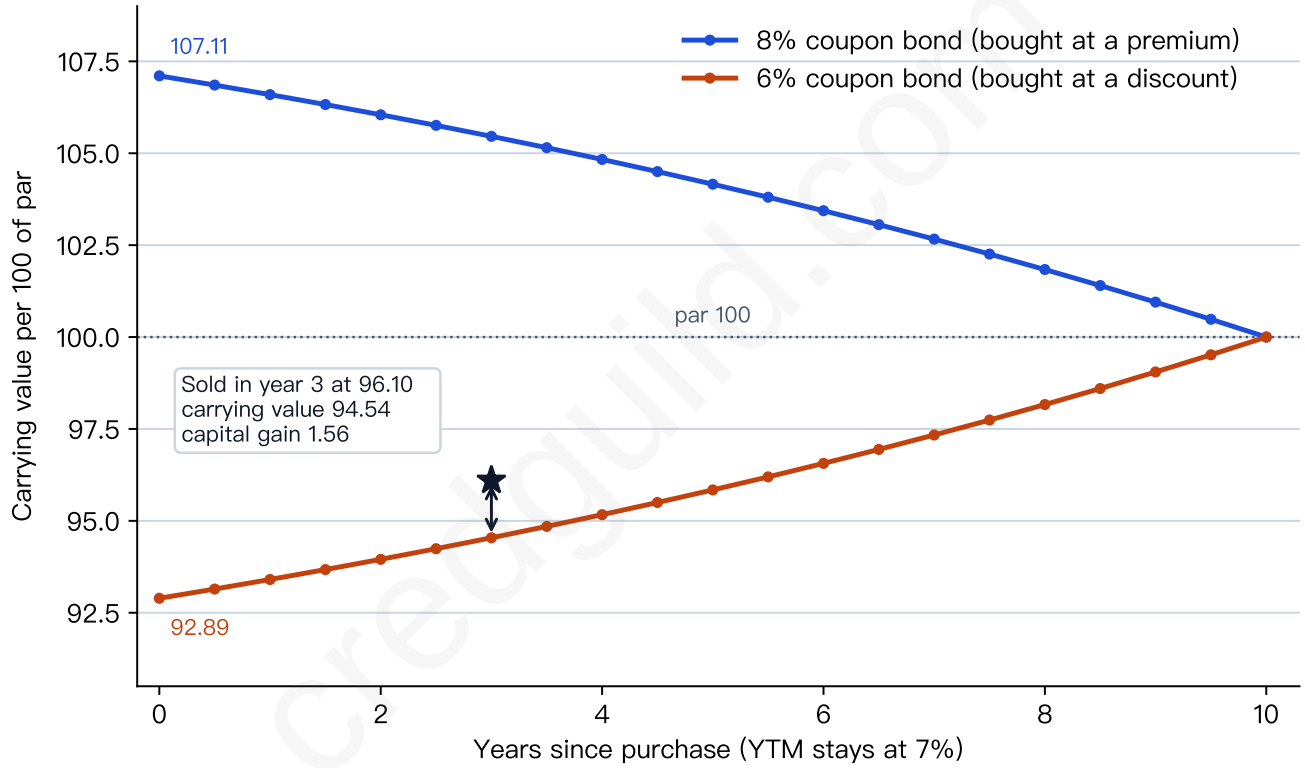
Horizon	Dominant risk
Short	Market price risk > reinvestment risk
Long (to maturity)	Reinvestment risk > market price risk

Other things equal, reinvestment risk is greater for bonds with higher coupons (more cash to reinvest) and longer maturities (a longer reinvestment period). A zero-coupon bond held to maturity has no reinvestment risk.

Capital gains and losses: carrying value

If the YTM does not change, a bond's price moves along its **constant-yield price trajectory** toward par: a discount bond's price rises toward par and a premium bond's price falls toward par. The price on that path at any date is the bond's **carrying value**: the purchase price plus the amortized discount (or minus the amortized premium). A capital gain or loss is measured as sale price minus carrying value, not sale price minus purchase price. Movement along the trajectory is part of interest income (it adds to a discount bond's interest and reduces a premium bond's), so it never counts as a capital gain or loss.

Example. A 10-year, 6% semiannual-coupon bond is bought at a YTM of 7%. Three years later (14 semiannual periods left) it is sold for 96.10. Carrying value: N = 14, I/Y = 3.5, PMT = 3, FV = 100, CPT PV = −94.54. Capital gain $= 96.10 - 94.54 = 1.56$ per 100 of par. A bond held to maturity, or sold at the purchase YTM, has no capital gain or loss.



Constant-yield price trajectories: carrying value of a premium and a discount bond as time passes

The figure shows both paths for 10-year bonds bought at a 7% YTM, together with the sale from the example.

A zero-coupon bond follows the same rule: the accretion toward face value is interest income, and a sale before maturity produces a capital gain or loss only if a change in YTM has moved the price away from the carrying value.

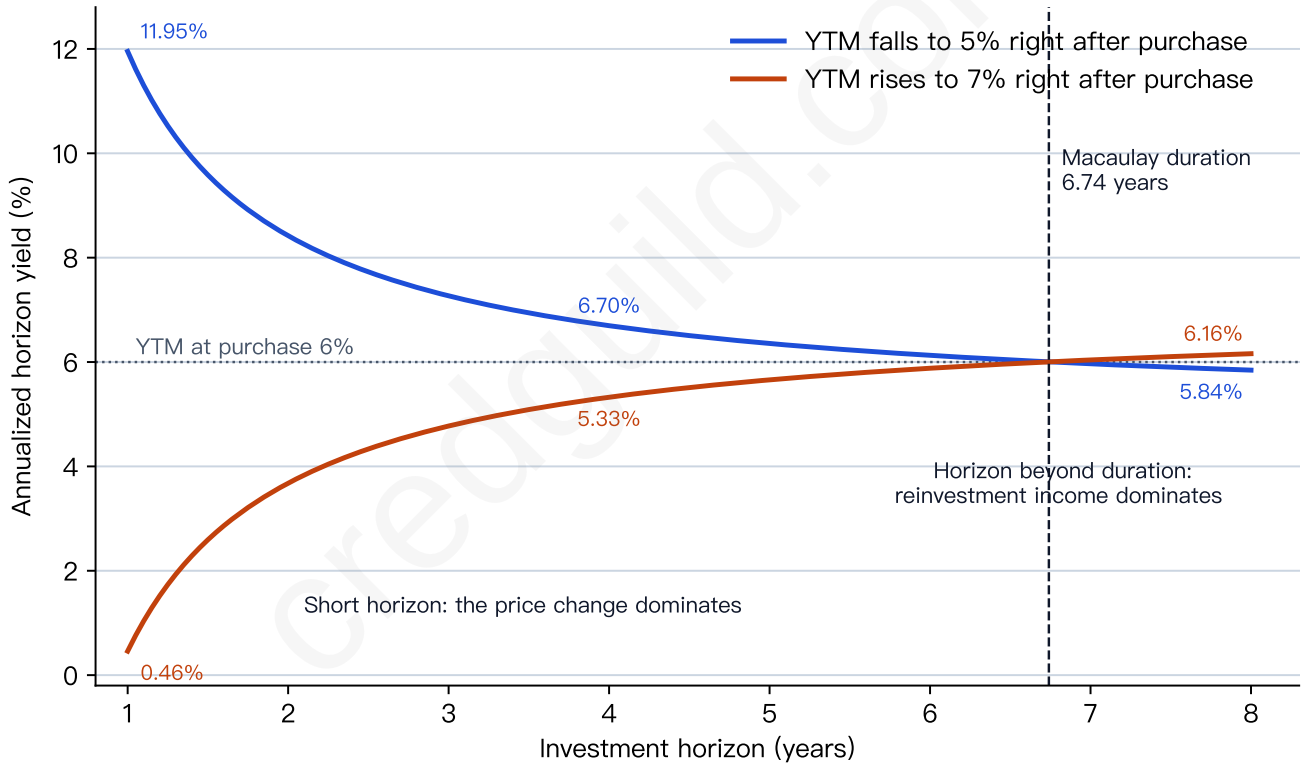
Common exam traps

- Assuming the YTM is earned whatever the reinvestment rate. Held to maturity with coupons reinvested at the coupon rate, a premium bond earns more than its YTM and a discount bond less.

- Measuring a capital gain against the purchase price or par. The 8% bond in the figure, bought at 107.11 and sold after three years at 106.00, shows a capital gain of $106.00 - 105.46 = 0.54$ even though it was sold below its purchase price. For the 10-year, 6% bond in the carrying-value example, measuring against the purchase price of 92.89 gives a gain of 3.21, and measuring against par gives a loss of 3.90, instead of a gain of 1.56.
- Expecting a rise in YTM to lower the return on a bond held to maturity. Only reinvestment income changes, so the return rises.

LOS 60.b — Holding period return, Macaulay duration and the investment horizon

Because price risk and reinvestment risk move in opposite directions, there is an investment horizon at which they roughly cancel. That horizon is the bond's **Macaulay duration**, the weighted average time until its cash flows are received (calculated under LOS 60.c). If the horizon equals the Macaulay duration, a one-time change in YTM right after purchase leaves the horizon yield approximately equal to the YTM at purchase: the gain (loss) on reinvestment income offsets the loss (gain) on the sale price.



Horizon yield against investment horizon after a one-time change in YTM right after purchase

The figure follows an 8-year, 5% annual-coupon bond bought at a 6% YTM (price 93.79, Macaulay duration 6.74 years) whose YTM moves to 5% or to 7% right after purchase. Over a one-year horizon the change in price dominates: the horizon yield is 11.95% if the YTM falls and 0.46% if it rises. As the horizon lengthens, the price effect shrinks because the bond is sold closer to maturity, when its price is nearer par. The reinvestment effect grows because more coupons are reinvested for longer. The two effects cancel at a horizon equal to the Macaulay duration, where both lines pass through 6%. Beyond that point the ranking reverses, and held to maturity the bond earns 6.16% if the YTM rose and 5.84% if it fell.

The **duration gap** is:

$$\text{Duration gap} = \text{Macaulay duration} - \text{investment horizon}$$

KEY CONCEPT

Duration gap	Horizon vs. MacDur	Main exposure	Hurt by
Positive duration gap	Horizon < MacDur	Market price risk	Rising rates
Zero	Horizon = MacDur	Risks roughly offset	—
Negative duration gap	Horizon > MacDur	Reinvestment risk	Falling rates

Example. An investor with a 7-year horizon holds bonds with a Macaulay duration of 5.5 years. The gap is $5.5 - 7 = -1.5$ years (negative). A fall in yields lowers the realized return, because reinvestment income falls by more than the sale price rises.

Common exam traps

- A horizon shorter than *maturity* does not by itself mean price risk dominates; compare the horizon with Macaulay duration.
- A negative duration gap means falling rates are the threat; it does not minimize reinvestment risk.
- Any nonzero duration gap leaves an exposure. Only a horizon equal to the Macaulay duration makes the two risks roughly offset.

LOS 60.c — Calculating and interpreting Macaulay duration

The (annual) Macaulay duration is a weighted average of the number of years until each promised cash flow is paid. Each weight is the present value of a cash flow (discounted at the YTM) divided by the bond's full price:

KEY CONCEPT

$$\text{MacDur} = \sum_{t=1}^{N} t \times w_t, \qquad w_t = \frac{CF_t/(1+r)^t}{\text{full price}}, \qquad \sum_t w_t = 1$$

Each cash flow contributes its weight times the number of years until it is paid. For a typical coupon bond, the final payment (last coupon plus principal) has the largest weight and contributes most to the duration.

Steps. (1) Price the bond at its YTM. (2) Discount each cash flow at the YTM. (3) Divide each present value by the price to get its weight; the weights sum to 1. (4) Multiply each weight by the time until its cash flow and add the products.

Example (the 4-year, 3% bond at 4%, price 96.370):

Year	Cash flow	PV at 4%	Weight
1	3	2.885	0.0299

Year	Cash flow	PV at 4%	Weight
2	3	2.774	0.0288
3	3	2.667	0.0277
4	103	88.045	0.9136

$\text{MacDur} = 0.0299(1) + 0.0288(2) + 0.0277(3) + 0.9136(4) = 3.82$ years.

For a semiannual-pay bond, the same calculation gives a duration in semiannual periods; divide by 2 to state it in years. The Macaulay duration of a zero-coupon bond equals its maturity; a coupon bond's is shorter than its maturity.

Interpretations

Measure	Meaning
Macaulay duration	Weighted average time to receipt of the cash flows; the horizon at which price risk and reinvestment risk offset
Modified duration (Reading 61)	Approximate percentage price change for a 1% change in YTM

Why the offset happens at this horizon: a zero-coupon bond held to its maturity has neither risk, because nothing is reinvested and nothing is sold early. Macaulay duration condenses a coupon bond's cash flows into one equivalent payment date, so an investor whose horizon matches that date is roughly in the position of a zero-coupon holder.

Common exam traps

- Macaulay duration is a time measure (years) and is the duration matched to the investment horizon. The "percentage price change per 1% yield change" reading belongs to modified duration.
- Macaulay duration is not the time needed to recover the principal. The principal comes back at maturity.
- Use present-value weights rather than undiscounted cash flows, and do not divide the result by $1 + \text{YTM}$ (that gives modified duration). For the 4-year, 3% bond in the example, undiscounted weights give 3.84 years and dividing by 1.04 gives 3.68, instead of 3.82 years.

EXAM SHORTCUTS

- For a bond held to maturity after a one-time change in YTM right after purchase, the realized return lies between the original YTM and the new reinvestment rate, so any answer outside that range can be ruled out.
- A zero-coupon bond's Macaulay duration equals its maturity, so it needs no weighting calculation.

BOTTOM LINE

- A fixed-rate bond's return comes from the promised coupons and principal, the income from reinvesting coupons, and any capital gain or loss on a sale before maturity.
- With every payment made on time and coupons reinvested at the YTM, the realized return equals the YTM at purchase whether the bond is kept until it matures or sold earlier at an unchanged YTM.

- Higher yields lower the sale price but raise reinvestment income; over a short horizon market price risk dominates, and over a long horizon reinvestment risk dominates.
- A capital gain or loss is the sale price minus the carrying value on the constant-yield price trajectory, so movement along that trajectory counts as interest income rather than as a gain or loss.
- When the investment horizon equals the bond's Macaulay duration, a one-time change in YTM right after purchase leaves the horizon yield approximately equal to the YTM at purchase.
- Duration gap = Macaulay duration − investment horizon; a positive gap leaves the investor exposed to rising rates through price risk, and a negative gap leaves the investor exposed to falling rates through reinvestment risk.
- Macaulay duration is the weighted average time until a bond's cash flows are received, with each weight equal to the cash flow's present value at the YTM divided by the full price; a coupon bond's Macaulay duration is shorter than its maturity.
- Other things equal, reinvestment risk is greater for bonds with higher coupons and longer maturities, and a zero-coupon bond held to maturity has none.

QUICK CHECK

Question 1

A wealth manager tells a client: "When you invest in a bond for a given horizon, your return comes from the coupons you collect plus the difference between what you paid and what you receive when you sell the bond or it matures." The wealth manager's description is:

A incomplete, because it leaves out interest earned on reinvested coupons.

B complete, because it captures all sources of bond return.

C incomplete, because a bond held to maturity produces no capital gain or loss.

Answer: A

A fixed-rate bond has three sources of return: coupon and principal payments, reinvestment income on the coupons, and any capital gain or loss if it is sold before maturity. The manager's description covers the coupons and the price/principal difference but omits the interest earned by reinvesting coupons over the horizon.

Why the other options are wrong

- **B.** The description omits reinvestment income, which can be a large part of the return on a long-horizon, high-coupon bond.
- **C.** It is true that a bond held to maturity has no capital gain or loss, but that is not what is wrong with the description. It already allows for the bond to mature at par.

Key takeaway Three sources of bond return: coupons and principal, reinvestment income, and capital gain or loss on a sale before maturity.

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Practice questions

Question 2

Dmitri Volkov bought a 15-year, 4% semiannual-coupon bond at a yield to maturity of 5.2%. Four years later, he sells it for 92.10 per 100 of par. Per 100 of par, Volkov realizes:

A a capital gain of 2.06.

B a capital gain of 4.49.

C a capital loss of 7.90.

Question 3

A pension fund must make a single payment in eight years. The trustees want any change in the income from reinvesting coupons to be roughly offset by an opposite change in the price at which the bonds are sold at the horizon. Which measure of the bonds' duration should they set equal to eight years?

A Modified duration.

B Effective duration.

C Macaulay duration.

Question 4

Rafael Quintero calculates a Macaulay duration of 3.46 for a four-year, option-free corporate bond. The best interpretation of 3.46 is that it measures:

A the bond's price sensitivity to a change in interest rates.

B the average time until the bond's cash flows are received.

C the time the investor needs to recover his principal.

Answers: 2. A · 3. C · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/interest-rate-risk-and-return/. The full bank has 23 questions on this reading.

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FIXED INCOME · READING 61

Macaulay Duration

What you'll learn

LOS 61.a Define, calculate, and interpret modified duration (exact and approximate), money duration, and the price value of a basis point (PVBP).

LOS 61.b Explain how maturity, coupon rate, yield level (and embedded options) affect a bond's duration and interest rate risk.

MODULE 61.1

Yield-Based Bond Duration Measures and Properties

This reading turns duration into measures of price sensitivity and explains what makes a bond's price more or less sensitive to yield changes. A candidate must be able to compute modified duration, approximate modified duration, money duration and the PVBP, estimate a price change from them, and explain how maturity, coupon, yield level and embedded options affect interest rate risk.

LOS 61.a — Modified duration, money duration and the PVBP

Modified duration

Modified duration (ModDur) converts Macaulay duration (Reading 60) into a measure of price sensitivity. Divide Macaulay duration by one plus the bond's periodic yield:

$$\text{ModDur} = \frac{\text{MacDur}}{1 + r} \quad (\text{annual-pay: } r = \text{YTM})$$

$$\text{ModDur} = \frac{\text{MacDur}}{1 + \text{YTM}/2} \quad (\text{semiannual-pay, YTM on a semiannual bond basis, durations in years})$$

Whenever the yield is positive, modified duration is *smaller* than Macaulay duration.

Modified duration is the approximate percentage change in price for a 1% (100 bp) change in YTM:

KEY CONCEPT

$$\%\Delta P \approx -\text{ModDur} \times \Delta\text{YTM}$$

Example. A 6-year, 4% annual-pay bond priced at 94.924 to yield 5% has a Macaulay duration of 5.435. $\text{ModDur} = 5.435/1.05 = 5.18$. If the YTM rises by 30 bp, the price should fall by about $5.18 \times 0.30\% = 1.55\%$. For a semiannual bond with MacDur 7.2 years and a 6% YTM, $\text{ModDur} = 7.2/1.03 = 6.99$.

The conversion also runs backward: $\text{MacDur} = \text{ModDur} \times (1 + r)$. This is the step needed when a question gives modified duration but asks about the duration gap (Reading 60), which uses Macaulay duration. *Example.* A semiannual-pay bond with a 5% YTM has a modified duration of 9.20, so its Macaulay duration is $9.20 \times$

$1.025 = 9.43$ years. An investor with a 7-year horizon has a duration gap of $9.43 - 7 = +2.43$ years and is exposed mainly to rising rates.

Approximate modified duration

Modified duration can be estimated by repricing the bond at a slightly lower and a slightly higher yield:

KEY CONCEPT

$$\text{ApproxModDur} = \frac{V_- - V_+}{2 \times V_0 \times \Delta\text{YTM}}$$

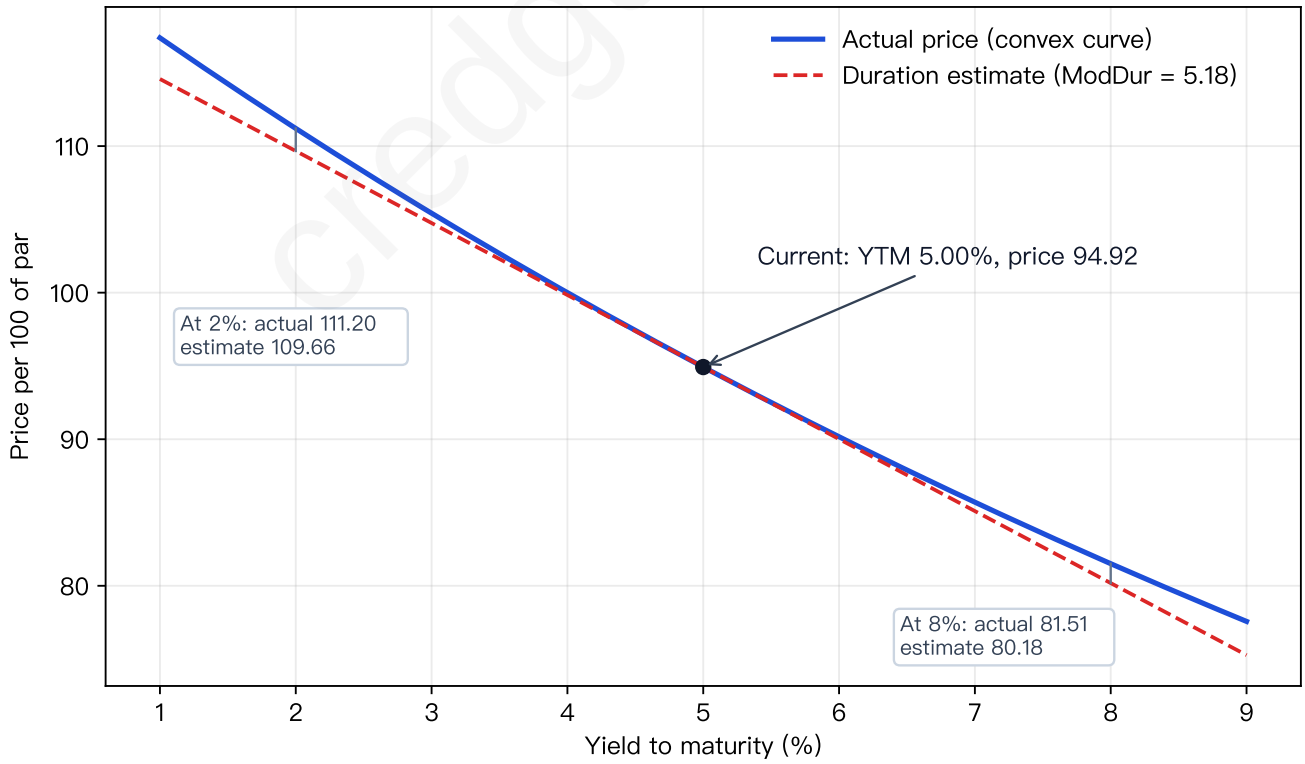
- V_- = price if the YTM falls by ΔYTM; V_+ = price if it rises by ΔYTM; V_0 = current price.
- The 2 averages the price rise and the price fall; V_0 turns the change into a percentage.

Geometrically, the straight line joining the two repriced points has almost the same slope as the tangent to the price-yield curve at the current price. Dividing the size of that slope by V_0 therefore approximates modified duration, and the approximation improves as the yield shift gets smaller.

For the 6-year bond: at 4.9% the price is 95.4172, at 5.1% it is 94.4345, so $\text{ApproxModDur} = (95.4172 - 94.4345)/(2 \times 94.9243 \times 0.001) = 0.9827/0.18985 = 5.18$.

Duration is a straight-line estimate

The true price-yield relationship of an option-free bond is **convex**. Duration draws a straight tangent line at the current yield, so it works well for small yield changes and less well for large ones. For large changes, duration alone underestimates the price increase when yields fall and overestimates the price decrease when yields rise.



Actual (convex) price-yield curve vs. the straight-line modified-duration estimate

Money duration and PVBP

Money duration (also called **dollar duration**) expresses sensitivity in currency units:

$$\text{Money duration} = \text{ModDur} \times \text{full price of the position}$$

$$\text{Money duration per 100 of par} = \text{ModDur} \times \text{full price per 100}$$

$$\Delta V \approx -\text{Money duration} \times \Delta\text{YTM}$$

The **price value of a basis point (PVBP)** is the currency change in a bond's full price for a 1 bp change in YTM. Reprice at ±1 bp and average:

$$\text{PVBP} = \frac{V_- - V_+}{2}, \qquad \text{PVBP} \approx \text{Money duration} \times 0.0001$$

When V_- and V_+ are prices per 100 of par, the PVBP of a position is $[(V_- - V_+)/2] \times \text{par value} \times 0.01$.

Example. For the 6-year bond, with the unrounded modified duration of 5.176, money duration per 100 of par $= 5.176 \times 94.924 = 491.3$. A $3 million par position is worth $2,847,720 and has a money duration of about $14.74 million, so a 1 bp rise cuts its value by about $1,474 (the PVBP).

Common exam traps

- Express ΔYTM as a decimal: 50 bp = 0.005. Entering it as 0.05 makes the duration 10 times too small, and entering it as 0.5 makes it 100 times too small. For the 6-year bond, entering the 10 bp shift as 0.01 gives 0.518 instead of 5.18.
- Do not forget the 2 in the approximate ModDur denominator, and use V_0 (not V_- or V_+) as the base. For the 6-year bond, leaving out the 2 gives 10.35, and using V_- as the base gives 5.15, instead of 5.18.
- For semiannual bonds, reprice with N = years × 2 and I/Y = YTM/2, but state the yield shift in annual terms. For the PVBP, a 1 bp shift in the annual yield moves the periodic rate by only 0.5 bp, and ΔYTM in the approximate ModDur formula is the annual change.
- Modified duration uses the YTM in the denominator. The coupon rate plays no part in the divisor. For the 6-year, 4% bond, dividing by 1.04 gives 5.23 instead of 5.18.
- Using par value instead of the full price misstates money duration whenever the bond does not trade at par. For the $3 million par position in the example, using par gives about $15.53 million instead of $14.74 million.

WORKED EXAMPLE: APPROXIMATE MODIFIED DURATION AND PVBP OF A SEMIANNUAL BOND

A 5-year bond pays a 4.4% coupon semiannually and yields 5.6% on a semiannual bond basis. It is valued on a coupon date, so its full price equals its quoted price. An investor holds $2,000,000 of par. Estimate the bond's modified duration by repricing it at yields 10 bp below and 10 bp above the current yield, then compute the position's money duration and PVBP.

Step 1. Current price: N = 10; I/Y = 2.8; PMT = 2.2; FV = 100; CPT PV = −94.8292.

Step 2. At a YTM of 5.5% the periodic rate is 2.75%: N = 10; I/Y = 2.75; PMT = 2.2; FV = 100; CPT PV = −95.2480. At 5.7% it is 2.85%: N = 10; I/Y = 2.85; PMT = 2.2; FV = 100; CPT PV = −94.4127.

Step 3. $\text{ApproxModDur} = (95.2480 - 94.4127)/(2 \times 94.8292 \times 0.001) = 0.8353/0.18966 = 4.404$. The shift enters as 0.001, the annual change, although each periodic rate moved by only 5 bp.

Step 4. Position value $= 2{,}000{,}000 \times 0.948292 = \$1{,}896{,}584$. Money duration $= 4.404 \times 1{,}896{,}584 = \$8{,}352{,}556$.

Step 5. PVBP $\approx 8{,}352{,}556 \times 0.0001 = \$835$.

Result. The approximate modified duration is 4.40, the position's money duration is about $8.35 million, and a 1 bp change in yield changes its value by about $835.

LOS 61.b — What drives interest rate risk

Other things equal:

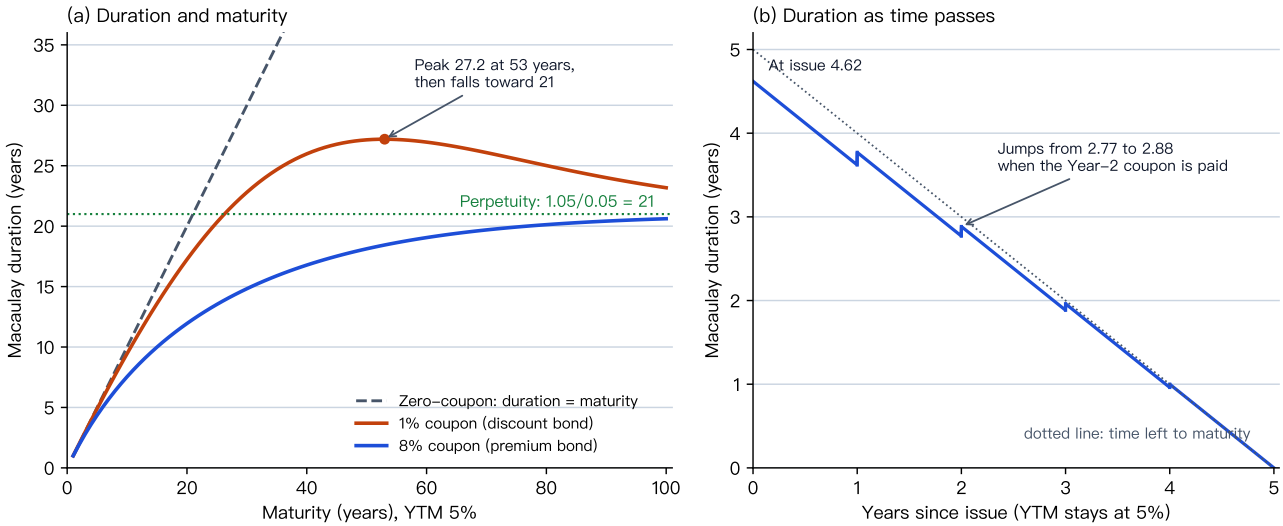
KEY CONCEPT

Feature	Effect on duration / interest rate risk	Why
Longer **maturity**	Usually higher	Distant cash flows are more sensitive to discount-rate changes
Higher **coupon rate**	Lower	More value comes from early cash flows
Higher **yield to maturity**	Lower	The price-yield curve is flatter at higher yields
Passage of time	Duration falls smoothly between coupon dates, then jumps up slightly on each coupon date	Time to the next payment resets to a full period

The slope of the price-yield curve is a picture of interest rate risk. When bonds on one chart have the same price at the current yield (for example, all at par), the one whose curve is steepest there has the highest modified duration.

- A **zero-coupon bond**'s Macaulay duration equals its maturity, so for the same maturity and yield it has more interest rate risk than any coupon bond.
- The maturity effect holds only usually: for **discount bonds** at long maturities, duration can fall as maturity rises, approaching the duration of a **perpetuity**, $(1 + \text{YTM})/\text{YTM}$. For example, at an 8% yield a perpetuity's Macaulay duration is $1.08/0.08 = 13.5$.
- A **floating-rate note (FRN)** resets its coupon to a **market reference rate (MRR)**, so it has little price risk. Its Macaulay duration is roughly the time until the next reset date. *Example.* An FRN resets every three months and one month of the current period has passed. Its Macaulay duration is about 2 months, or $2/12 = 0.17$ year, whatever its final maturity.

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Macaulay duration against maturity (a) and as time passes (b)

Panel (a) shows the maturity effect for annual-pay bonds at a 5% YTM, with the zero-coupon bond as the reference line. A premium bond's duration rises more slowly and approaches the perpetuity value of 21. A deep-discount bond's duration overshoots that value, peaking at about 27.2 for a 53-year maturity, and then declines toward 21. Panel (b) shows the saw-tooth path of a 5-year, 4% bond's Macaulay duration as time passes with its YTM unchanged.

Bonds with embedded options (preview of Reading 63)

A **callable bond** has less interest rate risk (lower duration) than an otherwise identical option-free bond. When yields fall, the call price acts as a ceiling on the bond's price. This limits price gains and exposes the investor to reinvestment risk if the bond is called. A **putable bond** has less price volatility at high yields, because the put becomes valuable and acts as a floor on the price. At low yields the put is worth little and the bond behaves like an option-free bond.

Common exam traps

- When bonds differ in more than one feature, weigh the drivers together. A large maturity difference usually outweighs a small coupon difference.
- Final maturity alone can mislead. An FRN with a long final maturity has very little duration, and a zero-coupon bond can have a higher duration than a coupon bond that matures somewhat later.

EXAM SHORTCUTS

- A floating-rate note's Macaulay duration is roughly the time to its next reset date, whatever its final maturity, so no cash flow weighting is needed.
- When bonds on one price-yield chart have the same price at the current yield, the bond whose curve is steepest at that yield has the highest modified duration.

BOTTOM LINE

- $\text{ModDur} = \text{MacDur}/(1 + r)$, where r is the periodic yield ($YTM/2$ for a semiannual-pay bond with durations in years), so with a positive yield modified duration is smaller than Macaulay duration.

- $\%\Delta P \approx -\text{ModDur} \times \Delta\text{YTM}$; this straight-line estimate works well for small yield changes, and for large changes it understates the price rise when yields fall and overstates the price fall when yields rise.
- $\text{ApproxModDur} = (V_- - V_+)/(2 \times V_0 \times \Delta\text{YTM})$, with ΔYTM entered as the annual yield change in decimal form.
- Money duration is modified duration times the full price of the position, $\Delta V \approx -\text{Money duration} \times \Delta\text{YTM}$, and the PVBP, the change in full price for a 1 bp yield change, is approximately money duration × 0.0001.
- Other things equal, a higher coupon rate or a higher YTM lowers duration, and a longer maturity usually raises it; for discount bonds at long maturities, duration can fall as maturity rises toward the perpetuity duration $(1 + \text{YTM})/\text{YTM}$.
- A zero-coupon bond's Macaulay duration equals its maturity, so for the same maturity and yield it has more interest rate risk than any coupon bond.
- A callable bond has a lower duration than an otherwise identical option-free bond, and a putable bond has less price volatility at high yields, where the put acts as a floor on its price.

QUICK CHECK

Question 1

A 7-year, option-free, annual-pay bond with a 5% coupon is priced at $944.18 for each $1,000 of face value. Its Macaulay duration is 6.0444. Its modified duration is *closest* to:

A 5.868.

B 5.757.

C 5.702.

Answer: C

Modified duration is Macaulay duration divided by one plus the bond's periodic yield. The YTM must first be found from the price; for an annual-pay bond the periodic yield is the YTM itself.

YTM (P/Y = C/Y = 1, END): N = 7, PV = −944.18, PMT = 50, FV = 1,000, CPT I/Y = 6.00.

$$\text{ModDur} = \frac{6.0444}{1.06} = 5.702$$

Why the other options are wrong

- **A.** 5.868 divides by $1 + 0.06/2 = 1.03$, a semiannual adjustment; this bond pays annually.
- **B.** 5.757 divides by one plus the coupon rate (1.05) instead of one plus the YTM (1.06).

Key takeaway ModDur divides by one plus the periodic yield (the coupon rate plays no part), and the periodic yield matches the payment frequency.

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Practice questions

Question 2

An analyst uses modified duration alone to estimate how an option-free bond's price will respond to a 150 basis point decline in its yield. Compared with the actual price change, the estimate will *most likely*:

A understate the price increase.

B overstate the price increase.

C match the price increase closely.

Question 3

A portfolio holds $6 million par value of an option-free bond with a full price of 98.75 per 100 of par and an annual modified duration of 5.4. If the bond's yield to maturity rises by 15 basis points, the estimated change in the value of the position, based on its money duration, is *closest* to:

A −$479,925.

B −$48,600.

C −$47,993.

Question 4

An option-free 20-year bond trades at a premium to par. If market interest rates rise, the bond's modified duration:

A increases.

B does not change.

C decreases.

Question 5

Relative to an otherwise identical option-free bond, which statement about a bond with an embedded call feature is *least accurate*? The call feature:

A caps the price gains the bondholder can earn when yields decline.

B exposes the bondholder to greater reinvestment risk.

C lengthens the bond's duration and so raises its price risk.

Answers: 2. A · 3. C · 4. C · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/macaulay-duration/. The full bank has 40 questions on this reading.

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FIXED INCOME · READING 62

Bond Convexity

What you'll learn

LOS 62.a Calculate and interpret convexity (cash-flow and approximate methods) and describe the convexity adjustment.

LOS 62.b Estimate a bond's percentage (or money) price change for a given yield change using duration and convexity.

LOS 62.c Calculate portfolio duration and convexity as weighted averages and explain the limitations of these measures.

MODULE 62.1

Yield-Based Bond Convexity and Portfolio Properties

This reading adds convexity to duration. It shows how convexity is measured, how the convexity adjustment improves a duration-based estimate of a bond's price change, and how duration and convexity are combined for a portfolio.

LOS 62.a — Convexity and the convexity adjustment

Modified duration provides a straight-line (first-order) estimate of how a bond's full price responds to a change in its yield to maturity. The true **price-yield relationship** of an option-free bond is curved. It is **convex** toward the origin, so the straight line lies below the actual curve. For small yield changes the gap is negligible; for large changes a duration-only estimate becomes noticeably wrong. **Convexity** measures the curvature of the price-yield relationship, and the **convexity adjustment** (a second-order term) corrects the duration estimate.

Convexity from the cash flows

Convexity can be built cash flow by cash flow, exactly like Macaulay duration. For the cash flow paid at period t, with periodic yield r (the annual YTM divided by the periods per year):

$$\text{convexity}_t = \frac{t(t+1)}{(1+r)^2}$$

The bond's convexity is the **weighted average** of these figures, where each weight is the cash flow's present value divided by the bond's price (the same weights used for Macaulay duration). If coupons are paid more than once a year, the result is in periods squared, so annualize by dividing by the periodicity squared (by $2^2 = 4$ for a semiannual bond, by $4^2 = 16$ for a quarterly bond).

Worked example. A 3-year, 5% annual-pay bond yields 7%, so its price is 94.751.

t	Cash flow	PV at 7%	Weight	$t(t+1)/1.07^2$
1	5	4.673	0.0493	1.747
2	5	4.367	0.0461	5.241
3	105	85.711	0.9046	10.481

Convexity $= 0.0493 \times 1.747 + 0.0461 \times 5.241 + 0.9046 \times 10.481 \approx 9.81$. No annualizing is needed because the bond pays annually.

Approximate convexity

A pricing model can reprice the bond after a yield decrease (V_-) and an equal yield increase (V_+):

KEY CONCEPT

$$\text{approximate convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta YTM)^2 V_0}$$

The numerator, $V_- + V_+ - 2V_0 = (V_- - V_0) - (V_0 - V_+)$, is how much larger the price rise is than the price fall. That gap exists only because the slope of the curve changes, which is what curvature means. The companion measure uses the same three prices: **approximate modified duration** $= (V_- - V_+)/(2V_0\,\Delta YTM)$. When the shifts are applied to a benchmark yield curve rather than to the bond's own YTM, the same formula gives **effective convexity** (Reading 63).

Worked example. $V_0 = 104.20$; after a 30 bp fall in yield the price is 105.48, after a 30 bp rise it is 102.95. Approximate convexity $= (105.48 + 102.95 - 208.40)/(0.003^2 \times 104.20) = 0.03/0.00093780 \approx 31.99$; approximate modified duration $= 2.53/(2 \times 104.20 \times 0.003) \approx 4.05$.

What drives convexity

The same features that raise duration raise convexity: a longer maturity, a lower coupon rate and a lower YTM all increase convexity. Between two bonds with the same duration, the one whose cash flows are more dispersed over time has more convexity (a barbell of cash flows is more convex than a bullet).

Positive and negative convexity

	Option-free bond (positive convexity)	Callable bond at low yields (negative convexity)
Shape	Curve bends away from the tangent line	Price is capped near the call price
Equal yield fall vs. rise	Price gain exceeds price loss	Price gain is smaller than price loss
As yields fall	Price rises at an increasing rate	Price rises at a decreasing rate

Callable bonds (and mortgage-backed securities, because of prepayments) exhibit **negative convexity** at low yields: the issuer is likely to call, so the price cannot rise much above the call price. At higher yields the call is unlikely and the callable bond behaves like an option-free bond with positive convexity. Option-free bonds, including government and zero-coupon bonds, always have positive convexity.

Common exam traps

- Forgetting to divide by periodicity squared when annualizing a semiannual convexity.
- Using $V_- - V_+$ (the duration numerator) in the convexity formula. With the prices in the example this gives about 2,698 instead of 31.99.

- A negative approximate/effective convexity is a legitimate answer for a callable bond.
- The larger the convexity, the larger the error of a duration-only estimate; short-term bonds have little convexity.

LOS 62.b — Price change using duration and convexity

KEY CONCEPT

$$\%\Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta YTM + \tfrac{1}{2} \times \text{AnnConvexity} \times (\Delta YTM)^2$$

The first term is the **duration effect** (sign opposite to the yield change). The second is the **convexity adjustment**; because $(\Delta YTM)^2$ is always positive, the adjustment is positive for both increases and decreases in yield when convexity is positive. Duration alone understates the price gain when yields fall and overstates the price loss when yields rise, and the positive convexity term improves both estimates. Enter ΔYTM as a decimal (40 bp = 0.0040).

Worked example. A bond has ModDur 5.8 and convexity 42, and its yield falls 80 bp: duration effect $= -5.8 \times (-0.008) = +4.64\%$; convexity effect $= 0.5 \times 42 \times 0.008^2 = +0.1344\%$; estimate $= +4.7744\%$.

How much the convexity term adds. A 10-year, 5% annual-pay bond priced at par has a modified duration of 7.72 and a convexity of 75.0. Its estimated and actual percentage price changes are:

Yield change	Actual	Duration only	Duration + convexity
+100 bp	−7.36%	−7.72%	−7.35%
−100 bp	+8.11%	+7.72%	+8.10%
+200 bp	−14.05%	−15.44%	−13.94%
−200 bp	+17.06%	+15.44%	+16.94%

A duration-only estimate is symmetric, but the actual price rises more than it falls. The convexity term is +0.375% for a 100 bp move and +1.50% for a 200 bp move in either direction. Because it grows with the square of the yield change, doubling the move roughly quadruples the error of a duration-only estimate.

Money convexity

Money convexity is the currency counterpart of money duration:

$$\text{MoneyCon} = \text{AnnConvexity} \times PV^{Full} \text{ of the position}$$

$$\Delta PV^{Full} \approx -(\text{MoneyDur} \times \Delta YTM) + \tfrac{1}{2} \times \text{MoneyCon} \times (\Delta YTM)^2$$

Maturity, par value and coupon do not enter the money convexity formula directly. Only the annual convexity and the full value of the position do. The money-based estimate and the percentage-based estimate are the same calculation expressed in different units, so they give equivalent estimated prices (apart from rounding).

Worked example. A position with full value \$3,900,000, ModDur 5.8 and convexity 42: MoneyDur = \$22,620,000; MoneyCon = \$163,800,000. For an 80 bp fall in yield: $22{,}620{,}000 \times 0.008 = \$180{,}960$ plus $0.5 \times 163{,}800{,}000 \times 0.008^2 = \$5{,}241.60$, a gain of about \$186,202 (4.7744% of \$3,900,000).

WORKED EXAMPLE: ESTIMATING A PRICE CHANGE FROM THREE PRICES

An 8-year, 4% annual-pay bond yields 4.40% and is priced at 97.351 on a coupon date, so its full price equals its flat price. A pricing model gives 98.996 if the yield falls 25 bp and 95.739 if it rises 25 bp. Estimate the percentage price change for a 75 bp rise in yield and the dollar change for a position of \$5,000,000 par.

Step 1. Approximate modified duration $= (98.996 - 95.739)/(2 \times 97.351 \times 0.0025) = 6.691$.

Step 2. Approximate convexity $= (98.996 + 95.739 - 2 \times 97.351)/(0.0025^2 \times 97.351) = 54.24$.

Step 3. For a 75 bp rise in yield: duration effect $= -6.691 \times 0.0075 = -5.0183\%$; convexity adjustment $= 0.5 \times 54.24 \times 0.0075^2 = +0.1526\%$; estimate $= -4.8657\%$.

Step 4. Repricing the bond at 5.15% gives 92.612, a change of −4.8680%. The duration-only estimate misses by about 0.15 percentage points; with the convexity adjustment the miss is about 0.002 percentage points.

Step 5. A position of \$5,000,000 par is worth $5{,}000{,}000 \times 0.97351 = \$4{,}867{,}550$, so its estimated loss is $0.048657 \times 4{,}867{,}550 \approx \$236{,}840$.

Common exam traps

- Leaving out the $\frac{1}{2}$, or subtracting the convexity term. For the 80 bp fall in the example, leaving it out gives +4.9088% and subtracting the term gives +4.5056%, against +4.7744%.
- Getting the sign of the duration effect wrong: yields down means price up.

LOS 62.c — Portfolio duration and convexity

Two approaches exist:

Approach	How	Comment
Aggregate cash flows	One duration/convexity from all portfolio cash flows, discounted at the portfolio's **cash flow yield**	Theoretically more correct, harder to use; cannot handle bonds whose cash flows are uncertain
Weighted average	$\text{portfolio duration} = \sum w_i D_i$, w_i = full market value of bond i / portfolio value	Used in practice: easier, and can use effective durations of bonds with embedded options

Portfolio convexity is weighted the same way, with weights based on full market values.

Worked example. Consider a portfolio holding \$3.5 million par of Bond P (full price 100.00, duration 3.2, convexity 12) and \$7.0 million par of Bond Q (full price 92.86, duration 8.6, convexity 88).

1. Market values: $3.5 \times 1.0000 = \$3.50$ million and $7.0 \times 0.9286 = \$6.50$ million, a total of \$10.00 million.

2. Weights: 35% in P and 65% in Q (par weights would give 33.3% and 66.7%).
3. Portfolio duration $= 0.35(3.2) + 0.65(8.6) = 6.71$; portfolio convexity $= 0.35(12) + 0.65(88) = 61.4$.

Limitations. The weighted-average measure is a valid estimate of the percentage value change per 1% change in yield only if every bond's yield changes by the same amount, which is a **parallel shift** of the yield curve. Real curves steepen, flatten and twist. Portfolio duration also inherits the straight-line limitation from LOS 62.a: without the convexity term it is accurate only for small yield changes.

To estimate a dollar change in portfolio value: $\Delta V \approx -D_{port} \times \Delta y \times V$.

Price value of a basis point. The **price value of a basis point (PVBP)** is the change in a bond's price when its yield changes by one basis point (0.01%): $\text{PVBP} \approx \text{modified duration} \times V \times 0.0001$. Convexity is ignored because a 1 bp change is too small for curvature to matter. The product of value and modified duration (without the 0.0001) is money duration.

Common exam traps

- Averaging durations equally, or with par-value weights, instead of with market-value weights. For bonds P and Q, equal weights give 5.90 and par weights give 6.80, against 6.71.
- Calling the weighted-average approach the theoretically correct one. It is the practical choice; the aggregate cash-flow approach is the theoretically correct one.

EXAM SHORTCUTS

- For an option-free bond the convexity adjustment is positive whether yields rise or fall.
- The convexity term grows with the square of the yield change, so doubling the change roughly quadruples it.

BOTTOM LINE

- Convexity measures the curvature of the price-yield relationship; for an option-free bond, duration alone understates the price gain when yields fall and overstates the price loss when yields rise.
- $\%\Delta PV^{Full} \approx -\text{AnnModDur} \times \Delta YTM + \frac{1}{2} \times \text{AnnConvexity} \times (\Delta YTM)^2$.
- Approximate convexity is $(V_- + V_+ - 2V_0)/[(\Delta YTM)^2 V_0]$.
- A longer maturity, a lower coupon and a lower yield raise convexity; with the same duration, more dispersed cash flows give more convexity.
- Callable bonds and mortgage-backed securities can have negative convexity at low yields; option-free bonds always have positive convexity.
- Money convexity is annual convexity × the full value of the position.
- Portfolio duration and convexity come either from the aggregate portfolio cash flows, the theoretically correct approach, or in practice as averages weighted by full market value, which give the portfolio's percentage change only for a parallel shift of the yield curve.

QUICK CHECK

Question 1

A 1.5-year option-free bond pays an 8% coupon semiannually and is priced to yield 6.00%. Using the present values of its cash flows as weights, its annualized convexity is *closest* to:

A 2.69.

B 5.37.

C 10.75.

Answer: A

Convexity is the present-value-weighted average of each cash flow's convexity, $t(t+1)/(1+r)^2$, using the periodic yield. For a semiannual bond the result is in half-years squared and must be divided by $2^2 = 4$ to annualize.

Periodic yield $r = 3\%$; cash flows 4, 4, 104.

PVs: $4/1.03 = 3.8835$; $4/1.03^2 = 3.7704$; $104/1.03^3 = 95.1747$; price $= 102.8286$.

Weights: 0.0378, 0.0367, 0.9256. Cash-flow convexities: $1 \times 2/1.03^2 = 1.8852$; $2 \times 3/1.03^2 = 5.6556$; $3 \times 4/1.03^2 = 11.3112$.

Periodic convexity $= 0.0378(1.8852) + 0.0367(5.6556) + 0.9256(11.3112) \approx 10.748$; annualized $= 10.748/4 \approx 2.69$.

Why the other options are wrong

- **B.** 5.37 divides by the periodicity (2) instead of the periodicity squared (4).
- **C.** 10.75 is the convexity in semiannual periods, before annualizing.

Key takeaway Annualize convexity by dividing by periodicity squared (4 for semiannual).

Practice questions

Question 2

Three option-free bonds are priced at the same yield to maturity. Which one is *most likely* to have the highest convexity?

A A 10-year bond with a 7% coupon.

B A 25-year bond with a 2% coupon.

C A 10-year bond with a 2% coupon.

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Question 3

Which statement about an option-free bond with positive convexity is *most accurate*?

A A given fall in yield raises its price by more than the same rise in yield lowers it.

B An increase in its yield leads to an increase in its price.

C Its price rises and falls at a faster rate than its yield changes.

Question 4

A portfolio holds the three option-free bonds shown below (prices per 100 of par).

Portfolio holdings

Bond	Par value ($)	Price (per 100 of par)	Duration
Bond 1	1,000,000	112	3.1
Bond 2	1,200,000	86	10.8
Bond 3	800,000	101	6.0

Using the weighted-average method, the portfolio's duration is *closest* to:

A 6.58.

B 6.63.

C 6.95.

Answers: 2. B · 3. A · 4. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/bond-convexity/. The full bank has 26 questions on this reading.

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FIXED INCOME · READING 63

Curve-Based and Empirical Fixed-Income Risk Measures

What you'll learn

LOS 63.a Explain why effective duration and effective convexity (measured against benchmark curve shifts) are the right risk measures for bonds with embedded options.

LOS 63.b Estimate a bond's percentage price change for a benchmark yield change using effective (or modified) duration and convexity.

LOS 63.c Define key rate duration and use it to measure sensitivity to nonparallel changes in the benchmark yield curve.

LOS 63.d Describe the difference between empirical duration and analytical duration.

MODULE 63.1

Curve-Based and Empirical Fixed-Income Risk Measures

This reading measures interest rate risk against the benchmark yield curve instead of a bond's own yield, which is needed when cash flows depend on future rates. A candidate must be able to compute and use effective duration and effective convexity, explain negative convexity in callable bonds, use key rate durations to measure shaping risk, and contrast empirical with analytical duration.

LOS 63.a — Why effective duration and effective convexity for bonds with embedded options

Yield-based measures (Macaulay duration, **modified duration**, **approximate modified duration**, convexity) assume the bond's cash flows are fixed and known. A bond with an **embedded option** does not satisfy that assumption:

- From the investor's perspective, a **callable bond** = option-free bond − a call option: the investor is short the embedded call and the issuer holds (is long) the call right; the issuer redeems early when rates fall.
- A **putable bond** = option-free bond + a long put held by the investor; the investor sells back when rates rise.
- A **mortgage-backed security (MBS)** behaves like a callable bond because borrowers can prepay.

Because the cash flows (and their timing) depend on the level and path of future interest rates, such a bond has no single well-defined yield (for example, yield to maturity versus yield to each call date). Interest rate risk is therefore measured against shifts in a **benchmark yield curve** (e.g., government par rates), with a pricing model producing the values V_- and V_+:

KEY CONCEPT

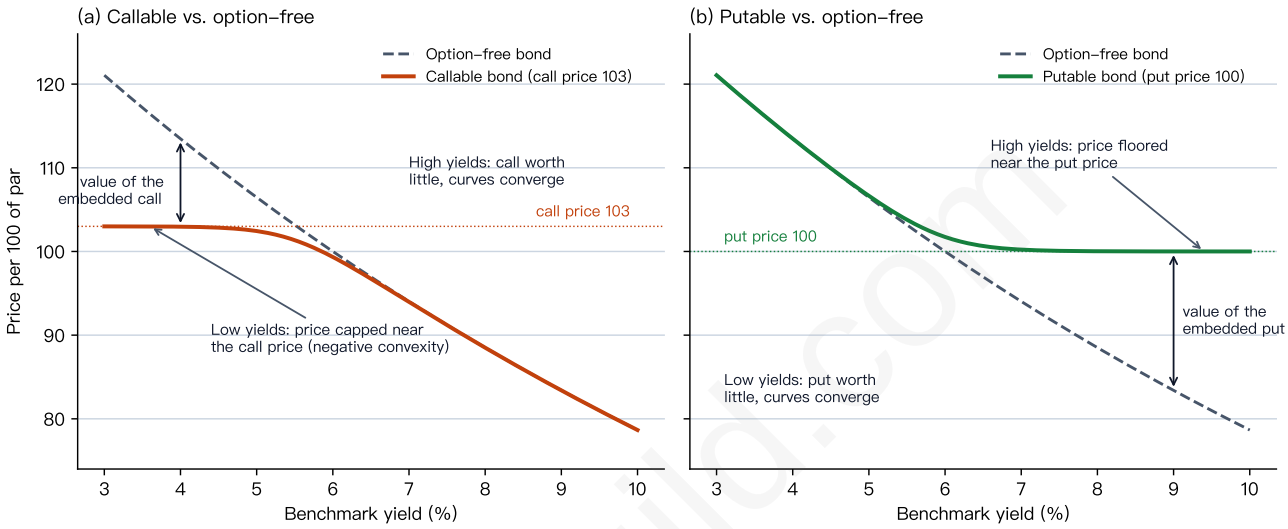
$$\text{effective duration (EffDur)} = \frac{V_- - V_+}{2V_0\,\Delta\text{Curve}} \qquad \text{effective convexity (EffCon)} = \frac{V_- + V_+ - 2V_0}{(\Delta\text{Curve})^2\,V_0}$$

Effective duration separates benchmark-rate risk from spread risk: it measures sensitivity to the benchmark curve only and holds the credit and liquidity spread constant. Modified duration makes no such distinction. Because effective duration is a curve-based measure, it can be used for option-free bonds too; for an option-

free bond, modified and effective duration are close but not identical unless the curve is flat (a parallel shift in the par curve moves the spot rates by different amounts).

Worked example. A callable bond is worth 99.10. A 50 bp parallel fall in the benchmark curve lifts its model value to 101.60; a 50 bp rise lowers it to 96.40.
EffDur $= (101.60 - 96.40)/(2 \times 99.10 \times 0.005) \approx 5.25$. EffCon $= (101.60 + 96.40 - 198.20)/(0.005^2 \times 99.10) \approx -80.7$, which is negative convexity.

Callable and putable bonds



Stylized price-yield curves of a callable bond (a) and a putable bond (b) against an otherwise identical option-free bond

In panel (a), the vertical gap between the curves is the value of the embedded call, which the investor has sold; it widens as yields fall. In panel (b) the gap is the value of the embedded put, which the investor owns; it widens as yields rise. At the other end of the yield range the option is worth little and each bond trades close to the option-free bond.

KEY CONCEPT

Feature	Callable bond	Putable bond
Option held by	Issuer	Investor
When the option matters	Low yields (call likely)	High yields (put likely)
Convexity	**Negative convexity** at low yields; positive at high yields	Always **positive convexity**
Effective duration vs. option-free bond	Lower at low yields (price capped near the call price)	Lower at high yields (price floored near the put price)

With negative convexity, the price gain from a fall in yield is smaller than the price loss from an equal rise in yield.

Common exam traps

- A negative effective convexity is the expected result for a callable bond or an MBS at low yields, not a calculation error.

LOS 63.b — Percentage price change for a change in the benchmark yield

The estimate has the same form as with yield-based measures:

$$\%\Delta PV^{Full} \approx -\text{EffDur} \times \Delta\text{Curve} + \tfrac{1}{2} \times \text{EffCon} \times (\Delta\text{Curve})^2$$

and for an option-free bond with yield-based measures, $\%\Delta PV \approx -\text{ModDur} \times \Delta YTM + \frac{1}{2} \times \text{Convexity} \times (\Delta YTM)^2$. The first term is the **duration effect**; the second is the **convexity effect** (positive for positive convexity, whatever the direction of the rate change). Convert basis points to decimals (35 bp = 0.0035).

Worked example. A bond has EffDur 7.1 and EffCon 62, and the benchmark curve falls 90 bp: duration effect $= +6.39\%$; convexity effect $= 0.5 \times 62 \times 0.009^2 = +0.2511\%$; estimate $= +6.6411\%$. With duration alone the estimate would be $+6.39\%$, too low.

Worked example (negative convexity). The callable bond from LOS 63.a has EffDur 5.25 and EffCon −80.7. For a 100 bp parallel shift the convexity effect is $0.5 \times (-80.7) \times 0.01^2 = -0.40\%$ in both directions. A rise in the benchmark curve gives $-5.25\% - 0.40\% = -5.65\%$; a fall gives $+5.25\% - 0.40\% = +4.85\%$, the asymmetry described in LOS 63.a.

To get a price, multiply the current price by $(1 + \%\Delta)$; to get a currency change, multiply the current value by $\%\Delta$.

Unlike yield-based measures, effective duration and convexity do not necessarily give better estimates for smaller rate changes, because factors other than the level of benchmark rates (credit spreads on a corporate bond, the principal outstanding on a mortgage) affect whether the option is exercised.

Common exam traps

- Forgetting the $\frac{1}{2}$, or giving the convexity effect the sign of the rate change instead of the sign of convexity. For the 90 bp fall in the example (EffDur 7.1, EffCon 62), leaving out the $\frac{1}{2}$ gives +6.8922%, and giving the convexity effect a negative sign gives +6.1389%, instead of +6.6411%.
- Sign errors in the duration effect: a fall in rates raises prices. In the same example, reversing the duration effect gives −6.1389% instead of +6.6411%.
- Using modified duration for a callable bond when an effective duration is given.

LOS 63.c — Key rate duration and shaping risk

Effective duration captures only **parallel shifts** of the benchmark curve. A **key rate duration** (also called a **partial duration**) is the sensitivity of a bond's or portfolio's value to a change in the benchmark yield at one specific maturity, holding all other yields constant. The key rate durations sum to the effective duration.

Key rate durations measure **shaping risk**: the effect of a nonparallel shift (steepening, flattening, twist) on a portfolio. The effect of each key rate's change is estimated separately and the effects are added:

$$\%\Delta V \approx -\sum_{k} \text{KeyRateDur}_k \times \Delta r_k$$

For a portfolio of cash flows, the key rate duration at a maturity is the modified duration of the cash flow at that maturity multiplied by its weight in the portfolio.

> *Worked example.* Half the portfolio is in a 2-year zero yielding 3%, half in a 7-year zero yielding 4.5% (annual compounding). KRD(2) $= 0.5 \times 2/1.03 = 0.971$; KRD(7) $= 0.5 \times 7/1.045 = 3.349$. If the 2-year yield rises 40 bp and the 7-year yield falls 10 bp: $-0.971(0.004) - 3.349(-0.001) = -0.388\% + 0.335\% \approx -0.05\%$.

The two key rate durations add up to $0.971 + 3.349 = 4.320$, the portfolio's effective duration. A parallel 40 bp rise would cost about $4.320 \times 0.40\% = 1.73\%$. The twist costs far less because the two key rates moved in opposite directions.

> *Example: same effective duration, different shaping risk.* The three portfolios below all have an effective duration of 5.00, so a parallel shift affects them equally.

Key rate durations of three portfolios with the same effective duration (illustrative)

Key rate maturity	Portfolio X	Portfolio Y	Portfolio Z
2-year	2.10	0.50	1.25
5-year	0.30	0.60	1.25
10-year	0.20	3.40	1.25
20-year	2.40	0.50	1.25
Sum = effective duration	5.00	5.00	5.00

Suppose only the 5-year and 10-year key rates rise by 20 bp. Each portfolio loses about $(\text{KRD}_5 + \text{KRD}_{10}) \times 0.20\%$: X loses $0.50 \times 0.20\% = 0.10\%$, Y loses $4.00 \times 0.20\% = 0.80\%$ and Z loses $2.50 \times 0.20\% = 0.50\%$. A manager expecting this change would prefer X, whose exposure sits at the maturities that do not move.

KEY CONCEPT

Measure	Yield change it captures
Macaulay duration	None directly: a weighted-average time in years, the horizon at which price and reinvestment risk offset
Modified duration	Change in the bond's own YTM
Money duration and PVBP	Change in the bond's own YTM, measured in currency units

Measure	Yield change it captures
Effective duration	Parallel shift in the benchmark curve
Key rate duration	Change in one maturity's benchmark rate
Empirical duration (LOS 63.d)	Benchmark changes as observed in historical prices, including any spread moves that came with them

LOS 63.d — Empirical versus analytical duration

Macaulay, modified and effective duration are **analytical durations**: they come from mathematical analysis of the bond's cash flows and assume, for credit-risky bonds, that the spread does not change when benchmark yields move (spread and benchmark changes are uncorrelated). **Empirical duration** is estimated statistically from observed data, that is, from how bond prices have actually responded to past changes in benchmark yields.

When the uncorrelated-spread assumption fails, empirical duration is more appropriate. In a **flight to quality**, government yields fall while credit spreads widen, so corporate bond prices rise less than analytical duration predicts (or even fall). The empirical duration of a corporate bond portfolio is then lower than its analytical duration. For a portfolio of government bonds, spreads are not a factor, so empirical and analytical durations should be similar and analytical duration remains appropriate.

Common exam traps

- Treating effective duration as empirical because it comes from a pricing model. It is analytical; only a duration estimated from historical price data (for example, by regression) is empirical.

EXAM SHORTCUTS

- For a portfolio of zero-coupon bonds with annual compounding, each key rate duration is the zero's portfolio weight times its maturity divided by one plus its yield, as in the 2-year and 7-year example, so no repricing is needed.

BOTTOM LINE

- Yield-based duration and convexity assume fixed, known cash flows, so callable bonds, putable bonds and mortgage-backed securities are measured with effective duration and effective convexity against shifts in a benchmark yield curve.
- $\text{EffDur} = (V_- - V_+)/(2V_0\,\Delta\text{Curve})$ and $\text{EffCon} = (V_- + V_+ - 2V_0)/[(\Delta\text{Curve})^2\,V_0]$, with V_- and V_+ produced by a pricing model.
- Effective duration measures sensitivity to the benchmark curve only and holds the credit and liquidity spread constant, while modified duration does not separate benchmark changes from spread changes.
- A callable bond has negative convexity and a lower effective duration than the option-free bond at low yields, where the call price caps its price; a putable bond always has positive convexity and has a lower effective duration at high yields, where the put price floors its price.
- $\%\Delta PV^{Full} \approx -\text{EffDur} \times \Delta\text{Curve} + \frac{1}{2} \times \text{EffCon} \times (\Delta\text{Curve})^2$, and the convexity effect takes the sign of the convexity whatever the direction of the rate change.

- A key rate duration is the sensitivity of value to the benchmark yield at one maturity with all other yields held constant; key rate durations sum to effective duration and measure shaping risk from nonparallel shifts.
- Analytical durations assume that credit spreads do not change when benchmark yields move, while empirical duration is estimated from historical prices; in a flight to quality a corporate bond portfolio's empirical duration is lower than its analytical duration, and for government bonds the two should be similar.

QUICK CHECK

Question 1

Effective duration is a better measure than modified duration of a bond's price sensitivity to interest rates when:

A the yield curve shifts in a nonparallel way.

B the bond has an embedded option.

C the bond has a long maturity and a low coupon rate.

Answer: B

Effective duration uses model values that reflect how an embedded option changes expected cash flows as rates move; modified duration ignores that effect. For option-free bonds the two measures are similar. Both, however, assume a parallel shift in the curve.

Why the other options are wrong

- **A.** Neither effective nor modified duration handles nonparallel shifts; key rate durations do.
- **C.** A long maturity and low coupon raise duration and convexity, but for an option-free bond modified duration is still appropriate.

Key takeaway Effective duration matters when cash flows depend on rates (embedded options).

Practice questions

Question 2

Interest rates fall by 65 basis points. Given a modified duration of 6.72 and a convexity of 55.30, the estimated percentage change in the bond's price is *closest* to:

A 4.48%.

B 4.37%.

C 4.25%.

Question 3

A key rate duration measures the sensitivity of a bond's value to a:

A parallel shift in the benchmark yield curve.

B change in the bond's expected cash flows.

C change in the benchmark yield at one specific maturity.

Question 4

To estimate the interest rate sensitivity of a corporate bond fund, an analyst regresses the fund's past returns on past changes in benchmark government yields. The resulting measure is *best* described as:

A modified duration.

B analytical duration.

C empirical duration.

Answers: 2. A · 3. C · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/curve-based-and-empirical-fixed-income-risk-measures/. The full bank has 34 questions on this reading.

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FIXED INCOME · READING 64

Credit Risk

What you'll learn

LOS 64.a Describe credit risk, the Cs of credit analysis, and how probability of default and loss given default combine into expected loss.

LOS 64.b Describe how credit ratings are used and the limitations of relying on them.

LOS 64.c Describe macroeconomic, market (liquidity) and issuer-specific factors that affect the level and volatility of yield spreads.

MODULE 64.1

Credit Risk

This reading defines credit risk and its components, explains what credit ratings do and do not tell an investor, and identifies what drives the level and volatility of yield spreads. A candidate must be able to compute expected loss and a fair credit spread from the probability of default and the recovery rate, split a yield spread into liquidity and credit parts, and estimate the price effect of a spread change.

LOS 64.a — Credit risk, probability of default and loss given default

Credit risk is the risk that a borrower does not make its promised interest or principal payments, which leaves fixed-income investors with a loss. A borrower that fails to service its debt is in **default**.

The Cs of credit analysis

Analysts sort the drivers of credit risk into two groups.

KEY CONCEPT

Bottom-up credit analysis (borrower-specific)	Top-down credit analysis (environment)
Capacity — the borrower's ability to make its debt payments on time (profitability, coverage, leverage)	**Conditions** — the general economic environment that affects every borrower's ability to pay
Capital — other resources that reduce the borrower's reliance on debt	**Country** — the geopolitical, legal and political setting that applies to the debt
Collateral — the value of assets pledged as security for the lender if the borrower defaults	**Currency** — exchange-rate moves that affect the ability to service foreign-currency debt
Covenants — the legal terms and conditions agreed between borrower and lenders in the bond issue	
Character — the borrower's integrity (for a company, its management) and commitment to pay	

Sources of repayment. At bottom, credit risk is the chance that the borrower's sources of repayment do not generate enough cash to service the debt.

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Borrower	Primary sources of repayment	Secondary sources	Credit risk rises with
Corporate issuer	Operating cash flows and investments. **Secured debt** also has the cash flows from the specific collateral pledged; **unsecured debt** has no such claim	Asset sales, divestitures of subsidiaries, new debt or equity	Weak economic and market conditions, stronger competition, low profitability, excessive debt
Sovereign	Taxes, tariffs and other fees	New borrowing, privatizations (sales of public assets)	A weak economy, political uncertainty, fiscal deficits, high debt relative to the size of the economy

An issuer that is **illiquid** (unable to raise cash to pay now) is not necessarily **insolvent** (assets worth less than debt), yet an illiquid issuer can still default.

A **cross-default clause** makes a default on one bond a default on all issues; a **pari passu** clause makes bonds of the same type rank equally in default. With both clauses in place, a default on any unsecured issue gives all unsecured holders a claim on the issuer's general assets. Secured holders get a claim on the general assets and on their pledged collateral, so they suffer a credit loss only if the collateral is worth less than the pari passu secured debt it backs.

Measuring credit risk

$$\text{expected loss} = \text{probability of default} \times \text{loss given default}$$

- **Probability of default (PD)**: the (usually annualized) probability that the borrower misses a payment of interest or principal.
- **Loss given default (LGD)**: the loss suffered if default occurs, as a money amount or as a rate (**LGD%**).
- **Recovery rate**: the proportion of the claim investors recover after default.
- **Loss severity**: the proportion not recovered: $1 - \text{recovery rate}$.
- **Expected exposure** (**exposure at default**): the money amount the investor is owed (principal plus accrued interest) net of available collateral value.

$$\text{LGD (in money)} = \text{expected exposure} \times (1 - \text{recovery rate})$$

$$\text{LGD\% (rate)} = \text{loss severity} = 1 - \text{recovery rate}$$

The **expected loss** in money terms is $\text{PD} \times \text{exposure} \times (1 - \text{RR})$, and the **expected loss rate** (per unit of exposure) is $\text{PD} \times (1 - \text{RR})$. Because expected loss is the compensation investors need for bearing default risk, the expected loss rate is used as an estimate of the fair **credit spread** (a rate, so no money amount enters):

KEY CONCEPT

$$\text{credit spread} \approx \text{PD} \times \text{LGD\%} = \text{PD} \times (1 - \text{recovery rate})$$

If the bond's actual spread over a risk-free benchmark is above this estimate, investors are more than fairly compensated; if it is below, they are under-compensated and should avoid the bond.

Exam convention: loss given default is often given as a rate (LGD%) and multiplied directly by the probability of default, which gives expected loss as a percentage. Current practice: LGD is stated either as a money amount or as a percentage of exposure at default, and the percentage form is the loss severity, $1 - \text{recovery rate}$.

Worked example. With a PD of 2% and a recovery rate of 60%, LGD% is 40% and expected loss $= 0.02 \times 0.40 = 0.80\%$. A bond trading at a spread of 1.10% more than covers this estimate.

Worked example (money terms). A bondholder is owed $5,000,000 of principal plus $100,000 of accrued interest, and collateral worth $1,500,000 is available to repay the claim.

1. Expected exposure $= 5{,}100{,}000 - 1{,}500{,}000 = \$3{,}600{,}000$.
2. With a 30% recovery rate, loss severity is 70%, so LGD $= 3{,}600{,}000 \times 0.70 = \$2{,}520{,}000$.
3. With an annual PD of 1.5%, expected loss $= 0.015 \times 2{,}520{,}000 = \$37{,}800$ a year.

What drives the inputs. PD depends on capacity metrics: high EBIT margin, high EBIT/interest coverage, low debt/EBITDA and high cash flow relative to debt all indicate a low PD. LGD depends on security and seniority: senior secured debt loses less in default than junior unsecured debt. Investment grade issuers have a lower PD than high-yield issuers (rating categories defined under LOS 64.b), but high-yield issuers often issue secured debt, so their LGD can be lower than that of an investment grade issuer's unsecured bonds. For unsecured investment grade debt the main worry is a rise in PD as the issuer's finances deteriorate.

Common exam traps

- Expected loss uses the loss severity, $1 - \text{recovery rate}$, not the recovery rate itself. A higher recovery rate lowers expected loss and spreads. In the first worked example, using the 60% recovery rate gives 1.20% instead of 0.80%.
- Yield volatility is an input to interest rate (price) risk. It plays no part in expected credit loss.

LOS 64.b — Uses and limitations of credit ratings

Credit rating agencies assign forward-looking ratings to issuers and to individual issues. Ratings are used to compare credit risk across issuers, industries and bond types; to track changing credit conditions and **credit migration risk** (the risk that a downgrade lowers a bond's value and triggers contractual clauses); and to satisfy regulatory, statutory or contractual rules. A credit-linked coupon, which rises when the issuer is downgraded (Reading 52), is one example of a contractual clause that a downgrade can trigger.

KEY CONCEPT

Scale (S&P and Fitch / Moody's)	Category
AAA/Aaa down to BBB−/Baa3	**Investment grade**
BB+/Ba1 and below	**Non-investment grade** (**high-yield**, "junk")
D (S&P, Fitch); Moody's lowest category C	In default

The full scale, notch by notch:

Credit rating scales of the three major agencies

Moody's	S&P	Fitch	Category
Aaa	AAA	AAA	Investment grade (highest rating)
Aa1, Aa2, Aa3	AA+, AA, AA−	AA+, AA, AA−	Investment grade
A1, A2, A3	A+, A, A−	A+, A, A−	Investment grade
Baa1, Baa2, Baa3	BBB+, BBB, BBB−	BBB+, BBB, BBB−	Investment grade
Ba1, Ba2, Ba3	BB+, BB, BB−	BB+, BB, BB−	Non-investment grade (high-yield)
B1, B2, B3	B+, B, B−	B+, B, B−	Non-investment grade (high-yield)
Caa1, Caa2, Caa3	CCC+, CCC, CCC−	CCC	Non-investment grade (high-yield)
Ca	CC	CC	Non-investment grade (high-yield)
C (also covers bonds in default)	C	C	Non-investment grade (high-yield)
(included in C)	D	D	In default

Limitations of ratings

- Ratings lag market pricing. Spreads and prices move faster than ratings, and two bonds with an identical rating may still trade at different yields, since ratings focus on expected loss while distressed-debt prices focus on default timing and recoveries.
- Some risks are hard to assess: litigation, natural disasters, environmental events, acquisitions and debt-financed buybacks. Agencies may disagree, producing **split ratings**.
- Agencies make mistakes, e.g., subprime mortgage securities before 2008–2009, and sudden defaults after corporate fraud.

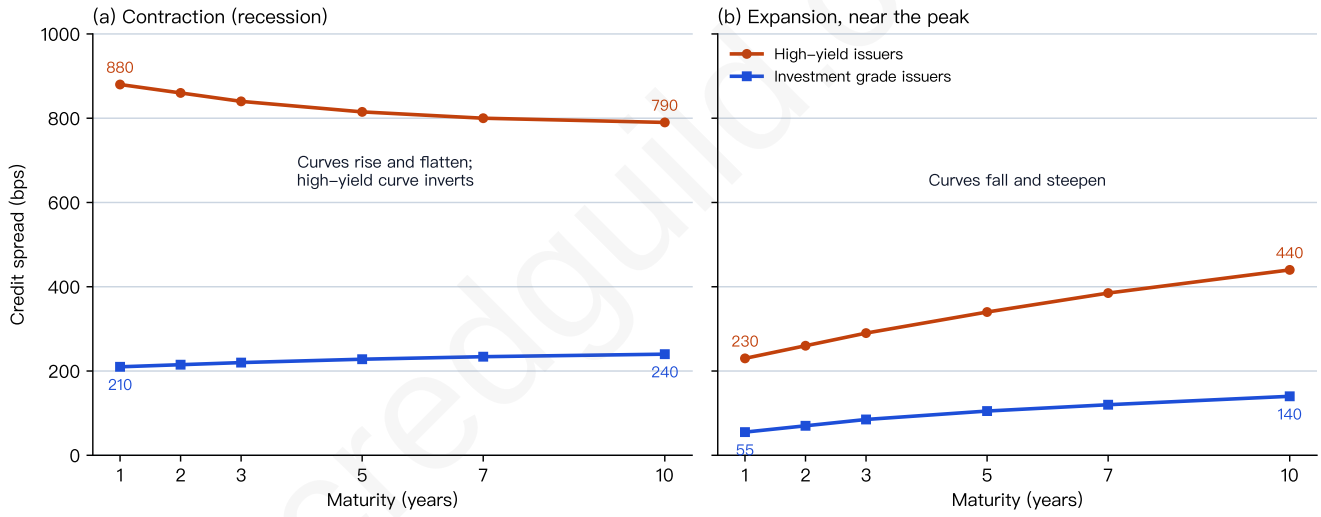
Investors should also carry out their own due diligence; those who correctly anticipate rating changes earn better results than those who trade only after a change is announced.

LOS 64.c — Factors that drive the level and volatility of yield spreads

Credit spread risk means the risk that yield spreads widen and prices of credit-risky bonds fall. For investment grade investors it is the main practical concern, because a sudden default is unlikely.

Macroeconomic factors

- Strong growth and high profits lower PD, so spreads narrow; a recession raises PD, so spreads widen.
- Investment grade spreads are lower than high-yield spreads (lower expected loss), and spreads usually rise with maturity because the probability of default grows over longer horizons. A **credit spread curve** plots credit spread against maturity, so it normally slopes upward.
- In a contraction, credit curves rise and flatten, and the high-yield curve may even invert. In an expansion, credit curves fall and steepen; they are lowest and steepest at the peak of the cycle.
- High-yield spreads are more dispersed across issuers and fluctuate more than investment grade spreads. In a crisis, investors sell risky assets and buy safe ones (**flight to quality**), so high-yield spreads can widen dramatically. Their bid–offer spreads widen more too, and the rise in risk aversion can spill over into wider bid–offer spreads on investment grade bonds.
- Why hold high-yield anyway: the larger spread, diversification (low or negative correlation with investment grade bonds), capital appreciation in recoveries, and, on some evidence, equity-like returns with lower volatility than equities.
- Other systematic factors that push spreads up: tighter regulation of dealers (higher cost of holding bond inventory), funding stresses that raise risk aversion, and heavy new issuance that is not matched by demand. Conversely, strong demand and light supply narrow spreads.



Stylized credit spread curves in a contraction (a) and near the peak of an expansion (b)

The figure sketches the credit spread curves of both rating groups in a contraction and near the peak of an expansion, with illustrative numbers.

Issuer-specific factors

An issuer with problems servicing its debt trades at a wider spread than the average for its rating; comparing an issuer's spread with its rating-category average highlights issuer-specific concerns.

Market factors: liquidity

Market liquidity risk is the risk that trading a bond is costly because the investor sells below (or buys above) the bond's fair midprice. It shows up in the dealers' **bid–offer spread**. It is higher for less actively traded bonds, lower-rated issuers and issuers with little debt outstanding. The two "spreads" differ: the **yield spread** is extra yield over the benchmark; the bid–offer spread is a price difference.

Splitting a yield spread. Compute the yield at the bid price and at the offer price; the difference is the **liquidity spread**, and the remainder of the yield spread is attributed to credit risk.

> *Worked example.* A 5-year, 4% annual-coupon bond is quoted 99.00 bid / 99.80 offer; the benchmark yields 2.80%.

- At the mid price of 99.40 the yield is 4.135%, so the yield spread is about 1.335% (133.5 bp).
- Bid yield (P/Y = C/Y = 1, END mode): `N = 5; PV = −99.00; PMT = 4; FV = 100; CPT I/Y = 4.226`. Offer yield, changing only the price: `PV = −99.80; CPT I/Y = 4.045`.
- Liquidity spread $\approx 4.226\% - 4.045\% = 0.181\%$, or 18.1 bp; credit component $\approx 133.5 - 18.1 = 115.4$ bp.

Building blocks of a corporate bond's yield (worked example above)

Building block	Size	How it is found
Benchmark yield	2.800%	yield on the government benchmark
• **Liquidity spread**	0.181%	yield at the bid price minus yield at the offer price
• **Credit spread**	1.154%	yield spread minus liquidity spread
= Yield spread	1.335%	bond yield at the mid price minus benchmark yield
Bond yield	4.135%	benchmark yield plus yield spread

Price impact of a spread change. Duration and convexity apply to spread changes just as to yield changes:

$$\%\Delta P_{\text{full}} \approx -\text{ModDur} \times \Delta\text{Spread} + \tfrac{1}{2}\,\text{Convexity} \times (\Delta\text{Spread})^2$$

E.g., ModDur 4.5, convexity 30, spread +40 bp: $-4.5(0.004) + 0.5(30)(0.004)^2 = -1.776\%$.

The formula can also be worked backward to find the spread change behind an observed price change. Start from the duration-only estimate, $-\%\Delta P/\text{ModDur}$, then test candidate values in the full formula. In the example, a 1.776% price fall gives a duration-only estimate of $1.776\%/4.5 = 0.395\%$, or 39.5 bp of widening. The convexity term cushions the fall, so the spread in fact widened by slightly more, 40 bp. For a price gain the error runs the other way: the duration-only estimate overstates how far the spread narrowed.

Because modified duration multiplies the spread change, a longer-duration bond loses more value than a shorter one for the same widening of its spread.

Common exam traps

- The liquidity spread is bid yield minus offer yield (the bid price is lower, so its yield is higher). Express it in the same units as the total spread before computing a share.
- An expected expansion, which narrows spreads, favors corporates over government bonds; an expected contraction favors government bonds.
- The convexity term is always added (positive for option-free bonds), whether spreads rise or fall. In the +40 bp example, subtracting it gives −1.824% instead of −1.776%.

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EXAM SHORTCUTS

- Bid and offer yields come from the same TVM entries with only PV changed, so the second yield needs one new entry and one compute.
- For an option-free bond, with the benchmark yield and all other factors unchanged, back out a spread change from a price change by starting from $-\%\Delta P/\text{ModDur}$; for a price fall the true widening is slightly larger than this, and for a price gain the true narrowing is slightly smaller.

BOTTOM LINE

- Bottom-up credit analysis looks at the borrower's capacity, capital, collateral, covenants and character, and top-down analysis looks at conditions, country and currency.
- $\text{Expected loss} = \text{PD} \times \text{LGD}$, where in money terms $\text{LGD} = \text{expected exposure} \times (1 - \text{recovery rate})$ and the loss severity is $1 - \text{recovery rate}$.
- Exam convention: loss given default is often given as a rate (LGD%) and multiplied directly by the probability of default to give expected loss as a percentage; current practice states LGD as a money amount or as a percentage of exposure at default, the percentage form being the loss severity.
- The expected loss rate, $\text{PD} \times (1 - \text{recovery rate})$, estimates the fair credit spread; an actual spread above it means investors are more than fairly compensated, and one below it means they are under-compensated.
- Ratings from AAA/Aaa down to BBB−/Baa3 are investment grade and ratings of BB+/Ba1 and below are non-investment grade (high-yield).
- Credit ratings lag market prices, struggle with risks such as litigation, natural disasters and debt-financed buybacks, and can be wrong, so ratings are no substitute for an investor's own due diligence.
- Spreads narrow when growth and profits are strong and widen in a recession; in a contraction credit curves rise and flatten, and in an expansion they fall and steepen.
- $\%\Delta P_{\text{full}} \approx -\text{ModDur} \times \Delta\text{Spread} + \frac{1}{2}\,\text{Convexity} \times (\Delta\text{Spread})^2$, and the liquidity spread inside a yield spread equals the yield at the bid price less the yield at the offer price.

QUICK CHECK

Question 1

To judge how likely Morrow Chemicals is to default, an analyst calculates its EBIT margin, its EBIT-to-interest expense ratio, and its debt-to-EBITDA ratio. This analysis is *most likely* an evaluation of the company's:

A capacity.

B collateral.

C covenants.

Answer: A

Capacity is the borrower's ability to make its debt payments on time. It is assessed with quantitative measures of profitability, coverage and leverage. A high EBIT margin, high interest coverage and low debt/EBITDA indicate strong capacity and a low probability of default.

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Why the other options are wrong

- **B.** Collateral analysis looks at the value of the specific assets pledged as security rather than at the issuer's income and leverage ratios.
- **C.** Covenant analysis reviews the legal terms and conditions of the bond agreement.

Key takeaway Ratio analysis of profits, coverage and leverage assesses capacity; pledged assets are collateral; legal terms are covenants.

Practice questions

Question 2

The trustees of the Linwood Teachers' Pension Plan propose to select corporate bonds using agency credit ratings alone. Which of the following is a limitation of relying solely on credit ratings?

A Agencies rate bond issuers but not individual bond issues.

B Bond prices and credit spreads can move well before a rating changes.

C Ratings let investors compare credit risk across industries and bond types.

Question 3

A portfolio manager at Corvel Asset Management expects the economy to move out of recession into a strong recovery over the coming year. Which change to her bond portfolio is *most appropriate*?

A Buy corporate bonds and sell government bonds.

B Sell corporate bonds and buy government bonds.

C Replace lower-rated corporate bonds with higher-rated ones.

Question 4

A corporate bond has a modified duration of 6.2 and a convexity of 52. If its credit spread widens by 60 basis points while benchmark yields are unchanged, the estimated percentage change in the bond's full price is *closest* to:

A −3.63%.

B −3.72%.

C −3.81%.

Answers: 2. B · 3. A · 4. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-risk/. The full bank has 20 questions on this reading.

FIXED INCOME · READING 65

Credit Analysis for Government Issuers

What you'll learn

LOS 65.a Explain the qualitative and quantitative factors in sovereign credit analysis and the special features of non-sovereign government issuers (agencies, supranationals, regional GO and revenue bonds).

MODULE 65.1

Credit Analysis for Government Issuers

This reading explains what to look for when assessing the credit of sovereign governments and of non-sovereign government issuers. A candidate must be able to name and apply the five qualitative and three quantitative factors of sovereign credit analysis, compute debt burden and debt affordability ratios, and compare general obligation bonds with revenue bonds.

LOS 65.a — Credit analysis of sovereign and non-sovereign government issuers

Sovereign (national government) debt

A national government services its **sovereign debt** mainly through its power to tax economic activity within its borders. Sovereign credit analysis therefore asks whether the country has the conditions for stable growth with low inflation. The evidence falls into five qualitative and three quantitative factors.

Five qualitative factors

KEY CONCEPT

Factor	What the analyst looks for
Institutions and policy	Rule of law, property rights, a culture of repaying debt, transparent and consistent data, business-friendly policy, political stability and peaceful relations with neighbors. The government's **willingness to pay** also counts, which matters because of **sovereign immunity** (bondholders usually cannot sue a government that refuses to pay)
Fiscal flexibility	Ability to raise taxes or cut spending to keep making debt payments
Monetary effectiveness	A credible central bank able to vary money supply and interest rates to support stable growth; an **independent central bank** is less likely to print money to pay government debt, lowering the risk of high inflation and a weak currency
Economic flexibility	Growth trends, income per capita, and diversity of the sources of growth
External status	Standing of the currency internationally. A **reserve currency** (one that central banks around the world widely hold in their foreign exchange reserves) gives a country more ability to issue debt to foreign investors in its own currency and to maintain larger budget deficits and debt; also geopolitical risk

Three quantitative factors

KEY CONCEPT

Factor	Stronger credit shown by
Fiscal strength	Low **debt burden** (debt/GDP, debt/revenue) and good **debt affordability** (low interest/GDP, low interest/revenue)
Economic growth and stability	High real GDP growth, large economy, high GDP per capita, low volatility of growth
External stability	High FX reserves relative to GDP and to external debt; low long-term external debt/GDP; low external debt due within 12 months relative to GDP

FX reserves typically build up in countries with a **current account surplus**. If exports are concentrated in one commodity, the sovereign's credit risk becomes tied to that commodity's price.

Worked example. A country has GDP of 2,000, government debt of 1,100, government revenue of 640 and annual interest cost of 38 (all in billions). Debt burden: $1{,}100/2{,}000 = 55\%$ of GDP; debt affordability: $38/640 = 5.9\%$ of revenue. A peer with debt/GDP of 50% but interest/revenue of 8% has the lighter debt burden yet the weaker debt affordability, so the two measures can point in different directions.

Non-sovereign government debt

Issuer type	Credit profile
Agencies (quasi-government entities, e.g., for infrastructure)	Backed by law with implicit government support; ratings usually close to the sovereign
Government sector banks / financing institutions (e.g., issuing green bonds)	Implied government support; ratings similar to the sovereign
Supranational issuers (e.g., the World Bank), set up by groups of sovereign governments	Ratings depend on the implicit support of the sponsoring governments and institutions
Regional governments (provinces, states, local governments — **municipal bonds** in the United States)	Analyzed on their own tax base or project cash flows

KEY CONCEPT

Regional government bonds are mostly of two kinds:

- **General obligation (GO) bonds**: unsecured bonds backed by the **full faith and credit** of the issuing government, i.e., by its taxing power.
- **Revenue bonds**: issued to finance a specific project (airport, toll bridge, hospital, power plant) and serviced from that project's revenues.

Regional governments, unlike sovereigns, have no monetary policy tools (they cannot print money) and usually must balance their operating budgets. Their ability to service GO debt depends ultimately on the local economy (the tax base). Revenue bonds often carry more credit risk than GO bonds, since repayment comes only from

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the single project.

Revenue bond analysis resembles corporate credit analysis: it focuses on the project's cash flows and the **debt-service coverage ratio**, which is revenue after operating costs divided by required interest and principal payments. The higher the ratio, the safer the bond.

Common exam traps

- Taxing power does not separate a sovereign from a state or city, since both levy taxes. The difference is that a sovereign that issues its own currency can print money.
- A revenue bond is not safer because a named project backs it. One project is a narrower source of repayment than a whole tax base.
- The direction of each ratio matters: a higher debt/GDP or interest/revenue ratio means weaker fiscal strength, while higher FX reserves relative to external debt mean stronger external stability.
- Three pairs of factor names look alike but belong to different groups: fiscal flexibility (qualitative) and fiscal strength (quantitative); economic flexibility (qualitative) and economic growth and stability (quantitative); external status (qualitative) and external stability (quantitative).

BOTTOM LINE

- A national government services its debt mainly through its power to tax, so sovereign credit analysis asks whether the country has the conditions for stable growth with low inflation.
- The five qualitative factors of sovereign credit are institutions and policy (including the government's willingness to pay, given sovereign immunity), fiscal flexibility, monetary effectiveness, economic flexibility and external status.
- The three quantitative factors are fiscal strength (a low debt burden and good debt affordability), economic growth and stability, and external stability (high FX reserves relative to GDP and to external debt, and low external debt relative to GDP).
- A higher debt/GDP or interest/revenue ratio means weaker fiscal strength, and debt burden and debt affordability can point in different directions for the same country.
- A country with a reserve currency can more easily issue debt to foreign investors in its own currency and can maintain larger budget deficits and debt.
- Agencies and government sector banks carry implied government support and ratings close to the sovereign's, while supranational ratings depend on the implicit support of the sponsoring governments and institutions.
- General obligation bonds are unsecured and backed by the issuing government's taxing power, while revenue bonds are serviced only from one project's revenues and often carry more credit risk.
- Regional governments cannot print money and usually must balance their operating budgets, and revenue bonds are analyzed like corporate bonds, with a higher debt-service coverage ratio (project revenue net of operating costs, divided by the interest and principal due) meaning a safer bond.

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QUICK CHECK

Question 1

Salvarra's central bank operates independently of the government and has a credible record of adjusting interest rates and the money supply to keep inflation low. In a sovereign credit analysis of Salvarra, this strength is best classified under which qualitative factor?

A Fiscal flexibility.

B External status.

C Monetary effectiveness.

Answer: C

Monetary effectiveness concerns the central bank's ability to vary the money supply and interest rates credibly to support stable growth. An independent central bank is less likely to print money to service government debt, which reduces the risk of high inflation and currency weakness.

Why the other options are wrong

- **A.** Fiscal flexibility is the government's ability to raise taxes or cut spending to keep up debt payments. That is a budget issue rather than a central bank issue.
- **B.** External status concerns the international standing of the currency (e.g., reserve currency status) and geopolitical risk.

Key takeaway Central bank credibility and independence belong to monetary effectiveness, room to tax or cut spending to fiscal flexibility, and reserve currency status to external status.

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Practice questions

Question 2

An analyst compares the fiscal strength of two sovereign issuers using the following data.

Selected fiscal data for two sovereign issuers (USD billions)

Item	Kestria	Varnholm
Nominal GDP	1,200	500
Government debt	780	290
Government revenue	420	145
Annual interest expense on government debt	29.4	13.05

Based on the debt burden (government debt to GDP) and debt affordability (interest expense to government revenue), which statement is *most accurate*?

A Kestria is stronger on both debt burden and debt affordability.

B Varnholm has the lighter debt burden, but Kestria has better debt affordability.

C Varnholm is stronger on both debt burden and debt affordability.

Question 3

The regional government of Varenna sets up a transit authority to build a light-rail line. The authority issues bonds whose interest and principal are to be paid from the line's fare revenue. The national government also guarantees the bonds as a backstop. These bonds are *best* described as:

A general obligation bonds.

B revenue bonds.

C sovereign bonds.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-analysis-for-government-issuers/. The full bank has 8 questions on this reading.

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FIXED INCOME · READING 66

Credit Analysis for Corporate Issuers

What you'll learn

LOS 66.a Describe the qualitative (business model, competition, business risk, governance) and quantitative factors used to assess a corporate borrower's creditworthiness.

LOS 66.b Calculate and interpret EBITDA, FFO, RCF and the profitability, coverage and leverage ratios used in credit analysis.

LOS 66.c Describe seniority rankings, secured versus unsecured debt, the priority of claims in bankruptcy, and how they affect issue ratings (notching).

MODULE 66.1

Credit Analysis for Corporate Issuers

This reading applies credit analysis to corporate issuers: the qualitative and quantitative factors, the financial ratios analysts use, and how seniority and collateral affect recovery and ratings. A candidate must be able to compute and interpret profitability, coverage and leverage ratios, rank debt by priority of claims, work out recoveries in a liquidation, and explain notching and structural subordination.

LOS 66.a — Qualitative and quantitative factors in corporate credit analysis

Qualitative factors

Factor	Signs of high credit quality
Business model	Stable, predictable cash flows; for long-dated debt, consider whether the model must change to stay competitive (change adds business risk)
Industry competition	Less intense competition, now and over the life of the debt
Business risk	Low risk of unexpected deviations from expected revenues and margins (from issuer, industry or external sources)
Corporate governance	Fair and legal treatment of debtholders, judged through covenants and accounting policies

Covenants. Unsecured investment-grade issuers typically have mainly **affirmative covenants** (comply with laws, maintain assets, pay taxes), so analysts must judge the risk that management adds debt that dilutes existing lenders. High-yield issuers usually also have **negative covenants** restricting dividends and new debt. For them, analysts look for past evidence of favoring shareholders over lenders, such as a debt-financed buyback that led to a downgrade.

Accounting policies. Fraud is the obvious concern. Other warning signs about management's character include aggressive revenue recognition, heavy off-balance-sheet financing, a strong preference for capitalizing rather than expensing, and frequent changes of auditor or CFO.

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Quantitative factors

Analysts forecast financial statements and cash flows to see what drives the **probability of default** and **loss given default** over the economic cycle. Investors in unsecured investment-grade debt worry mostly about the probability of default rising. Investors in secured high-yield debt also focus on loss given default, because default is more likely for high-yield issuers. Inputs can be **top-down** (macro cycle, industry size, market share, event risk), **bottom-up** (issuer revenue, costs, assets, liabilities, cash flows), or a **hybrid** of both.

All else equal, credit quality is higher with strong operating profits and recurring revenues, low leverage, high coverage of debt service by income, and high liquidity for short-term obligations.

In a bottom-up analysis these features come from the three financial statements: revenue and operating profit from the income statement, leverage and liquidity from the balance sheet, and the cash available to service debt from the cash flow statement. The profitability, coverage and leverage ratios in LOS 66.b measure the first three features.

LOS 66.b — Financial ratios used in credit analysis

Ratios let analysts judge a company's creditworthiness, follow trends over time and compare the company with industry averages and peers. The main groups are profitability, coverage and leverage.

Building blocks

KEY CONCEPT

Measure	Definition
EBITDA	Operating income + depreciation and amortization. Drawback: ignores capital expenditures and working capital needs
Cash flow from operating activities (CFO)	Net income + noncash charges − increase in working capital (from the cash flow statement)
Funds from operations (FFO)	Net income from continuing operations + depreciation + amortization + deferred taxes + other noncash items (like CFO but excludes working capital changes)
Free cash flow (FCF)	CFO − fixed asset expenditures + net interest expense; the discretionary cash that could go to providers of financing once all the company's obligations are met
Retained cash flow (RCF)	Operating cash flow (CFO, FFO or another chosen measure) − dividends

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Key ratios

KEY CONCEPT

Type	Ratio	Higher credit quality shown by
Profitability	**EBIT margin** = EBIT / revenue	Higher
Coverage	**EBIT to interest expense**	Higher
Leverage	**Debt to EBITDA**	Lower
Leverage	**RCF to net debt** = RCF / (debt − cash and marketable securities)	Higher

Example. A company reports (USD millions) revenue 4,000; EBIT 600; D&A 200; interest expense 75; total debt 2,400; cash 400; CFO 700; dividends 150.

- EBIT margin $= 600/4{,}000 = 15.0\%$
- EBIT/interest $= 600/75 = 8.0\times$
- Debt/EBITDA $= 2{,}400/(600 + 200) = 3.0\times$
- RCF/net debt $= (700 - 150)/(2{,}400 - 400) = 27.5\%$

Reading the results. Suppose a peer reports an EBIT margin of 11.0%, EBIT/interest of 9.0×, debt/EBITDA of 2.2× and RCF/net debt of 36%. The company is more profitable than the peer but more heavily indebted: its coverage is slightly weaker, its debt/EBITDA is higher and its RCF/net debt is lower. Mixed results like these are common, so the analyst weighs profitability against leverage instead of relying on any single ratio.

A multi-year average ratio can be taken as the simple average of the yearly ratios. Dividing total debt by total EBITDA over the same years gives a slightly different figure.

LOS 66.c — Seniority, secured vs unsecured debt, priority of claims and ratings

A bond's place in the **priority of claims** is its **seniority ranking**. **Secured debt** is backed by **collateral** and ranks ahead of **unsecured debt**, which is a general claim on the issuer's assets and cash flows.

KEY CONCEPT

Rank (highest first)	Category
1	**First lien / first mortgage**
2	**Senior secured (second lien)**
3	**Junior secured**
4	**Senior unsecured**
5	**Senior subordinated**
6	**Subordinated**

Rank (highest first)	Category
7	**Junior subordinated**

Debt in the same category ranks **pari passu** (equal priority). Any part of a secured claim not covered by its collateral ranks pari passu with senior unsecured claims.

Example. A company is liquidated. Its senior secured lenders are owed $200 million and their collateral sells for $150 million. Senior unsecured bondholders are owed $300 million and subordinated noteholders $100 million. After the collateral is sold, $270 million of other assets remain.

1. The secured lenders take the $150 million from their collateral. Their $50 million shortfall joins the senior unsecured claims.
2. The senior unsecured pool is $50 + 300 = \$350$ million, and it shares the $270 million pro rata: $270/350 = 77.1\%$ of each claim.
3. Nothing is left for the subordinated notes or the shareholders (under strict priority).

The secured lenders recover $150 + 0.771 \times 50 = \$188.6$ million (94.3%), the senior unsecured bondholders 77.1%, and the subordinated noteholders nothing. Recovery rates fall, and credit risk rises, with each step down in seniority. In practice, strict priority is often not applied. Junior creditors and even shareholders may receive something before seniors are paid in full, because a negotiated settlement shortens a costly bankruptcy during which asset values tend to erode.

Issuer vs issue ratings. The **issuer credit rating**, called the **corporate family rating (CFR)**, reflects the borrower's overall creditworthiness and is typically based on its senior unsecured debt. **Issue credit ratings** apply to specific bonds (individual debt obligations). Issue credit ratings are also called **corporate credit ratings (CCRs)**. Setting an issue rating above or below the issuer rating is **notching**. Seniority and covenants (including collateral) drive it.

Issue feature	Likely notching
Secured, strong protective covenants (e.g., limits on additional debt)	Upward (e.g., one notch above the CFR)
Subordinated, weak protection	Downward

Structural subordination. In a holding company group, a subsidiary's covenants may block cash transfers to the parent until the subsidiary's own debt is serviced. The subsidiary's bonds then have the first claim on the subsidiary's cash flows. The parent's bonds are therefore **effectively subordinated** to them with respect to those cash flows, even though they are not formally junior. Rating agencies consider structural subordination when notching.

Notching is more common for lower-rated issuers. When the probability of default is high, differences in expected recovery between issues matter more. For highly rated issuers they matter little, so the issues may not be notched at all.

Common exam traps

- EBITDA adds D&A to operating income (EBIT). Dividing debt by EBIT overstates leverage. In the example, debt/EBIT gives 4.0× instead of a debt/EBITDA of 3.0×.

- FFO starts from net income from continuing operations. It does not start from CFO and does not subtract capex or dividends.
- Debt/EBITDA is the one key ratio for which a lower value means stronger credit. Higher earnings, all else equal, lower it and raise the other three.
- RCF/net debt uses net debt. Dividing RCF by total debt, without subtracting cash and marketable securities, understates the ratio. In the example, dividing by total debt gives 22.9% instead of 27.5%.
- Second lien is still secured debt and ranks above all unsecured debt; subordinated debt ranks below senior unsecured.
- Bonds of the same seniority rank pari passu whatever their maturity or coupon, so a 3-year and a 20-year senior unsecured bond of one issuer have the same priority of claims in bankruptcy.
- Notching can be upward or downward.
- The rating typically based on senior unsecured debt is the issuer rating (CFR), not an issue rating.

BOTTOM LINE

- The qualitative factors in corporate credit analysis are the business model, industry competition, business risk and corporate governance, the last judged through covenants and accounting policies.
- All else equal, credit quality is higher with strong operating profits and recurring revenues, low leverage, high coverage of debt service by income and high liquidity for short-term obligations.
- EBITDA is operating income plus depreciation and amortization, FFO starts from net income from continuing operations and adds back noncash items, and RCF is the chosen operating cash flow measure minus dividends.
- Higher EBIT margin, EBIT/interest expense and RCF/net debt indicate higher credit quality, while for debt/EBITDA a lower value indicates higher credit quality; net debt is debt minus cash and marketable securities.
- The priority of claims runs first lien, senior secured (second lien), junior secured, senior unsecured, senior subordinated, subordinated, junior subordinated; debt in the same category ranks pari passu, and any secured claim not covered by collateral ranks with senior unsecured claims.
- Recovery rates fall and credit risk rises with each step down in seniority, although in practice strict priority is often not applied because a negotiated settlement shortens a costly bankruptcy.
- The issuer credit rating (corporate family rating) is typically based on senior unsecured debt, and issue ratings are notched above or below it according to seniority and covenants, with notching more common for lower-rated issuers.
- Under structural subordination, a subsidiary's own bonds have the first claim on its cash flows, so the parent's bonds are effectively subordinated to them for those cash flows even though they are not formally junior.

QUICK CHECK

Question 1

All else equal, which of the following would indicate *higher* credit quality for a corporate issuer?

A Higher leverage

B Higher liquidity

C Lower coverage of debt service

Answer: B

Other things equal, companies have higher credit quality when they have strong operating profits and recurring revenues, low leverage, high coverage of debt service by income, and high liquidity to meet short-term debt payments. Higher liquidity means more cash and liquid assets to meet obligations that fall due soon.

Why the other options are wrong

- **A.** Higher leverage means greater reliance on debt in the capital structure, which points to lower credit quality.
- **C.** Lower coverage means income covers interest and principal payments less comfortably, which points to lower credit quality.

Key takeaway Signs of stronger corporate credit: strong, recurring profits; low leverage; high coverage; high liquidity.

Practice questions

Question 2

Tamworth Mills is being liquidated. It has three classes of debt outstanding: senior subordinated notes, first mortgage bonds and senior unsecured notes. Which list ranks them from highest to lowest priority of claims?

A First mortgage bonds, senior unsecured notes, senior subordinated notes.

B Senior unsecured notes, first mortgage bonds, senior subordinated notes.

C First mortgage bonds, senior subordinated notes, senior unsecured notes.

Question 3

How does an issuer credit rating *most likely* differ from an issue credit rating?

A An issuer credit rating reflects the borrower's overall creditworthiness.

B An issue credit rating is always set below the issuer credit rating.

C An issue credit rating is the one based on senior unsecured debt.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/credit-analysis-for-corporate-issuers/. The full bank has 13 questions on this reading.

FIXED INCOME · READING 67

Fixed-Income Securitization

What you'll learn

LOS 67.a Explain the benefits of securitization for originators, investors, economies and financial markets, and the risks to ABS investors.

LOS 67.b Describe the securitization process and the roles of the seller, SPE (issuer/trust), servicer and trustee.

MODULE 67.1

Fixed-Income Securitization

This reading explains how securitization turns pools of loans into asset-backed securities and who takes part in the process. A candidate must be able to describe the benefits of securitization to originators, investors and the wider economy, and explain the roles of the seller, the SPE, the servicer and the trustee and why the SPE is bankruptcy remote.

LOS 67.a — Benefits of securitization

KEY CONCEPT

Securitization turns a pool of loans or receivables into tradable securities:

1. An **originator** (a bank making loans, or a company extending credit to customers) builds a pool of debt-based assets.
2. It sells the pool, called the **collateral**, to a separate legal entity, the **special purpose entity (SPE)**.
3. The SPE issues **asset-backed securities (ABS)** whose payments come from the borrowers' loan repayments, and sells them to investors such as pension and fixed-income funds.

Securitization links investors directly with the borrowers in the pool and takes the originator out of the intermediation chain.

The word "issuer" is used in two ways. In lists of the benefits of securitization, "issuers" are the originators. The legal issuer of the ABS is the SPE.

KEY CONCEPT

Who benefits	Benefit
Originators (issuers)	**Increased business activity**: sale proceeds are re-lent, so a bank can lend more than its own balance sheet would allow **Improved profitability**: fees for originating the loans and for selling them to the SPE **Lower capital reserves**: selling the loans removes their credit risk from the bank's balance sheet, so regulators require less capital **Improved liquidity**: illiquid loan portfolios can be sold

Who benefits	Benefit
Investors	**Tailored risk and return** to match their needs (e.g., long-horizon investors in long mortgage pools)
	Access to loan pools without the expertise to originate and service loans
	Liquidity: ABS are easier to sell than the underlying loans
Economies and financial markets	**Decreased liquidity risk** and **improved market efficiency** (tradable ABS allow equilibrium prices)
	Lower financing costs for originators than issuing their own debt or equity
	Lower leverage for originators, which can grow without adding debt to their balance sheets

Risks to ABS investors. Cash flows are uncertain in timing and size; for example, borrowers may prepay at an unexpected rate. The credit risk of the collateral also passes through to ABS investors, who bear defaults in the pool. A system-wide buildup of this risk contributed to the 2007–2009 financial crisis.

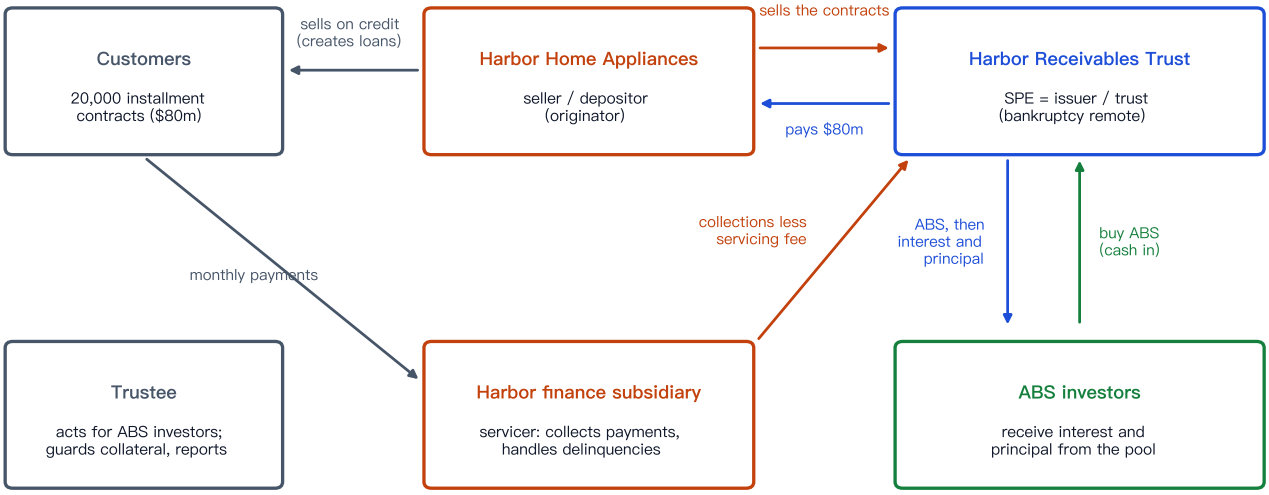
LOS 67.b — The securitization process and its parties

Example. Harbor Home Appliances sells washing machines on installment credit. It bundles 20,000 installment contracts worth $80 million and sells them for $80 million to Harbor Receivables Trust, an SPE created solely to buy the contracts and issue ABS. Harbor keeps collecting the monthly payments through a finance subsidiary.

KEY CONCEPT

Party	Role in the example
Seller / **depositor** (originator)	Harbor Home Appliances creates the loans and sells the pool to the SPE; it is the firm raising funds through the securitization
Issuer / **trust** (**SPE**, also **special purpose vehicle (SPV)** or special purpose company)	Harbor Receivables Trust buys the loans and issues ABS to investors
Servicer	Harbor's finance subsidiary collects payments, sends delinquency notices, and repossesses and sells collateral when borrowers default
Trustee	An independent (**disinterested**) party that safeguards the collateral and cash flows due to ABS holders and reports to them

The seller and servicer are often, but not always, the same company. The figure traces the flows in the example.



Structure of the Harbor Home Appliances securitization

Flow of cash. Collections of principal and interest on the pool first pay the **servicing fee** and other administrative fees. The rest goes to ABS investors as interest and principal, so investors receive less than the total payments collected from the pool.

Bankruptcy remote. The SPE is a **separate legal entity** that owns the collateral outright, so the assets are legally isolated from the seller. A decline in the seller's finances, even its bankruptcy, does not affect the ABS holders' claim on the collateral, and the seller's creditors cannot reach the pool. The SPE is therefore **bankruptcy remote**. In return, ABS holders have no claim on the seller's other assets. This separation can allow senior ABS to carry a higher credit rating than the seller's own corporate bonds, whose holders are exposed to all of the seller's risks.

Key documents. Besides the indenture and covenants, a securitization relies on the **purchase agreement** (the terms on which the SPE buys the collateral) and the **prospectus** (fees paid to servicers and administrators, and how collateral cash flows are distributed to ABS investors).

Common exam traps

- Removing liabilities from the balance sheet is not a benefit of securitization. Only assets (loans, receivables) are sold; deposits and bonds stay with the originator.
- Securitization does not make loans more collectible or protect investors from defaults in the pool. Credit enhancement may reduce that risk but does not eliminate it.
- The SPE's independence from the seller is legal: it is a separate legal entity that has bought the collateral outright. An SPE sponsored or even owned by the seller can still be bankruptcy remote if the structure isolates the assets; being a subsidiary or a joint venture is not what the requirement asks for.
- ABS do not rank "senior" to the seller's bonds in the seller's bankruptcy. They sit outside it, and their higher rating comes from the SPE structure and any credit enhancement.
- The servicer collects from borrowers; the trustee protects investors' interests.

BOTTOM LINE

- In a securitization, an originator sells a pool of loans or receivables, the collateral, to a special purpose entity, which issues asset-backed securities paid from the borrowers' loan repayments.
- Securitization lets originators increase business activity, earn fees, hold lower capital reserves and turn illiquid loan portfolios into cash.
- Investors gain risk and return tailored to their needs, access to loan pools without the expertise to originate and service loans, and securities that are easier to sell than the underlying loans.
- For economies and financial markets, securitization decreases liquidity risk, improves market efficiency and gives originators lower financing costs and lower leverage.
- ABS investors bear uncertainty in the timing and size of cash flows, for example from unexpected prepayments, and the credit risk of the collateral passes through to them.
- The seller (depositor) originates and sells the loans, the SPE (issuer or trust) buys them and issues the ABS, the servicer collects from borrowers, and the trustee safeguards the collateral and cash flows for ABS holders.
- The SPE owns the collateral outright as a legally separate entity, which makes it bankruptcy remote: the seller's creditors cannot reach the pool, and ABS holders have no claim on the seller's other assets.
- Collections from the pool pay the servicing fee and other administrative fees first, so ABS investors receive less than the total payments collected.

QUICK CHECK

Question 1

Kestrel Savings Bank funds its mortgage lending mainly with customer deposits. It sells $400 million of its mortgages to a special purpose entity, which pays for them with the proceeds of an ABS issue. As a direct result of the sale, which item on Kestrel's balance sheet is *least likely* to change?

A Mortgage loans.

B Customer deposits.

C Cash.

Answer: B

Securitization sells financial assets, not liabilities. Kestrel gives up $400 million of mortgage loans and receives $400 million of cash, so both of these asset lines change. The deposits that funded the loans stay on the balance sheet, because the SPE buys the loans and takes on none of the bank's obligations to its depositors. The bank benefits from the cash it can re-lend and from the lower capital it must hold once the loans are sold; removing deposits or other liabilities is not a benefit of securitization.

Effect of the sale on Kestrel's balance sheet:

- Mortgage loans: $-\$400$ million (sold to the SPE).
- Cash: $+\$400$ million (sale proceeds).
- Customer deposits and other liabilities: no change.

Why the other options are wrong

- **A.** The mortgages are the assets sold to the SPE, so this line falls by $400 million.

- **C.** The SPE pays $400 million for the mortgages, so cash rises by that amount until the bank re-lends it.

Key takeaway In a securitization the originator swaps loans for cash on the asset side; its deposits and other liabilities are not transferred.

Practice questions

Question 2

Brenlow Bank sells a pool of its auto loans to a newly created trust, which issues asset-backed securities (ABS). The senior ABS receive a higher credit rating than Brenlow's own senior bonds. The *most likely* reason is that:

A ABS are always investment grade, whereas bank bonds need not be.

B the trust is a special purpose entity that is bankruptcy remote from Brenlow.

C the ABS rank senior to Brenlow's bonds if Brenlow goes bankrupt.

Question 3

An investor buys ABS backed by a pool of consumer loans originated by Farrow Bank. Which of the following risks does the investor *most likely* bear?

A The credit risk of Farrow Bank's other lending businesses.

B The credit risk of the borrowers in the collateral pool.

C The risk that Farrow Bank's creditors seize the collateral.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/fixed-income-securitization/. The full bank has 9 questions on this reading.

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FIXED INCOME · READING 68

Asset-Backed Security (ABS) Instrument and Market Features

What you'll learn

LOS 68.a Describe the characteristics and risks of covered bonds (including hard-bullet, soft-bullet and conditional pass-through) and how they differ from other ABS.

LOS 68.b Describe internal credit enhancement structures: overcollateralization, excess spread and subordination (credit tranching).

LOS 68.c Describe the cash flows and risks of non-mortgage ABS, such as credit card ABS and solar ABS.

LOS 68.d Describe collateralized debt obligations (CBOs, CLOs), their collateral manager, cash flows and risks.

MODULE 68.1

Asset-Backed Security (ABS) Instrument and Market Features

This reading compares covered bonds with other asset-backed securities, explains the internal credit enhancements that protect ABS investors, and describes non-mortgage ABS and collateralized debt obligations. A candidate must be able to contrast covered bonds with typical ABS, allocate collateral losses through a tranche waterfall, and describe credit card ABS, solar ABS and CLOs.

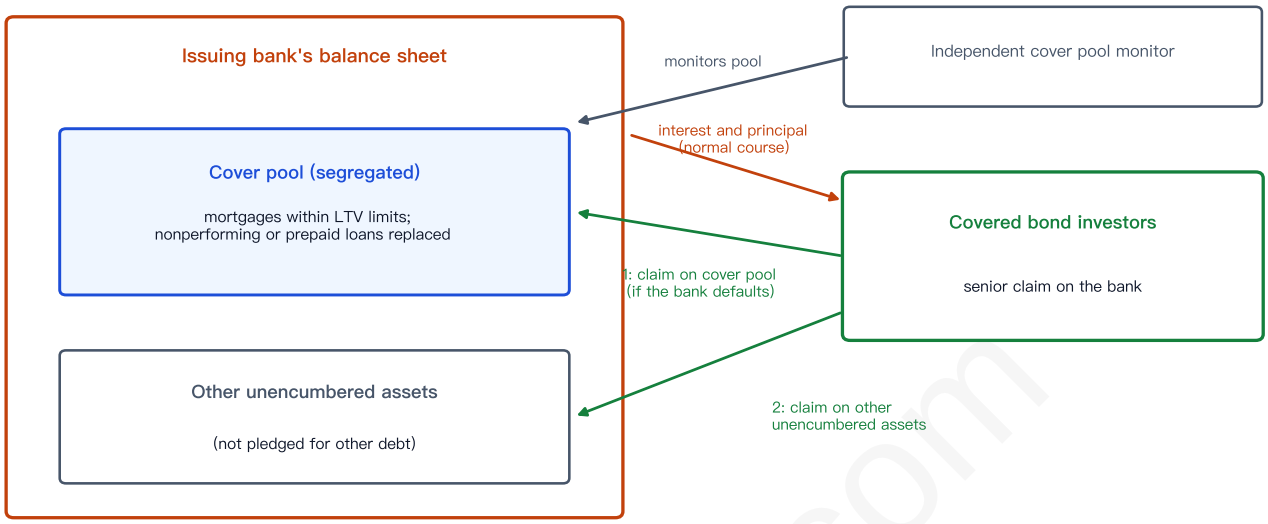
LOS 68.a — Covered bonds versus other ABS

Covered bonds are senior debt obligations of financial institutions (mainly European, Asian and Australian banks) backed by a segregated pool of assets, the **cover pool**, which usually consists of mortgages. No SPE is created, so the cover pool stays on the issuer's balance sheet. Because the bond is the bank's own debt, in the normal course the bank pays interest and principal from its general resources, and the cover pool is security that bondholders can turn to if the bank defaults.

KEY CONCEPT

Feature	Covered bond	Typical ABS
Where the assets sit	On the issuer's balance sheet (segregated)	Sold to a bankruptcy-remote SPE
Investor recourse	**Dual recourse**: the cover pool **and** the issuer's other **unencumbered assets**	The collateral pool only
Asset pool	**Dynamic**: the issuer must replace or add to nonperforming or prepaid assets so the pool keeps supporting the promised payments	Fixed at issuance
Credit tranching	Typically none	Common
Issuer's capital relief	None (assets stay on balance sheet)	Yes

Feature	Covered bond	Typical ABS
Yield	Generally **lower** than comparable ABS (higher credit quality)	Higher



Dual recourse, no SPE, dynamic pool: why covered bonds usually yield less than comparable ABS (illustrative).

How a covered bond is structured

Covered bond investors have further protection from loan-to-value limits on the mortgages in the pool, from overcollateralization (pool value above the bonds outstanding), and from monitoring of the pool by a third party.

If the issuer misses a payment, the outcome depends on the bond's terms:

KEY CONCEPT

Type	Treatment
Hard-bullet covered bond	In default as soon as a scheduled payment is missed; payments to bondholders are accelerated
Soft-bullet covered bond	The original maturity can be postponed by as much as a year, delaying default and acceleration
Conditional pass-through covered bond	Converts to a pass-through bond at maturity if payments remain due; later recoveries from the cover pool are passed through to investors

LOS 68.b — Internal credit enhancement

Internal credit enhancements are features built into the ABS structure to absorb defaults in the collateral pool.

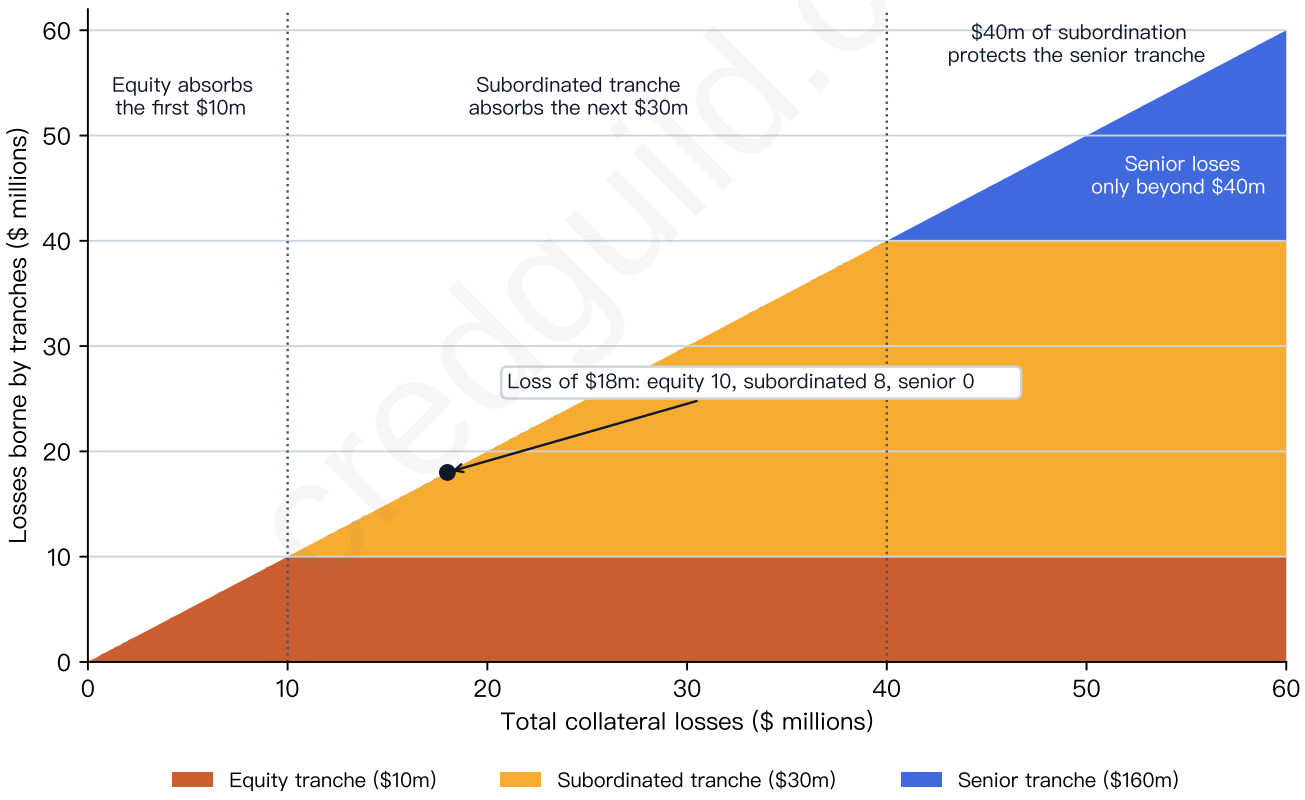
KEY CONCEPT

1. **Overcollateralization**: the collateral value exceeds the ABS face value. For example, if collateral of $250 million backs ABS of $225 million, the $25 million excess can absorb losses of $25/250 = 10\%$ of the collateral value before any ABS investor loses money.

2. **Excess spread**: the collateral earns more interest than the ABS coupons require, and the surplus builds a reserve that absorbs losses.
3. **Subordination** (**credit tranching**): the ABS is split into **tranches** with different priority. Junior tranches absorb losses first, up to their principal, which protects the senior tranches. The larger the subordinated share, the greater the protection for the senior tranche.

This loss-absorption order is a **waterfall structure**: each junior tranche receives only what is left after the more senior tranches are paid. The lowest-ranking tranche has a residual claim and is often called the **equity tranche**. Credit tranching and the bankruptcy-remote SPE together can allow senior tranches to be rated above the originator.

Example. A deal backed by $200 million of collateral has a senior tranche of $160 million, a subordinated tranche of $30 million and an equity tranche of $10 million. Collateral losses are $18 million. The equity tranche absorbs the first $10 million and is wiped out. The subordinated tranche absorbs the other $8 million, so its face value falls from $30 million to $22 million. A subordinated tranche investor's coupon is paid on the reduced face value, so it falls to $22/30 = 73.3\%$ of its previous amount. The senior tranche loses nothing.



Allocation of collateral losses across the senior, subordinated and equity tranches

The figure extends the example to any level of collateral losses: the senior tranche is hit only once losses exceed the $40 million of subordination, which is 20% of the $200 million collateral.

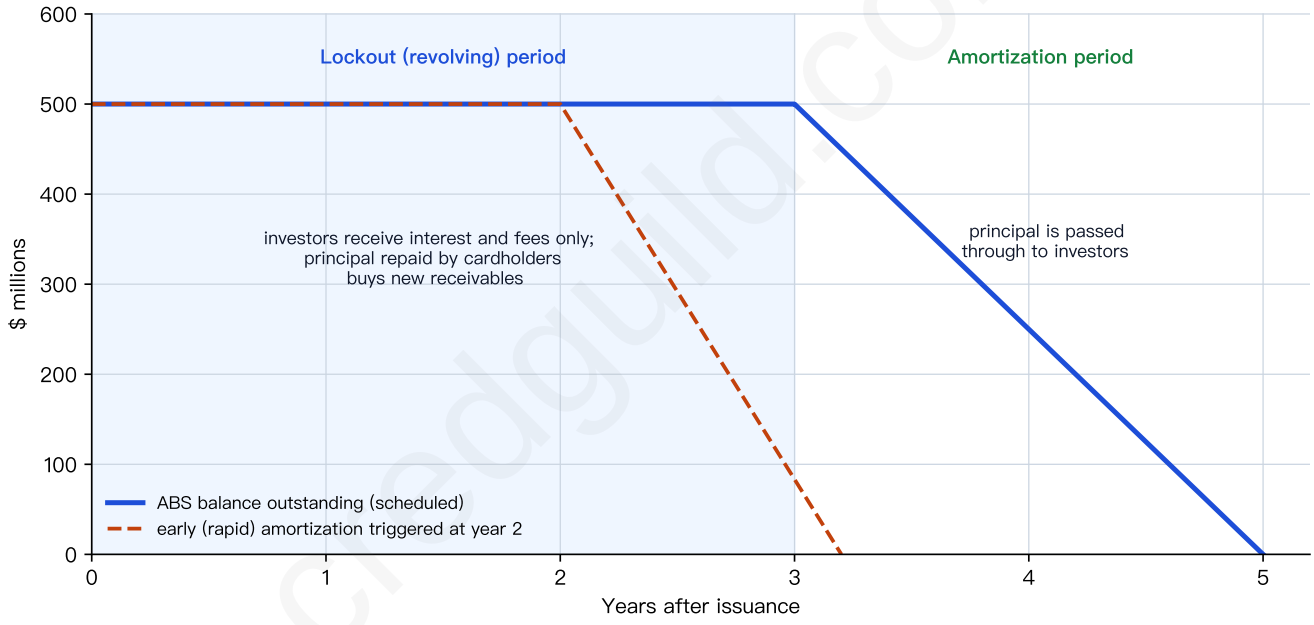
Credit tranching redistributes default risk. **Time tranching**, covered with MBS, redistributes prepayment risk.

LOS 68.c — Non-mortgage ABS

Any asset that generates future cash flows can be securitized: business loans, accounts receivable and auto loans, and even music royalties or franchise license payments. A key distinction is whether the loans are **amortizing** (scheduled principal paydown) or **nonamortizing** (no schedule).

Credit card ABS

- Backed by credit card receivables of banks, retailers and others. The collateral is nonamortizing, though cardholders may repay principal at will.
- Cash flows consist of finance charges (interest), fees (membership, late payment) and principal. Rates may be fixed or floating and are typically subject to a cap.
- During the **lockout period** (**revolving period**), investors receive only interest and fees. Principal repaid by cardholders buys new receivables, which keeps the pool size stable, so investors face no prepayment risk during this period.
- After the lockout, the **amortization period** passes principal through to investors. An **early (rapid) amortization provision** accelerates principal repayment if needed to protect credit quality.



Credit card ABS: balance outstanding in the lockout and amortization periods (illustrative)

Solar ABS

- Backed by loans to homeowners to install solar energy systems, made by specialist finance companies or the finance arms of solar energy companies. The asset class draws investors who weigh environmental, social and governance (**ESG**) factors.
- The loans are secured on the system itself or on the home as a junior mortgage. Borrowers usually have good credit and save on energy bills, so credit risk is thought to be low, although the asset class has not yet been tested through a full credit cycle.
- Internal credit enhancement is common. Many deals have a **pre-funding period** during which the trust invests newly raised funds in additional solar loans, building a larger, more diversified pool.

LOS 68.d — Collateralized debt obligations

A **collateralized debt obligation (CDO)** is a structured security that an SPE issues against a pool of debt obligations.

Type	Collateral
Collateralized bond obligation (CBO)	Corporate and emerging market debt
Collateralized loan obligation (CLO)	**Leveraged loans**: senior secured bank loans to companies with high existing debt or a poor credit history

A CDO differs from ordinary ABS in having a **collateral manager**, who actively buys and sells securities in the pool to generate the cash needed to pay investors. Ordinary ABS rely on a static pool. CDOs are tranched like ABS, and the structure must still offer an attractive return to the equity tranche after paying the senior tranches.

Example. A CLO buys \$400 million of leveraged loans yielding 8% and funds them with \$360 million of debt tranches at an average cost of 5.5% plus a \$40 million equity tranche. Collateral income is $0.08 \times 400 = \$32.0$ million and interest on the debt tranches is $0.055 \times 360 = \$19.8$ million, which leaves \$12.2 million, a 30.5% return on the equity before fees. If defaults cost \$6 million in a year, the residual falls to \$6.2 million and the equity return roughly halves to 15.5%, while the debt tranches are still paid in full. The equity tranche earns the gap between the collateral yield and the cost of the debt tranches, magnified by leverage, and it absorbs the first losses.

Since the 2007–2009 crisis, CDOs have become simpler, requirements for collateral quality have become more stringent, and most are now backed by leveraged loans, so the CLO is the most common form.

CLO type	How investors are paid
Cash flow CLO	From the cash flows of the underlying collateral
Market value CLO	From trading the collateral at market value
Synthetic CLO	Exposure is created with **credit derivative** contracts; the trust does not own the collateral

CLO collateral is subject to protective tests: coverage of payments to investors by collateral cash flows; overcollateralization limits for each tranche (a breach diverts cash to buy more collateral or to repay the most senior tranche); diversification; and limits on CCC rated holdings.

Common exam traps

- A covered bond differs from a typical ABS on three points that are easy to reverse: it gives recourse to the issuer, uses no SPE, and has a dynamic rather than fixed pool. It usually yields less.
- A hard bullet means immediate default; a soft bullet allows a maturity extension of up to a year.
- The lockout period belongs to credit card ABS; the pre-funding period belongs to solar ABS.
- The collateral manager distinguishes a CDO. Tranching and the use of an SPE are common to ABS in general.

EXAM SHORTCUTS

- Under subordination, the senior tranche loses nothing until collateral losses exceed the total of the tranches ranked below it, so smaller losses need only be allocated among the junior tranches.

BOTTOM LINE

- Covered bonds are senior debt of financial institutions backed by a segregated cover pool, usually mortgages, that stays on the issuer's balance sheet because no SPE is created.
- Covered bond investors have dual recourse to the cover pool and to the issuer's unencumbered assets, the cover pool is dynamic, and covered bonds generally yield less than comparable ABS.
- When an issuer misses a payment, a hard-bullet covered bond defaults at once and payments are accelerated, a soft-bullet one lets the maturity slip by up to a year, and a conditional pass-through one becomes a pass-through security at maturity if payments are still owed.
- Internal credit enhancements are overcollateralization, excess spread and subordination, under which junior tranches absorb losses first, up to their principal, to protect the senior tranches.
- Credit tranching redistributes default risk, while time tranching redistributes prepayment risk.
- Credit card ABS are backed by nonamortizing receivables; during the lockout (revolving) period investors receive only interest and fees, and principal repaid by cardholders buys new receivables.
- Solar ABS are backed by loans to homeowners to install solar energy systems, and many deals have a pre-funding period in which the trust adds new solar loans to the pool.
- A CDO differs from ordinary ABS by having a collateral manager who actively buys and sells securities in the pool, and the CLO, backed by leveraged loans, is now its most common form.

QUICK CHECK

Question 1

Compared with otherwise similar asset-backed securities, covered bonds issued by Eskdale Bank *most likely* offer investors:

A recourse to the issuing bank.

B higher yields.

C lower credit quality.

Answer: A

Covered bondholders have dual recourse: a claim on the cover pool and a claim on the issuing bank's other unencumbered assets. Ordinary ABS investors can look only to the collateral held by the SPE. The extra recourse, together with overcollateralization and loan-to-value limits, gives covered bonds higher credit quality and lower yields than comparable ABS.

Why the other options are wrong

- **B.** Because covered bonds are safer, investors accept lower yields than on comparable ABS.
- **C.** Dual recourse and a dynamic, overcollateralized cover pool give covered bonds higher credit quality than comparable ABS.

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Key takeaway A covered bond has dual recourse, no SPE and a dynamic cover pool, which give it higher credit quality and a lower yield than comparable ABS.

Practice questions

Question 2

Halden Card Trust issued asset-backed securities backed by credit card receivables, and the securities are still in their lockout period. Which event would *most likely* cause the ABS holders to start receiving principal payments ahead of schedule?

A Cardholders repay a larger share of their balances each month.

B A trigger in the early (rapid) amortization provision is breached.

C The interest rate charged on the card balances reaches its cap.

Question 3

Which feature *most clearly* distinguishes a collateralized debt obligation (CDO) from most other asset-backed securities?

A It is issued through a special purpose vehicle.

B A collateral manager actively trades the securities in its collateral pool.

C It is structured with senior and subordinated tranches.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/asset-backed-security-abs-instrument-and-market-features/. The full bank has 17 questions on this reading.

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FIXED INCOME · READING 69

Mortgage-Backed Security (MBS) Instrument and Market Features

What you'll learn

LOS 69.a Define prepayment risk (contraction and extension risk) and describe how time tranching reallocates it.

LOS 69.b Describe the features of residential mortgage loans that are securitized: prepayment penalties, recourse, LTV and DTI, agency vs non-agency RMBS.

LOS 69.c Describe mortgage pass-through securities and CMOs (sequential pay, PAC and support, Z, IO/PO, floating-rate tranches) and their cash flows and risks.

LOS 69.d Describe the characteristics and risks of commercial MBS, including DSCR, LTV, call protection and balloon risk.

MODULE 69.1

Mortgage-Backed Security (MBS) Instrument and Market Features

This reading covers mortgage-backed securities: prepayment risk, the features of residential mortgage loans, pass-throughs and CMOs, and commercial MBS. A candidate must be able to explain contraction and extension risk, compute LTV, DTI, WAC, WAM and DSCR, and describe how sequential pay, PAC, support, IO and PO tranches reallocate prepayment risk.

LOS 69.a — Prepayment risk and time tranching

Prepayments are principal repayments above the scheduled amortization of a mortgage, usually because borrowers sell their homes or refinance. MBS are priced on an assumed prepayment rate, and **prepayment risk** is the risk that actual prepayment speeds differ from that assumption. The **average life** of an MBS is the average time until its principal is received, weighted by the amount of principal paid at each date. Faster prepayments shorten it and slower prepayments lengthen it.

Example. A borrower's scheduled monthly payment is $1,500, of which $900 is interest and $600 is scheduled principal. If the borrower pays $2,300, the extra $800 is a prepayment, and the loan balance falls by $1,400 instead of $600.

KEY CONCEPT

Risk	What happens	Typical trigger	Why it hurts investors
Contraction risk	Prepayments **faster** than expected; cash arrives sooner, average life shortens	Interest rates **fall** (refinancing)	Principal must be reinvested at lower rates; MBS prices rise less than option-free bonds when rates fall (**negative convexity**, like a callable bond)

Risk	What happens	Typical trigger	Why it hurts investors
Extension risk	Prepayments **slower** than expected; average life lengthens	Interest rates **rise**	Cash flows are received later and discounted over more periods at a higher rate

Time tranching creates bond classes with different maturities. It reallocates prepayment risk among them without removing it. Tranches that mature first offer more protection against extension risk; longer tranches offer more protection against contraction risk. **Sequential pay tranching** (LOS 69.c) is the standard example.

LOS 69.b — Features of residential mortgage loans

A **residential mortgage loan** is secured by residential real estate. The lender's legal claim on the property is a **first lien**: if the borrower defaults, the lender can take possession of the property and sell it, a process called **foreclosure**.

- A **prepayment penalty** is an extra payment by a borrower who prepays when rates have fallen. It reduces the lender's prepayment risk. Prepayment penalties are common in Europe and rare in the United States.
- With a **recourse loan**, the lender can also claim the borrower's other assets for any shortfall after foreclosure. With a **nonrecourse loan**, the lender can look only to the property. When a home is worth less than the loan (an **underwater mortgage**, or **negative equity**), a nonrecourse borrower has a stronger incentive to **strategically default**, walking away even though they could pay. Most mortgages in Europe are recourse loans. US rules differ from state to state, but most US loans are nonrecourse.
- The **loan-to-value ratio (LTV)** is the loan divided by the property value. A lower LTV means more borrower equity, which lowers default risk and raises recovery.
- The **debt-to-income ratio (DTI)** is monthly debt payments divided by monthly gross pretax income. A lower DTI means lower default risk.

Example. A $360,000 loan on a $450,000 home has an LTV of 80%. At 5.5% for 30 years with monthly payments, the payment is $2,044.04. On a financial calculator in END mode with P/Y = C/Y = 1: `N = 360, I/Y = 5.5/12, PV = 360,000, FV = 0, CPT PMT = −2,044.04` (the loan received is positive, the payment negative). With gross income of $96,000 a year ($8,000 a month), DTI $= 2{,}044.04/8{,}000 = 25.6\%$.

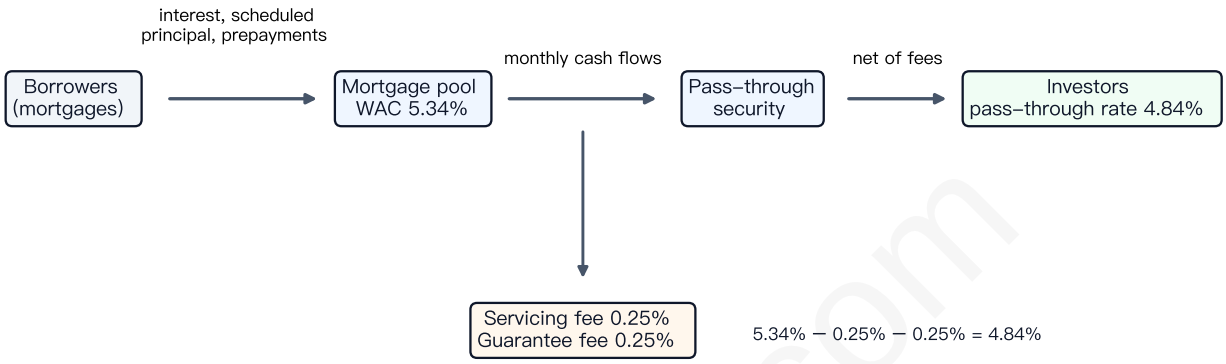
Prime loans are made to borrowers whose credit record is strong and whose LTV and DTI are both low. **Subprime loans** involve weaker credit, a higher DTI or LTV, or a lower-priority lien.

Agency RMBS (U.S.) are guaranteed by the government or by a **government-sponsored enterprise (GSE)**. A GSE is legally separate from the government, so securities it guarantees carry only the GSE's own guarantee, not the **full faith and credit** of the government. The collateral must meet minimum underwriting standards. **Non-agency RMBS** are issued by private entities such as banks without these guarantees and rely on credit enhancement (insurance, letters of credit, tranching, private guarantees). They are widely blamed for helping to trigger the 2007–2009 financial crisis: senior tranches rated investment grade, backed by pools of subprime loans, took large losses when many of those loans defaulted. New issuance almost stopped in the years after the crisis as regulation tightened.

LOS 69.c — Mortgage pass-throughs and CMOs

A **mortgage pass-through security** is a claim on the cash flows of a mortgage pool, net of fees.

- The **weighted average maturity (WAM)** and **weighted average coupon (WAC)** weight each mortgage's maturity or rate by its share of the pool's outstanding balance.
- Investors receive the pool's monthly cash flows minus servicing and guarantee or insurance fees, so the **pass-through rate** (net coupon) is below the WAC.



Cash flows of a mortgage pass-through (pool WAC 5.34%; fees illustrative)

With w_i the share of mortgage i in the pool's outstanding balance:

$$\text{WAC} = \sum_i w_i \times \text{mortgage rate}_i \qquad \text{WAM} = \sum_i w_i \times \text{remaining term}_i$$

$$\text{pass-through rate} = \text{WAC} - \text{fees}$$

> *Example.* A pool holds balances of 150, 250 and 100 (total 500) at rates of 4.8%, 5.4% and 6.0%, with remaining terms of 18, 26 and 12 years.

$$\text{WAC} = \frac{150(4.8) + 250(5.4) + 100(6.0)}{500} = 5.34\%,$$

$$\text{WAM} = \frac{150(18) + 250(26) + 100(12)}{500} = 20.8 \text{ years}$$

Collateralized mortgage obligations (CMOs) are backed by pass-throughs and mortgage pools. They split the cash flows into tranches with different exposure to prepayment risk. The total prepayment risk of the collateral is unchanged; it is reallocated to suit investors with different concerns (some worry mainly about extension risk, others about contraction risk). This widens the market for securitized mortgages and may lower funding costs.

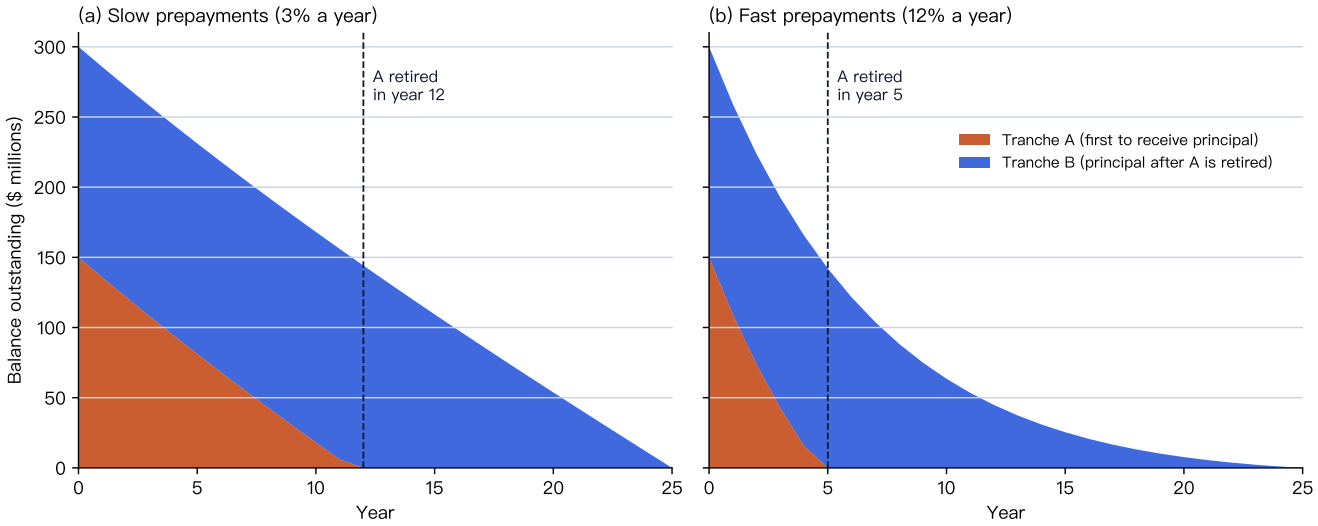
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KEY CONCEPT

CMO structure	How it works / who it suits
Sequential pay	All principal (scheduled and prepaid) goes to the first ("short") tranche until it is retired, then to the next. This is the time tranching of LOS 69.a, so the short tranche suits investors most concerned about extension risk
Planned amortization class (PAC)	Makes scheduled payments as long as prepayments stay within a range, reducing **both** contraction and extension risk
Support tranche	Absorbs the variability: receives excess principal when prepayments are fast and has its principal payments curtailed when they are slow, so PAC schedules are met. Bears high **prepayment risk**. If speeds move outside the range, the protection breaks down
Z-tranche (accrual / accretion bond)	Receives no cash interest during an accrual period; interest is added to principal (e.g., $200 at 4% becomes $208). Typically the lowest-ranking tranche
Principal-only (PO)	Pays only principal (a zero-coupon-like security bought at a discount). Its return is the gap between the price paid and the principal received, so the sooner principal arrives, the higher the return: it **benefits from falling rates and faster prepayments**
Interest-only (IO)	Pays only interest on the principal still outstanding. Faster prepayments shrink that balance and so the total interest received: it is **hurt by falling rates and faster prepayments**
Floating-rate / inverse floater	Coupon tied to a market reference rate (often capped/floored); an inverse floater's coupon rises as rates fall
Residual	Most junior, like an ABS equity tranche

Example: sequential pay. A $300 million pool of 6%, 25-year mortgages backs Tranche A ($150 million) and Tranche B ($150 million). All principal, scheduled and prepaid, goes to A until A is retired. If 3% of the balance is prepaid each year, A is paid off in year 12. If 12% is prepaid each year, A is paid off in year 5, and B starts receiving principal seven years earlier. Fast prepayments shorten A's life sharply (contraction risk), and slow prepayments leave B waiting much longer for its principal (extension risk). Tranche A has the shorter expected average life, so it suits investors who prefer shorter-term securities.

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Tranche balances in a two-tranche sequential-pay CMO at slow and fast prepayment speeds

Example: PAC and support. A PAC tranche is scheduled to receive $20 million of principal this year, and the support tranche has $40 million outstanding. The PAC schedule therefore holds as long as the pool's principal for the year is between $20 million and $60 million.

Pool principal this year	PAC tranche receives	Support tranche receives
$24 million (slow)	$20 million	$4 million
$40 million (as expected)	$20 million	$20 million
$58 million (fast)	$20 million	$38 million
$15 million (below the range)	$15 million, $5 million short of schedule	$0

Inside the range the support tranche takes all the variation. In the last row it has nothing left to give up, so the PAC tranche falls short. Above $60 million the support tranche would be paid off, and the excess principal would go to the PAC tranche ahead of schedule.

LOS 69.d — Commercial mortgage-backed securities

CMBS are backed by mortgages on income-producing property (multifamily, industrial, retail, office, health care, hotels). Pools contain fewer loans than RMBS pools, sometimes just one, so diversification is lower. A pool may hold loans from one lender or several loans to one borrower. In an RMBS pool of thousands of small, similar home loans, one default barely affects investors; in a CMBS pool, the default of a single large loan can. U.S. CMBS coupons are typically fixed; European CMBS usually float. The loans are repaid by property investors who depend on tenants' rents, and the pool's regular income, the **weighted average proceeds from the mortgages (WAMP)**, plays the role the WAC plays for RMBS. Commercial mortgages are **nonrecourse loans**, so the lender's recovery depends on the property and its income. Analysis therefore focuses on the credit risk of the property rather than that of the borrower. CMBS are tranched so that credit losses are absorbed by the lowest-priority tranches first, in sequence.

KEY CONCEPT

$$\text{DSCR} = \frac{\text{net operating income}}{\text{debt service}} \qquad \text{LTV} = \frac{\text{loan amount}}{\text{appraised value}}$$

Net operating income (NOI) is rental income minus cash operating expenses and a replacement reserve. A higher **debt service coverage ratio (DSCR)** and a lower LTV indicate better credit quality. For example, NOI of $1.3 million and debt service of $1.0 million give a DSCR of $1.30\times$; a $12 million loan on a $16 million property has an LTV of 75%.

Call protection (protection against prepayment) is a key difference from RMBS:

- At the loan level: a **prepayment lockout** (typically 2–5 years), **prepayment penalty points** (1 point = 1% of the principal prepaid), and **defeasance** (the borrower buys government securities that replicate the remaining loan payments, freeing the property from the lien).
- At the structure level: sequential pay tranching of the pool.

Balloon payments. Commercial loans are usually not fully amortizing, which leaves a **balloon payment** at maturity. If the borrower cannot refinance it, the lender may extend the loan in a workout, so **balloon risk** is a form of extension risk.

Common exam traps

- Contraction and extension describe the average life, not the stated maturity: contraction comes with falling rates and faster prepayments, extension with rising rates and slower prepayments.
- Residential mortgages in agency RMBS pools can usually be prepaid freely; CMBS loans typically carry call protection.
- A support tranche's high risk is prepayment risk in both directions, not credit risk; with agency collateral its credit risk is low.
- Prepayment speed changes the timing of cash flows, not the net coupon; the gap between the WAC and the pass-through rate comes from fees.

BOTTOM LINE

- Contraction risk is the risk that prepayments run faster than expected, typically when rates fall, which shortens the average life and forces reinvestment at lower rates; extension risk is the risk that they run slower, typically when rates rise, which lengthens it.
- Time tranching reallocates prepayment risk among bond classes without removing it, and the tranches that mature first give more protection against extension risk.
- A lower loan-to-value ratio and a lower debt-to-income ratio mean lower default risk, and a borrower with an underwater nonrecourse loan has a stronger incentive to default strategically.
- A GSE is legally separate from the government, so agency RMBS it guarantees carry only the GSE's own guarantee, while non-agency RMBS rely on credit enhancement.
- A pass-through's net coupon is below the pool's weighted average coupon because servicing and guarantee or insurance fees are deducted, and the WAC and WAM weight each mortgage by its share of the outstanding balance.

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- A PAC tranche makes its scheduled payments as long as prepayments stay within a range, which reduces both contraction and extension risk, while the support tranche absorbs the variation and bears high prepayment risk.
- A principal-only security benefits from falling rates and faster prepayments, while an interest-only security is hurt by them.
- Commercial mortgages are nonrecourse, so CMBS analysis focuses on the property: a higher $\text{DSCR} = \text{NOI}/\text{debt service}$ and a lower LTV indicate better credit quality, call protection is common, and balloon risk is a form of extension risk.

QUICK CHECK

Question 1

A sequential-pay CMO has two tranches. Class J receives all principal payments until it is fully repaid, and principal then goes to Class K. Relative to Class K, Class J has:

A less contraction risk and more extension risk.

B more contraction risk and more extension risk.

C more contraction risk and less extension risk.

Answer: C

In a sequential-pay structure the first tranche receives all scheduled principal and prepayments first. If prepayments speed up, it is repaid sooner (more contraction risk); if they slow, it is still first in line, so it is well protected against extension, which falls mainly on the later tranche.

Why the other options are wrong

- **A.** This is the profile of the later tranche (Class K), which waits longest for principal.
- **B.** Time tranching reallocates prepayment risk: the short tranche gains extension protection in exchange for bearing more contraction risk, so it cannot have more of both.

Key takeaway First-pay tranche: contraction risk. Last-pay tranche: extension risk.

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Practice questions

Question 2

The Linwell pass-through security is backed by the three mortgages shown below. Servicing and guarantee fees total 0.50% a year.

Mortgages in the Linwell pass-through pool

Mortgage	Current balance ($ thousands)	Mortgage rate	Remaining term (years)
1	120	5.00%	22
2	80	6.50%	15
3	200	5.75%	27

The pass-through rate on the security is *closest* to:

A 5.18%.

B 5.25%.

C 5.68%.

Question 3

When comparing the loans backing different CMBS pools, which combination of credit ratios *most likely* indicates better credit quality?

A Higher DSC ratios and lower LTV ratios.

B Lower DSC ratios and higher LTV ratios.

C Higher DSC ratios and higher LTV ratios.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/fixed-income/mortgage-backed-security-mbs-instrument-and-market-features/. The full bank has 16 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Derivatives

Exam weight 6–9% · 10 readings

What forwards, futures, swaps and options are and why issuers and investors use them, how no-arbitrage pricing and replication set their values, put-call parity and the one-period binomial model.

Reading 70	Derivative Instrument and Derivative Market Features
Reading 71	Forward vs Futures
Reading 72	Derivative Benefits, Risks, and Issuer and Investor Uses
Reading 73	Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives
Reading 74	Pricing and Valuation of Forward Contracts and for an Underlying With Varying Maturities
Reading 75	Pricing and Valuation of Futures Contracts
Reading 76	Interest Rate Swap
Reading 77	Pricing and Valuation of Options
Reading 78	Put Call Parity
Reading 79	Binomial Option Pricing Model

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DERIVATIVES · READING 70

Derivative Instrument and Derivative Market Features

What you'll learn

LOS 70.a Define a derivative, identify its underlying, contract price, size and settlement date, and describe long/short exposure, cash vs physical settlement, and hedging vs speculation.

LOS 70.b Describe derivative markets and contrast exchange-traded (standardized, centrally cleared) with OTC (customized, dealer) derivatives, including the central clearing mandate.

MODULE 70.1

Derivatives Markets

This reading defines derivatives, their basic features and their uses, and compares exchange-traded derivatives with those traded over the counter. A candidate must be able to compute the payoff of a forward to each side, distinguish hedging from speculation and deliverable from cash-settled contracts, and contrast exchange-traded and OTC markets, including the role of central clearing.

LOS 70.a — What a derivative is and its basic features

A **derivative** is a contract whose value comes from (is *derived* from) the value of something else at a specified future date. That something else is the **underlying**: most often a stock or bond price, a stock or bond index, or an interest rate, but also a currency, a commodity, credit, or even the weather, a cryptocurrency or longevity.

Every derivative contract spells out four basic features:

KEY CONCEPT

Feature	Meaning	Example (the forward below)
Underlying	Asset or variable that drives the contract's value	Shares of a listed company
Contract price (e.g., **forward price**)	Price at which the future trade will take place	$62 per share
Contract size	Quantity of the underlying covered	300 shares
Settlement date (maturity date)	Date on which the exchange (or payment) occurs	Three months from today

The forward price is normally chosen so that the contract is worth zero to both parties at initiation, and neither side pays anything to enter it.

Long and short exposure

- The buyer of a forward agrees to buy the underlying. She has **long exposure** and gains when the spot price at settlement is above the forward price.

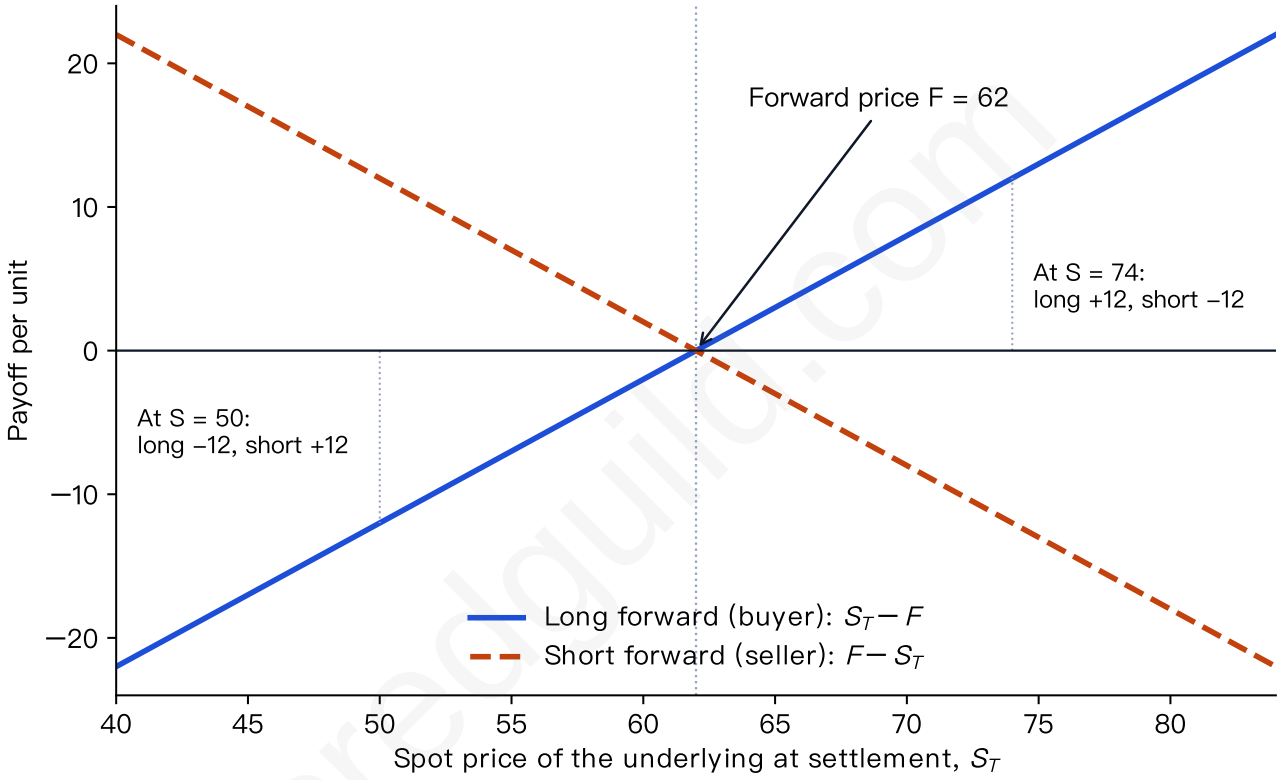
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- The seller agrees to deliver the underlying. He has **short exposure** and gains when the spot price at settlement is below the forward price.
- One side's gain equals the other side's loss (a zero-sum outcome before costs).

KEY CONCEPT

$$\text{Payoff to long at settlement} = S_T - F \qquad \text{Payoff to short} = F - S_T$$

Because neither side pays anything to enter the contract, this payoff is also each party's profit or loss on the forward.



Payoff at settlement of a long and a short forward with a forward price of 62

Example. A forward commits one party to buy 300 shares at \$62 in three months.

- If the spot price at settlement is \$74, the buyer pays $300 \times 62 = \$18{,}600$ for shares worth $300 \times 74 = \$22{,}200$ and gains $\$3{,}600$. The seller loses $\$3{,}600$.
- If the spot price at settlement is \$50, the buyer loses $300 \times (62 - 50) = \$3{,}600$ and the seller gains the same amount.

Deliverable versus cash-settled

- A **deliverable contract** requires the underlying to be exchanged for the contract price at settlement.
- A **cash-settled contract** exchanges only the net gain or loss: the losing side pays $|S_T - F| \times \text{size}$ to the winning side. With a settlement price of \$50 in the example above, the buyer pays the seller $\$3{,}600$ in cash instead of paying $\$18{,}600$ for shares worth $\$15{,}000$. Ignoring trading costs, the two methods are

economically the same.

Hedging versus speculating

- **Hedging**: a party with an existing risk uses a derivative to offset it, fully or partially. A holder of shares who sells them forward locks in the sale price.
- **Speculating**: a party with no existing exposure takes one on through the derivative, which increases its risk.
- A useful rule for hedgers: take the derivative position now that matches the trade the party must make later in the cash market. A firm that will receive foreign currency will have to sell it, so it sells the currency forward. A firm that must pay foreign currency buys it forward.

Why use derivatives instead of a cash market trade?

- Exposure can be obtained with little cash, so the position is highly leveraged.
- Transaction costs can be much lower than for the equivalent cash trade.
- Entering the position may move the price of the underlying less than an equivalent cash trade would.

Underlyings and the main derivative types

A higher interest rate benefits the buyer of an interest rate forward but hurts the buyer of a bond forward, because bond prices fall when rates rise. Commodities may be hard (mined or extracted, such as gold and oil) or soft (grown, such as coffee and cattle). Currency forwards are quoted as price currency per unit of base currency: USD/EUR 1.10 means one euro costs 1.10 US dollars. The long side agrees to buy the base currency (here, euros) at the forward rate and pays the price currency; the short side sells the base currency.

The main derivative types are forwards, futures, options and swaps. A **call option** lets its buyer purchase the underlying on a later date at a price set today, with no obligation to do so; a **put option** gives its buyer the same kind of right to sell. In a plain **interest rate swap**, the two sides exchange periodic payments on the same notional amount: one pays a fixed rate, and the other pays a floating rate set from a market reference rate (MRR). The fixed-rate payer is in effect borrowing at a fixed rate to hold a floating-rate bond. In a **credit default swap**, one party makes fixed periodic payments, and the other pays only if the issuer of the underlying credit instrument defaults.

LOS 70.b — Derivative markets: exchange-traded versus OTC

Exchange-traded derivatives

Futures, some options and some other derivatives trade on organized exchanges; the largest by trading volume are the National Stock Exchange of India, B3 (Brazil) and CME Group (United States). The exchange writes **standardized contracts** (size, underlying, dates) and the rules of trading. Exchange members (**market makers** or dealers) mainly earn the **bid–ask spread** rather than speculating.

The **central clearinghouse (CCH)** steps in between the two traders and becomes the buyer to every seller and the seller to every buyer (**novation**). It therefore guarantees that each side's obligations will be performed. To back that guarantee, it collects deposits from both sides when a trade opens and further deposits when a position loses value, which keeps **counterparty credit risk** very low.

Standardization makes exchange contracts more liquid, more transparent and cheaper to trade, and it makes it easy to exit a position by taking an offsetting one. It also simplifies **clearing** (recording the trade and handling payments) and **settlement** (final delivery or payment at maturity).

Over-the-counter (OTC) dealer markets

Forwards, most swaps and some options are customized contracts that dealers create in a market with no central trading venue, the **over-the-counter (OTC) market**. Size, underlying, settlement date and delivery method can be tailored, and the trade can be kept private. Without a central clearinghouse, each side bears the other's counterparty credit risk. Dealers trade with end users and also with each other to lay off their exposure to price changes in the underlying.

Since 2008, a **central clearing mandate** has required many swaps to be cleared through a **central counterparty (CCP)**. Trades reported on a **swap execution facility (SEF)** are replaced by two trades with the CCP. This lowers counterparty risk but concentrates it in the CCP.

KEY CONCEPT

Feature	Exchange-traded	OTC (no central clearing)
Terms	Standardized	Customized
Where traded	Centralized exchange, through members	Dealer network, no central location
Liquidity	Higher	Lower
Transparency	High; trades known to exchange and regulators	Lower
Regulation	Exchange rules; more regulated	Traditionally less regulated; post-2008 rules now cover many OTC derivatives
Counterparty risk	Minimal (CCH guarantee plus deposits)	Each side faces the other
Collateral deposits	Required from both sides (exchange/CCH margin)	No exchange-imposed margin; depends on the contract, the counterparties and regulation
Trading costs	Lower	Higher
Clearing and settlement	Simpler	More difficult

OTC trades that fall under the central clearing mandate have lower counterparty risk and more disclosure, but they remain customizable dealer contracts.

Exam convention: exchange-traded derivatives are standardized, liquid, transparent and margined through the clearinghouse, while OTC derivatives are customized, less transparent, largely unregulated, not subject to collateral deposit requirements and carry more counterparty risk. Current practice: trading outside a CCP does not mean trading outside regulation or without collateral. Bilateral collateral is set by the contract and the counterparties, and for many covered counterparties uncleared-swap margin rules require both initial and variation margin.

Common exam traps

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- A derivative's value depends on an underlying. Being a contract, or trading in a particular place, does not make an instrument a derivative.
- A forward has zero value at initiation, but the forward price is not zero.
- Selling forward is not automatically a hedge. The same trade hedges an existing exposure but is speculation for a party that has no such exposure.
- The clearinghouse does not hold an insurance policy or stabilize prices of the underlying. It guarantees performance by being the counterparty to both sides.
- Illiquidity, customized terms and bilateral counterparty risk describe OTC contracts outside central clearing. Exchange-traded futures and options are standardized, liquid and backed by the clearinghouse.

EXAM SHORTCUTS

- A hedger takes now the derivative position that matches the trade it must make later in the cash market: a party that will have to sell the underlying sells forward, and a party that will have to buy it buys forward.

BOTTOM LINE

- A derivative is a contract whose value is derived from an underlying, and every derivative specifies four features: its underlying, contract price, settlement date and contract size.
- The forward price is normally set so that the contract is worth zero to both parties at initiation; at settlement the long receives $S_T - F$ and the short receives $F - S_T$, so one side's gain is the other's loss.
- A deliverable contract exchanges the underlying for the contract price, a cash-settled contract exchanges only the net gain or loss, and ignoring trading costs the two are economically the same.
- Hedging uses a derivative to offset an existing risk, while speculating takes on an exposure the party did not have, so the same trade can be a hedge for one party and speculation for another.
- Compared with a cash market trade, a derivative gives highly leveraged exposure for little cash, can cost much less to trade and may move the underlying's price less.
- For exchange-traded derivatives, the central clearinghouse becomes the buyer to every seller and the seller to every buyer (novation), guaranteeing performance and collecting deposits from both sides, which keeps counterparty credit risk very low.
- Exam convention: exchange-traded derivatives are standardized, liquid, transparent and margined through the clearinghouse, while OTC derivatives are customized, less transparent, largely unregulated, carry more counterparty risk and are not subject to collateral deposit requirements; current practice: an OTC trade outside a CCP still has collateral set by the contract and the counterparties, and for many covered counterparties uncleared-swap margin rules require initial and variation margin.
- Since 2008, a central clearing mandate has required many swaps to be cleared through a central counterparty, which lowers counterparty risk but concentrates it in the CCP.

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QUICK CHECK

Question 1

Harlow Capital agrees with a dealer to buy 200 shares of Pemberton Mills in three months at a forward price of $48.00 per share. The contract is cash settled. On the settlement date, Pemberton shares trade at $44.50. The settlement of the contract is:

A The dealer pays Harlow $700.

B Harlow pays the dealer $700.

C Harlow pays the dealer $9,600.

Answer: B

In a cash-settled forward only the gain or loss changes hands. Harlow, the buyer (long), agreed to pay $48.00 for shares now worth $44.50, so it has a loss and pays the difference to the dealer (the short).

$$\text{Payoff to long} = (S_T - F) \times \text{shares} = (44.50 - 48.00) \times 200 = -\$700$$

The long (Harlow) has a $700 loss and pays it to the short (the dealer). Economically this equals delivery: Harlow would pay $9,600 for shares it could sell for $200 \times 44.50 = \$8{,}900$.

Why the other options are wrong

- **A.** This reverses the direction. The buyer of a forward loses when the spot price at settlement is below the forward price, so the buyer pays the seller.
- **C.** $9,600 is the full forward price ($200 \times \$48$) that Harlow would pay in a deliverable contract in exchange for the shares. With cash settlement, only the net difference is paid.

Key takeaway Cash settlement transfers only $|S_T - F| \times$ size, from the losing side to the winning side.

Practice questions

Question 2

A client asks her adviser how exchange-traded derivatives differ from contracts arranged with dealers. Which of the adviser's statements about exchange-traded derivatives is *least accurate*?

A They are backed by a central clearinghouse.

B They are traded by exchange members at a central exchange.

C They are generally illiquid.

Question 3

A portfolio manager is explaining to a trustee why an interest rate swap held by the fund is classified as a derivative. The defining feature she should point to is that the instrument's value:

A is derived from the value of an underlying asset, rate, or index.

B is set out in a contract between two counterparties.

C is determined in trading that takes place away from an organized exchange.

Answers: 2. C · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/derivative-instrument-and-derivative-market-features/. The full bank has 7 questions on this reading.

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DERIVATIVES · READING 71

Forward vs Futures

What you'll learn

LOS 71.a Define swaps, credit default swaps, and call and put options, and describe the rights and obligations of each party.

LOS 71.b Calculate the value at expiration and the profit of long and short call and put positions, including breakeven and maximum gain/loss.

LOS 71.c Contrast forward commitments (forwards, futures, most swaps) with contingent claims (options, credit default swaps).

MODULE 71.1

Forwards and Futures

This reading describes the main derivative instruments, forwards, futures, swaps, credit default swaps and options, and how option values and profits are measured at expiration. A candidate must be able to track a futures margin account through the daily mark-to-market, compute net swap payments, compute option values, profits and breakevens, and classify a derivative as a forward commitment or a contingent claim.

LOS 71.a — Forward and futures contracts

Forward contracts

A **forward contract** is a private agreement in which one party (the buyer, **long**) is obligated to purchase a physical or financial asset from the other (the seller, **short**) at a price fixed today, on a specific future **settlement date**. Both sides are obligated, and neither pays anything at initiation. The long profits if the underlying's price at settlement is above the forward price; the short profits if it is below.

Futures contracts

A **futures contract** is essentially a forward that is standardized and traded on an exchange. The main differences are:

KEY CONCEPT

Feature	Forward	Futures
Terms	Customized	Standardized
Market	OTC dealer market	Organized exchange
Liquidity / secondary market	Limited	Liquid
Regulation and transparency	Lower	Higher
Counterparty credit risk	Each side bears it	Minimal: **clearinghouse** guarantee
Gains and losses	Usually settled only at maturity	Settled **daily** (mark-to-market)

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Margin and the daily mark-to-market

Futures **margin** is collateral (cash or acceptable securities) that both buyer and seller must post. It protects the clearinghouse. Unlike margin on a stock purchase, it is not a loan, so no interest is charged.

KEY CONCEPT

- **Initial margin** is the amount that must be deposited before a trade. Its size is set at about the largest price move expected in one day on the contract's value.
- **Maintenance margin** is the minimum balance the account must keep. The exchange sets both margin requirements.
- In the **mark-to-market** (marking to market), every account is credited with gains and debited with losses at the end of each day, based on the change in the settlement price.
- The **settlement price** is the average price of trades during a closing period at the end of the session. The exchange sets the length of that period.
- A **margin call** occurs if the balance falls below the maintenance margin. The trader must then deposit enough to bring the account back to the initial margin, or the position is closed out. In an equity margin account, by contrast, the investor only has to restore the maintenance level.

Daily cash settlement of gains and losses, together with the clearinghouse, minimizes counterparty credit risk on futures. Forward contracts typically are not marked to market daily, although they can be when a central clearing party is used.

Example. A trader buys one crude oil futures contract on 1,000 barrels at $78.00. Initial margin is $6,000 and maintenance margin is $4,500.

Day	Settlement price	Daily gain/loss (long)	Balance before deposit	Deposit	Ending balance
0	78.00	—	6,000	—	6,000
1	76.80	$-1.20 \times 1{,}000 = -1{,}200$	4,800	0 (above 4,500)	4,800
2	75.90	$-0.90 \times 1{,}000 = -900$	3,900	$6{,}000 - 3{,}900 = 2{,}100$	6,000

On Day 2 the balance of $3,900 is below $4,500, so the deposit must restore the initial margin of $6,000. The short's account is credited with the same amounts.

Price limits

Many futures have **price limits**: the exchange caps how far the settlement price may move from the previous day's. Trades outside the limits are not allowed, so if the equilibrium price lies beyond a limit, trading stops. Some exchanges use **circuit breakers** that halt trading briefly when a limit is hit.

Common exam traps

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- A margin call restores the account to the initial margin. The maintenance margin is only the trigger. On Day 2 of the crude oil example, restoring only the maintenance level would need $600 instead of the required $2,100.
- The settlement price is an average over a closing window. It is not the last trade of the day.

BOTTOM LINE

- A forward is a private, customized agreement that obligates both parties, with nothing paid at initiation; the long profits if the underlying's price at settlement is above the forward price, and the short profits if it is below.
- A futures contract is a standardized, exchange-traded forward whose gains and losses are settled daily and whose performance the clearinghouse guarantees, which keeps counterparty credit risk minimal.
- Futures margin is collateral posted by both buyer and seller, not a loan; initial margin must be deposited before a trade, and maintenance margin is the minimum balance the account must keep.
- When a futures account falls below the maintenance margin, the margin call requires a deposit that restores it to the initial margin, or the position is closed out.
- The settlement price used for the daily mark-to-market is the average price of trades during a closing period, and price limits cap how far it may move from the previous day's settlement price.

QUICK CHECK

Question 1

Laszlo Bencze holds a long futures position. The initial margin on his account is $3,800 and the maintenance margin is $2,900. After today's mark-to-market, his account balance is $2,650. If Bencze wants to keep the position open, the deposit he must make is *closest* to:

A $250.

B $1,150.

C $2,900.

Answer: B

Because the balance ($2,650) has fallen below the maintenance margin ($2,900), Bencze receives a margin call. In futures trading the deposit must restore the account to the initial margin level, otherwise the position is closed out.

Balance $= \$2{,}650 < \$2{,}900$ maintenance margin $\Rightarrow$ margin call.

$$\text{Deposit} = \text{initial margin} - \text{current balance} = 3{,}800 - 2{,}650 = \$1{,}150$$

Why the other options are wrong

- **A.** $250 only restores the account to the maintenance margin $(2{,}900 - 2{,}650)$. That is the rule for equity margin accounts. Futures accounts must be restored to the initial margin.
- **C.** $2,900 is the maintenance margin itself. There is no rule requiring a deposit equal to the maintenance margin; the deposit is whatever brings the balance back to the initial margin.

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Key takeaway After a futures margin call, the account is topped up to the initial margin whenever the balance drops below the maintenance margin.

MODULE 71.2

Swaps and Options

LOS 71.a — Swaps, credit default swaps and options

Interest rate swaps

A **swap** is an agreement to exchange a series of payments on several settlement dates. In a plain **fixed-for-floating interest rate swap**, one party pays a fixed rate (the **swap rate**) and the other pays a floating **market reference rate (MRR)**, both on the same **notional principal**. On each date the two amounts are netted, and only the **net payment** is made, by the party that owes more. The floating rate for a period is set at the start of that period and paid at its end, so the first payment is known when the swap begins.

The swap rate is chosen to give the swap a value of zero when it starts. A swap is equivalent to a **series of forward contracts** on the floating rate. Each forward settles on one swap settlement date, with the fixed rate playing the role of the forward price. Interest rate forwards often settle at the start of the period, paying the present value of the end-of-period swap amount. Swaps trade in dealer markets, so counterparty credit risk exists unless a central counterparty is used; a central counterparty may also require the parties to post margin and make mark-to-market payments.

Example. A 1-year swap has a notional principal of $40 million, quarterly payments and a fixed rate of 2.50%, so every fixed payment is $40{,}000{,}000 \times 0.025/4 = \$250{,}000$. The MRRs that set the four floating payments turn out as follows:

Quarter	MRR set at start of quarter	Floating payment (end of quarter)	Net payment
1	2.10% (known at inception)	$40{,}000{,}000 \times 0.021/4 = \$210{,}000$	Fixed-rate payer pays $40,000
2	2.30%	$230,000	Fixed-rate payer pays $20,000
3	2.80%	$280,000	Fixed-rate payer receives $30,000
4	3.10%	$310,000	Fixed-rate payer receives $60,000

The net payment changes direction once MRR rises above the fixed rate.

A firm with a floating-rate loan can pay fixed and receive floating in a swap to convert its liability to a fixed rate. The floating receipts cover the loan's interest, so the swap hedges the uncertainty about future rates. As rate expectations change after inception, the swap takes on positive value for one side and negative value for the other.

Credit default swaps (CDS)

In a **credit default swap**, the **protection buyer** makes a series of fixed periodic payments to the **protection seller**. The seller pays only if the reference security suffers a **credit event** (default, bankruptcy, involuntary restructuring), and the payment covers the resulting drop in the security's value. The periodic payments reflect the credit spread, which compensates for the expected loss from default (probability of default times the loss if default occurs). The buyer is effectively buying insurance against default; a holder of the risky bond can hedge its default risk this way. The seller earns the credit spread and takes on risk similar to owning the reference bond.

Options

- A **call option** gives its buyer (**holder**) the **right, but not the obligation**, to buy the underlying at the **exercise price** (**strike price**) until or at expiration. The **call writer** (seller) must sell if the holder exercises.
- A **put option** gives its holder the right to sell the underlying at the exercise price, with no obligation to do so. The **put writer** must buy if the holder exercises.
- Options have a cost at initiation: the buyer pays the **option premium** to the writer.
- A **European option** can be exercised only at expiration.

LOS 71.b — Value at expiration and profit for long and short option positions

With S_T the underlying price at expiration and X the exercise price:

KEY CONCEPT

$$c_T = \max(0,\ S_T - X) \qquad p_T = \max(0,\ X - S_T)$$

$$\text{Profit to buyer} = \text{value at expiration} - \text{premium paid}$$

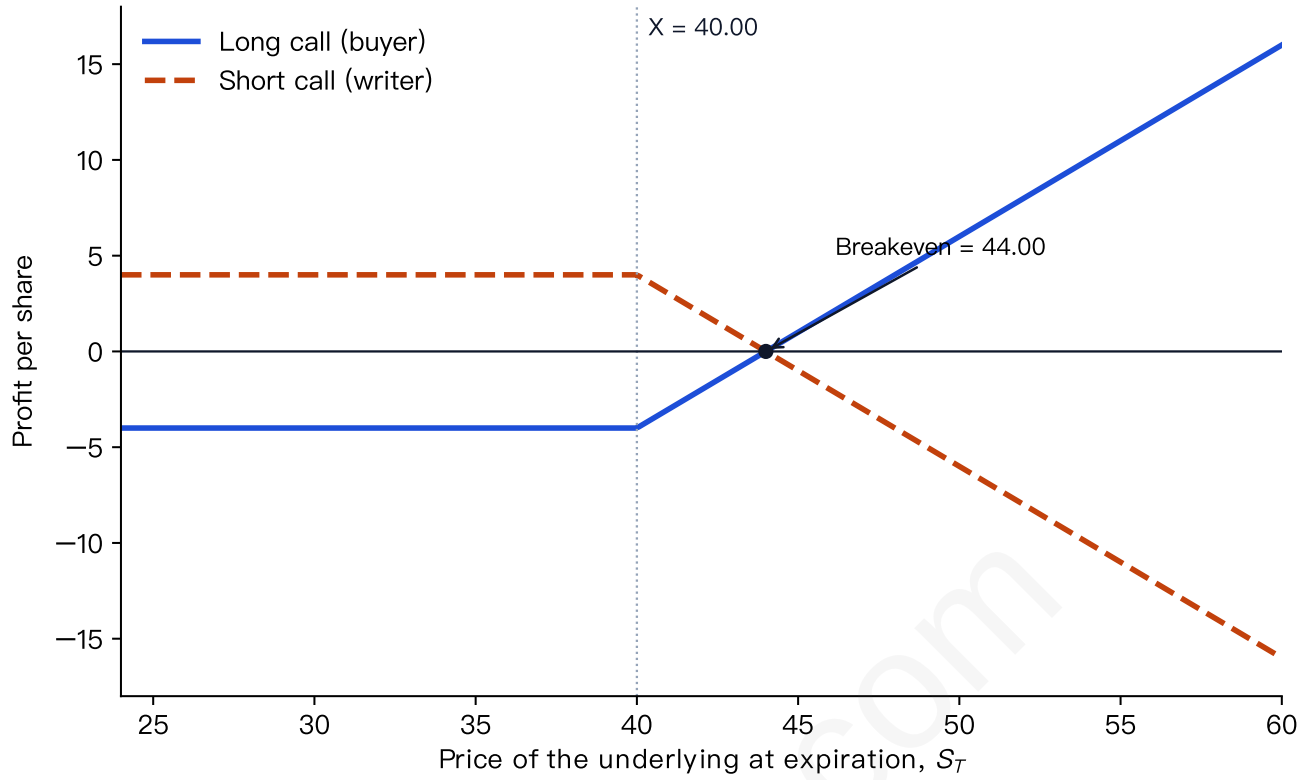
$$\text{Profit to writer} = \text{premium received} - \text{value at expiration}$$

An option's value can never be negative, because the holder simply lets an out-of-the-money option expire. A call is **in the money** when $S > X$ (by $S - X$), and a put is in the money when $S < X$ (by $X - S$). Otherwise the option is **out of the money**, or at the money when $S = X$.

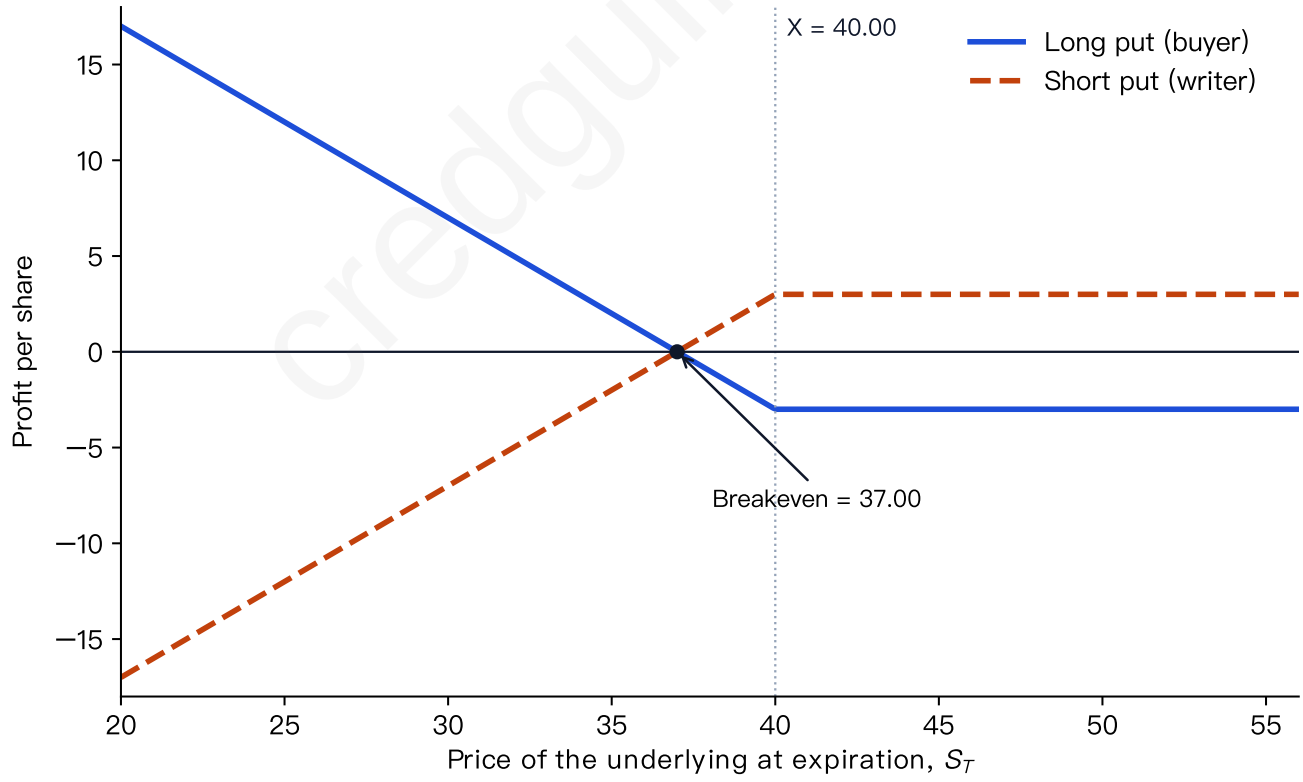
KEY CONCEPT

Position	Max gain	Max loss	Breakeven S_T	Exposure to underlying
Long call	Unlimited	Premium	$X + c_0$	Long
Short call	Premium	Unlimited	$X + c_0$	Short
Long put	$X - p_0$	Premium	$X - p_0$	Short
Short put	Premium	$X - p_0$	$X - p_0$	Long

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Profit at expiration of a long and a short call (exercise price 40, premium 4)



Profit at expiration of a long and a short put (exercise price 40, premium 3)

Example. A call and a put both have $X = \$40$. The call premium is \$4 and the put premium is \$3.

S_T	Long call	Short call	Long put	Short put
\$47	$7 - 4 = +3$	-3	$0 - 3 = -3$	$+3$
\$34	$0 - 4 = -4$	$+4$	$6 - 3 = +3$	-3

Options are a **zero-sum game**: the writer's profit is always the buyer's loss, and vice versa. The call holder exercises whenever $S_T > X$, even if the premium is not fully recovered, because exercising reduces the loss.

LOS 71.c — Forward commitments versus contingent claims

- A **forward commitment** is a legally binding promise to perform an action in the future (buy, sell, or exchange payments). Forwards, futures and most swaps are forward commitments. The underlying can be an asset, an index, a portfolio or a rate.
- A **contingent claim** is a claim whose payoff is contingent on whether a particular event happens. Options fall into this category (the event being the underlying finishing above or below the strike), and so do credit default swaps (the event being a credit event).

KEY CONCEPT

	Forward commitment	Contingent claim
Obligation	Both parties obligated	Holder has a right; writer is obligated if exercised
Cost at initiation	Usually zero value	Buyer pays a premium
Payoff profile	Linear (symmetric)	One-sided (asymmetric)
Examples	Forwards, futures, interest rate swaps	Calls, puts, credit default swaps

Common exam traps

- A put holder has the right to sell; a put writer has the obligation to buy. A call holder has the right to buy and never an obligation.
- A CDS is called a swap, but it is a contingent claim.
- An option is a contingent claim whatever its underlying. An option on a futures contract is a contingent claim even though the futures contract itself is a forward commitment.
- Profit is value minus premium. When a question asks only for the value at expiration, the premium is not subtracted. At $S_T = \$47$ in the example, the call's value at expiration is \$7; the \$3 figure, after the \$4 premium, is its profit.
- The breakeven for both call parties is $X + c_0$; for both put parties it is $X - p_0$.
- Exposure follows the direction that profits, not the option type. A short call has short exposure and a short put has long exposure.

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- Both sides of a call break even at $X + c_0$ and both sides of a put at $X - p_0$, so the writer's breakeven needs no separate calculation.

BOTTOM LINE

- In a plain fixed-for-floating interest rate swap, the fixed and floating amounts on the same notional principal are netted and only the party that owes more pays; each floating rate is set at the start of its period and paid at the end.
- An interest rate swap works like a set of forwards on the floating rate, one for each settlement date with the fixed rate as the forward price, and the swap rate is chosen to give the swap zero value when it starts.
- In a credit default swap, the protection buyer makes fixed periodic payments and the protection seller pays only if a credit event occurs, covering the drop in the reference security's value.
- $c_T = \max(0,\ S_T - X)$ and $p_T = \max(0,\ X - S_T)$; the buyer's profit is the value at expiration minus the premium paid, and the writer's profit is the reverse.
- A long call and a short put have long exposure to the underlying, while a long put and a short call have short exposure.
- A forward commitment (forwards, futures, most swaps) obligates both parties and has a linear payoff, while a contingent claim (options, credit default swaps) gives the holder a right, costs the buyer a premium and has a one-sided payoff.

QUICK CHECK

Question 2

Bergen Pension Fund owns bonds issued by Holloway Freight. It enters a credit default swap as the buyer of credit protection on those bonds. Under the swap, the fund:

A issues a note whose payments come from the cash flows of the Holloway bonds.

B exchanges the total return on the Holloway bonds for a fixed or floating rate.

C makes a series of periodic payments to the protection seller.

Answer: C

In a credit default swap the protection buyer makes a series of fixed periodic payments to the protection seller. In return, the seller pays an amount that offsets the loss in value if the reference bond suffers a credit event such as default. The fund is effectively buying insurance on its Holloway bonds.

Why the other options are wrong

- **A.** A note funded by cash flows from an underlying bond is not a CDS. The CDS protection buyer issues nothing; it pays periodic premiums and receives a payment only after a credit event.
- **B.** Exchanging the bond's total return for a fixed or floating rate describes a different credit derivative. In a CDS the buyer pays fixed periodic premiums and receives a payment only after a credit event.

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Key takeaway CDS: protection buyer pays periodic premiums; protection seller pays only if a credit event occurs.

Practice questions

Question 3

Tobias Grant buys a call option on Wexford Bio shares with an exercise price of $36.00 for a premium of $3.40, when the shares trade at $38.90. At expiration the shares are priced at $37.10. Grant's net profit or loss on the call is *closest* to:

A −$3.40.

B −$2.30.

C $2.30.

Question 4

A trainee lists features of forward commitments. Which item on her list is *least accurate*?

A The underlying can be a stock index or a portfolio.

B Each party is legally obligated to act at a future date.

C Either party can walk away at settlement without penalty.

Answers: 3. B · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/forward-vs-futures/. The full bank has 31 questions on this reading.

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DERIVATIVES · READING 72

Derivative Benefits, Risks, and Issuer and Investor Uses

What you'll learn

LOS 72.a Describe the benefits (risk transfer, information discovery, operational advantages, market efficiency) and risks (implicit leverage, basis, liquidity, counterparty and systemic risk) of derivatives.

LOS 72.b Compare how issuers (cash flow, fair value and net investment hedges under hedge accounting) and investors use derivatives.

MODULE 72.1

Uses, Benefits, and Risks of Derivatives

This reading weighs the benefits of derivatives against their risks and shows how issuers and investors use them. A candidate must be able to explain the risk-management, information and operational benefits of derivatives, measure the implicit leverage of a margined position, identify basis, liquidity, counterparty and systemic risk, and classify a corporate hedge as a cash flow, fair value or net investment hedge.

LOS 72.a — Benefits and risks of derivatives

Benefits

1. **Changing, transferring and managing risk.** Without buying or selling any cash market security, a user can raise or lower exposure to an index, hedge the currency risk of expected receipts or payments, or turn a floating-rate liability into a fixed-rate one. Derivatives also create exposures unavailable in cash markets. Buying puts on a stock puts a floor under its sale price, and buying calls gives upside exposure with the loss limited to the premium.
2. **Information discovery.** Option prices depend on observable inputs (price of the underlying, exercise price, time to expiration, interest rates) and one unobservable input, expected volatility. Option prices therefore reveal the market's **implied volatility**. Futures and forward prices indicate expected prices of the underlying, and interest rate futures across maturities reveal expected future rates, even the expected number of central bank rate moves.
3. **Operational advantages.**
   - **Ease of short selling**: selling a forward or futures contract is simpler than borrowing an asset to short it, especially where borrowing is hard or short sales are restricted.
   - **Lower transaction costs**: commodity exposure needs no transport, storage or insurance, and entering a swap costs far less than buying back a floating-rate note and replacing it with a new fixed-rate issue.
   - **Greater leverage**: little cash is needed for a large exposure.
   - **Greater liquidity**: the small cash requirement makes very large trades easy.
4. **Improved market efficiency.** Low costs, leverage, liquidity and easy shorting make it cheaper to exploit mispricing, which pushes prices toward fair value.

Risks

Leverage appears on both lists. The small cash requirement behind the greater leverage and liquidity listed as benefits is also the source of the implicit leverage risk in the table below.

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KEY CONCEPT

Risk	What it means	Example
Implicit leverage	A small cash deposit controls a large exposure, so small price moves cause large percentage changes in the deposit; opaque **structured securities** may hide the risk	Margin of 8% of contract value: a 2% adverse price move wipes out $2/8 = 25\%$ of the margin
Basis risk	The derivative's underlying or its settlement date does not match the position being hedged, so the hedge is imperfect	Hedging a 40-stock portfolio with index futures; hedging a harvest due on one date with futures settling weeks later
Liquidity risk	Cash flows on the hedge do not match the cash flows of the hedged position	A producer short futures must meet margin calls when prices rise, before the higher-priced crop is sold
Counterparty credit risk	The other side may fail to pay what it owes	Option **buyer** is exposed (it may be owed the payoff); option **writer** is not (it already has the premium); both sides of a forward can be exposed
Systemic risk	Excessive speculation can destabilize markets and institutions	Addressed by regulation such as mandatory central clearing of swaps

With margin equal to a fraction m of contract value, a price change of $\Delta P\%$ changes the margin deposit by approximately

KEY CONCEPT

$$\%\Delta\text{margin} = \frac{\Delta P\%}{m} \qquad \text{leverage} = \frac{1}{m}$$

Margins of 3% to 12% imply leverage of roughly 33:1 to 8:1.

Futures carry three protections against counterparty risk: initial margin, daily mark-to-market, and the clearinghouse guarantee. Forwards may have none of these protections unless the contract provides for collateral or central clearing.

LOS 72.b — How issuers and investors use derivatives

Issuers (corporations)

Nonfinancial companies, called issuers in this context, use derivatives to manage the effect of prices, rates and currencies on asset and liability values and on earnings. Where allowed, **hedge accounting** lets a company recognize gains and losses on a qualifying hedge in the same period as the offsetting change in the hedged item. Hedges are classified by purpose:

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KEY CONCEPT

Hedge type	Purpose	Typical examples
Cash flow hedge	Reduces variability of future cash flows	Currency forwards on expected foreign-currency receipts; a swap converting a **floating-rate** liability to fixed rate (interest payments become certain)
Fair value hedge	Offsets changes in the balance-sheet value of an asset or liability	Swap in which the firm pays floating/receives fixed to stabilize the fair value of **fixed-rate** debt (in effect a floating-rate liability with much lower duration); forwards sold against inventory carried at market value
Net investment hedge	Reduces volatility of the reported (domestic-currency) value of a **foreign subsidiary's equity**	Currency forwards or futures on the subsidiary's functional currency

The class depends on what is being hedged. Forward contracts appear in all three rows. An interest rate swap can fall in either of the first two, depending on the debt it hedges and which side of the swap the firm takes.

Example. A food company with an outstanding floating-rate bank loan enters a swap paying 4.1% fixed and receiving MRR. Its interest cash flows become fixed, so this is a cash flow hedge. A different company that reports its 6% fixed-rate bonds at fair value enters a swap paying MRR and receiving 6% fixed. When rates change, the gain or loss on the swap offsets the change in the bonds' fair value, so this is a fair value hedge.

Investors

Investors use forward commitments and contingent claims to hedge, modify or increase exposure. For example, they can:

- buy commodity (e.g., silver) forwards to gain price exposure with little or no initial cash;
- lengthen a bond portfolio's duration by entering a swap as fixed-rate receiver and floating-rate payer, which is like issuing floating-rate debt to buy a fixed-rate bond;
- raise equity exposure temporarily by buying index futures, cut it by selling index futures, or protect the downside while keeping the upside by buying index puts.

Common exam traps

- Ease of shorting and implied volatility information are benefits of derivatives.
- The writer of an option bears no counterparty risk once the premium is received; the buyer does.
- Basis risk comes from a mismatch in the underlying or in the dates. Volatility alone does not create it.

BOTTOM LINE

- Derivatives let users change, transfer and manage risk without buying or selling cash market securities, and they create exposures, such as a floor under a sale price, that cash markets do not offer.

- Option prices reveal the market's implied volatility, and futures and forward prices indicate the expected prices of their underlyings, including expected future interest rates.
- Easier short selling, lower transaction costs, greater leverage and greater liquidity make it cheaper to exploit mispricing, which improves market efficiency.
- With margin equal to a fraction m of contract value, a price change of $\Delta P\%$ changes the margin deposit by about $\Delta P\%/m$, so leverage is $1/m$.
- Basis risk arises when the derivative's underlying or settlement date does not match the hedged position, and liquidity risk arises when the hedge's cash flows do not match those of the hedged position.
- An option buyer bears counterparty credit risk while the writer, who already holds the premium, does not, and both sides of a forward can be exposed.
- A cash flow hedge reduces the variability of future cash flows, a fair value hedge offsets changes in the balance-sheet value of an asset or liability, and a net investment hedge reduces the volatility of the reported value of a foreign subsidiary's equity.
- A firm with floating-rate debt that pays fixed in a swap has a cash flow hedge, while a firm with fixed-rate debt carried at fair value that receives fixed and pays floating has a fair value hedge.

QUICK CHECK

Question 1

A manager wants short exposure to a commodity but finds that borrowing physical inventory for a short sale is impractical. Which feature of derivatives is *most useful* to the manager?

A They eliminate basis risk whenever a derivative is used as a hedge.

B Short exposure is generally as easy to establish as long exposure.

C They prevent implicit leverage from magnifying gains and losses on a position.

Answer: B

Selling a forward or futures contract usually creates short exposure without borrowing and selling the physical commodity. This operational advantage is especially important for assets that are difficult to short in the cash market.

Why the other options are wrong

- **A.** A derivatives hedge can still have basis risk when its underlying or settlement date does not match the exposure.
- **C.** Derivatives often provide substantial implicit leverage; they do not remove it.

Key takeaway Derivatives can make short exposure practical when a cash market short sale is difficult or impossible.

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Practice questions

Question 2

Which of the following hedges is *most* exposed to basis risk?

A A gold refiner sells gold forwards that match the quantity, grade and delivery date of a sale it has already agreed.

B A Canadian importer buys a US dollar forward for the amount and settlement date of a dollar invoice it must pay.

C A manager of 40 regional bank stocks sells forwards on a broad national equity index equal to the portfolio's value.

Question 3

Marlowe Industries carries its fixed-rate notes payable at fair value and applies hedge accounting. To reduce swings in the reported value of the notes as interest rates change, it enters an interest rate swap in which it pays a floating rate and receives a fixed rate. This hedge is best classified as a:

A fair value hedge.

B cash flow hedge.

C net investment hedge.

Answers: 2. C · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/derivative-benefits-risks-and-issuer-and-investor-uses/. The full bank has 11 questions on this reading.

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DERIVATIVES · READING 73

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

What you'll learn

LOS 73.a Explain how arbitrage and replication determine derivative prices, and identify the arbitrage when a forward is mispriced.

LOS 73.b Distinguish the forward price from the expected future spot price, and explain how the cost of carry (financing, holding costs and benefits) sets the no-arbitrage forward price, including with continuous compounding.

MODULE 73.1

Arbitrage, Replication, and Carrying Costs

This reading explains how derivatives are priced by arbitrage and replication, and how the costs and benefits of holding the underlying set a forward price. A candidate must be able to compute a no-arbitrage forward price with discrete or continuous compounding, including carrying costs, benefits and currency interest rates, and describe the trades that exploit a mispriced forward.

LOS 73.a — Arbitrage and replication in derivative pricing

Risky assets are usually valued as the present value of expected cash flows at a risk-adjusted rate. Derivatives are instead priced with a **no-arbitrage condition**.

- **Arbitrage** means simultaneously buying an asset (or portfolio) and selling, at a higher price, another with identical future payoffs in every state. It locks in a risk-free gain with no net investment.
- Arbitrage opportunities may be rare, but when they appear they are exploited rapidly. This forces assets with identical payoffs to the same price (the **law of one price**). Small mispricings can persist when the gain is smaller than the transaction costs of capturing it.
- **Replication** means building a portfolio of cash market positions (the underlying plus risk-free borrowing or lending) that has the same payoff as the derivative for every possible value of the underlying. The derivative must cost the same as its replicating portfolio. A position that combines the derivative with the opposite replicating position has a certain payoff, so the relevant discount rate is the **risk-free rate**, and no assumption about investors' risk preferences is needed.

Replicating forwards (underlying with no costs or benefits of holding)

Position	Replicating portfolio	Payoff at T
Long forward at $F_0(T)$	Borrow S_0 at R_f and buy the underlying	$S_T - S_0(1+R_f)^T$
Short forward at $F_0(T)$	Short the underlying and lend the proceeds at R_f	$S_0(1+R_f)^T - S_T$

Matching these payoffs with the forward's payoff $S_T - F_0(T)$ gives the **no-arbitrage forward price**, which is the **future value of the spot price** at the risk-free rate:

KEY CONCEPT

$$F_0(T) = S_0(1 + R_f)^T$$

Owning the asset and selling it forward locks in $F_0(T)$: the combined position has a certain payoff, so it must earn the risk-free rate.

Exploiting a mispriced forward

KEY CONCEPT

Situation	Forward is...	Arbitrage	Profit at T
$F_0(T) > S_0(1 + R_f)^T$	Too high	**Sell the forward**, borrow S_0 and **buy the underlying**	$F_0(T) - S_0(1 + R_f)^T$
$F_0(T) < S_0(1 + R_f)^T$	Too low	**Buy the forward**, **short the underlying** and invest the proceeds	$S_0(1 + R_f)^T - F_0(T)$

Arbitrage trading pushes the forward price back toward its no-arbitrage level. The overpriced case can also be read as a return comparison: buying the asset at S_0 and delivering it at $F_0(T)$ earns more than the risk-free rate, so borrowing at R_f to fund the purchase leaves a riskless surplus.

Example. A share paying no dividends trades at \$50, and the 1-year risk-free rate is 6%. The no-arbitrage forward price is $50 \times 1.06 = \$53.00$. If a dealer quotes \$54.00, sell the forward, borrow \$50 and buy the share. In one year, deliver the share for \$54 and repay $50 \times 1.06 = \$53$, a riskless profit of \$1.00 with no initial outlay. If the quote is instead \$52.20, the forward is too cheap: buy the forward, short the share and lend the \$50 proceeds at 6%. In one year, collect \$53.00 from the loan, pay \$52.20 for the share under the forward and return it to close the short, a riskless profit of \$0.80.

LOS 73.b — Spot versus expected future price, and the cost of carry

The no-arbitrage forward price is not a forecast of the expected future spot price. It is the cost of buying the underlying today and carrying it to T: the financing cost at R_f, plus other costs of holding, minus the benefits of holding.

- **Costs of holding** arise mostly for commodities: **storage**, **insurance** and spoilage. Their present value is $PV_0(\text{cost})$. For financial assets these costs are negligible.
- **Benefits of holding** may be monetary (**dividends**, **coupon interest**) or non-monetary. The non-monetary benefit is the **convenience yield**, the advantage of physically holding a commodity, such as being able to sell into a temporary shortage, especially when the asset is hard to short. Their present value is $PV_0(\text{benefit})$.

KEY CONCEPT

$$F_0(T) = [S_0 + PV_0(\text{cost}) - PV_0(\text{benefit})]\,(1 + R_f)^T$$

Equivalently, the forward price is $S_0(1+R_f)^T$ plus the future value of the holding costs minus the future value of the benefits. A $2 dividend paid at settlement, for instance, lowers the forward price by exactly $2.

- With net costs (costs greater than benefits), the forward price is above $S_0(1+R_f)^T$; with net benefits, it is below.
- Carrying costs and benefits affect the forward price at initiation and the forward's value during its life. At expiration the value is simply $S_T - F_0(T)$.

A stated annual rate r with continuous compounding corresponds to an effective annual rate of $e^r - 1$. Over T years, $FV = S\,e^{rT}$ and $PV = S\,e^{-rT}$. With continuously compounded annual rates (risk-free r, cost c, and benefit b such as a dividend yield):

$$F_0(T) = S_0\,e^{rT} \qquad F_0(T) = S_0\,e^{(r+c)T} \qquad F_0(T) = S_0\,e^{(r+c-b)T}$$

The first applies with no costs or benefits, the second with costs only, and the third with both.

Compared with the spot price itself, what matters is the total carry, financing included: $F_0(T) = S_0$ when $r + c = b$, $F_0(T) > S_0$ when $r + c > b$, and $F_0(T) < S_0$ only when the benefits exceed financing plus storage costs.

Examples.

- Discrete: spot 200, $PV_0(\text{cost}) = 3$, $PV_0(\text{benefit}) = 1$, $R_f = 5\%$, $T = 0.5$. Then $F_0 = (200 + 3 - 1)(1.05)^{0.5} = 202 \times 1.024695 = 206.99$.
- Continuous: an index at 5,000, dividend yield 2%, risk-free rate 4%, $T = 0.5$. Then $F_0 = 5{,}000\,e^{(0.04-0.02)(0.5)} = 5{,}000\,e^{0.01} = 5{,}050.25$.

Currency forwards

For an exchange rate quoted as price currency per unit of base currency, the no-arbitrage forward rate reflects the interest rate differential:

$$F = S_0 \times \left(\frac{1+R_{\text{price}}}{1+R_{\text{base}}}\right)^T \quad \text{or, continuously compounded,} \quad F = S_0\,e^{(r_{\text{price}}-r_{\text{base}})T}$$

Here R_{price} and R_{base} are annual effective risk-free rates and T is in years. For a one-year forward ($T = 1$), the discrete formula reduces to $S_0(1+R_{\text{price}})/(1+R_{\text{base}})$. If the rates are instead stated as returns over the forward's own horizon, they are used without the exponent.

The currency with the higher interest rate trades at a forward discount, which offsets its higher interest income. Otherwise, borrowing in one currency and investing in the other would earn an arbitrage profit. If the quoted forward rate (price/base) is above the no-arbitrage rate, borrow the price currency, convert it to the base currency at spot, invest at the base-currency rate and sell the base currency forward. If the quoted rate is below it, reverse every leg. Rates converted to continuous compounding ($r = \ln(1+R)$) give the same forward rate.

> *Example.* The spot rate is 1.2500 USD/GBP (US dollars per pound), and the 1-year risk-free rates are 4% in US dollars and 5% in pounds (annual effective). The 1-year no-arbitrage forward rate is $1.2500 \times 1.04/1.05 = 1.2381$ USD/GBP. The pound, the base currency, has the higher interest rate, so it buys fewer dollars forward than at spot. A dealer quote of 1.2450 would be too high: borrow dollars, buy pounds at spot, invest them at 5% and sell the pounds forward at 1.2450.

Change (other things equal)	Effect on no-arbitrage forward price
Higher risk-free rate	Increase
Higher storage or insurance costs	Increase
Higher dividends, coupons or convenience yield	Decrease
Longer time to settlement (positive net carry)	Increase

Common exam traps

- Convenience yield is a benefit, so it lowers the forward price even though it is non-monetary. Storage and insurance are costs. In the discrete example, adding the benefit of 1 as if it were a cost gives 209.04 instead of 206.99.
- Derivative pricing assumes that arbitrage is exploited quickly. It does not assume that investors are risk neutral or that arbitrage never occurs.

EXAM SHORTCUTS

- A cash benefit paid at settlement lowers the forward price by exactly its amount, so a $2 dividend paid at settlement needs no discounting or compounding.
- The currency with the higher interest rate trades at a forward discount, so the direction of the forward rate relative to spot can be read from the two interest rates before any calculation.

BOTTOM LINE

- Arbitrage locks in a risk-free gain with no net investment, so assets with identical payoffs in every state must sell for the same price (the law of one price).
- A derivative must cost the same as the portfolio of the underlying and risk-free borrowing or lending that replicates its payoff, so it is priced with the risk-free rate and no assumption about investors' risk preferences.
- For an underlying that has neither carrying costs nor benefits, the no-arbitrage forward price is the future value of the spot price, $F_0(T) = S_0(1 + R_f)^T$.
- If $F_0(T) > S_0(1 + R_f)^T$, sell the forward, borrow S_0 and buy the underlying; if $F_0(T) < S_0(1 + R_f)^T$, buy the forward, short the underlying and invest the proceeds.
- The forward price is the cost of buying the underlying and carrying it to settlement, $F_0(T) = [S_0 + PV_0(\text{cost}) - PV_0(\text{benefit})](1 + R_f)^T$, and it is not a forecast of the expected future spot price.
- With continuously compounded annual rates for the risk-free rate r, costs c and benefits b, $F_0(T) = S_0\, e^{(r+c-b)T}$.
- For an exchange rate quoted as price currency per unit of base currency, $F = S_0[(1 + R_{\text{price}})/(1 + R_{\text{base}})]^T$, and the higher-rate currency trades at a forward discount.
- Other things equal, a higher risk-free rate or higher storage and insurance costs raise the forward price, and higher dividends, coupons or convenience yield lower it.

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QUICK CHECK

Question 1

The no-arbitrage approach used to value derivatives relies on the assumption that:

A any arbitrage opportunities that appear are exploited quickly.

B investors are risk neutral.

C arbitrage opportunities never arise in actual markets.

Answer: A

Derivative valuation assumes that any arbitrage opportunity is quickly traded away when it appears. That trading drives assets with identical future cash flows to the same price. It does not require that arbitrage never happens, and it makes no assumption about investors' attitude to risk.

Why the other options are wrong

- **B.** No-arbitrage pricing discounts at the risk-free rate because a position combining the derivative with its replicating portfolio (one long, the other short) has a certain payoff. The result does not depend on investors' risk preferences, so no assumption that investors are risk neutral is needed.
- **C.** Arbitrage opportunities can and do arise (small ones may even persist when transaction costs exceed the gain); the assumption is that they are quickly eliminated.

Key takeaway No-arbitrage pricing assumes that arbitrage is competed away quickly, so identical payoffs have identical prices.

Practice questions

Question 2

A share of Halvorsen Robotics, which pays no dividends, trades at $80.00, and the 1-year risk-free rate is 4%. A dealer quotes a 1-year forward price on the share of $82.50. Assuming no transaction costs, which strategy earns an arbitrage profit?

A Buy the forward, sell the share short, and invest the proceeds at 4%.

B Sell the forward and borrow at 4% to buy the share.

C None, because the forward price is above the spot price.

Question 3

An analyst compares forwards on two commodities with the same spot price and the same time to settlement. Commodity P has no costs or benefits of holding. For commodity Q, storage and insurance costs exceed its convenience yield. Compared with the forward price on P, the no-arbitrage forward price on Q will be:

A Lower.

B Higher.

C The same.

Answers: 2. A · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/arbitrage-replication-and-the-cost-of-carry-in-pricing-derivatives/. The full bank has 12 questions on this reading.

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DERIVATIVES · READING 74

Pricing and Valuation of Forward Contracts and for an Underlying With Varying Maturities

What you'll learn

LOS 74.a Explain how the value and price of a forward contract are determined at initiation, during its life and at expiration, including holding costs and benefits.

LOS 74.b Explain how implied forward interest rates are derived from spot rates and how forward rate agreements (FRAs) are settled and used.

MODULE 74.1

Forward Contract Valuation

This reading separates the price of a forward from its value and applies forward pricing to interest rates through implied forward rates and forward rate agreements. A candidate must be able to value a forward at initiation, during its life and at expiration, compute implied forward rates from spot rates, and compute the settlement payment on an FRA.

LOS 74.a — Value and price of a forward: at initiation, during its life, at expiration

A forward has both a price and a value, and the two are different things:

- The **forward price** $F_0(T)$ is the price written into the contract at which the underlying will change hands at T. It is fixed at initiation and does not change.
- The **value** of the forward is what the position is worth to one party at a point in time. It is normally zero at initiation and then moves with the spot price.

For an underlying with no costs or benefits of holding, priced at its no-arbitrage level $F_0(T) = S_0(1 + R_f)^T$:

KEY CONCEPT

Time	Value of the long forward	Comment
Initiation $(t = 0)$	$V_0 = S_0 - F_0(T)(1 + R_f)^{-T} = 0$	Neither party pays to enter
During the life $(0 < t < T)$	$V_t = S_t - F_0(T)(1 + R_f)^{-(T-t)}$	Spot price minus the **present value of the forward price**
Expiration $(t = T)$	$V_T = S_T - F_0(T)$	Spot minus forward price

The value to the short is the negative of the value to the long, because one party's gain is the other's loss.

The long can lock in $V_t = S_t - PV(F_0)$ by shorting the asset at S_t and investing $F_0(T)(1 + R_f)^{-(T-t)}$ in a zero-coupon bond. At T, the bond pays $F_0(T)$, which buys the asset under the forward, and the asset is delivered to close the short. The position is fully offset, and the cash left over today is $S_t - F_0(T)(1 + R_f)^{-(T-t)}$.

With costs and benefits of holding (the general case):

KEY CONCEPT

$$V_t(T) = [S_t + PV_t(\text{cost}) - PV_t(\text{benefit})] - F_0(T)(1+R_f)^{-(T-t)}$$

Example. A 1-year forward on a non-dividend-paying asset was entered at $F_0 = 100$. Three months later, with 9 months remaining, the spot price is 104 and $R_f = 5\%$.

$$V_t = 104 - \frac{100}{1.05^{0.75}} = 104 - 96.41 = 7.59$$

The long has a gain of 7.59 and the short a loss of 7.59. Two common errors give different answers: $104 - 100 = 4.00$ forgets to discount the forward price, and $(104 - 100)/1.05^{0.75}$ wrongly discounts the spot price too.

WORKED EXAMPLE: VALUING A FORWARD ON A DIVIDEND-PAYING SHARE DURING ITS LIFE

Eight months ago an investor went long a 1-year forward on a share at a forward price of 62.40. Today the share trades at 65.10, a dividend of 1.20 will be paid in 2 months, before the forward settles, and the annual risk-free rate is 4.5%. Compute the value of the long forward today and its value to the short.

Step 1. Four months remain, so $T - t = 4/12$ year.

Step 2. Present value of the dividend, a benefit of holding the share: $1.20/1.045^{2/12} = 1.19$.

Step 3. Present value of the forward price: $62.40/1.045^{4/12} = 61.49$.

Step 4. $V_t = (65.10 - 1.19) - 61.49 = 2.42$.

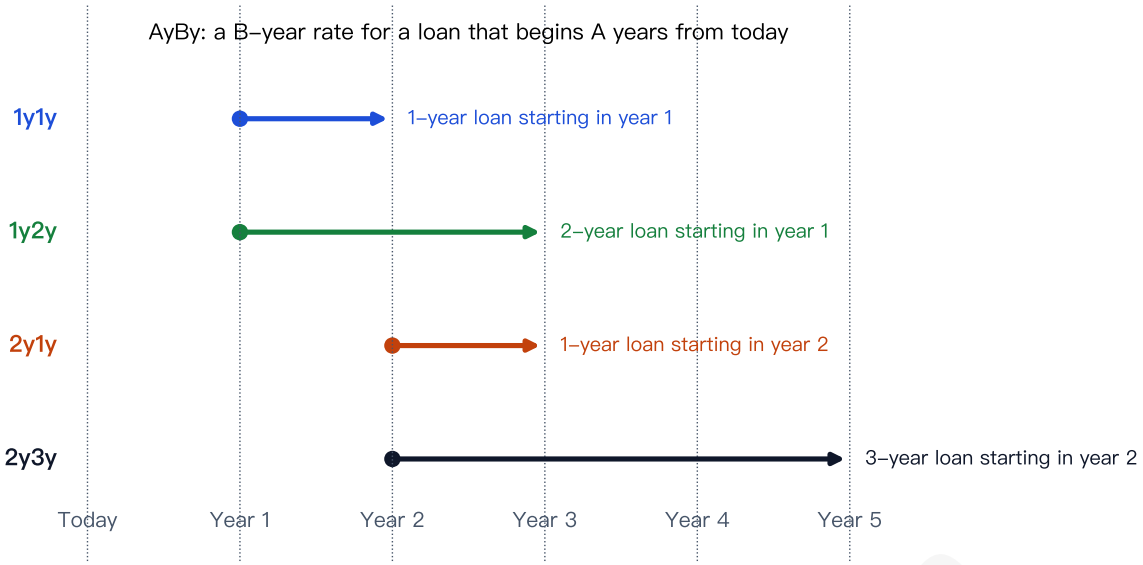
Result. The long forward is worth 2.42 today, and the short forward is worth −2.42.

LOS 74.b — Forward interest rates and forward rate agreements (FRAs)

Forward rate notation

A **forward rate** is an interest rate for a loan that starts in the future. AyBy (or $F_{A,B}$) is the rate for a B-year loan beginning A years from today. Thus **1y1y** is a 1-year loan starting in one year, and **2y3y** is a 3-year loan from year 2 to year 5. Money market versions use months: **3m6m** is a 6-month rate three months from now.

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Forward rate notation AyBy on a timeline

Implied (no-arbitrage) forward rates

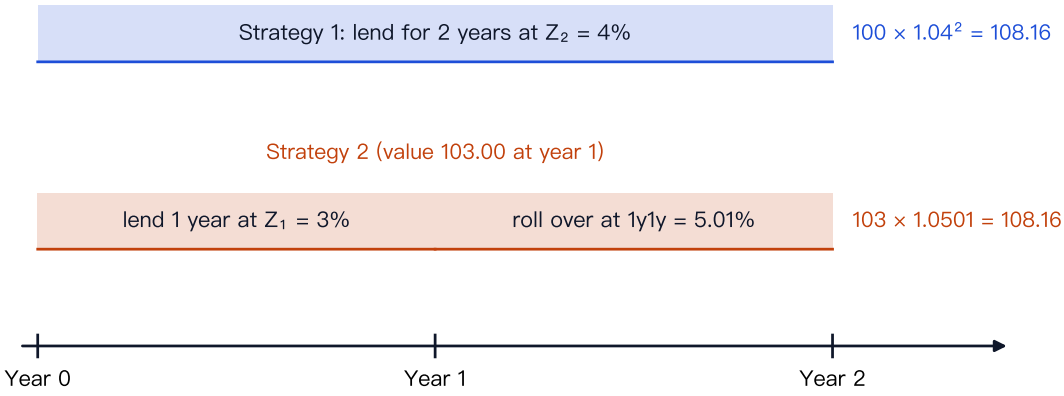
With **spot rates** (zero-coupon yields) Z_n, the **implied forward rate** makes two strategies earn the same: investing to the forward date and rolling over at the forward rate, or investing for the whole period at the longer spot rate.

KEY CONCEPT

$$(1+Z_{A+B})^{A+B} = (1+Z_A)^A\,(1+F_{A,B})^B \quad \text{so} \quad F_{A,B} = \left[\frac{(1+Z_{A+B})^{A+B}}{(1+Z_A)^A}\right]^{1/B} - 1$$

Example. With $Z_1 = 3\%$ and $Z_2 = 4\%$, $F_{1,1} = 1.04^2/1.03 - 1 = 5.01\%$. With zero rates only out to year 3, one can derive 1y1y, 1y2y and 2y1y, but no forward rate that ends after year 3.

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No arbitrage: both strategies end with the same amount, so $(1.04)^2 = (1.03)(1 + 1y1y)$

Implied 1y1y forward rate from spot rates of 3% (one year) and 4% (two years)

Money market rates use simple interest on a 360-day year, and the same logic applies with day-count adjustments (d = days in each period):

$$1 + Z_{\text{long}} \tfrac{d_{\text{long}}}{360} = \left(1 + Z_{\text{short}} \tfrac{d_{\text{short}}}{360}\right)\left(1 + F \tfrac{d_{\text{long}} - d_{\text{short}}}{360}\right)$$

Example. The 3-month MRR is 2.0% and the 9-month MRR is 2.6%. Then $1 + 0.026 \times \frac{270}{360} = 1.0195$ and $1 + 0.020 \times \frac{90}{360} = 1.0050$, so the implied 3m6m rate is $\left(\frac{1.0195}{1.0050} - 1\right) \times \frac{360}{180} = 2.89\%$.

Forward rate agreements (FRAs)

A **forward rate agreement (FRA)** lets a party lock in an interest rate today for borrowing or lending that begins in the future. It is a forward commitment, so both parties are obligated.

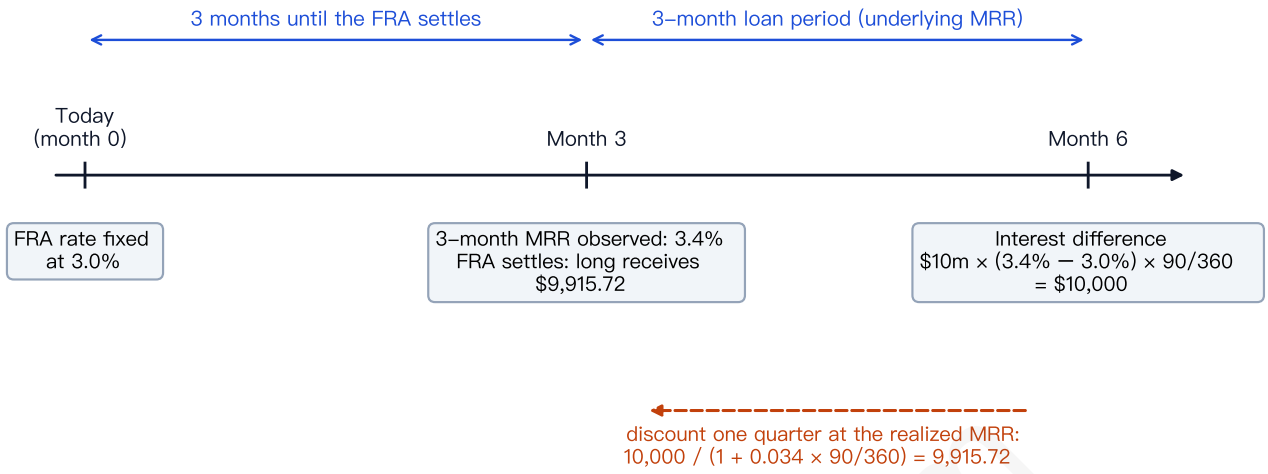
- The long (FRA buyer) pays the fixed contract rate and receives the market reference rate (**MRR**) on a notional principal. It gains if MRR rises. A future borrower goes long to hedge.
- The short (FRA seller) receives fixed and pays MRR. A future lender goes short.
- Only the net amount is paid. Settlement happens at the start of the loan period, so the interest difference, which relates to the end of the period, is discounted at the realized MRR:

$$\text{Payment to long} = \frac{\text{Notional} \times (\text{MRR} - \text{FRA rate}) \times \frac{\text{days}}{360}}{1 + \text{MRR} \times \frac{\text{days}}{360}}$$

Example. A 3-month forward on 3-month MRR (a 3m3m FRA) has a notional principal of $10 million and an FRA rate of 3.0%. Three months later, 3-month MRR is 3.4%.

1. Interest difference for the loan period, due at its end (month 6): $10{,}000{,}000 \times (0.034 - 0.030) \times 0.25 = \$10{,}000$.
2. Discount it to the settlement date (month 3) at the realized MRR: $10{,}000/(1 + 0.034 \times 0.25) = 10{,}000/1.0085 = \$9{,}915.72$.

3. MRR is above the FRA rate, so the long receives $9,915.72 from the short.



Timeline of a 3m3m FRA: notional $10 million, FRA rate 3.0%, realized 3-month MRR 3.4%

FRAs are used mainly by financial institutions to manage the rate sensitivity of their assets and liabilities. An FRA is equivalent to a **single-period swap**. A multi-period **interest rate swap** is a series of FRAs and is used mainly by issuers and investors.

Instrument	What it does
FRA	Locks in one future period's rate (obligation)
Interest rate swap	Exchanges fixed for floating over many periods
Interest rate option	Right, not obligation, to a rate (contingent claim)

Common exam traps

- The forward contract's value is usually zero at initiation; its price is the contract price.
- FRA settlement is discounted at the realized MRR, not at the FRA rate, and it is not paid undiscounted. In the 3m3m example, discounting at the 3.0% FRA rate gives $9,925.56 and paying undiscounted gives $10,000, instead of $9,915.72.
- In AyBy, the first number is when the loan starts and the second is how long it lasts.

BOTTOM LINE

- The forward price is written into the contract at initiation and does not change, while the forward's value is normally zero at initiation and then moves with the spot price.
- For an underlying with no costs or benefits of holding, a long forward is worth $V_t = S_t - F_0(T)(1 + R_f)^{-(T-t)}$ during its life and $S_T - F_0(T)$ at expiration, and the short's value is the negative of the long's.
- With costs and benefits of holding, $V_t(T) = [S_t + PV_t(\text{cost}) - PV_t(\text{benefit})] - F_0(T)(1 + R_f)^{-(T-t)}$.

- In AyBy notation, the first number is when the loan starts and the second is how long it lasts, so 2y3y is a 3-year loan from year 2 to year 5.
- $F_{A,B} = [(1 + Z_{A+B})^{A+B} / (1 + Z_A)^A]^{1/B} - 1$, the rate that makes rolling over at the forward rate earn the same as investing at the longer spot rate.
- The long in an FRA pays the fixed FRA rate and receives MRR, so it gains when MRR rises; a future borrower hedges by going long and a future lender by going short.
- The FRA payment to the long is $\text{Notional} \times (\text{MRR} - \text{FRA rate}) \times \frac{\text{days}}{360}$ discounted at $1 + \text{MRR} \times \frac{\text{days}}{360}$, because it is paid at the start of the loan period.
- A multi-period interest rate swap is a series of FRAs, so each FRA amounts to a swap with only one period.

QUICK CHECK

Question 1

Eight months ago, Selma Varga entered a 1-year long forward contract on a share that has no dividends or other costs or benefits of holding, at a forward price of $55.00. The share now trades at $57.00 and the annual risk-free rate is 3%. The value of Varga's forward position is *closest* to:

A $1.98.

B $2.00.

C $2.54.

Answer: C

For an underlying that is costless to hold and pays nothing, a long forward's value before expiration is the spot price less the present value of the forward price. Four months remain, so the forward price is discounted for 4/12 of a year.

$$V_t = S_t - \frac{F_0(T)}{(1 + R_f)^{T-t}} = 57.00 - \frac{55.00}{1.03^{4/12}} = 57.00 - \frac{55.00}{1.009902} = 57.00 - 54.46 = \$2.54$$

Why the other options are wrong

- **A.** $1.98 is the present value of the difference, $(57 - 55)/1.03^{4/12}$. It wrongly discounts the spot price as well as the forward price.
- **B.** $2.00 is the undiscounted difference between spot and forward prices. That would be the value only at expiration.

Key takeaway Discount only the forward price, for the time remaining $(T - t)$.

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Practice questions

Question 2

Annual-pay zero-coupon yields are 2.4% for 1 year, 2.8% for 2 years, and 3.1% for 3 years. The implied 1-year forward 2-year rate (1y2y), stated as an annual rate, is *closest* to:

A 3.45%.

B 3.70%.

C 7.02%.

Question 3

A company expects to invest surplus cash for three months, starting four months from today. It wants to lock in the return on that future investment with an FRA. The company should enter the FRA as the party that:

A pays fixed and receives MRR.

B receives fixed and pays MRR.

C receives both fixed and MRR.

Answers: 2. A · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/pricing-and-valuation-of-forward-contracts-and-for-an-underlying-with-varying-maturities/. The full bank has 9 questions on this reading.

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DERIVATIVES · READING 75

Pricing and Valuation of Futures Contracts

What you'll learn

LOS 75.a Compare the price and value of forwards and futures over their lives, and interpret interest rate futures quotes and basis point value.

LOS 75.b Explain why forward and futures prices differ (daily settlement and the correlation of interest rates with futures prices) and the convexity bias of FRAs versus interest rate futures.

MODULE 75.1

Futures Valuation

This reading compares the pricing and valuation of forwards and futures, including interest rate futures, and explains why their prices can differ. A candidate must be able to describe how daily settlement changes a futures contract's price and value, compute an interest rate futures price and its basis point value, and relate the forward–futures price difference to the correlation between interest rates and futures prices and to convexity bias.

LOS 75.a — Value and price of forwards versus futures

Forwards (no mark-to-market). The forward price stays fixed for the whole life of the contract. The forward's value changes as the spot price moves, and the entire gain or loss is paid only at settlement, as the difference between the spot price and the unchanged forward price.

Futures (daily mark-to-market). At the end of each day, the gain or loss since the previous **settlement price** is credited to or debited from each margin account, and the contract is then treated as if it had been re-entered at the new settlement price. As a result:

- the futures price changes every day, because it is reset to the latest settlement price; and
- the value of the futures position goes back to zero after each daily settlement.

Example. A trader is long one futures contract on 100 units bought at 250.

Day	Settlement price	Mark-to-market cash flow (long)	Contract price after MTM	Value after MTM
0	250	—	250	0
1	253	$+3 \times 100 = +300$ credited	253	0
2	247	$-6 \times 100 = -600$ debited	247	0

Over the two days the long has received \$300 and paid \$600, a net $-\$300 = (247 - 250) \times 100$. A forward entered at 250 and settled at 247 would pay the same total, ignoring interest. What differs is the timing: futures gains and losses arrive day by day, and that timing is the source of the forward–futures price differences in LOS 75.b.

A forward that is marked to market daily, for example under a central clearing arrangement, behaves the same way.

Interest rate futures

Interest rate futures are the exchange-traded counterparts of **forward rate agreements (FRAs)**, but they are quoted on a price basis:

KEY CONCEPT

$$\text{futures price} = 100 - (100 \times \text{MRR}_{A,\,B-A})$$

A quote of 95.80 implies an MRR of 4.20%. The price moves opposite to the rate, so a long futures position gains when rates fall.

The **basis point value (BPV)** is the change in contract value for a 1 bp change in the rate:

KEY CONCEPT

$$\text{BPV} = \text{notional principal} \times \text{period (in years)} \times 0.0001$$

For a 6-month rate on \$2,000,000, $\text{BPV} = 2{,}000{,}000 \times 0.5 \times 0.0001 = \$100$ per basis point.

LOS 75.b — Why forward and futures prices can differ

The key pricing difference is the **daily settlement** of futures gains and losses. Gains above initial margin can be withdrawn and reinvested, and losses that take the account below the maintenance margin must be funded. Forwards usually generate no cash flows until settlement.

KEY CONCEPT

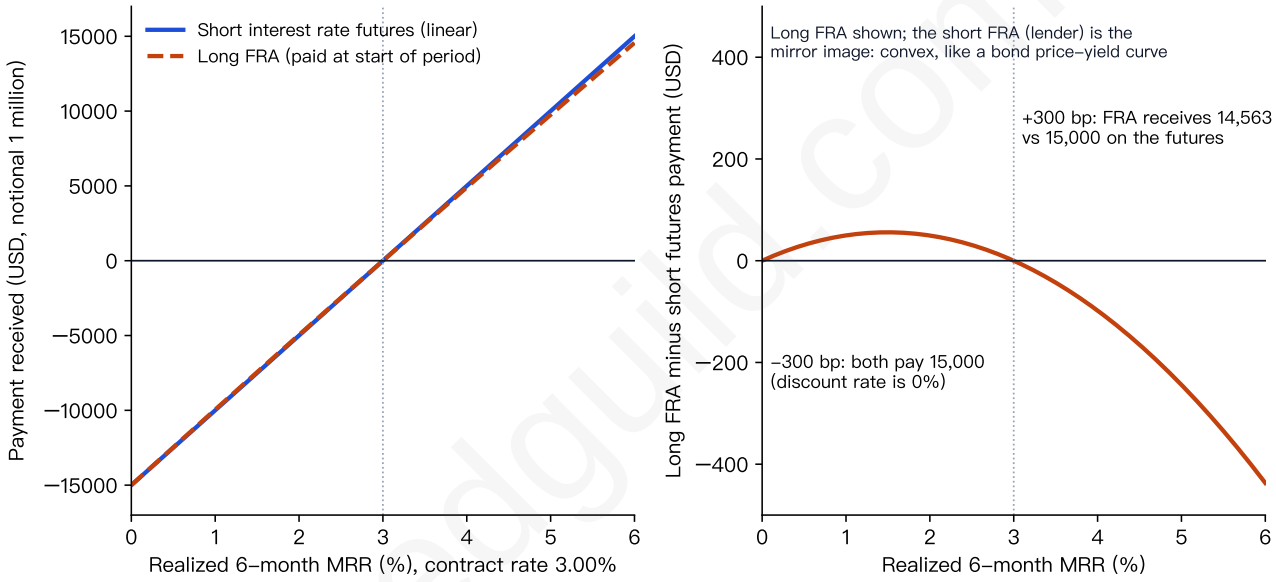
Correlation between interest rates and the futures price	Effect of daily settlement on a long futures	Which is more attractive to a long?	Price relationship
Positive	Cash received when rates are high (reinvested at high rates); cash paid when rates are low (cheap to finance)	**Futures**	Futures price > forward price
Negative	Cash paid when rates are high (costly); cash received when rates are low (poor reinvestment)	**Forward**	Futures price < forward price
Zero or constant rates	No systematic effect	Indifferent	Equal

These differences do not create arbitrage opportunities. They reflect the value of the mark-to-market cash flows. In practice the differences are usually too small to observe, because most forwards are short-dated, funds are available near the risk-free rate, and many dealers post margin.

Convexity bias in interest rate forwards versus futures

An FRA settles at the start of the loan period, so its payment is the interest difference discounted at the realized MRR. Interest rate futures pay an undiscounted, linear amount (BPV per basis point). A like-for-like comparison sets the FRA long, which gains when rates rise, against a short futures position, whose value also rises when rates rise because futures prices then fall.

- If rates rise, the FRA long receives the interest saving discounted at a higher realized rate, so it receives slightly less than the futures payment.
- If rates fall by the same amount, the FRA long pays an amount discounted at a lower rate, which is larger than what it would receive for the rise.
- The FRA payoff is therefore a nonlinear function of the rate: a rise in rates changes it by less than an equal fall does. From the side of the FRA short (the lender, whose position resembles a bond), the curve is convex, like a bond's price–yield curve. The long's payoff is the mirror image (concave). This curvature is the FRA's **convexity**. Futures payoffs are linear and have no convexity.



Settlement payment for a rise in MRR: long FRA (discounted, nonlinear) versus the equivalent short interest rate futures position (linear)

This **convexity bias** grows with the length of the period and can make forward and futures prices differ noticeably for longer-term rates.

Common exam traps

- For futures, both the price and the value change each day, and the value is reset to zero. For forwards, the price is fixed and the value fluctuates.
- Positive correlation between rates and futures prices favors the long futures; negative correlation favors the long forward.
- The FRA exhibits convexity; the futures contract is linear.

BOTTOM LINE

- A forward's price stays fixed and its whole gain or loss is paid at settlement, while a futures contract is marked to market daily, so its price resets to each settlement price and its value returns to zero after each daily settlement.
- Ignoring interest, the daily futures cash flows add up to the same total as the payoff of a forward entered at the same price; what differs is the timing.
- An interest rate futures price is quoted as $100 - (100 \times \text{MRR})$, so a long futures position gains when rates fall.
- $\text{BPV} = \text{notional principal} \times \text{period (in years)} \times 0.0001$ gives the change in an interest rate futures contract's value for a 1 bp change in the rate.
- A positive correlation between rates and futures prices makes futures more attractive to a long, so the futures price is above the forward price; a negative correlation makes the forward more attractive and the futures price lower; with zero correlation or constant rates the two are equal.
- Forward–futures price differences reflect the value of the mark-to-market cash flows and do not create arbitrage opportunities, and in practice they are usually too small to observe.
- Because an FRA's settlement is discounted at the realized MRR, its payoff is nonlinear in the rate while a futures payoff is linear; this convexity bias grows with the length of the period and can make prices differ noticeably for longer-term rates.

QUICK CHECK

Question 1

A trader buys one futures contract on 50 tonnes of cocoa at a price of 4,120 per tonne. At the end of the day, the settlement price is 4,180. After the daily mark-to-market, which statement is *most accurate*?

A The contract price and contract value both change.

B The contract price is unchanged, but the contract value changes.

C The contract price changes, and the contract value returns to zero.

Answer: C

The daily mark-to-market credits the trader's account with the day's gain and resets the futures price to the new settlement price. Because the gain has already been paid, a contract at the new price of 4,180 has a value of zero again.

Gain credited $= (4{,}180 - 4{,}120) \times 50 = 3{,}000$.

New contract price $= 4{,}180$; value after mark-to-market $= 0$.

Why the other options are wrong

- **A.** The price does change, but the change in value is only temporary. Once the gain is settled in cash, the value returns to zero.
- **B.** This describes a forward without mark-to-market, whose price is fixed while its value moves. A futures price is reset every day.

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Key takeaway A futures price resets daily and its value goes back to zero; a forward price is fixed while its value fluctuates.

Practice questions

Question 2

Analyst Farah Idris observes that when the price of a commodity rises, short-term interest rates also tend to rise. For an investor seeking long exposure, which statement is *most accurate* about a long futures contract compared with an otherwise equivalent forward that has no mark-to-market?

A The long futures contract is more attractive than the forward.

B The long forward contract is more attractive than the futures.

C Both contracts are equally attractive.

Question 3

A fund manager is long 8 three-month MRR futures contracts, each on a notional principal of $1,000,000. Over the day, the futures price falls from 96.40 to 96.25. The change in the value of the manager's position is:

A A loss of $12,000.

B A loss of $3,000.

C A gain of $3,000.

Answers: 2. A · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/pricing-and-valuation-of-futures-contracts/. The full bank has 7 questions on this reading.

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DERIVATIVES · READING 76

Interest Rate Swap

What you'll learn

LOS 76.a Describe how an interest rate swap is equivalent to a series of FRAs at the swap rate (values summing to zero at inception) and how it can be replicated with cash market positions.

LOS 76.b Contrast the price of a swap (the par swap rate, fixed at initiation) with its value (zero at inception, then changing with expected future rates).

MODULE 76.1

Swap Valuation

This reading treats an interest rate swap as a series of forward rate agreements and separates a swap's price, its fixed rate, from its value. A candidate must be able to explain how the FRAs embedded in a swap differ from market FRAs, replicate a swap with cash market positions, compute the par swap rate from spot rates, and value a swap after initiation.

LOS 76.a — Swaps compared with a series of forward contracts

In a plain **fixed-for-floating interest rate swap**, one party pays a fixed rate and the other pays a floating **market reference rate (MRR)** on the same **notional principal**. On each settlement date only the **net payment** changes hands.

A swap as a series of FRAs

Take a 1-year swap with quarterly settlements. The floating rate for each quarter is set at the start of the quarter, so:

- The first net payment (end of quarter 1) is already known at initiation. It depends on today's MRR and the swap's fixed rate.
- Each later payment (quarters 2, 3 and 4) equals the payoff of a **forward rate agreement** in which the fixed-rate payer is long at a contract rate equal to the **swap fixed rate**. From the fixed-rate payer's perspective, each payment is $(\mathrm{MRR}_n - F) \times \frac{1}{4} \times \mathrm{notional}$.
- The timing differs slightly. A standard FRA on the 90-day MRR is cash-settled at the start of the underlying quarter, when the rate is set, and pays the present value of the interest difference: $(\mathrm{MRR}_n - F) \times \frac{1}{4} \times \mathrm{notional}$ discounted for one quarter at MRR_n. The FRAs that replicate the swap's payments on days 180, 270 and 360 therefore settle on days 90, 180 and 270. Growing each discounted settlement forward one quarter at MRR_n gives the swap's net payment at the end of the quarter, so the amounts are equivalent and only the payment date differs.

A swap is therefore equivalent to a series of FRAs. Like an FRA, it can produce gains or losses for either side, and each party bears the other's counterparty credit risk unless a central counterparty is used. The one important difference is in the contract rates:

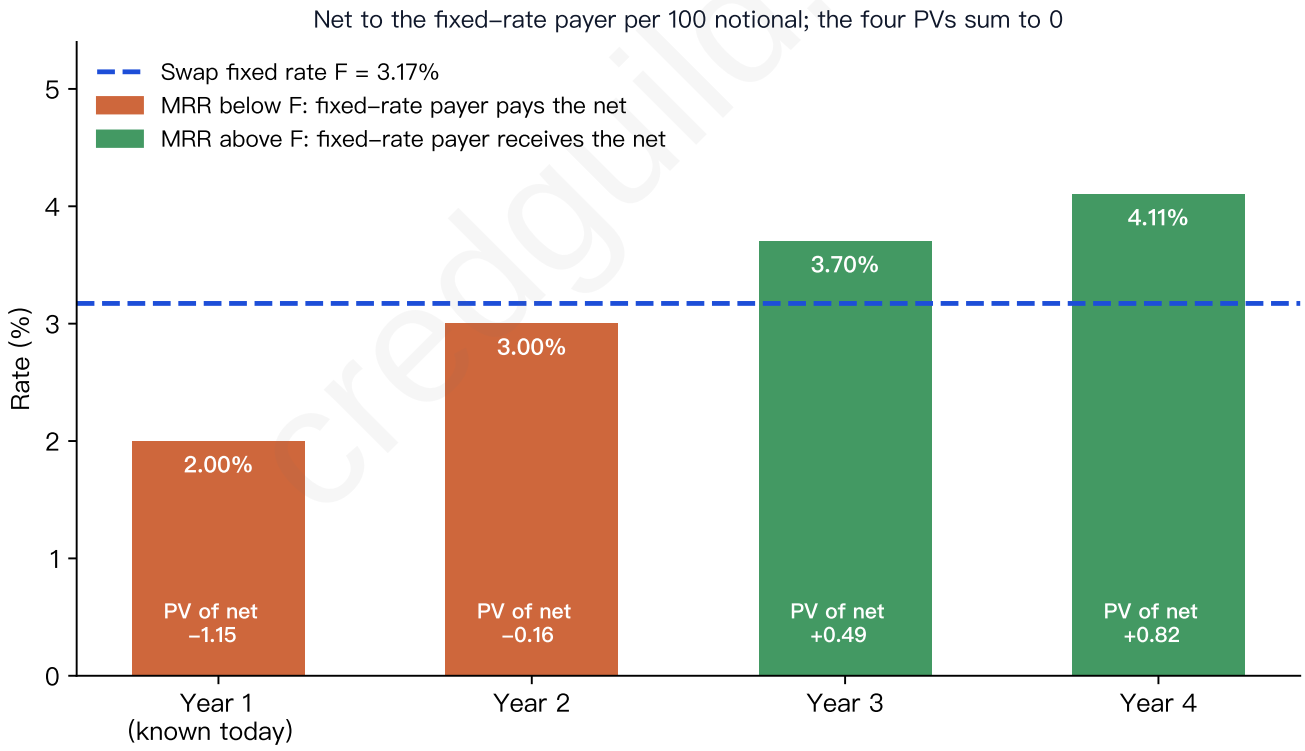
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KEY CONCEPT

	Series of market FRAs	FRAs embedded in a swap
Contract rates	Each FRA has its own no-arbitrage rate, so each starts at **zero value**	**All** use the same rate: the swap fixed rate
Value of each forward at initiation	Zero	Generally **not** zero: some positive, some negative
Total value at initiation	Zero	The values **sum to zero**

With an upward-sloping curve, the early FRAs (swap rate above the forward rate) have negative value to the fixed-rate payer and the later ones have positive value. In total they offset.

Example. For an annual-pay 4-year swap with spot rates of 2.0%, 2.5%, 2.9% and 3.2%, the implied 1-year MRRs are 2.00%, 3.00%, 3.70% and 4.11% (for year 2, $1.025^2/1.02 - 1 = 3.00\%$). The fixed rate that gives the swap zero value, the par swap rate derived in LOS 76.b, is 3.17%. Per 100 of notional, the present values of the net payments to the fixed-rate payer are −1.15, −0.16, +0.49 and +0.82, which sum to zero. For year 1, $(2.00 - 3.17)/1.02 = -1.15$.



An annual-pay 4-year swap as a series of forward contracts that all use one fixed rate

Replicating a swap with cash market positions

Swap position	Cash market replication
Fixed-rate payer (receives floating)	Borrow at a fixed rate (issue a fixed-rate bond) and invest the proceeds at the floating rate (buy a floating-rate note)
Floating-rate payer (receives fixed)	Borrow at a floating rate (issue a floating-rate note) and invest the proceeds in a **fixed-rate bond**

The floating-rate payer's net cash flows are also equivalent to a series of FRAs (the other side of the fixed-rate payer's FRAs). Those FRAs would all carry the swap rate, so they would not be zero-value FRAs.

LOS 76.b — Price versus value of a swap

- The **price** of a swap is its fixed rate, the **par swap rate**, written into the contract at initiation. It does not change over the swap's life.
- The **value** of the swap is zero at initiation, because the par swap rate is chosen so that the fixed leg and the expected floating leg have equal present values. Neither party pays to enter.
- After initiation, the value changes as expected future MRRs change.

Solving for the par swap rate

Let S_n be the spot rate for period n and MRR_n the floating rate for period n implied by those spot rates (the first equals the current MRR). The par swap rate F (per period) satisfies

$$\sum_{n=1}^{N} \frac{\text{MRR}_n}{(1+S_n)^n} = \sum_{n=1}^{N} \frac{F}{(1+S_n)^n}$$

A floating-rate note paying the implied rates and then returning its principal is worth par. Per unit of notional, the floating payments alone are therefore worth $1 - 1/(1+S_N)^N$, and the equation simplifies to

KEY CONCEPT

$$F = \frac{1 - \dfrac{1}{(1+S_N)^N}}{\displaystyle\sum_{n=1}^{N} \frac{1}{(1+S_n)^n}}$$

Example. An annual-pay 2-year swap faces spot rates $S_1 = 1.5\%$ and $S_2 = 2.5\%$. The discount factors are $1/1.015 = 0.985222$ and $1/1.025^2 = 0.951814$, so $F = (1 - 0.951814)/(0.985222 + 0.951814) = 0.048186/1.937036 = 2.49\%$. As a check, the implied 1y1y rate is $1.025^2/1.015 - 1 = 3.51\%$, and $PV(\text{floating}) = 0.015 \times 0.985222 + 0.0351 \times 0.951814 = 0.048186$, which matches $PV(\text{fixed})$.

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Value during the life

$$\text{Value to fixed-rate payer} = PV(\text{expected floating payments}) - PV(\text{remaining fixed payments})$$

Example. Take the 2-year swap above with a notional principal of $50 million and a fixed rate of 2.49%. At the first settlement the fixed-rate payer pays the net $50{,}000{,}000 \times (0.0249 - 0.015) = \$495{,}000$. Just after that payment, 1-year MRR is 3.90%, above the 3.51% implied at initiation, and it sets the final floating payment. One net payment remains, due in a year: $50{,}000{,}000 \times (0.0390 - 0.0249) = \$705{,}000$ to the fixed-rate payer. Its value today is $705{,}000/1.039 = \$678{,}537$ to the fixed-rate payer and $-\$678{,}537$ to the floating-rate payer. Had MRR come in at the expected 3.51%, the remaining payment would be worth $50{,}000{,}000 \times (0.0351 - 0.0249)/1.0351 = \$492{,}706$, about what the fixed-rate payer paid at the first settlement. The rise in MRR above that expectation is the fixed-rate payer's gain and the floating-rate payer's loss.

KEY CONCEPT

Change in expected future MRRs after initiation	Value to fixed-rate payer	Value to floating-rate payer	Swap price (fixed rate)
Increase	Rises (positive)	Falls (negative)	Unchanged
Decrease	Falls (negative)	Rises (positive)	Unchanged

This value is the basis for mark-to-market payments on centrally cleared swaps.

Common exam traps

- A floating-rate payer's position is like issuing floating-rate debt and buying a fixed-rate bond, a long-duration position that gains when rates fall.
- The par swap rate is neither the simple average of the spot rates nor the longest spot rate. For the 4-year swap in the example, the simple average gives 2.65% and the longest spot rate 3.20%, against a par swap rate of 3.17%.

EXAM SHORTCUTS

- The par swap rate needs only the spot-rate discount factors, $F = (1 - \text{last discount factor})/\text{sum of the discount factors}$, so the implied floating rates do not have to be computed first.

BOTTOM LINE

- An interest rate swap is equivalent to a series of FRAs in which the fixed-rate payer is long at the swap fixed rate, and its first net payment is already known at initiation.
- Market FRAs each carry their own no-arbitrage rate and start at zero value, while the FRAs embedded in a swap all use the swap fixed rate, so their individual values are generally not zero but sum to zero.

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- A fixed-rate payer's position replicates borrowing at a fixed rate and investing the money at the floating rate, and a floating-rate payer's position replicates borrowing at a floating rate and putting the money into a fixed-rate bond.
- A swap's price is its fixed par swap rate, which is set at initiation and does not change, while its value is zero at initiation and then changes as expected future MRRs change.
- $F = \left[1 - 1/(1+S_N)^N\right] / \sum_{n=1}^{N} 1/(1+S_n)^n$, the fixed rate per period that makes the fixed leg and the expected floating leg equal in present value.
- Value to the fixed-rate payer = PV(expected floating payments) − PV(remaining fixed payments), so a rise in expected MRRs after initiation gives the fixed-rate payer a gain and the floating-rate payer an equal loss.

QUICK CHECK

Question 1

An analyst breaks a fixed-for-floating interest rate swap into a series of forward contracts (FRAs), each with a contract rate equal to the swap's fixed rate. For this series to replicate a swap entered at market terms, the forward contracts must have:

A values that are each equal to zero at swap initiation.

B values that sum to zero at swap initiation.

C values that sum to zero at swap expiration.

Answer: B

Because every FRA in the series uses the same contract rate (the swap fixed rate), the individual FRAs will generally not be zero-value contracts: some have positive and some negative value. A swap entered at market terms has zero value at initiation, so the FRA values must add up to zero at that date.

Why the other options are wrong

- **A.** Zero-value FRAs would each need their own no-arbitrage contract rate. With a single common rate, individual values are generally non-zero; only their sum is zero.
- **C.** The zero-sum condition applies at initiation, when the swap is priced to have zero value. At expiration, the values reflect realized rates and need not sum to zero.

Key takeaway A swap is a series of off-market FRAs whose values sum to zero at inception.

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Practice questions

Question 2

Six months after Orsolya Farms entered a 5-year interest rate swap as the fixed-rate payer, expected future MRRs have risen. Which statement is *most accurate*?

A The swap's value to Orsolya has fallen; the swap price is unchanged.

B The swap's value to Orsolya has risen; the swap price has also risen.

C The swap's value to Orsolya has risen; the swap price is unchanged.

Question 3

Rhea Kapoor wants to use cash market transactions to reproduce the net cash flows of the floating-rate payer in a plain vanilla interest rate swap. She could best do so by:

A taking out a floating-rate loan and buying a fixed-rate bond.

B taking out a fixed-rate loan plus a strip of zero-value FRAs.

C taking out a floating-rate loan plus a strip of zero-value FRAs.

Answers: 2. C · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/interest-rate-swap/. The full bank has 6 questions on this reading.

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DERIVATIVES · READING 77

Pricing and Valuation of Options

What you'll learn

LOS 77.a Explain an option's exercise (intrinsic) value, moneyness (in, at, out of the money) and time value.

LOS 77.b Contrast no-arbitrage pricing of forward commitments with replication-based pricing of contingent claims, including the upper and lower bounds for European option values.

LOS 77.c Identify the six factors that determine an option's value and the direction in which each affects call and put values.

MODULE 77.1

Option Valuation

This reading covers the building blocks of option pricing: moneyness, exercise value and time value, the no-arbitrage bounds on European option values, and the factors that move option values. A candidate must be able to compute exercise value, time value and the lower and upper bounds of European calls and puts, and state how each of the six factors changes call and put values.

LOS 77.a — Moneyness, exercise value and time value

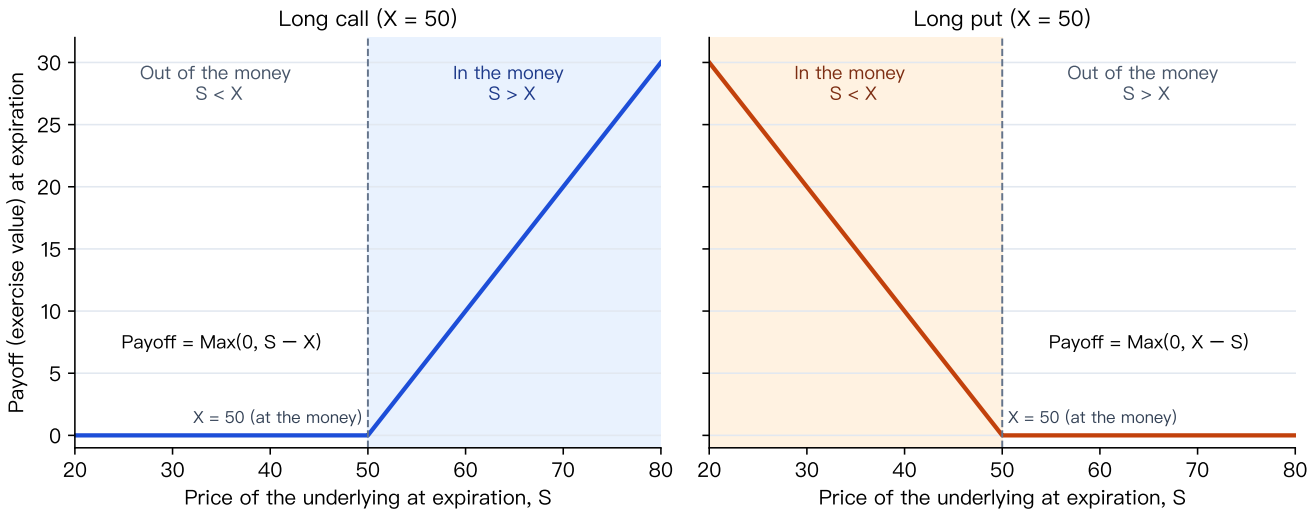
Moneyness describes what the holder would receive from exercising an option right now. With S = current price of the underlying and X = exercise (strike) price:

KEY CONCEPT

	Call	Put
In the money	$S > X$	$S < X$
At the money	$S = X$	$S = X$
Out of the money	$S < X$	$S > X$

An in-the-money option would produce a positive payoff if exercised immediately. An out-of-the-money option would not, so the holder would simply not exercise it.

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Payoff at expiration of a long call and a long put, both with exercise price 50

The **exercise value** (also called **intrinsic value**) is the amount by which the option is in the money, floored at zero:

$$\text{Call exercise value} = \max(0,\ S - X) \qquad \text{Put exercise value} = \max(0,\ X - S)$$

Before expiration an option normally trades above its exercise value. The difference is the **time value** (sometimes called speculative value):

$$\text{Option premium} = \text{exercise value} + \text{time value}, \quad \text{so} \quad \text{time value} = \text{premium} - \text{exercise value}$$

Three consequences follow:

- For an out-of-the-money or at-the-money option, exercise value is zero, so the entire premium is time value.
- For an in-the-money option, exercise value is positive, so time value is less than the premium.
- Time value comes from the chance that the underlying moves favorably before expiration. It generally shrinks as expiration approaches and is zero at expiration, when the option is worth exactly its exercise value (zero if at or out of the money). Time value is typically positive but can be negative: a deep in-the-money European put can trade below $X - S$, because its holder must wait until expiration to receive X.

> *Example.* A stock trades at \$83. An 80-strike call is \$3 in the money (exercise value \$3), and an 80-strike put is \$3 out of the money (exercise value \$0). If the call's premium is \$5.40, its time value is $5.40 - 3.00 = \$2.40$. If the put's premium is \$0.85, all \$0.85 is time value.

LOS 77.b — Arbitrage and replication for contingent claims

Forward commitments (forwards, futures, swaps) are priced so that their value at initiation is zero to both parties, and both sides face essentially unlimited gains or losses. Options (**contingent claims**) differ in two ways. The buyer pays a premium, so the value at initiation is positive, and the payoffs are one-sided, because the buyer's loss is limited to the premium. Options are therefore priced with a **replication** approach. The option's

payoff is combined with a position in the underlying and a **pure discount bond** (risk-free zero-coupon bond), and no-arbitrage requires that a portfolio whose payoff is never negative must itself have a value of zero or more.

Two such portfolios give the lower bounds:

- *Call.* Buy the call, buy a pure discount bond that pays X at T, and short one share. The payoff at T is $c_T + X - S_T$: zero if $S_T \geq X$ and $X - S_T > 0$ otherwise. Because it is never negative, $c_0 - S_0 + X(1 + R_f)^{-T} \geq 0$, so $c_0 \geq S_0 - X(1 + R_f)^{-T}$.
- *Put.* Buy the put, buy one share, and borrow $X(1 + R_f)^{-T}$. The payoff at T is $p_T + S_T - X$: zero if $S_T \leq X$ and $S_T - X > 0$ otherwise. Hence $p_0 + S_0 - X(1 + R_f)^{-T} \geq 0$, so $p_0 \geq X(1 + R_f)^{-T} - S_0$.

Combined with the fact that no option can have a negative value, this yields boundary values for European options at time t (expiration at T, R_f = risk-free rate):

KEY CONCEPT

Option	Minimum value	Maximum value
European call	$c_t \geq \max[0,\ S_t - X(1 + R_f)^{-(T-t)}]$	S_t
European put	$p_t \geq \max[0,\ X(1 + R_f)^{-(T-t)} - S_t]$	$X(1 + R_f)^{-(T-t)}$

Nobody pays more than the share price for the right to buy the share. A European put can never pay more than X, and that payment cannot arrive before expiration, so its ceiling is the present value of X. The lower bounds use the present value of the exercise price because a European option cannot be exercised early.

Example. $S_0 = 83$, $X = 80$, six months to expiration and $R_f = 4\%$. $PV(X) = 80/1.04^{0.5} = 78.45$. The lower bound for the call is $83 - 78.45 = 4.55$; the lower bound for the put is $\max(0,\ 78.45 - 83) = 0$.

LOS 77.c — Factors that determine option values

Six factors determine option values. Their effects, other things equal, are:

KEY CONCEPT

Increase in …	Call value	Put value
Price of the underlying	Increase	Decrease
Exercise price	Decrease	Increase
Risk-free rate	Increase	Decrease
Volatility of the underlying	Increase	Increase
Time to expiration	Increase	Usually increase (exceptions for some European puts)
Costs and benefits of holding the asset	Benefits (dividends, interest, convenience yield): decrease. Storage costs: increase	Benefits: increase. Storage costs: decrease

- *Underlying price and exercise price.* These drive exercise value directly: a call gains as S rises or X falls, and a put gains as S falls or X rises. At expiration only these two matter, because time value is zero. Interest rates and volatility affect only time value.
- *Risk-free rate.* A call holder pays X in the future, and a higher rate lowers the present value of that payment, so the call is worth more. A put holder receives X in the future, and a higher rate lowers the present value of that receipt, so the put is worth less. A falling rate therefore lowers calls and raises puts.
- *Volatility.* Volatility makes both calls and puts valuable, because the downside is capped while the potential upside widens. With no volatility, the time value that comes from possible favorable price moves would disappear. Exam convention: option values then equal their exercise values. Current practice: pricing models still discount the exercise price of a European option. With zero volatility the underlying grows at the risk-free rate, so a European call on a non-dividend-paying share is worth $S - PV(X)$ when $S > PV(X)$.
- *Time to expiration.* More time effectively means more volatility, so calls and most puts are worth more. A deep in-the-money European put, however, can be worth less with a longer life, because its upside is almost capped and the payment of X is received later, at a lower present value. The effect is more likely for a put that is deeper in the money, when the risk-free rate is higher, and when the time to expiration is longer.
- *Holding benefits and costs.* Benefits such as a dividend lower the price of the underlying, which hurts calls and helps puts. Storage costs work the other way.

A **long call** profits when the underlying rises, so its profit is positively correlated with the price. A **long put** profits when the underlying falls, so its profit is negatively correlated with the price. Short positions have the opposite sensitivity: a **short put** gains value when the underlying price rises and loses value when volatility increases or, for most puts, when time to expiration increases.

Example. An investor is long a put and long a call on the same share. If the risk-free rate falls, the call loses value and the put gains value, so only one of the two positions benefits. If the investor were instead long the put and short the call, both positions would gain.

Common exam traps

- Moneyness compares the exercise price with the current price. It says nothing about whether the option is now worth more than the holder paid for it.
- The European bounds use $PV(X)$. The put's upper bound is $PV(X)$, not X, and using X in the lower bounds gives the exercise value from immediate exercise instead. In the six-month example, using X gives a call lower bound of 3.00 instead of 4.55 and a put upper bound of 80 instead of 78.45.
- The risk-free rate moves call and put values in opposite directions, while volatility moves them in the same direction.
- A longer life does not always raise a European put's value.

BOTTOM LINE

- A call is in the money when $S > X$ and a put when $S < X$; either is at the money when $S = X$.
- Exercise value is $\max(0,\ S - X)$ for a call and $\max(0,\ X - S)$ for a put, and time value is the premium minus the exercise value, so the whole premium of an option with zero exercise value is time value.
- Time value generally shrinks as expiration approaches and is zero at expiration; it is typically positive but can be negative for a deep in-the-money European put.

- European option bounds: $\max[0,\ S_t - X(1+R_f)^{-(T-t)}] \le c_t \le S_t$ and $\max[0,\ X(1+R_f)^{-(T-t)} - S_t] \le p_t \le X(1+R_f)^{-(T-t)}$.
- Other things equal, a higher underlying price or a lower exercise price raises a call's value and lowers a put's, a higher risk-free rate raises calls and lowers puts, and higher volatility raises both.
- A longer time to expiration raises a call's value and usually a put's, but a deep in-the-money European put can be worth less with a longer life, more likely when it is deeper in the money, the risk-free rate is higher and the time to expiration is longer.
- Benefits of holding the underlying, such as dividends, lower call values and raise put values, while storage costs do the opposite.
- At expiration only the underlying price and the exercise price matter, because time value is zero; interest rates and volatility affect only time value.

QUICK CHECK

Question 1

Hana Novak owns two options on the same stock: a call with an exercise price of $62 and a put with an exercise price of $70. If the stock currently trades at $66:

A neither option is in the money.

B only the call option is in the money.

C both options are in the money.

Answer: C

The call is in the money because it lets Novak buy at $62 a stock worth $66 ($S - X = 4$). The put is in the money because it lets her sell at $70 a stock worth $66 ($X - S = 4$).

Why the other options are wrong

- **A.** Each option would give a positive payoff if exercised now, so both are in the money.
- **B.** The call is in the money, but so is the put, because its exercise price is above the market price.

Key takeaway Check each option separately: call ITM if $S > X$, put ITM if $S < X$. With different strikes both can be in the money at once.

Practice questions

Question 2

Which statement about a European put option is *most accurate*?

A Its exercise value is the stock price less the option's exercise price.

B Its time value is the sum of its market price and its exercise value.

C Its exercise value is the exercise price less the stock price, if positive.

Question 3

Compared with forward commitments, why must a different no-arbitrage approach be used to price options?

A Options have a positive value at initiation and one-sided payoffs.

B Options cannot be replicated with the underlying and a bond.

C Options are valued with a risk-adjusted discount rate only.

Question 4

Tomás Reyes has bought a European call option and written a European put option on the same equity index. Holding all other factors constant, an increase in the risk-free interest rate will increase the value of:

A neither of his positions.

B only one of his positions.

C both of his positions.

Answers: 2. C · 3. A · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/pricing-and-valuation-of-options/. The full bank has 21 questions on this reading.

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DERIVATIVES · READING 78

Put Call Parity

What you'll learn

LOS 78.a Explain put–call parity for European options (fiduciary call = protective put) and use it to build synthetic stocks, calls, puts and bonds.

LOS 78.b Explain put–call forward parity for European options and the option view of a firm's equity and debt (credit risk as a put).

MODULE 78.1

Put-Call Parity

This reading derives put–call parity and put–call forward parity from no-arbitrage and applies option thinking to a firm's equity and debt. A candidate must be able to use parity to value an option or build a synthetic position, identify and exploit a parity violation, and explain why shareholders hold a call on the firm's assets and debtholders are short a put.

LOS 78.a — Put–call parity

Two portfolios, both built with European options on the same underlying with the same exercise price X and the same expiration T:

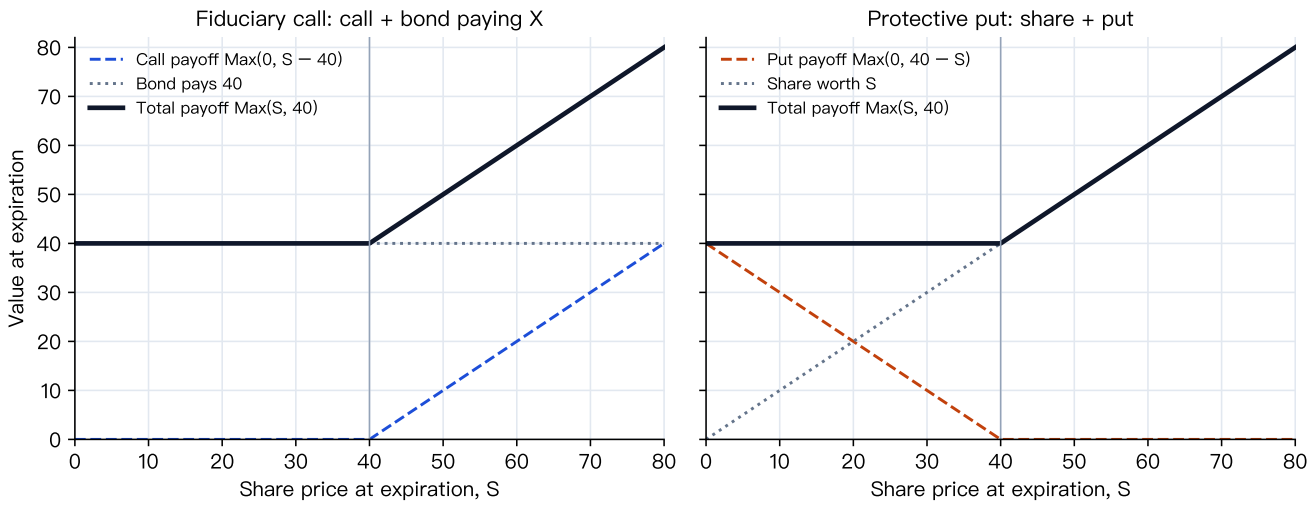
- **Fiduciary call** = long call + a risk-free **pure-discount bond** that pays X at expiration.
- **Protective put** = long one share of the underlying + long put.

Payoffs at expiration:

	$S_T \leq X$	$S_T > X$
Fiduciary call	call 0 + bond X = X	call $S_T - X$ + bond X = S_T
Protective put	share S_T + put $X - S_T$ = X	share S_T + put 0 = S_T

Both pay $\max(S_T, X)$ in every state (see the figure below), so no-arbitrage forces them to cost the same today. This is **put–call parity**:

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Both portfolios pay 40 if S ⩽ 40 and S if S > 40, so $c + X(1 + Rf)^{-T} = S + p$.

Fiduciary call and protective put with exercise price 40: identical payoffs at expiration

KEY CONCEPT

$$c + X(1 + R_f)^{-T} = S + p$$

Synthetic securities

Rearranging parity isolates any one instrument; a **+** sign means a long position and a **−** sign a short position (a short bond = borrowing the PV of X; a long bond = lending it):

KEY CONCEPT

Target (synthetic equivalent)	Replicating portfolio
Synthetic stock: $S = c - p + X(1 + R_f)^{-T}$	long call, short put, long bond
Synthetic put: $p = c - S + X(1 + R_f)^{-T}$	long call, short stock, long bond
Synthetic call: $c = S + p - X(1 + R_f)^{-T}$	long stock, long put, short bond (borrow)
Synthetic bond: $X(1 + R_f)^{-T} = S + p - c$	long stock, long put, short call

Parity in this form holds for **European** options with the same exercise price and expiration on an underlying that pays no income and has no holding costs during the option's life.

Arbitrage. If one side of parity is cheap relative to the other, buy the cheap side and sell the expensive side. Example: if a put trades *below* $c - S + PV(X)$, buy the put and sell the synthetic put: write the call, buy the stock and borrow $PV(X)$. The difference is an arbitrage profit received today.

Example. $S = 36$, $X = 35$, 3 months to expiration, $R_f = 6\%$, put price 1.10. The call implied by parity is

$$c = S + p - \frac{X}{(1 + R_f)^T} = 36 + 1.10 - \frac{35}{1.06^{0.25}} = 37.10 - 34.49 = 2.61$$

If the call is quoted at 3.00 instead, it is expensive relative to the synthetic call worth 2.61. Write the call for 3.00 and build the synthetic call: buy the share for 36, buy the put for 1.10 and borrow 34.49. The synthetic call costs 2.61, so 0.39 is received today. At expiration the share plus the put is worth $\max(S_T, 35)$, the loan repayment is 35 and the written call costs $\max(0, S_T - 35)$. These net to zero in every state, so the 0.39 is an arbitrage profit.

Common exam traps

- Confusing a fiduciary call (call + bond paying X) with a **covered call** (stock + short call) or with call + stock.
- Getting the bond sign wrong: a synthetic call *borrows* (short bond); a synthetic put *lends* (long bond).
- Discounting X at the wrong horizon, or forgetting to discount it at all. In the 3-month example, leaving the 35 undiscounted gives a call value of 2.10, and discounting it for a full year gives 4.08, instead of 2.61.

LOS 78.b — Put–call forward parity and options in corporate finance

A long **forward contract** with price $F_0(T)$ plus a pure-discount bond paying $F_0(T)$ at T replicates the underlying: at T the bond proceeds exactly pay for the asset. The forward costs nothing at initiation, so this **synthetic asset** costs $F_0(T)(1 + R_f)^{-T}$. Substituting it for S in put–call parity gives **put–call forward parity** (at initiation of the forward):

KEY CONCEPT

$$F_0(T)(1 + R_f)^{-T} + p_0 = c_0 + X(1 + R_f)^{-T}$$

Equivalently,

$$p_0 - c_0 = [X - F_0(T)]\,(1 + R_f)^{-T}$$

So a long put plus a short call (same X) has the same value as a risk-free bond that pays $X - F_0(T)$ at expiration. The option pair matches the bond in **value**, not in **payoff**: at expiration it pays $X - S_T$, which is uncertain. The long forward (payoff $S_T - F_0(T)$) removes that uncertainty, so options plus forward pay the fixed amount $X - F_0(T)$; because the forward costs nothing at initiation, adding it leaves the value unchanged. The components are the call, the put, the PV of the exercise price and the **present value of the forward price**. The spot price of the underlying does not appear.

Example. One-year forward price 62, $X = 60$, $R_f = 5\%$: $p_0 - c_0 = (60 - 62)/1.05 = -1.90$, so the call must be worth 1.90 more than the put.

Option view of a firm (equity and debt)

Consider a firm financed by equity and one zero-coupon debt issue with face value D; firm value V = equity + debt.

- Because of limited liability, **shareholders hold a call option** on the firm's assets with exercise price D: if $V_T > D$ they pay off the debt and keep the rest; if $V_T < D$ they default and walk away.

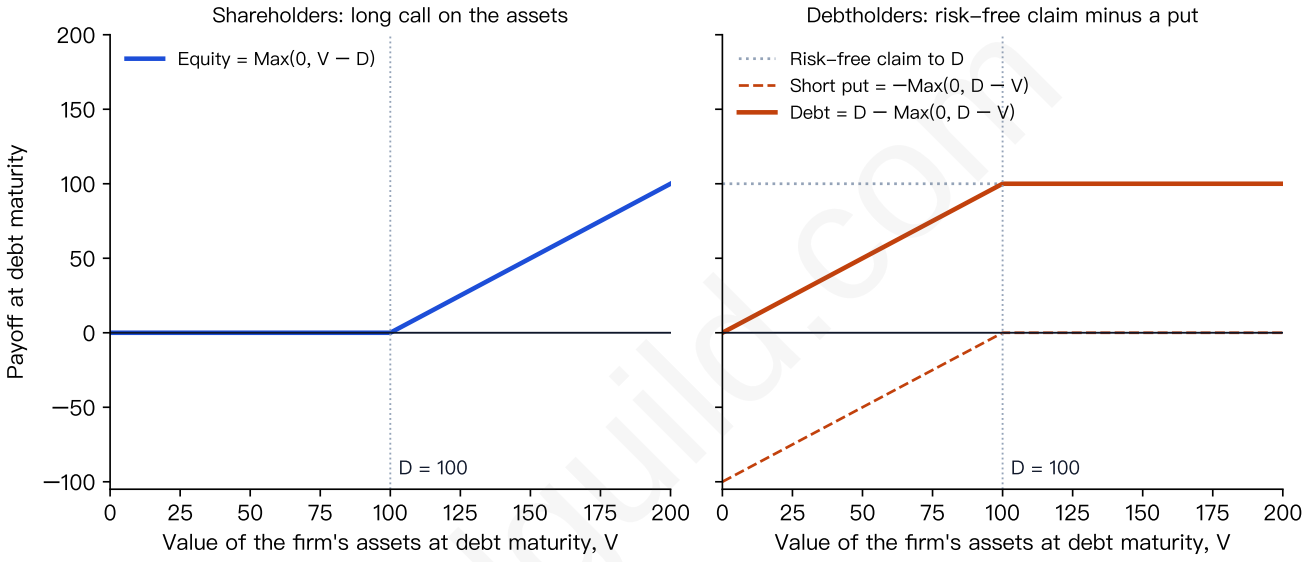
- Equivalently, shareholders own the assets plus a **long put** on the assets with exercise price D. Defaulting amounts to "putting" the assets to the debtholders.
- **Debtholders** hold a risk-free claim to D plus a **short put** written to the shareholders. That put is the firm's **credit risk** (default risk): the debt is worth less than $PV(D)$ by the value of the put.

$$V_0 = c_0 + PV(D) - p_0$$

Here c_0 is the equity and $PV(D) - p_0$ is the risky debt.

At debt maturity the two claims pay

$$\text{Equity}_T = \max(0,\ V_T - D) \qquad \text{Debt}_T = \min(V_T,\ D) = D - \max(0,\ D - V_T)$$



Payoffs at debt maturity to shareholders and debtholders of a firm with zero-coupon debt of face value 100

Example. With $D = 100$: if $V_T = 130$, shareholders repay 100 and keep 30. If $V_T = 70$, they default, equity is worth 0 and debtholders receive 70, which is the risk-free 100 minus the put payoff of 30.

Common exam traps

- Saying the credit-risk put is written by the shareholders. The debtholders are short the put and the shareholders hold it.
- Including the spot price in put–call forward parity; the forward price (in PV terms) replaces it.
- Using the future value, rather than the present value, of the forward price. In the one-year example, this gives $p_0 - c_0 = -2.00$ instead of −1.90.
- Claiming that long put + short call alone replicates the bond's payoff; the long forward is also needed.

BOTTOM LINE

- A fiduciary call (long call plus a bond paying X) and a protective put (share plus long put) both pay $\max(S_T, X)$, so $c + X(1 + R_f)^{-T} = S + p$.

- Put–call parity in this form holds for European options with the same exercise price and expiration on an underlying with no income and no holding costs during the option's life.
- Rearranging parity gives synthetic positions: a synthetic call is long the stock, long the put and short the bond (borrowing $PV(X)$), and a synthetic put is long the call, short the stock and long the bond (lending $PV(X)$).
- When one side of parity is cheap relative to the other, buying the cheap side and selling the expensive side earns the price difference today as an arbitrage profit.
- Put–call forward parity replaces the spot price with the present value of the forward price, $F_0(T)(1+R_f)^{-T} + p_0 = c_0 + X(1+R_f)^{-T}$, so $p_0 - c_0 = [X - F_0(T)](1+R_f)^{-T}$.
- For a firm financed by equity and zero-coupon debt with face value D, shareholders hold a call on the firm's assets with exercise price D, or equivalently the assets plus a long put.
- Debtholders hold a risk-free claim to D plus a short put written to the shareholders; that put is the firm's credit risk, and $V_0 = c_0 + PV(D) - p_0$.

QUICK CHECK

Question 1

A portfolio manager wants to reproduce the expiration payoff of a European put without trading any puts. She can trade the stock, a European call with the same strike and expiration, and a risk-free pure-discount bond that pays the exercise price X at expiration. Based on put–call parity, which combination should she hold?

A Short the stock, long the call, short a pure-discount bond that pays X.

B Short the stock, long the call, long a pure-discount bond that pays X.

C Long the stock, short the call, short a pure-discount bond that pays X.

Answer: B

Solving put–call parity for the put gives $p = c - S + X(1+R_f)^{-T}$: a long call, a short stock position and an investment of $PV(X)$ in the risk-free bond (a long bond position). At expiration the combination pays exactly $\max(0, X - S_T)$.

Why the other options are wrong

- **A.** Borrowing (short bond) has the wrong sign; the put equation adds $PV(X)$, so the bond is held long.
- **C.** This is the negative of the synthetic put (the position of someone who has sold a put synthetically).

Key takeaway Put side of parity: $p = c - S + PV(X)$, so hold a long call, short stock and a long bond.

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Practice questions

Question 2

Castellan Group is financed with common equity and a single issue of zero-coupon debt. When options theory is applied to this capital structure, the firm's credit risk can be modeled as a:

A put option written by the debtholders and held by the shareholders.

B call option written by the debtholders and held by the shareholders.

C call option written by the shareholders and held by the debtholders.

Question 3

Shares of Brisa Networks trade at $73.00. A 6-month European call on the shares with an exercise price of $70.00 is priced at $6.20, and the annual risk-free rate is 4%. The shares pay no dividends. Based on put–call parity, the value of a 6-month European put with the same exercise price is *closest* to:

A $1.84.

B $3.20.

C $10.56.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/put-call-parity/. The full bank has 14 questions on this reading.

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DERIVATIVES · READING 79

Binomial Option Pricing Model

What you'll learn

LOS 79.a Value a call or put with a one-period binomial model by building a hedged (riskless) portfolio of the option and the underlying using the hedge ratio.

LOS 79.b Describe risk neutrality: value an option as the expected payoff under risk-neutral probabilities, discounted at the risk-free rate.

MODULE 79.1

Binomial Model for Option Values

This reading values options with a one-period binomial model, first by building a riskless hedge portfolio and then by risk-neutral valuation. A candidate must be able to compute a hedge ratio and use it to value a call or a put, compute risk-neutral probabilities, and explain why option values do not depend on investors' risk preferences.

LOS 79.a — One-period binomial model: the hedge (replication) approach

A **binomial model** assumes that over one period the underlying moves to one of just two values. Valuing an option on it requires:

- the value of the underlying today, S_0;
- the option's exercise price X (the option expires at the end of the period);
- the size of the moves: the up-move return $R^u = S_1^u/S_0$ and down-move return $R^d = S_1^d/S_0$ (these capture the underlying's volatility);
- the **risk-free rate** over the period.

The valuation needs neither the real-world probabilities of an up-move or down-move nor a risk-adjusted discount rate.

Call: long h shares, short one call

Choose the number of shares h so that the portfolio is worth the same after either move ($V_1^u = V_1^d$):

KEY CONCEPT

$$h\,S_1^u - c_1^u = h\,S_1^d - c_1^d \quad \Rightarrow \quad h = \frac{c_1^u - c_1^d}{S_1^u - S_1^d}$$

h is the **hedge ratio**, the number of shares held per option written. Because the end-of-period value is certain, it must earn the risk-free rate: $V_0 = V_1/(1 + R_f)$. With $V_0 = hS_0 - c_0$:

$$c_0 = hS_0 - V_0$$

Put: long h shares, long one put

For a put the hedge portfolio is **long** both the put and the shares (the put gains when the stock falls):

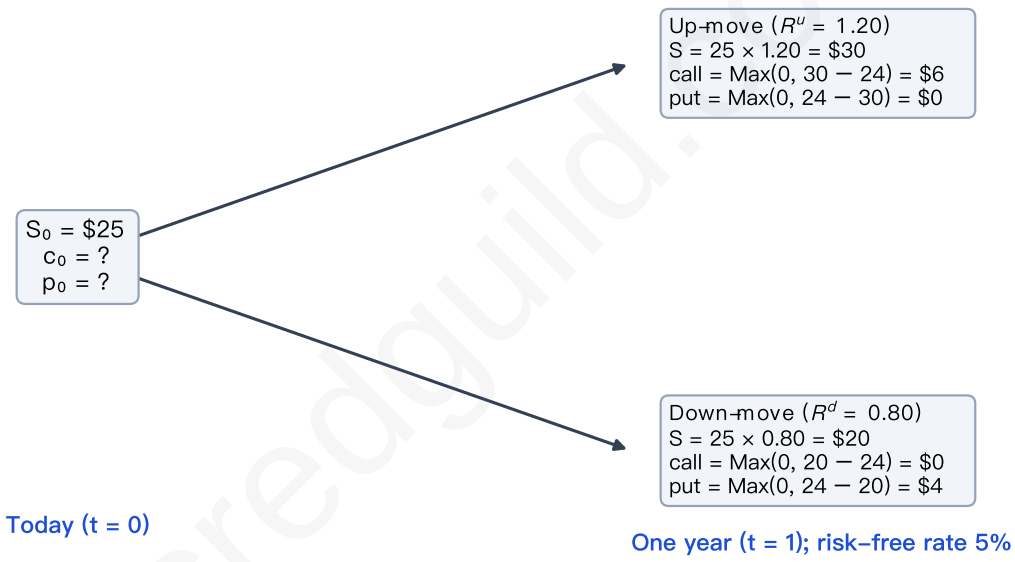
$$h = \frac{p_1^d - p_1^u}{S_1^u - S_1^d}, \qquad V_0 = hS_0 + p_0 \;\Rightarrow\; p_0 = V_0 - hS_0$$

KEY CONCEPT

The hedge approach in steps:

1. Find the two end-of-period prices of the underlying, S_0R^u and S_0R^d, and the option payoff at each.
2. Compute the hedge ratio h with the call or put formula.
3. Value the hedge portfolio at either node to get V_1; both nodes give the same number.
4. Discount V_1 one period at the risk-free rate to get V_0.
5. Back out the option value from V_0 and hS_0.

Example.



One-period binomial tree for a stock at $25 with call and put payoffs (X = $24)

$S_0 = 25$; after one year $S_1^u = 30$ ($R^u = 1.20$) or $S_1^d = 20$ ($R^d = 0.80$); $X = 24$; $R_f = 5\%$.

Call: $c_1^u = 6$ and $c_1^d = 0$, so $h = 6/(30 - 20) = 0.60$. $V_1 = 0.60(30) - 6 = 12 = 0.60(20)$. $V_0 = 12/1.05 = 11.429$. $c_0 = 0.60(25) - 11.429 = 3.57$.

Put: $p_1^u = 0$ and $p_1^d = 4$, so $h = 4/10 = 0.40$. $V_1 = 0.40(30) + 0 = 12 = 0.40(20) + 4$. $V_0 = 11.429$. $p_0 = 11.429 - 0.40(25) = 1.43$.

Check with put–call parity: $c_0 - p_0 = 3.57 - 1.43 = 2.14 = 25 - 24/1.05$.

The hedge portfolio is a pricing device; no cash needs to change hands to form it. Practical models split the option's life into many short periods, which gives many possible prices at expiration; the one-period tree shows the method.

Common exam traps

- Discounting the hedge ratio, or compounding it, at the risk-free rate. h is a pure ratio of payoff range to price range.
- Using a short put in the put hedge (the put hedge is long put + long shares).

LOS 79.b — Risk neutrality

Because the hedged portfolio is riskless, option value does not depend on investors' risk preferences. The option can therefore be valued *as if* investors were risk-neutral: weight the two payoffs by **risk-neutral probabilities**, then discount that expected payoff using the risk-free rate.

KEY CONCEPT

$$\pi_U = \frac{1 + R_f - R^d}{R^u - R^d}, \qquad \pi_D = 1 - \pi_U$$

$$c_0 = \frac{\pi_U\, c_1^u + \pi_D\, c_1^d}{1 + R_f}, \qquad p_0 = \frac{\pi_U\, p_1^u + \pi_D\, p_1^d}{1 + R_f}$$

The steps: (1) compute the option payoffs after the up- and down-move; (2) take their probability-weighted average using π_U and π_D; (3) discount one period at the risk-free rate.

Trees are often built with the down-move factor equal to the reciprocal of the up-move factor ($R^d = 1/R^u$), so an up-move of 1.25 pairs with a down-move of 0.80.

At Level I the risk-neutral probabilities are typically given; what matters is using them to weight the payoffs and discounting at the risk-free rate. π_U and π_D are **pseudo-probabilities**. They are not the actual probabilities of the moves; they come from the no-arbitrage hedge, which is why discounting at the risk-free rate is correct. In the example above, $\pi_U = (1.05 - 0.80)/(1.20 - 0.80) = 0.625$, so $c_0 = 0.625 \times 6/1.05 = 3.57$, the same value as the hedge approach gives.

The name comes from what these probabilities do to the underlying. Weighted by π_U and π_D, the underlying's expected return equals the risk-free rate: $\pi_U R^u + \pi_D R^d = 1 + R_f$. In the example, $0.625 \times 30 + 0.375 \times 20 = 26.25 = 25 \times 1.05$: under these probabilities the share is expected to earn exactly the 5% risk-free rate.

WORKED EXAMPLE: VALUING A PUT BOTH WAYS

A share trades at 40. Over one year it will either rise by a factor of 1.25 or fall by a factor of 0.80, and the 1-year risk-free rate is 3%. Value a 1-year European put with an exercise price of 42 using risk-neutral probabilities, and confirm the value with the hedge approach.

Step 1. End-of-year prices: $40 \times 1.25 = 50$ and $40 \times 0.80 = 32$. Put payoffs: $p_1^u = 0$ and $p_1^d = 42 - 32 = 10$.

Step 2. $\pi_U = (1.03 - 0.80)/(1.25 - 0.80) = 0.23/0.45 = 0.5111$ and $\pi_D = 1 - 0.5111 = 0.4889$.

Step 3. $p_0 = (0.5111 \times 0 + 0.4889 \times 10)/1.03 = 4.889/1.03 = 4.75$.

Step 4. Hedge check: $h = (10 - 0)/(50 - 32) = 0.5556$. The portfolio of 0.5556 shares and one put is worth $0.5556 \times 50 + 0 = 27.78$ after an up-move and $0.5556 \times 32 + 10 = 27.78$ after a down-move.

Step 5. $V_0 = 27.78/1.03 = 26.97$, so $p_0 = 26.97 - 0.5556 \times 40 = 26.97 - 22.22 = 4.75$.

Result. Both approaches value the put at 4.75.

Approach	Uses	Discount rate
Hedge (replication) portfolio	hedge ratio h, certain end value	risk-free
Risk-neutral valuation	π_U, π_D from move sizes and R_f	risk-free

Common exam traps

- Weighting payoffs by actual (real-world) probabilities.
- Forgetting to discount the expected payoff. For the call in the example, this gives 3.75 instead of 3.57.
- Discounting at a risk-adjusted rate instead of the risk-free rate.

EXAM SHORTCUTS

- For European options with the same exercise price and expiration on an underlying with no income or holding costs, once the call is valued the put follows from put–call parity, $p_0 = c_0 - S_0 + X/(1 + R_f)$, with no second hedge calculation.

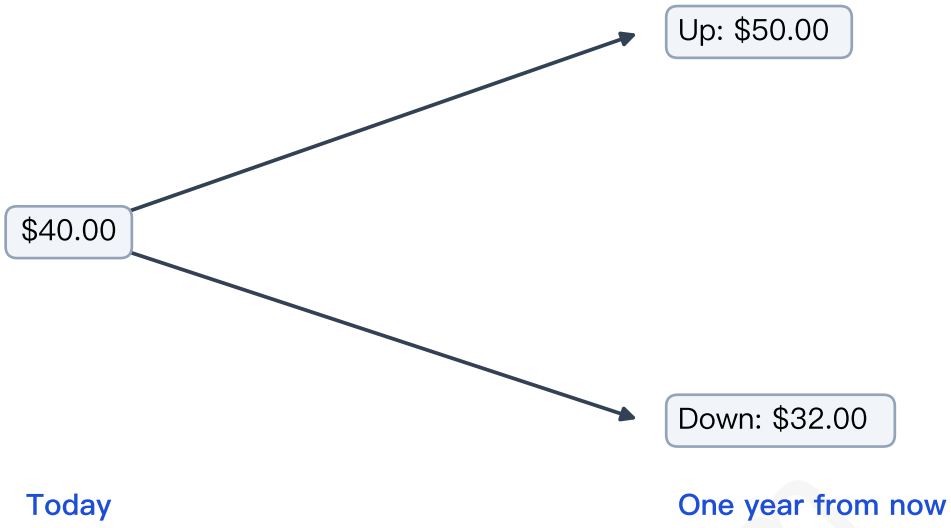
BOTTOM LINE

- A one-period binomial model values an option from the current underlying price, the exercise price, the up-move and down-move factors and the risk-free rate; it needs neither real-world probabilities nor a risk-adjusted discount rate.
- For a call, the hedge portfolio is long h shares and short one call, with $h = (c_1^u - c_1^d)/(S_1^u - S_1^d)$; its certain end value is discounted at the risk-free rate and $c_0 = hS_0 - V_0$.
- For a put, the hedge portfolio is long h shares and long one put, with $h = (p_1^d - p_1^u)/(S_1^u - S_1^d)$ and $p_0 = V_0 - hS_0$.
- $\pi_U = (1 + R_f - R^d)/(R^u - R^d)$ and $\pi_D = 1 - \pi_U$, and an option's value is its payoffs weighted by π_U and π_D and discounted one period at the risk-free rate.
- Risk-neutral probabilities are pseudo-probabilities derived from the no-arbitrage hedge, under which the underlying's expected return equals the risk-free rate; they are not the actual probabilities of the moves.
- The hedge approach and risk-neutral valuation both discount at the risk-free rate and give the same option value.

QUICK CHECK

Question 1

Shares of Solvay Ridge follow the one-period binomial tree shown below. The one-year risk-free rate is 4%.



One-period binomial tree for Solvay Ridge shares

An analyst wants to write one-year European calls on the shares with an exercise price of $44 and hold shares so that the combined position has the same value at expiration whatever the share price. The number of shares to hold per call written is *closest* to:

A 0.32.

B 0.33.

C 0.35.

Answer: B

The hedge ratio equals the spread of the call's payoffs divided by the spread of the share prices. It is a ratio of payoffs, so no discounting or compounding at the risk-free rate is involved.

Call payoffs: $c_1^u = \max(0,\ 50 - 44) = 6$; $c_1^d = \max(0,\ 32 - 44) = 0$.

$$h = \frac{c_1^u - c_1^d}{S_1^u - S_1^d} = \frac{6 - 0}{50 - 32} = \frac{6}{18} = 0.333$$

Check: up-move value $= 0.333(50) - 6 = 10.67$; down-move value $= 0.333(32) - 0 = 10.67$. The portfolio of 0.333 shares per short call is worth 10.67 in both states.

Why the other options are wrong

- **A.** 0.32 divides the hedge ratio by 1.04 ($0.3333/1.04 = 0.3205$); the risk-free rate is used to value the portfolio and plays no part in computing h.
- **C.** 0.35 multiplies the hedge ratio by 1.04 ($0.3333 \times 1.04 = 0.3467$); again, h is not adjusted for interest.

Key takeaway $h = (c_1^u - c_1^d)/(S_1^u - S_1^d)$. The risk-free rate enters only when the certain portfolio value is discounted.

Practice questions

Question 2

Shares of Keswick Marine trade at $60. In one year they will be worth either $78 after an up-move or $51 after a down-move. The one-year risk-free rate is 3%, and the risk-neutral probability of an up-move is 0.40. An analyst estimates the actual probability of an up-move at 0.55. Using risk-neutral valuation, the value of a one-year European call on the shares with an exercise price of $65 is *closest* to:

A $5.05.

B $5.20.

C $6.94.

Question 3

In a one-period binomial model, a call is valued by combining one short call with h shares of the underlying so that the portfolio's value at the end of the period is the same after an up-move or a down-move. Rearranging this relationship, a position that replicates a long call consists of:

A buying h shares and also lending at the risk-free rate.

B selling h shares short and lending at the risk-free rate.

C buying h shares, financed partly by borrowing at the risk-free rate.

Answers: 2. A · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/derivatives/binomial-option-pricing-model/. The full bank has 8 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Alternative Investments

Exam weight 6–9% · 7 readings

How alternative investments differ from stocks and bonds, how the funds are structured and paid, how their returns are measured, and the main types: private equity and debt, real estate and infrastructure, natural resources, hedge funds and digital assets.

Reading 80	Alternative Investment Features, Methods, and Structures
Reading 81	Hurdle Rate
Reading 82	Mezzanine Financing
Reading 83	Real Estate and Infrastructure
Reading 84	Natural Resources
Reading 85	High Water Mark
Reading 86	Introduction to Digital Assets

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ALTERNATIVE INVESTMENTS · READING 80

Alternative Investment Features, Methods, and Structures

What you'll learn

LOS 80.a Describe what alternative investments are, why investors hold them, their typical features and their three broad categories.

LOS 80.b Compare fund investing, co-investing and direct investing on control, fees, diversification, minimum size and expertise.

LOS 80.c Describe limited partnership structures and fee terms: management fees, performance fees, hurdles, catch-up, high-water marks, waterfalls and clawbacks.

MODULE 80.1

Alternative Investment Structures

This reading defines alternative investments, the reasons investors hold them and how they differ from traditional investments. It compares fund investing, co-investing and direct investing, and explains limited partnership structures, management and performance fees, hurdle rates, catch-up clauses, high-water marks, waterfalls and clawbacks, including how to calculate the fee under each hurdle.

LOS 80.a — What counts as an alternative investment

Traditional investments are long-only holdings of cash and publicly traded stocks and bonds. Anything outside that box is an **alternative investment**. Most alternatives are actively managed. They fall into three broad categories: private capital, real assets and hedge funds.

Why investors add them (perceived benefits)

- **Diversification**: returns have relatively low correlations with those of stocks and bonds, so adding them can reduce portfolio risk.
- Higher potential returns: investors may earn an illiquidity premium, and markets for some alternatives may be less efficient than public markets.

Typical features compared with traditional investments

KEY CONCEPT

Feature	Alternatives	Traditional
Manager knowledge	More specialized	Less specialized
Correlation with stocks/bonds	Relatively low	—
Liquidity of assets held	**Less liquid**	Liquid
Investor time horizon	Longer	Shorter/flexible

Feature	Alternatives	Traditional
Size of commitment	Larger	Smaller
Leverage	Used by some vehicles (LBO funds borrow heavily; hedge funds may use leverage and derivatives)	Long-only holdings of cash, stocks and bonds
Regulation and transparency	Less	More
Fees	Higher	Lower
Structures	Limited partnerships and other special legal forms, with management and performance fees	Mutual funds, directly held securities

Resulting characteristics

- Investment structures designed to let the manager invest directly in the underlying assets.
- **Information asymmetry** between managers and investors, which funds try to offset with incentive-based fees.
- Difficulty appraising performance: historical return and volatility data are scarce and less reliable.
- Correlations that are low *on average* can rise sharply during periods of economic stress.

Three broad categories

1. **Private capital**: **private equity** and **private debt**. Private equity funds hold shares of unlisted firms, or buy listed firms with the aim of delisting them; many targets sit in the mature or declining phase of their industry. Most private equity funds are **leveraged buyout** (LBO) funds, which finance the purchase of established firms largely with debt. **Venture capital** funds finance new firms with no track record, in their start-up and early phases. Private debt funds make direct loans, provide **venture debt** to young firms, or buy **distressed debt** of borrowers that are struggling with payments or are in bankruptcy.
2. **Real assets**: real estate, natural resources, infrastructure and other real assets.
   - Real estate covers residential and commercial property and real estate-backed debt. It can be held through outright or leveraged ownership of a building, a single property loan, private or listed securities whose collateral is a pool of properties or mortgages, or a limited partnership.
   - Natural resources are commodities, farmland and timberland. Commodity exposure can come from physical holdings, commodity derivatives (some funds track a commodity index with futures) or shares of commodity producers. Farmland earns lease income or income from crops and livestock; timberland earns cash flows from harvesting trees.
   - Infrastructure means long-lived assets that provide public services: economic infrastructure (roads, airports, utility grids) and social infrastructure (schools, hospitals). Governments often finance and build infrastructure, but more recently public-private partnerships have also taken large stakes in such projects, and an asset may later return to public ownership.
   - Other real assets include collectibles (art), intangibles (patents) and **digital assets** (cryptocurrencies).
3. **Hedge funds**: pooled vehicles open to qualified investors that may use leverage, short positions and derivatives. Despite the name, they do not necessarily hedge.

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COMMON EXAM TRAPS

- Read the direction of a comparison. A question on how *traditional* investments differ wants the mirror image of the alternatives column: lower fees, greater liquidity, less specialized managers.
- Diversification rests on *low* correlation with stocks and bonds. An option that cites high correlation as a benefit is wrong, even though correlations can rise in a crisis.

LOS 80.b — Fund investing, co-investing and direct investing

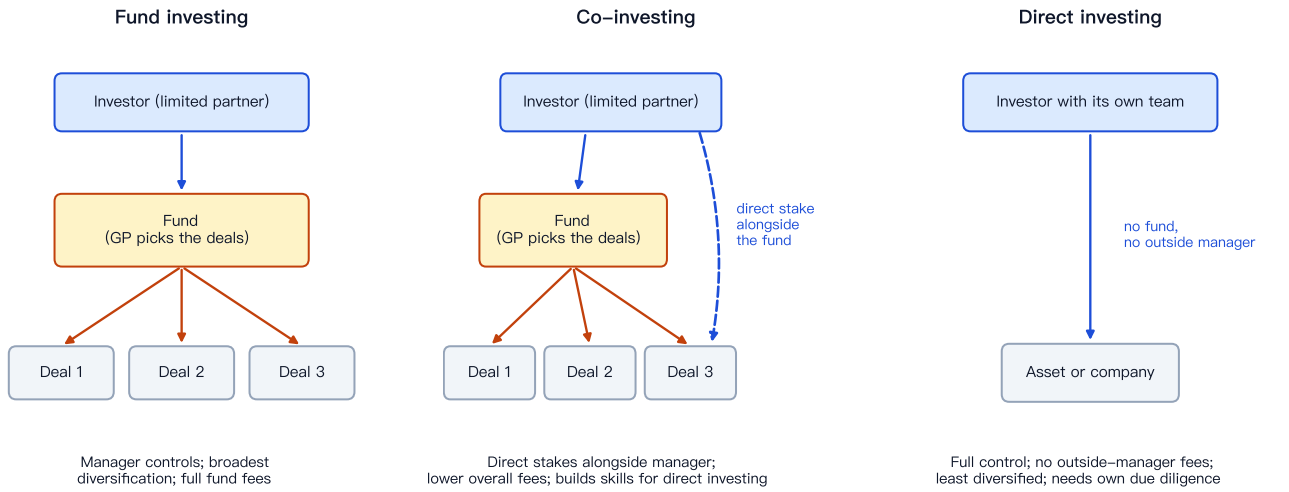
KEY CONCEPT

Method	What the investor does	Control	Fees	Diversification	Minimum size / expertise
Fund investing	Pools money with other investors; a fund manager selects and manages the assets	Manager controls	Management + incentive fees	Broadest	Lowest amounts; manager does the due diligence
Co-investing	Invests in the fund *and* has the right to invest directly alongside the manager in some deals	Investor takes selected direct stakes alongside the manager	Lower overall than pure fund investing	Moderate	Larger amounts; builds skills for direct investing
Direct investing	Buys assets itself (e.g., a sovereign wealth fund buying farmland or a private company)	Full control	No outside-manager fees	Least	Highest amounts; must do its own due diligence

A fund's **term sheet** sets out its investment policy, fee structure and the requirements investors must meet to participate.

The fee saving in co-investing comes from the part of the money that goes into a deal directly rather than through the fund, while the investor still relies on the manager to find and evaluate the deal. For the manager, allowing co-investment can bring in more capital and widen the scope and diversification of the fund.

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Fund investing, co-investing and direct investing: where the investor's money goes

COMMON EXAM TRAPS

- A small investor who wants diversification, or one with no in-house deal team, belongs in fund investing, because the manager does the due diligence.
- Direct investing wins on control and fees but loses on diversification, minimum size and required expertise. The cheapest method is the wrong answer for an investor without scale or staff.

LOS 80.c — Ownership and compensation structures

Limited partnerships. The **general partner (GP)** is the fund manager: it makes all investment decisions and bears the partnership's liabilities, including its debt. The **limited partners (LPs)** are the investors: they own shares in proportion to their contributions, have no say in management and no liability beyond their investment. LP interests are usually limited to **accredited investors**, who have the wealth to bear significant risk and the sophistication to understand it, and a partnership typically caps the number of LPs it admits. LPs often commit an amount up front and contribute it over time as the GP calls capital.

- The **limited partnership agreement** holds the rules of the partnership.
- A **side letter** gives terms to one LP that others do not get (e.g., an **excusal right** to skip a capital call). A **most-favored-nation clause** requires that special terms granted to other LPs also be offered to this LP.
- A **master limited partnership (MLP)** is publicly traded; MLPs are common in natural resources and real estate.

Management fee. Typically 1%–2% a year, earned regardless of performance.

- Hedge funds: a percentage of **assets under management (AUM)**, usually the fund's NAV.
- Private equity funds: a percentage of **committed capital** rather than invested capital. Committed capital is **drawn down** over roughly three to five years, at the manager's discretion; undrawn commitments are **dry powder**. Charging on committed capital removes the incentive to invest quickly rather than selectively.

A fee quoted as "m and p" means an $m\%$ management fee and a $p\%$ performance fee, so "2 and 20" is a 2% management fee and a 20% incentive fee.

Calculating the performance fee

Performance fee. The **performance fee** (also **incentive fee** or **carried interest**) is a share of profits. It is usually paid only after a **hurdle rate** (**preferred return**) is reached: the minimum return the fund must earn before the GP receives any performance fee. Below the hurdle, the GP earns no performance fee at all.

- **Soft hurdle rate**: once the hurdle is met, the fee applies to the *whole* gain.
- **Hard hurdle rate**: the fee applies only to the gain *above* the hurdle.
- **Catch-up clause**: after LPs receive the hurdle return, the GP receives 100% of the next slice of gains until it has received its full share of all gains so far; after that, gains are split normally. The clause favors the GP.
- Performance fees are usually settled once a year. The base is typically the year's gain in the fund's investments net of the management fee and other charges, such as consulting and monitoring fees billed to portfolio companies. A fund's terms may instead compute the performance fee independently of the management fee, on the full gain.
- **High-water mark**: no performance fee on gains that only recover earlier losses. The fee is paid only on value above the highest previous net-of-fees value, so investors are never charged twice for the same gain. Each investor can have a different high-water mark.

With a performance fee rate p, a hurdle h and a fund return R at or above the hurdle (no fee if $R < h$):

$$\text{Fee}_{\text{soft hurdle}} = p \times R \qquad \text{Fee}_{\text{hard hurdle}} = p \times (R - h) \qquad \text{full catch-up reached at } R = \frac{h}{1-p}$$

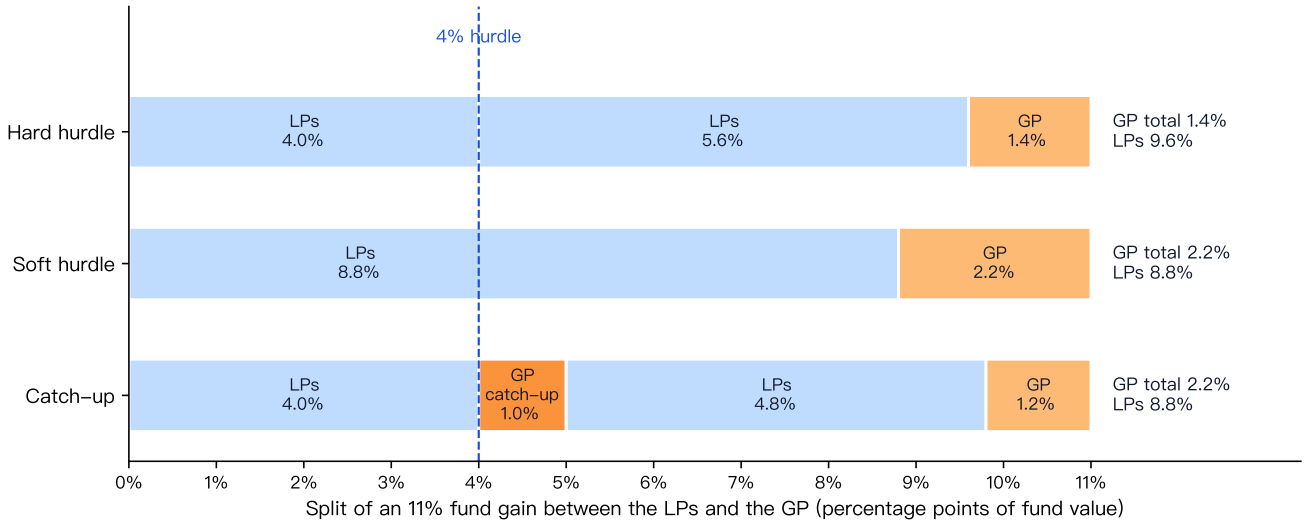
With a full catch-up, the GP receives $R - h$ while R is between h and $h/(1-p)$, and $p \times R$ once R is above that point.

> *Example.* A fund earns 11% in a year with a 20% performance fee and a 4% hurdle.

$$\text{Soft hurdle: } 0.20 \times 11\% = 2.2\% \qquad \text{Hard hurdle: } 0.20 \times (11\% - 4\%) = 1.4\%$$

With a catch-up instead, LPs get the first 4%, the GP gets 100% of the next slice until it holds 20% of the total. That total is $4\%/0.80 = 5\%$, so the GP's catch-up is 1%. The remaining $11\% - 5\% = 6\%$ is split 80/20, which gives the GP another 1.2%. The GP's total is $1\% + 1.2\% = 2.2\%$, exactly 20% of 11%.

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Splitting an 11% gain with a 4% hurdle and a 20% performance fee

The catch-up gives the same result as a soft hurdle once the gain is large enough to complete the catch-up, here 5%. Between the 4% hurdle and 5%, the GP receives less than a soft hurdle would pay. Had the fund earned 4.6%, the GP would receive the whole 0.6% above the hurdle, less than its full 20% share of 0.92%.

Waterfalls and clawbacks

Waterfalls. A waterfall sets how realized profits are split between the GP and the LPs:

KEY CONCEPT

	Deal-by-deal waterfall (American waterfall)	Whole-of-fund waterfall (European waterfall)
When the GP is paid	As each investment is sold	Only after LPs get back 100% of their capital plus the hurdle
Who it favors	GP	LPs

The deal-by-deal waterfall favors the GP because carried interest is paid on each profitable exit before the LPs have received all of their contributed capital plus the hurdle return.

A **clawback provision** lets LPs recover incentive fees paid on earlier gains that are later reversed by losses on other deals. For example, if the GP took fees on early winners, the clawback makes sure the LPs still receive their agreed share (e.g., 80%) of the fund's total profit. A clawback is most valuable with a deal-by-deal waterfall.

COMMON EXAM TRAPS

- "Invested capital" is the usual wrong answer for a private equity fund's management fee base.
- Check whether the stated hurdle is soft or hard before computing the fee; the two give different answers from the same inputs. In the example, applying a hard hurdle as if it were soft gives a fee of 2.2% instead of 1.4%.
- Sort provisions by whom they favor. GP: catch-up clause, deal-by-deal (American) waterfall. LPs: high-water mark, clawback, whole-of-fund (European) waterfall.

- A high-water mark stops new fees until earlier losses are recovered; a clawback takes back fees already paid. Options often swap the two.

BOTTOM LINE

- Alternative investments are everything outside long-only holdings of cash and publicly traded stocks and bonds, and they fall into three broad categories: private capital, real assets and hedge funds.
- Investors add alternatives for diversification, because their returns have relatively low correlations with stocks and bonds, and for higher potential returns from an illiquidity premium and less efficient markets, although correlations that are low on average can rise sharply under economic stress.
- Compared with traditional investments, alternatives have more specialized managers, less liquid assets, longer horizons, larger commitments, less regulation and transparency, higher fees and scarce, less reliable return data.
- Fund investing gives the broadest diversification with the lowest amounts and leaves due diligence to the manager, co-investing lowers fees overall by putting part of the money directly into deals alongside the manager, and direct investing gives full control with no outside-manager fees but the least diversification and the highest size and expertise requirements.
- In a limited partnership the GP makes all investment decisions and bears the partnership's liabilities, while LPs own shares in proportion to their contributions, have no say in management and no liability beyond their investment.
- The management fee, typically 1%–2% a year regardless of performance, is charged on AUM for hedge funds and on committed capital for private equity funds.
- The performance fee is paid only once a hurdle rate is met, on the whole gain under a soft hurdle and only on the gain above the hurdle under a hard hurdle, and a high-water mark bars fees on gains that merely recover earlier losses.
- A deal-by-deal (American) waterfall and a catch-up clause favor the GP, a whole-of-fund (European) waterfall, a high-water mark and a clawback favor the LPs, and a clawback is most valuable with a deal-by-deal waterfall.

QUICK CHECK

Question 1

A trustee new to the investment committee of a family foundation asks what sets alternative investments apart as a group. Which statement is *most accurate*?

A They trade in markets that tend to be more efficient than those for listed stocks and bonds.

B They cover investments other than long-only holdings of cash and publicly traded stocks and bonds.

C Their returns tend to be highly correlated with the returns on traditional investments.

Answer: B

Alternative investments are defined by exclusion: traditional investments are long-only positions in cash and publicly traded stocks and bonds, and everything else is an alternative. The group includes private capital, real assets such as real estate, infrastructure and natural resources, and hedge funds, which may use short positions, leverage and derivatives.

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Why the other options are wrong

- **A.** The reverse holds. Markets for some alternatives are thought to be less efficient than public stock and bond markets, which is one reason investors hope to earn higher returns from them.
- **C.** Alternatives are valued for their relatively low correlation with stocks and bonds, which is the source of their diversification benefit. Correlations can rise in periods of stress, but they are not typically high.

Key takeaway Traditional means long-only cash, listed stocks and listed bonds. Anything outside that definition is an alternative investment. Alternatives tend to have relatively low correlation with traditional assets, and some trade in markets that may be less efficient.

Practice questions

Question 2

A large insurer is weighing whether to buy stakes in private companies itself instead of committing money to a private equity fund. Compared with fund investing, direct investing *most likely* offers:

A lower fees.

B broader diversification.

C smaller minimum investment amounts.

Question 3

A foundation holds stakes in both a hedge fund and a private equity fund. The management fee charged by the *hedge fund* is *most likely* calculated as a percentage of:

A assets under management.

B committed capital.

C invested capital.

Question 4

The partnership agreement of Halvorsen Growth Partners provides that, each time a portfolio company is sold, the profit on that sale is split between the general partner and the limited partners immediately, without waiting for the rest of the portfolio to be exited. This arrangement is *best* described as a:

A whole-of-fund waterfall.

B deal-by-deal waterfall.

C clawback provision.

Answers: 2. A · 3. A · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/alternative-investments/alternative-investment-features-methods-and-structures/. The full bank has 26 questions on this reading.

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ALTERNATIVE INVESTMENTS · READING 81

Hurdle Rate

What you'll learn

LOS 81.a Describe why alternative investment performance is hard to appraise: life-cycle timing (J-curve, IRR, money multiple), leverage, valuation (fair value hierarchy) and fees.

LOS 81.b Calculate and interpret alternative investment returns before and after management fees, performance fees, hurdle rates and high-water marks.

MODULE 81.1

Performance Appraisal and Return Calculations

This reading explains how to appraise alternative investments given the J-curve, leverage, hard-to-value assets and complex fee terms, and how IRR and MOIC differ. It then covers redemption terms and index biases and shows how to calculate returns before and after management and performance fees, with hurdles, high-water marks, fund-of-funds fees and waterfalls.

LOS 81.a — Appraising the performance of alternative investments

Alternative investments carry risks that an unleveraged, long-only stock or bond portfolio does not. Returns should, as far as possible, be judged with these in mind:

1. Timing of cash flows over the fund's life cycle
2. Use of leverage
3. Valuation of assets that may have no observable market price
4. Complexity of fees, taxes and accounting

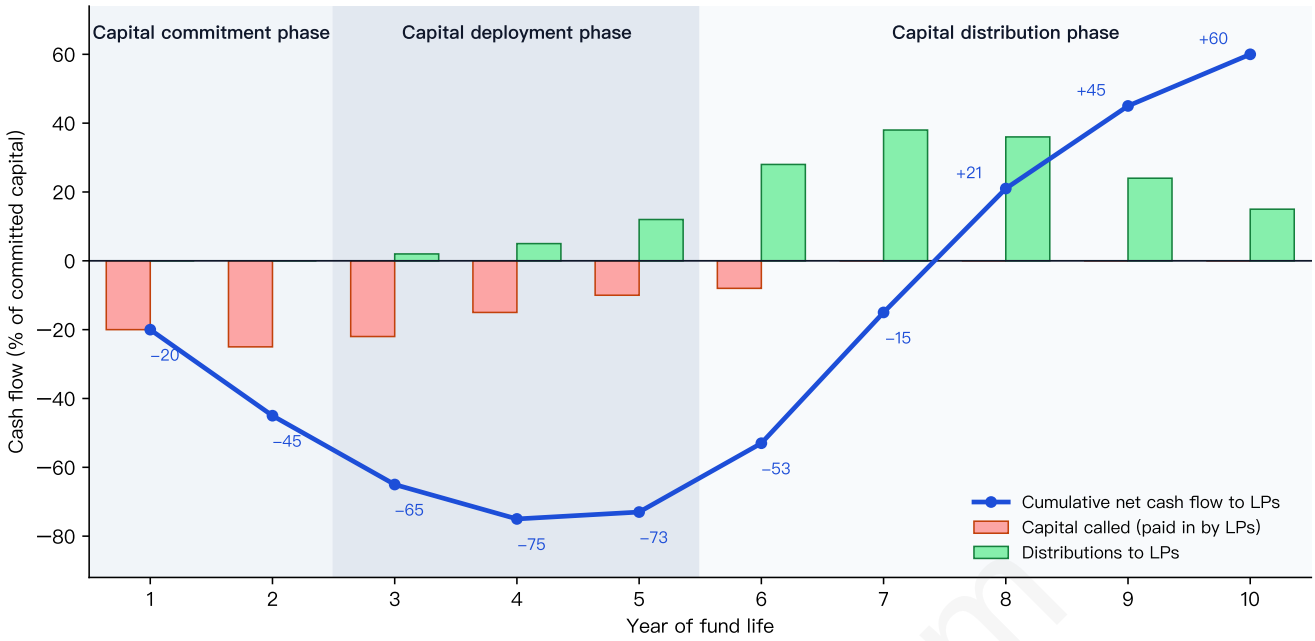
Life cycle and the J-curve

KEY CONCEPT

Phase	What happens	Typical returns
Capital commitment phase	Manager finds deals and makes **capital calls** on LPs' commitments	Negative
Capital deployment phase	Capital is invested; manager works with the portfolio companies or projects	Still negative (especially for start-ups or troubled firms being turned around), but improving
Capital distribution phase	Investments produce income and are exited	Positive and rising, and may level off near the end of the fund's life

Plotting cumulative returns, or the LPs' cumulative net cash flow, against time gives the **J-curve effect**: an early dip followed by a steep rise. The dip comes from the timing of cash flows. Capital calls come first, and distributions start only after the investments begin producing income or are sold.

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J-curve: capital calls come first and distributions later (commitment of 100)

Because the manager controls the timing and size of cash flows, the **IRR** (a money-weighted return) over the fund's life is the most appropriate performance measure. Its drawback is that it depends on assumed rates: a financing cost for the capital the investor contributes and a reinvestment rate for the distributions the investor receives. A simpler measure is the **multiple of invested capital** (MOIC), also called the **money multiple**:

$$\text{MOIC} = \frac{\text{total capital returned} + \text{value of remaining assets}}{\text{total capital paid in}}$$

It ignores timing, so it is considered somewhat naive.

> *Example.* In the fund shown in the figure, LPs pay in 100 in total and receive 160 in distributions, with nothing left in the fund at the end, so MOIC $= 160/100 = 1.6\times$. The IRR on the same annual cash flows is about 11.3%. If every distribution arrived two years later, MOIC would still be $1.6\times$, but the IRR would fall to about 7.6%. The money multiple cannot see the delay; the IRR can.

Leverage

Hedge funds in particular may borrow through margin financing from **prime brokers** or gain leverage through derivatives. If a fund with equity V_0 borrows V_B at rate r_B and earns r on the whole portfolio:

$$\text{leveraged return} = \frac{r(V_0 + V_B) - r_B V_B}{V_0}$$

> *Example.* $V_0 = \$40$ million, $V_B = \$20$ million, $r = 7\%$, $r_B = 3\%$.
> $(0.07 \times 60 - 0.03 \times 20)/40 = (4.2 - 0.6)/40 = 9.0\%$, versus 7% unleveraged.

Rearranging the formula shows where the extra return comes from:

KEY CONCEPT

$$\text{leveraged return} = r + \frac{V_B}{V_0}(r - r_B)$$

Leverage adds the spread between the portfolio return and the borrowing rate, scaled by the ratio of borrowing to equity. In the example, $7\% + 0.5 \times (7\% - 3\%) = 9\%$. The same ratio works against the fund when r is below r_B: if the portfolio returned -4%, the fund would return $-4\% + 0.5 \times (-4\% - 3\%) = -7.5\%$.

Some strategies use leverage because the pricing anomalies they exploit are too small to earn a meaningful return without it. Leverage also brings risks. **Margin calls** can force sales at bad prices, large forced sales can push prices down further, and lenders may cut off further borrowing.

Valuation: the fair value hierarchy

Level	Inputs	Examples
Level 1	Quoted prices in active markets	Exchange-traded securities
Level 2	Observable inputs, directly or indirectly (models)	Many derivatives
Level 3	Unobservable inputs; few or no transactions	Private equity, real estate

Level 3 values can stay near cost for long periods and may not reflect exit values. Because these values change little, reported returns are smoothed, so risk and correlation with traditional investments look lower than they really are. Exam convention: stale Level 3 values can also make reported returns appear higher (for example, when write-downs of assets that have lost value are delayed). Current practice: staleness by itself only smooths returns; a lag can hold back a gain as well as delay a write-down, so the direction of any effect on the average return is not fixed.

Fees are negotiable

Fee terms are negotiated and can differ by investor and by when the investor committed. As a result, different investors in the same fund can earn different returns.

COMMON EXAM TRAPS

- "Improving" in the deployment phase does not mean positive. Returns typically turn positive only in the distribution phase.
- Stale Level 3 values make a fund look *less* volatile and *less* correlated with public markets, not more.

LOS 81.b — Returns before and after fees

Redemption and fee terms

Weak returns tend to trigger redemption requests, which, like margin calls, can force a fund to sell at bad prices and at bad times. Early returns are usually negative (the J-curve), so funds (hedge funds in particular) restrict early redemptions:

- **Lockup period**: time after the initial investment during which LPs cannot redeem, or can redeem only with significant fees.

- **Notice period**: time (typically 30–90 days) the fund has to meet a redemption request, so positions can be reduced in an orderly way.
- **Redemption fees** offset the manager's transaction costs. A **gate** lets the manager restrict redemptions temporarily.
- **Side letters** contain investor-specific terms that differ from the standard offering documents.
- **Founders class shares**: better fee or liquidity terms given to early investors.
- **Either-or fees**: each year the investor pays the *greater* of the management fee and the incentive fee. For example, under terms of 1% (management) or 30% (incentive), the investor pays 1% unless the incentive fee works out higher; the terms may also deduct a management fee paid from a later year's incentive fee.

"2 and 20" for single funds and "1 and 10" for funds of funds were once common, but fees are under competitive pressure. Larger commitments can negotiate lower fees, and investors can trade off lower fees against better liquidity (shorter lockup and notice periods). Hurdle rates, the choice of a hard or soft hurdle, and catch-up terms can also be negotiated.

Index biases

Index returns tell an investor little about alternatives as a group: no two funds are structured alike, and the funds in an index may be at very different stages of their life cycles. Comparing funds of the same **vintage year** deals with the second problem. **Survivorship bias** (failed funds drop out) is especially large for hedge funds, since by some estimates more than a quarter of them fail within their first three years; it overstates returns and understates risk. **Backfill bias** (managers add only their successful funds, with their history) may magnify the effect.

After-fee return mechanics

The simplest case has a management fee m on end-of-period assets, a performance fee p on the total gain and no hurdle:

$$\text{total fees} = mV_1 + \max\left[0,\ p(V_1 - V_0)\right] \qquad r_{\text{after fees}} = \frac{V_1 - V_0 - \text{total fees}}{V_0}$$

Each of these provisions must be stated in the fund's terms (or the question); none should be assumed:

KEY CONCEPT

Provision	Effect on the performance fee
Net of management fee	Fee base = gain minus management fee
Independent of management fee	Fee base = full gain
Soft hurdle rate	Once the hurdle is met, fee on the whole gain
Hard hurdle rate	Fee only on the gain above the hurdle
High-water mark	Fee only on value above the highest previous net-of-fees value
Management fee on beginning vs ending assets	Changes the management fee amount

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Steps for an after-fee return.

1. Compute the management fee on the stated asset base (beginning or end of the period).
2. Find the performance fee base: the gain above the high-water mark, or above the beginning value if there is none, after deducting the management fee if the performance fee is net of it.
3. Apply the hurdle, measured from the same reference value (the high-water mark or the beginning value). Under a soft hurdle, the whole base earns the fee once the return clears the hurdle, and no fee is due if it does not. Under a hard hurdle, subtract the hurdle amount from the base.
4. Multiply the base by the performance fee rate. The fee is zero if the base is zero or negative.
5. Add the two fees, subtract them from the ending value and compute the return on the beginning value, as in the formula above.

> *Example.* A $60 million fund charges "2 and 20", with the management fee on year-end assets and the incentive fee net of the management fee. The gross return is 10%.

- $V_1 = 60 \times 1.10 = \$66$ million; management fee $= 0.02 \times 66 = \$1.32$ million
- Incentive fee $= 0.20 \times (66 - 60 - 1.32) = \$0.936$ million
- After-fee return $= (66 - 1.32 - 0.936)/60 - 1 = 6.24\%$

> *Example (high-water mark with a hurdle).* A fund starts at $40 million. It charges a 1% management fee on beginning-of-year assets and a 20% performance fee on gains net of the management fee, with a 5% hurdle and a high-water mark.

- Year 1: assets fall to $36.4 million before fees. The management fee is $0.01 \times 40 = \$0.4$ million, so the year ends at $36.0 million. No performance fee is due, and the high-water mark stays at $40 million.
- Year 2: assets rise to $44.36 million before fees. The management fee is $0.01 \times 36 = \$0.36$ million, which leaves $44.0 million. The gain above the high-water mark is $44.0 - 40.0 = \$4.0$ million, a 10% return measured from $40 million.
- Soft hurdle: 10% clears 5%, so the fee is $0.20 \times 4.0 = \$0.8$ million. The year ends at $43.2 million, an after-fee return of $43.2/36.0 - 1 = 20.0\%$.
- Hard hurdle: the fee applies only above $40 \times 1.05 = \$42$ million, so it is $0.20 \times (44.0 - 42.0) = \$0.4$ million. The year ends at $43.6 million, a return of $43.6/36.0 - 1 = 21.1\%$.

Without the high-water mark, the soft-hurdle fee would be charged on the whole $44.0 - 36.0 = \$8.0$ million rise, which is $1.6 million. Half of that fee would be charged on the $4.0 million that only recovers the Year 1 loss.

Fund of funds. An investor pays two layers of fees: the underlying funds' fees (already reflected in their net values) plus the fund of funds' own fees on top.

> *Example.* An investor puts $30 million into a fund of funds that charges "1 and 10", with both fees computed independently on year-end values. The fund of funds places $18 million with one hedge fund and $12 million with another. A year later the two stakes are worth $20.4 million and $13.2 million, net of the hedge funds' own fees.

- Gross year-end value $= 20.4 + 13.2 = \$33.6$ million
- Management fee $= 0.01 \times 33.6 = \$0.336$ million; incentive fee $= 0.10 \times (33.6 - 30.0) = \$0.36$ million

- Net value $= 33.6 - 0.336 - 0.36 = \$32.904$ million, a return of $32.904/30 - 1 = 9.68\%$

Holding the same two stakes directly would have returned $33.6/30 - 1 = 12\%$. The gap is the fund of funds' layer of fees.

Waterfalls and clawback. Under a deal-by-deal (American-style) waterfall, the GP earns a fee on each profitable exit even if other deals lose money. Under a whole-of-fund (European-style) waterfall, the fee is based on the net gain across all deals. A clawback lets LPs recover the excess so that total fees equal the agreed share of the fund's net gain.

Example. A fund with a 20% carried interest puts $50 million into each of two companies. One is sold for $70 million and the other for $40 million in the same year. Deal-by-deal, the GP earns $0.20 \times 20 = \$4$ million on the winner and nothing on the loser. Whole-of-fund, the net gain is $110 - 100 = \$10$ million and the fee is $2 million. A clawback returns $4 - 2 = \$2$ million to the LPs. The risk for LPs is greatest when the winners are sold before the losers: a deal-by-deal fee is paid on the early exit, and only a clawback recovers it once the later loss is realized. Exam convention: if the winning deal is sold in an earlier year than the losing deal, a performance fee computed year by year makes the whole-of-fund result the same as the deal-by-deal result. Current practice: a whole-of-fund waterfall pays the GP no carried interest until LPs have received all contributed capital (plus any hurdle), so an early winner on its own does not trigger a fee.

COMMON EXAM TRAPS

- Subtracting fee percentages from the return (e.g., "10% − 2% − 20%") gives a wrong answer; the fees are money amounts computed on assets and gains.
- Charging the incentive fee on the gross gain when the terms say net of the management fee (or the reverse) gives a distractor close to the key. In the "2 and 20" example, a fee on the full $6 million gain is $1.2 million and gives 5.80% instead of 6.24%.
- After a loss year, the prior year-end value is the wrong base for a fund with a high-water mark; the fee starts only above the previous peak. In the high-water mark example, measuring from $36.0 million gives a soft-hurdle fee of $1.6 million and a return of 17.8% instead of $0.8 million and 20.0%.

EXAM SHORTCUTS

- Delaying distributions without changing their amounts leaves MOIC unchanged but lowers the IRR, so when only timing changes, MOIC can be ruled out as the measure that reflects it.

BOTTOM LINE

- Over a fund's life returns are negative in the capital commitment phase, still negative but improving in the deployment phase and positive in the distribution phase, the J-curve, which makes the IRR the most appropriate measure, while MOIC ignores timing.
- The leveraged return is $r + (V_B/V_0)(r - r_B)$, so borrowing adds return when the portfolio earns more than the borrowing rate and deepens losses when it earns less, and margin calls can force sales at bad prices.
- Level 3 values rest on unobservable inputs and change little, which smooths reported returns so risk and correlation with traditional investments look lower than they are; exam convention: stale values can also make returns look higher; current practice: staleness by itself only smooths, and the direction of

any effect on the average return is not fixed.
- Lockup periods, notice periods, redemption fees and gates restrict redemptions, and under either-or fees the investor pays the greater of the management fee and the incentive fee each year.
- Survivorship bias, especially large for hedge funds, overstates index returns and understates risk, backfill bias may magnify it, and comparing funds of the same vintage year deals with differences in life-cycle stage.
- An after-fee return requires the stated asset base for the management fee, whether the performance fee is net of or independent of the management fee, the type of hurdle and any high-water mark, none of which should be assumed.
- An investor in a fund of funds pays the underlying funds' fees, already reflected in their net values, plus the fund of funds' own fees on top.
- Under a deal-by-deal (American) waterfall the GP earns a fee on each profitable exit even if other deals lose money, a whole-of-fund (European) waterfall bases the fee on the net gain across all deals, and a clawback lets LPs recover the excess.

QUICK CHECK

Question 1

Two private equity funds each invested all of their investors' capital at inception and each made a single distribution. Fund Aldgate took in $60 million; at the end of Year 4 it distributed $84 million, and its remaining holdings were then valued at $24 million. Fund Brixton took in $50 million; at the end of Year 8 it distributed $70 million, and its remaining holdings were then valued at $20 million. Which statement is *most* accurate?

A Each fund's MOIC is 1.40, because only distributed capital counts as returned.

B Each fund's MOIC is 1.80, so the two funds earned the same annualized return.

C Each fund's MOIC is 1.80, but Aldgate earned the higher annualized return.

Answer: C

The multiple of invested capital (MOIC) divides the total capital returned plus the value of the remaining assets by the total capital paid in. Both funds have a MOIC of 1.80. MOIC ignores the timing of cash flows, however. Aldgate reached 1.80 times its capital in four years and Brixton needed eight, so Aldgate's annualized return (its IRR) is much higher.

$$\text{MOIC} = \frac{\text{capital returned} + \text{value of remaining assets}}{\text{capital paid in}}$$

Aldgate: $(84 + 24)/60 = 1.80$. Brixton: $(70 + 20)/50 = 1.80$.

With one outflow at inception and all value measured at the end of year n, the annualized return (IRR) is $\text{MOIC}^{1/n} - 1$:

Aldgate: $1.80^{1/4} - 1 = 15.8\%$ a year. Brixton: $1.80^{1/8} - 1 = 7.6\%$ a year.

Why the other options are wrong

- **A.** 1.40 counts only the distributions ($84/60$ and $70/50$). MOIC also includes the value of the holdings that have not yet been realized.
- **B.** The MOICs are equal, but MOIC says nothing about how long the capital was tied up. The same multiple earned over four years and over eight years implies annualized returns of 15.8% and 7.6%.

Key takeaway MOIC is simple to compute and interpret but ignores timing; the IRR accounts for when cash flows occur and is the preferred measure when the manager controls that timing.

Practice questions

Question 2

Brackwater Credit Partners, a hedge fund whose lockup period has expired for all investors, has a 10-day redemption notice period. It holds a few thinly traded corporate bonds, and each position is a large part of that bond's outstanding issue. In one month two issuers default, the fund's net asset value falls by 12%, its prime broker makes margin calls, and investors ask to redeem 40% of the partnership interests. The *most likely* result is that:

A the 10-day notice period gives the fund enough time to sell 40% of its holdings in an orderly way.

B forced sales are likely to push prices lower, deepen losses and prompt further redemption requests.

C prices received will be largely unaffected, since redemptions are paid only after the notice period.

Question 3

A private equity fund invests $60 million in Ashby Robotics and $40 million in Corran Freight. In the same year, Ashby is sold for $81 million and Corran is liquidated for $34 million. The general partner's carried interest is 20%, profits are distributed under a deal-by-deal (American-style) waterfall, and there is no clawback provision. The limited partners' return on the $100 million, after carried interest, is *closest* to:

A 15.0%.

B 12.0%.

C 10.8%.

Answers: 2. B · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/alternative-investments/hurdle-rate/. The full bank has 13 questions on this reading.

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ALTERNATIVE INVESTMENTS · READING 82

Mezzanine Financing

What you'll learn

LOS 82.a Explain private equity categories (LBO, MBO/MBI, venture capital stages, minority/PIPE), exit routes, and risk/return features.

LOS 82.b Explain the types of private debt (direct lending, unitranche, mezzanine, venture, distressed) and their risk and return characteristics.

LOS 82.c Describe the diversification benefits of private capital and the importance of vintage year and the business cycle.

MODULE 82.1

Private Capital

This reading covers private capital: the main private equity strategies and exit routes, the types of private debt, the diversification private capital offers and the importance of vintage years.

LOS 82.a — Private equity: categories, exits, risk and return

Private capital is funding for companies that is not raised in public markets. It covers **private equity**, which is equity capital raised outside the public markets, and private debt (LOS 82.b). Private equity funds typically buy stakes in private companies, buy public companies in order to take them private (leveraged buyout funds), or back companies early in their lives (venture capital funds). The companies a private equity fund invests in are its **portfolio companies**.

Leveraged buyouts

A **leveraged buyout (LBO)** fund buys a public company with debt financing a large percentage of the purchase price, taking it private. The fund then tries to improve operations so that the company's cash flows can service and repay the acquisition debt.

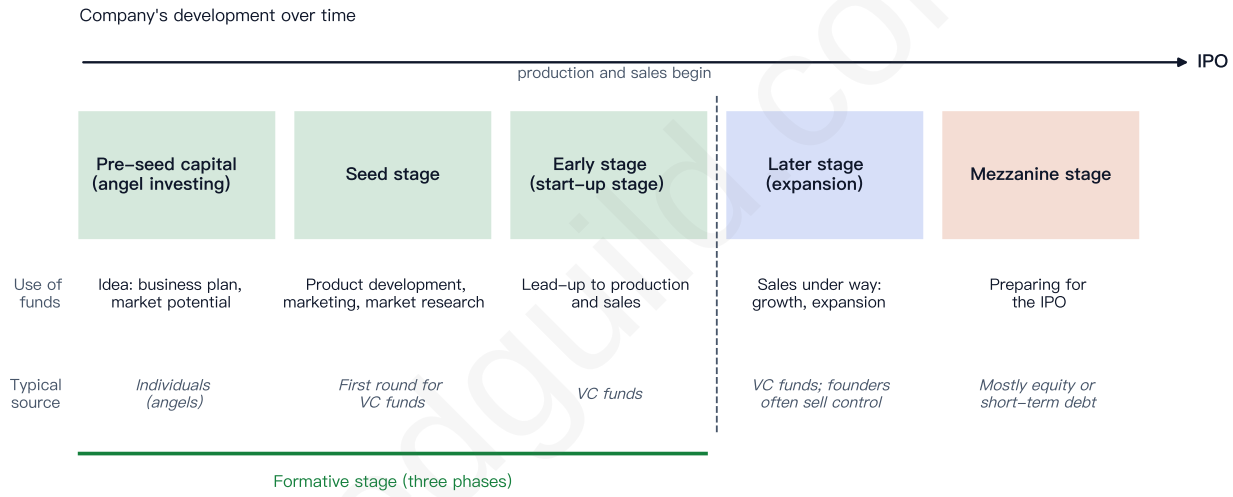
LBO type	Who runs the company afterwards
Management buyout (MBO)	The *existing* management team, which takes part in the purchase
Management buy-in (MBI)	A *new* management team brought in by the private equity manager to replace the current one

Venture capital

Venture capital (VC) funds back young companies, usually taking common equity, but sometimes **convertible debt** or **convertible preferred stock**. Convertibles align the interests of the VC investor and the founders while giving the VC investor a higher claim in liquidation. VC investors are hands-on: they often sit on boards or fill key roles. Risk is high, and returns can be very large.

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Stage	Use of the money	Typical source or form
Formative stage: **pre-seed capital** / **angel investing**	Idea stage: business plans, assessing market potential	Individuals ("angels"), small amounts
Formative stage: **seed stage** (**seed capital**)	Product development, marketing and market research	Usually the first round in which VC funds take part
Formative stage: **early stage** (**start-up stage**)	Running the business in the lead-up to production and sales	VC funds
Later stage (**expansion venture capital**)	Growth after production and sales have begun (expansion, product improvement, marketing)	VC funds; founders often sell control
Mezzanine stage (**mezzanine-stage financing**)	Preparing the company for an **initial public offering (IPO)**	Mostly equity or short-term debt



Venture capital stages along a company's development path

"Mezzanine stage" describes timing (just before the IPO), not the type of security. **Mezzanine financing** is a separate term for hybrid debt–equity securities such as convertibles.

Minority equity investing

Minority equity investing (developmental capital) buys a less-than-controlling stake to fund growth or restructuring. When the company is public, this is a **private investment in public equity (PIPE)**, a private offering of the company's shares to institutional investors. A PIPE lets the company raise capital faster and more cheaply than a public offering, with fewer disclosures. The company stays listed.

Exit strategies (average holding period of about five years)

Exit	Buyer / mechanism	Notes
Trade sale	A **strategic buyer**, such as a competitor, via direct sale or auction	Buyer often pays a premium for synergies; faster and cheaper than an IPO; management may resist; few buyers
IPO	Shares sold to the public with underwriters	Most common listing route; usually the highest price and more visibility; high transaction and compliance costs, and the market may receive it poorly; best for large firms in growing industries with stable finances and a clear strategy
Direct listing	Shares listed without underwriters	Lower cost than an IPO because there are no underwriters. Exam convention: a direct listing floats existing shares and raises no new capital for the company. Current practice: some exchanges also permit a *primary* direct listing, in which the company sells new shares.
Special purpose acquisition company (SPAC)	"Blank check" company that raised cash to buy a private firm	Must complete an acquisition within a stated period or return the cash. More flexible than an IPO, less uncertainty about the portfolio company's valuation, and access to investors who deal regularly with the sponsors; drawbacks include dilution from SPAC shares and warrants, a gap between the SPAC's value and the target's value, deal risk, stockholder overhang and growing regulatory scrutiny
Recapitalization	Portfolio company issues debt to pay a dividend	Not a true exit: the fund keeps control but extracts cash
Secondary sale	Another private equity firm or group of investors	
Write-off/liquidation	None	Loss is recognized

Risk and return

Private equity has historically returned more than public equity, with a higher standard deviation (illiquidity and leverage risk). Indexes are self-reported and subject to **survivorship bias** and **backfill bias**, which overstate returns. Infrequent valuation of portfolio companies biases measured volatility and correlation with other assets downward.

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COMMON EXAM TRAPS

- A question about the first round in which a VC *fund* invests wants the seed stage. Angel money comes earlier, but from individuals.
- A company with a finished product but no production or sales yet is at the early stage; later-stage (expansion) capital starts only once sales have begun.
- Selling to a competitor is a trade sale and selling to another private equity firm is a secondary sale. Options often swap the two.
- A PIPE leaves the company listed, while an LBO takes it private.

LOS 82.b — Private debt

Private debt is debt that investors provide directly to private entities. Its main types:

Type	Features	Relative risk
Direct lending	Loans made directly to private companies without an intermediary; usually **senior and secured**, with covenants. Exam convention: a **leveraged loan** is a loan made by a private debt fund that has borrowed money to fund its lending, so the fund's loan portfolio is leveraged. Current practice: in wider market usage (for example, CLO collateral), a leveraged loan is a loan to a highly indebted or low-rated borrower	Lowest of these five types
Unitranche debt	Secured and unsecured classes blended into one loan with a blended rate; ranks between senior and subordinated debt	Moderate
Mezzanine debt	Subordinated to senior secured debt, often unsecured; may carry **warrants** or conversion rights	High
Venture debt	Lending to start-ups that are not yet profitable; often convertible or paired with warrants; lets founders keep control	High
Distressed debt	Debt of **mature** companies in trouble (default, bankruptcy). Some investors look for sound companies with temporary cash flow problems and wait for recovery; others buy the debt to take an active role in restructuring or turning the company around	High

Private debt typically offers a higher return than traditional bonds to compensate for more risk (default, illiquidity). It adds diversification because its returns have relatively low correlation with traditional assets. Rates usually float over a reference rate such as SOFR. Senior private debt has steadier yields; mezzanine debt has more upside. Investing in private debt requires specialized knowledge of the debt's structure, the borrower's life cycle phase and, for secured loans, the underlying assets.

COMMON EXAM TRAPS

- In a comparison with traditional bonds, private debt offers the higher expected return. Options that give it greater liquidity or lower risk reverse the comparison.
- Warrants and conversion rights point to the riskier types (venture and mezzanine debt). Senior secured direct lending seldom carries them.

- The issuers of distressed debt are mature firms, not start-ups.

LOS 82.c — Diversification and vintage years

Private capital offers some diversification. Correlations of private capital index returns with public markets are reported at roughly 0.63–0.83.

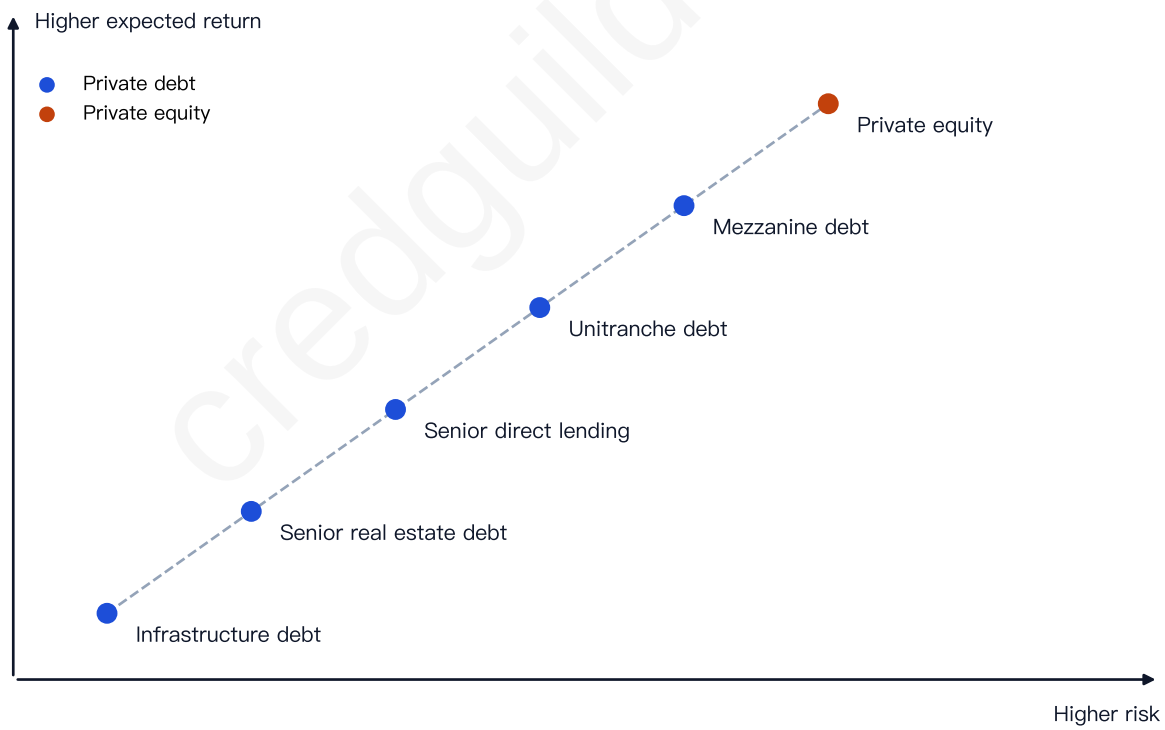
KEY CONCEPT

A fund's **vintage year** is the year it makes its **first investment**. This is not necessarily the year capital is first committed, because committed money may be deployed later. Vintage-year conditions matter a lot:

- Funds that start investing in a **business cycle expansion** tend to do better if they specialize in early-stage companies (venture capital).
- Funds that start investing in a **business cycle contraction** tend to do better if they specialize in distressed companies.

Investors should therefore **diversify across vintage years**.

Risk/return ranking (lowest to highest): infrastructure debt < senior real estate debt < senior direct lending < unitranche debt < mezzanine debt < private equity.



Risk and return across private capital categories

COMMON EXAM TRAPS

- Options that date a fund by its first commitment or its first exit are wrong; only the first investment sets the vintage year.

- A strong venture capital track record points to a vintage in an expansion.

EXAM SHORTCUTS

- Place a VC round by whether sales have begun: formative stages come before production and sales, later-stage capital comes after, and mezzanine-stage financing prepares for an IPO.

BOTTOM LINE

- An LBO fund buys a public company largely with debt and takes it private; an MBO keeps the existing management and an MBI brings in a new team.
- Venture capital runs from pre-seed (angel) and seed capital, the first round with VC funds, through early-stage and later-stage financing to mezzanine-stage financing before an IPO.
- Exit routes are a trade sale, an IPO, a direct listing, a SPAC, a secondary sale and a write-off; a recapitalization extracts cash but is not a true exit.
- A PIPE is a private sale of a listed company's shares, and the company stays listed.
- Among private debt types, direct lending carries the lowest risk, unitranche debt moderate risk, and mezzanine, venture and distressed debt high risk.
- Private equity indexes overstate returns through survivorship and backfill bias, and infrequent valuation understates volatility and correlation.
- A fund's vintage year is the year of its first investment, and investors should diversify across vintage years.

QUICK CHECK

Question 1

A private equity firm acquires Ashcombe Textiles in a leveraged buyout. As part of the deal, the firm removes Ashcombe's current executives and installs a leadership team it has recruited from outside the company. This transaction is *best* described as a:

A management buyout.

B management buy-in.

C private investment in public equity.

Answer: B

In a management buy-in (MBI), the private equity manager replaces the company's existing management with a new team. In a management buyout, the existing team takes part in the purchase.

Why the other options are wrong

- **A.** In a management buyout, the *existing* management team participates in the purchase and stays on.
- **C.** A PIPE is a minority investment in a company that stays public. It is not a buyout of the whole company.

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Key takeaway MBO = existing managers take part in the purchase; MBI = outside managers are brought in.

Practice questions

Question 2

A private equity fund exits its stake in Norquay Logistics by selling the company to Tidewell Freight, a larger rival in the same industry that expects cost synergies from the deal. This exit is *best* described as a(n):

A secondary sale.

B trade sale.

C initial public offering.

Question 3

To make fair comparisons, a consultant groups private equity funds by vintage year, so that each group holds funds that began putting capital to work under similar market conditions. A fund's vintage year is the year in which it:

A receives its first capital commitment.

B makes its first investment.

C exits its first investment.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/alternative-investments/mezzanine-financing/. The full bank has 19 questions on this reading.

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ALTERNATIVE INVESTMENTS · READING 83

Real Estate and Infrastructure

What you'll learn

LOS 83.a Explain real estate investment forms (public/private, debt/equity), direct vs. indirect investment, REIT types and real estate strategies.

LOS 83.b Explain the risk, return and diversification characteristics of real estate investments.

LOS 83.c Explain infrastructure categories, investment routes, cash-flow arrangements and greenfield/brownfield/secondary-stage assets.

LOS 83.d Explain the risk, return and diversification characteristics of infrastructure investments.

MODULE 83.1

Real Estate

This reading describes the forms of real estate investment, the benefits and drawbacks of direct and indirect ownership, REITs and the core to opportunistic strategies, and how real estate holdings range from bond-like to equity-like. It then describes infrastructure assets, their cash-flow arrangements and the greenfield, brownfield and secondary-stage categories, and compares their risk and return.

LOS 83.a — Features of real estate investments

Real estate earns **rental income** and may appreciate in price. Property divides into two main types, single-family residential and commercial. Within commercial property the four largest categories are office, rental residential (multifamily and single-family detached), retail (shopping) and industrial (warehouse and distribution).

The four basic forms (the real estate quadrant)

Investments are classified on two dimensions: public vs. private and debt vs. equity.

KEY CONCEPT

	Equity	Debt
Private	Direct ownership: sole ownership, joint ventures, limited partnerships (a GP manages the property and LPs such as pension plans invest). Indirect ownership: real estate funds, private REITs	Mortgage debt, construction loans, mezzanine debt
Public	Publicly traded shares: construction, operating and development companies, public **REIT** shares, UCITS, mutual funds and ETFs	Residential **MBS**, commercial **CMBS**, CMOs, covered bonds, mortgage REITs, mortgage ETFs

Each of the four forms has its own risk, expected return, regulation, legal issues and market structure.

- An **equity investor** owns the property (or shares of an owner) and controls decisions on borrowing, management and exit. Equity value = property value − outstanding debt.
- A **debt investor** is a lender. Its claim is secured by the property and ranks ahead of equity in default.

- Private investments are large, indivisible and illiquid. Public securities let investors split ownership without splitting the property, which gives liquidity and diversification across many properties. Direct private ownership needs property-management expertise; REITs and partnerships are professionally managed.

Direct investment

KEY CONCEPT

Benefits	Drawbacks
Control over purchase, financing, improvements, tenants and timing of sale	**Illiquidity** and **price opacity** (properties trade rarely, so current prices are hard to observe)
Diversification (returns imperfectly correlated with stocks and bonds)	Complexity of managing property
Tax benefits (noncash depreciation and interest deductions)	Need for specialized knowledge of current market conditions
	High initial capital; **concentration risk** with few properties

Indirect investment and REITs

Indirect routes include limited partnerships and joint ventures, where the investor holds a stake in a partnership that owns the property, and publicly traded securities. The quadrant lists these partnerships under private direct ownership because the partnership itself owns the property. **REITs** are exempt from double taxation, which makes them a popular way to hold income-producing real estate. They have specialized managers and report GAAP metrics such as EPS; some also report net asset value. Exam convention: REITs trade on exchanges, so unlike mutual funds they face no redemption or liquidation risk; investors exit by selling shares to other investors. Current practice: a REIT keeps its tax exemption only if it distributes most of its taxable income, and **non-traded REITs** also exist; they have no exchange liquidity, and their redemption programs may be limited or suspended.

- **Equity REITs** own property; **mortgage REITs** lend or buy MBS/CMBS; **hybrid REITs** do both.

Strategies (from least to most risky equity strategy)

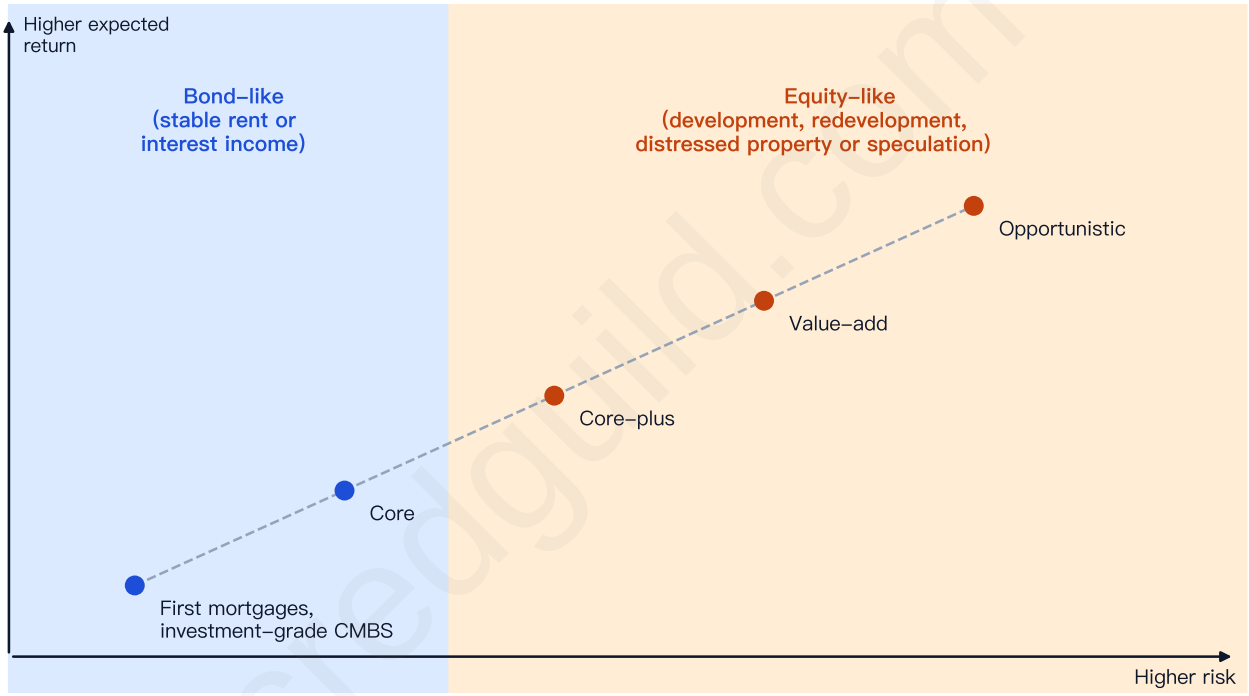
KEY CONCEPT

Strategy	What it does	Typical structure
Core	High-quality commercial and residential property; stable income	Open-end, indefinite life
Core-plus	A little more risk: modest development/redevelopment	Closed-end, finite life
Value-add	Larger-scale development/redevelopment	Closed-end, finite life

Strategy	What it does	Typical structure
Opportunistic	Large-scale redevelopment and repurposing, distressed property, market speculation	Closed-end, finite life

LOS 83.b — Investment characteristics of real estate

Real estate returns come from rental income, from price appreciation, or from both. The more a holding depends on steady income from leases, the more it behaves like a bond; the more it depends on gains from development or rising prices, the more it behaves like equity. First mortgages and investment-grade CMBS are the least risky real estate investments and behave much like bonds. Core strategies come next and are also bond-like, because their rent is stable and comes from many tenants. Core-plus, value-add and opportunistic strategies take on more risk in turn and are equity-like.



Real estate investments from bond-like to equity-like

- The diversification and liquidity that publicly traded REITs offer come with a cost: REIT returns are more correlated with equities than direct real estate returns are, and the correlation rises in steep market downturns.
- Even so, adding real estate improves the risk-return profile of a stock-and-bond portfolio.

Example. A closed-end fund buys an empty 1970s office block to convert into apartments. It has little current income and a large development budget, so it is an equity-like, value-add or opportunistic strategy rather than a bond-like core strategy.

COMMON EXAM TRAPS

- "Stable income from many tenants" and "open-end" describe core strategies. Both are wrong answers for a strategy built on development or redevelopment.

- Classify by the security held, not by the collateral: CMBS and mortgage REIT shares are public debt even though the underlying loans are private mortgages.
- The extra risk from owning a building directly, compared with listed stocks and bonds, is **liquidity risk**. Market risk is shared with traditional assets.

BOTTOM LINE

- Real estate investments are classified as public or private and as debt or equity; an equity investor owns the property and controls borrowing, management and exit decisions, while a debt investor's claim is secured by the property and ranks ahead of equity in default.
- Direct real estate gives control, diversification and tax benefits, but brings illiquidity, price opacity, management complexity, a need for specialized market knowledge, high initial capital and concentration risk.
- Exam convention: exchange-traded REITs face no redemption or liquidation risk, because investors exit by selling shares; current practice: a REIT keeps its tax exemption only if it distributes most of its taxable income, and non-traded REITs exist whose redemption programs may be limited or suspended.
- Equity real estate strategies run from least to most risky as core (high-quality property, stable income, open-end), core-plus, value-add and opportunistic (closed-end, finite life).
- The more a holding relies on steady lease income the more it behaves like a bond, and the more it relies on development or rising prices the more it behaves like equity, with first mortgages and investment-grade CMBS the least risky.
- Publicly traded REITs offer diversification and liquidity but are more correlated with equities than direct real estate, with the correlation rising in steep downturns, although adding real estate still improves a stock-and-bond portfolio's risk-return profile.

QUICK CHECK

Question 1

A pension fund buys a portfolio of commercial mortgage-backed securities. Within the basic four-quadrant classification of real estate investments, this holding is *best* classified as:

A public equity.

B private debt.

C public debt.

Answer: C

Real estate investments are classified as public or private, and as debt or equity. CMBS are traded securities that give a claim on pools of commercial mortgages, so they are public debt.

Why the other options are wrong

- **A.** Public equity would be shares of an equity REIT or a real estate company, which are ownership claims rather than claims on mortgages.
- **B.** Private debt would be a mortgage or loan held directly. CMBS are traded securities.

Key takeaway Debt vs. equity = lender vs. owner; public vs. private = traded securities vs. privately held interests.

MODULE 83.2

Infrastructure

LOS 83.c — Features of infrastructure

Infrastructure assets are long-lived, capital-intensive assets that provide public services.

KEY CONCEPT

Category	Examples
Economic infrastructure: transportation	Roads, airports, ports, railways
Economic infrastructure: utilities and energy	Gas distribution, electricity generation and distribution, **waste disposal and treatment**
Economic infrastructure: information and communication technology	**Telecom towers**, cable systems
Social infrastructure	**Prisons**, **schools**, **health care facilities** (hospitals)

Economic infrastructure supports economic activity: it moves people and goods, supplies energy and water, and carries data. Social infrastructure serves people directly, for example through education, health care and justice.

Ways to invest. Build the asset and sell or lease it to the government or operate it; buy an existing asset from the government to lease back or operate; or join a **public-private partnership**.

Cash-flow arrangements

- **Availability payments**: paid for making the asset available, regardless of use.
- **Usage-based payments**: e.g., highway tolls.
- **Take-or-pay arrangements**: the buyer must pay a minimum price for an agreed volume, whether or not it takes delivery.

Stage of the asset

KEY CONCEPT

Type	Meaning	Cash-flow pattern
Greenfield investment	Assets still **to be constructed**; typically follows **build-operate-transfer (BOT)**	Outflows while building, rising inflows once operating, then transfer to a government or third party
Brownfield investment	Expanding or privatizing an **existing** asset, e.g., a **sale-leaseback** with the government	Existing cash flows, though a newly privatized asset or sale-leaseback may have a short operating record

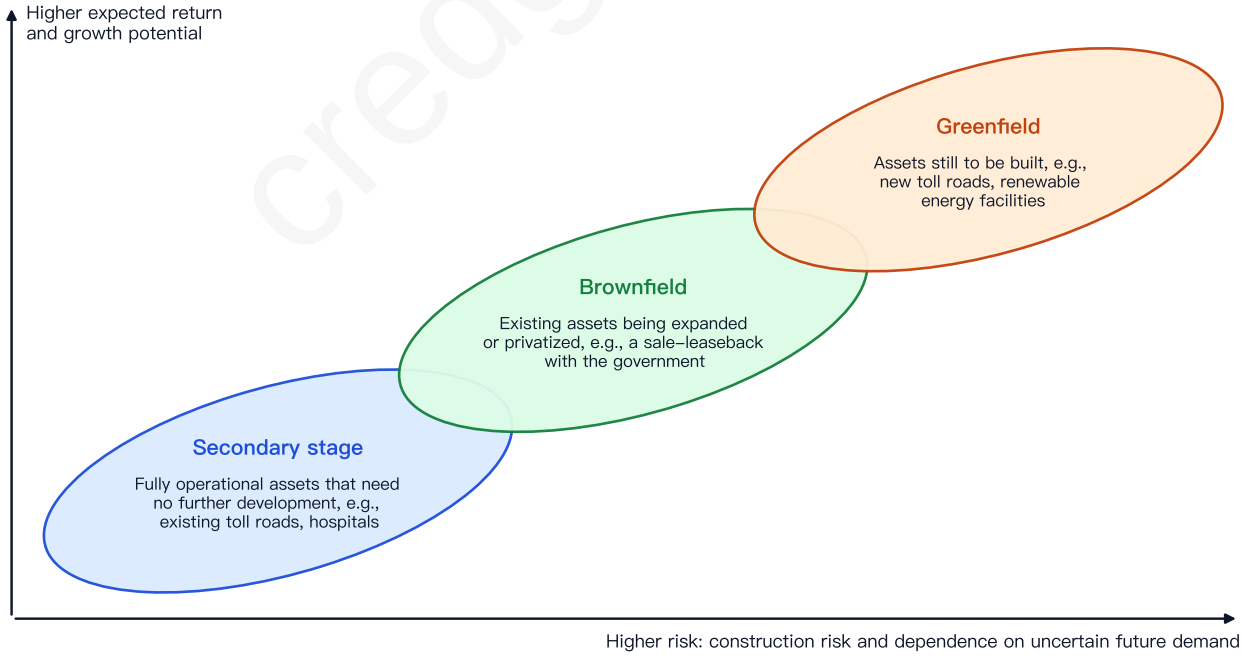
Type	Meaning	Cash-flow pattern
Secondary-stage investment (a type of brownfield investment)	Fully operational facility that needs **no further development**	Most predictable

Direct infrastructure investment, such as owning the asset or holding a first mortgage on it, is illiquid because the assets are huge and long-lived. Investors who want more liquidity can use **MLPs**, ETFs, mutual funds or private equity funds. The listed universe is small and concentrated in a few asset types. Infrastructure debt can be privately placed or publicly traded.

LOS 83.d — Investment characteristics of infrastructure

KEY CONCEPT

	Secondary-stage / brownfield	Greenfield
Cash flows	Stable, relatively high current yield	Uncertain; lower near-term yield
Growth potential	Little	Greater
Risk and expected return	Lower; secondary-stage assets (e.g., existing toll roads and hospitals) are the least risky and offer the lowest return	Higher (e.g., new toll roads, renewable energy facilities)



Infrastructure investments by stage of the asset

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- Long-term contracts and barriers to entry make equity cash flows stable and give low correlation with public equities. Infrastructure debt tends to be safe and less cyclical.
- Greenfield projects in developing economies are risky but have produced attractive long-run returns as incomes rise.
- Risks: **regulatory risk**, financial leverage, cash flows below expectations, **construction risk** (building projects) and **operational risk** (privately operated assets).
- Suitable for long-term institutions: pension plans, life insurers, sovereign wealth funds.

How cash-flow arrangements shift risk. The contract behind the revenue matters as much as the asset. Availability payments are made for making the infrastructure available, take-or-pay arrangements require the buyer to pay a minimum price for an agreed volume, and usage-based payments such as tolls depend on how much the asset is used. Projects that rely on revenue from uncertain future demand tend to be the riskiest. A toll road built to serve a city that has not yet grown is therefore riskier than a hospital leased to a health authority on an availability basis.

Example. A consortium plans a new solar farm that has not yet been built. It is a greenfield project with construction risk, early cash outflows, and the most risk and upside. Buying an existing, fully operating water-treatment plant would be a secondary-stage, lower-risk investment.

COMMON EXAM TRAPS

- A sale-leaseback of an existing asset is brownfield. Only an asset still to be built is greenfield.
- When an asset is fully operational and needs no further development, "secondary stage" is the more precise answer than "brownfield", even though it is a type of brownfield investment.
- Public use does not make an asset social infrastructure. Waste treatment, power grids and telecom towers are economic infrastructure; prisons, schools and health care facilities are social.
- An infrastructure ETF, mutual fund or MLP is an indirect investment; holding the first mortgage on a toll road counts as direct.

BOTTOM LINE

- Economic infrastructure covers transportation, utilities and energy (including waste disposal and treatment) and information and communication technology (including telecom towers), while social infrastructure covers prisons, schools and health care facilities.
- Infrastructure revenue comes from availability payments made regardless of use, usage-based payments such as tolls, or take-or-pay arrangements with a minimum price for an agreed volume, and the projects carrying the most risk are usually those whose revenue depends on demand that is still uncertain.
- A greenfield investment is an asset still to be constructed, typically under build-operate-transfer; a brownfield investment expands or privatizes an existing asset, for example through a sale-leaseback; and a secondary-stage investment is a fully operational brownfield asset needing no further development.
- Secondary-stage and brownfield assets have stable cash flows, a relatively high current yield, little growth potential and lower risk and return, with secondary-stage assets the least risky, while greenfield assets have uncertain cash flows, more growth potential and higher risk and expected return.

- Direct infrastructure investment is illiquid because the assets are huge and long-lived, and MLPs, ETFs, mutual funds and private equity funds offer indirect exposure with more liquidity.
- Infrastructure risks include regulatory risk, financial leverage, cash flows below expectations, construction risk and operational risk, and the assets suit long-term institutions such as pension plans, life insurers and sovereign wealth funds.

QUICK CHECK

Question 2

Which of the following is *least likely* to be classified as social infrastructure?

A Schools.

B Waste treatment facilities.

C Health care facilities.

Answer: B

Waste treatment facilities are utility assets, which are part of economic infrastructure. Schools and health care facilities are social infrastructure.

Why the other options are wrong

- **A.** Schools are social infrastructure.
- **C.** Health care facilities are social infrastructure.

Key takeaway Utilities (power, gas, water and waste treatment) are economic infrastructure.

Practice questions

Question 3

Which type of infrastructure investment typically offers the highest risk and the highest potential return?

A Greenfield.

B Secondary stage.

C Brownfield.

Answers: 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/alternative-investments/real-estate-and-infrastructure/. The full bank has 12 questions on this reading.

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ALTERNATIVE INVESTMENTS · READING 84

Natural Resources

What you'll learn

LOS 84.a Explain the features, cash flows and investment characteristics of raw land, timberland and farmland.
LOS 84.b Describe commodity sectors, ways to gain commodity exposure, and futures pricing (cost of carry, convenience yield, contango and backwardation).
LOS 84.c Analyze the sources of risk, return and diversification for natural resource investments, including inflation hedging.

MODULE 84.1

Farmland, Timberland, and Commodities

This reading compares raw land, farmland and timberland, describes the commodity sectors and the ways to gain commodity exposure, and shows how a futures price follows from the spot price and the cost of carry, including contango and backwardation. It then covers the sources of commodity risk and return and the diversification and inflation-hedging role of natural resources.

LOS 84.a — Raw land, timberland and farmland

Natural resources include **raw land**, land used to grow crops or timber, and **commodities**. Exposure can be direct or through commingled vehicles (ETFs, REITs, limited partnerships, LLCs), as well as through derivatives such as commodity futures and swaps.

Common features of the three kinds of land:

- All are illiquid, and value is driven mainly by location. For farmland and timberland, value also depends on access to transport and markets, water and soil quality.
- Each type can produce lease income as well as gains from rising prices; farmland and timberland add income from what they produce (crops, livestock, timber).

KEY CONCEPT

	Raw land	Farmland	Timberland
Current cash flow	Little or none: no output, only possible lease income (value mainly from appreciation or development)	Most steady: annual crop sales or lease income	Yes, but the owner chooses *when* to harvest
Typical owners	Institutions	Mostly individuals (farmland REITs open access to retail investors)	Institutions; lot sizes much larger than farmland

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	Raw land	Farmland	Timberland
Special features	None	Crops must be harvested within a short window; farmers often **short commodity futures** to hedge the harvest	Harvest timing is flexible: trees keep growing if prices are low; expertise available through **timberland investment management organizations (TIMOs)**

Financing choices for farmland and timberland are narrower than for residential or commercial real estate and consist mainly of bank loans or direct private debt. Because crops and trees absorb carbon, they appeal to investors with an ESG climate focus.

Risks. Low liquidity, weather-dependent cash flows, and losses from natural disasters.

LOS 84.b — Commodities and how to gain exposure

Sectors. Commodity sectors are **metals** (base/industrial, such as copper and aluminum, and precious, such as gold and silver), **agricultural products** (grains, livestock, coffee) and **energy** (crude oil, natural gas, coal). Contracts also specify **grade** (quality) and **delivery location**. Governments influence supply and demand. They may subsidize food crops for consumers or support prices for farmers, control access to extractable resources, or take part in production themselves. Climate-related regulation can reduce demand for fossil fuels and increase demand for lithium, cobalt, nickel and similar minerals.

Ways to gain exposure. Derivatives are the most common route:

Vehicle	Key points
Physical commodity	Possible (e.g., gold, grain) but has storage and transport costs
Futures, forwards, options on futures	Exchange-traded futures have no counterparty risk, because the clearinghouse guarantees performance
Exchange-traded products (ETPs): ETFs and ETNs	Trade on exchanges like shares and can track prices or indexes. Exam convention: ETPs, both ETFs and ETNs, suit investors who are limited to buying equity shares. Current practice: an **ETN** is an unsecured debt note of its issuer and carries issuer credit risk, so an equity-only investor would normally need its mandate to allow debt before holding one; an **ETF**'s fund shares raise no such problem
Managed futures funds (e.g., run by **commodity trading advisers**, CTAs)	Actively managed; may be sector-focused or diversified; structured as LPs (hedge-fund-like fees and investor restrictions) or as publicly traded mutual-fund-style shares (lower minimums, more liquidity)
Separately managed accounts (SMAs)	For large investors that want customized portfolios
Specialized funds	Focus on one commodity sector, such as oil and gas, grains, precious metals or industrial metals; can use any of the structures above

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Commodity valuation: cost of carry

Holding a commodity today costs financing and storage (warehousing, insurance, spoilage), but it also gives the **convenience yield**. This is the nonmonetary benefit of having the physical good available to use over the life of the contract. A futures buyer does not have the good until delivery and so gives up this benefit, which is why the convenience yield lowers the futures price while financing and storage costs raise it.

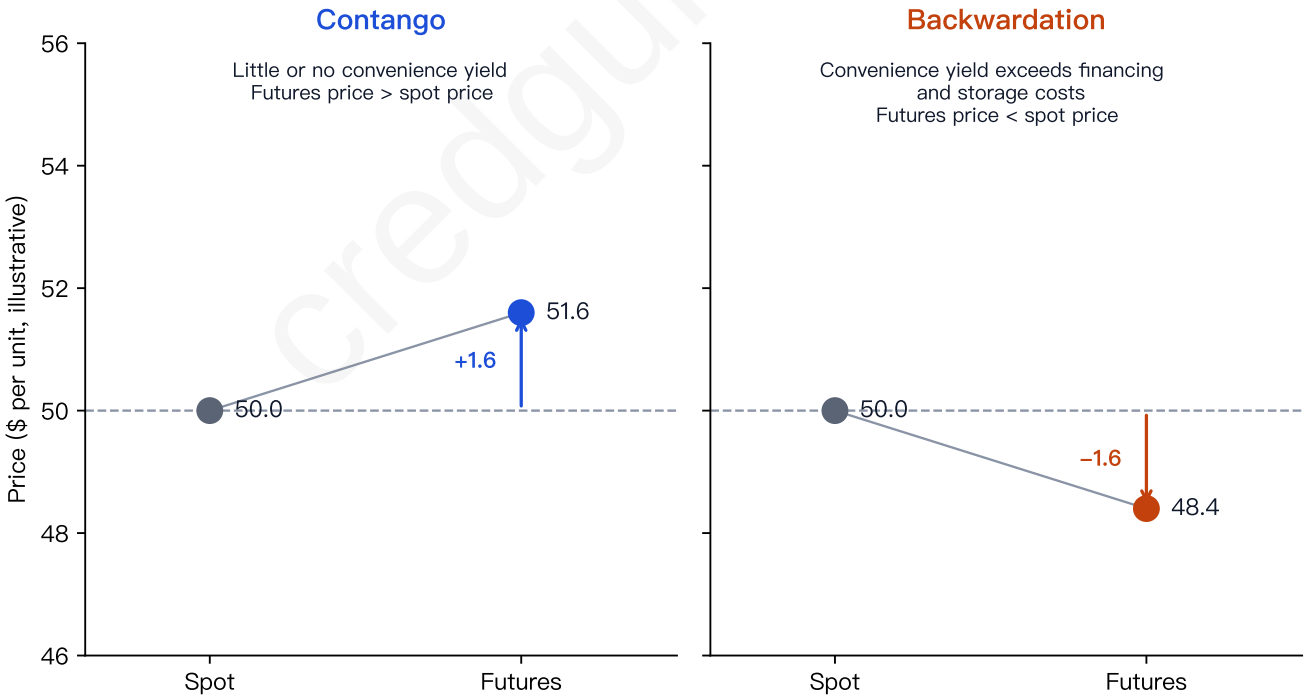
KEY CONCEPT

$$\text{futures price} \approx \text{spot price} \times (1 + r_f) + \text{storage costs} - \text{convenience yield}$$

Equivalently, futures price = spot price + **net cost of carry**.

KEY CONCEPT

Situation	Net cost of carry	Futures vs. spot	Market classification	Effect on long-only investors
Little or no convenience yield	Positive	Futures > spot	**Contango**	Lowers returns
Convenience yield > storage (and financing) costs	Negative	Futures < spot	**Backwardation**	Raises returns



Futures prices compared with spot: contango and backwardation (illustrative prices)

To classify a market from a list of prices, compare each futures price with the spot price: futures above spot indicate contango, and futures below spot indicate backwardation. This spot comparison is separate from the slope of the futures term structure, which describes how futures prices change across maturities; futures can all

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be above spot without rising at every maturity. When storage costs and the convenience yield are equal, they cancel, so the futures price exceeds the spot price by the financing cost alone whenever the risk-free rate is positive.

> *Example.* Copper spot \$9.00/lb, 3% financing for the period, storage \$0.12, convenience yield \$0.05. $F \approx 9.00 \times 1.03 + 0.12 - 0.05 = \$9.34$, which is above spot, so the market is in contango.

LOS 84.c — Sources of risk, return and diversification

Prices come from supply and demand. Demand depends on the value to end users and on global economic conditions. Supply depends on production, storage costs and inventories.

- Supply is **inelastic in the short run** (drilling wells or planting crops takes time), so prices can swing sharply over the economic cycle or after supply shocks such as natural disasters.
- Weather and plant disease drive agricultural output. Poor harvests mean high prices; bumper crops mean low prices.
- Extraction costs rise as producers turn to more expensive methods or more remote sites.
- Forecasters watch inventories, production forecasts, government policy and economic growth.

Return and risk history

In recent decades:

Asset	Returns	Volatility
Commodities	Higher than global stocks or bonds	Higher than global stocks or bonds
Timberland and farmland	Higher average returns	Lower than global stocks, similar to global bonds

Speculators who correctly anticipate short-term commodity price moves can earn high returns over short periods.

Farmland and timberland returns come from selling what the land produces and from changes in land values. Timberland values also move with the lumber price outlook and with the volume of timber already cut. Farmland returns depend on crop prices and on crop quality and quantity.

Portfolio benefits

- Diversification: historically low correlation with global equities and bonds.
- **Inflation hedge**: commodity prices tend to move with inflation, so commodities have tended to outperform stocks and bonds when inflation is high and to underperform when it is low.
- Simply holding a commodity produces no current income; the holder gains only if the price rises.
- Commodity prices are especially sensitive to geopolitical and weather risks.

COMMON EXAM TRAPS

- Options that call convenience yield a cash payment, or say it raises the futures price, confuse it with income or with storage costs. In the copper example, adding the \$0.05 convenience yield instead of subtracting it gives \$9.44 instead of \$9.34.

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- A *negative* net cost of carry means backwardation and higher long-only returns. Pairing negative carry with contango is the usual wrong statement.
- The inflation hedge rests on a *positive* correlation between commodity prices and inflation. An option that cites a negative correlation is wrong.
- "Current income" is the wrong answer when asked for a benefit of holding commodities.
- Timberland's flexibility to delay the harvest is not shared by farmland, whose crops must be harvested within a short window.

EXAM SHORTCUTS

- When storage costs equal the convenience yield they cancel, so with a positive risk-free rate the futures price exceeds the spot price by the financing cost alone and the market is in contango.

BOTTOM LINE

- Raw land, farmland and timberland are illiquid and valued mainly by location; raw land produces little or no current cash flow, farmland has the steadiest income, and timberland lets the owner choose when to harvest, whereas farm crops must be harvested within a short window.
- Derivatives are the most common route to commodity exposure, and exchange-traded futures carry no counterparty risk because the clearinghouse guarantees performance.
- Exam convention: commodity ETPs, both ETFs and ETNs, suit investors limited to buying equity shares; current practice: an ETN is an unsecured debt note carrying issuer credit risk, so an equity-only mandate would normally have to allow debt before holding one.
- The futures price is approximately spot × $(1 + r_f)$ + storage costs − convenience yield, the convenience yield being the nonmonetary benefit of holding the physical good, which lowers the futures price.
- A positive net cost of carry puts futures above spot (contango) and lowers long-only returns, while a negative net cost of carry puts futures below spot (backwardation) and raises them.
- Commodity supply is inelastic in the short run, so prices can swing sharply over the economic cycle or after supply shocks such as natural disasters.
- In recent decades commodities have had higher returns and higher volatility than global stocks or bonds, while timberland and farmland have had higher average returns with volatility below global stocks and similar to global bonds.
- Commodities have historically offered diversification through low correlation with global equities and bonds and an inflation hedge because their prices tend to move with inflation, but holding them produces no current income.

QUICK CHECK

Question 1

An investor in land-based natural resources wants the most regular annual cash flow. Which of the following is *most likely* to provide it?

A Timberland.

B Farmland.

C Raw land.

Answer: B

Farmland produces income every year from leasing the land or from selling crops, which must be harvested within a short window. It therefore has the steadiest cash flows of the three.

Why the other options are wrong

- **A.** Timberland produces cash flow, but the owner decides when to harvest: it can let the trees keep growing when prices are low. Its cash flows are therefore less regular.
- **C.** Raw land produces no output to sell. It may earn some lease income, but its return comes mainly from price appreciation, so it offers the least regular cash flow.

Key takeaway Cash-flow steadiness: farmland > timberland > raw land.

Practice questions

Question 2

For a particular commodity, the convenience yield outweighs the storage and financing costs of holding the physical good. In this situation:

A the spot price is below the futures price.

B long-only futures investors see lower returns.

C the forward curve exhibits backwardation.

Question 3

Which of the following is *least likely* to be a benefit of adding commodities to a portfolio of stocks and bonds?

A Current income.

B Diversification.

C Inflation hedge.

Answers: 2. C · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/alternative-investments/natural-resources/. The full bank has 11 questions on this reading.

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ALTERNATIVE INVESTMENTS · READING 85

High Water Mark

What you'll learn

LOS 85.a Explain what distinguishes hedge funds from other asset classes and vehicles, including their main strategy families and liquidity terms.

LOS 85.b Describe the legal forms and vehicles for hedge fund investing: partnerships, commingled funds, SMAs, master-feeder structures and funds of funds.

LOS 85.c Analyze hedge fund return sources, risk-adjusted comparison, index biases and diversification benefits.

MODULE 85.1

Hedge Funds

This reading describes what distinguishes hedge funds from other vehicles, their four strategy families and their liquidity and fee terms, including the high-water mark. It then covers their legal forms, separately managed accounts, master-feeder structures and funds of funds, and explains where hedge fund returns come from, how index biases distort reported performance and how to compare funds on a risk-adjusted basis.

LOS 85.a — What makes a hedge fund different, and the main strategy families

A **hedge fund** is a privately offered pool of capital. Access is normally limited to investors who clear a wealth or sophistication threshold (accredited, qualified or institutional investors). Because it is not sold to the general public, it faces far lighter regulation than a mutual fund or ETF, and the manager enjoys a broad mandate: long and short positions, leverage, derivatives, and securities across many markets. Hedge funds began as long/short vehicles meant to earn positive returns in any market, but they now pursue a wide variety of strategies.

Key features:

- **Return drivers:** hedge funds try to profit from market inefficiencies (mispricings) and from volatility rather than from simply owning the market.
- **Holdings:** mostly liquid assets, chiefly the traditional asset classes of debt and equity, along with derivatives. Leverage and derivatives mean the fund's risk/return profile can look very different from the assets it holds.
- **Performance evaluation:** usually on an absolute (total) return or risk-adjusted basis instead of against a conventional benchmark, because the strategies do not map neatly onto any market index.
- **Alignment:** managers often co-invest their own money. Incentive fees are commonly subject to a **high-water mark**, so a fee is earned only on value above the fund's previous peak.
- **Versus other vehicles:** hedge funds are privately held. Exam convention: mutual funds, REITs and ETFs are all publicly traded. Current practice: private (non-traded) REITs also exist, so the contrast holds for publicly offered REITs. Compared with private equity, hedge funds hold more liquid assets, have a shorter horizon and allow periodic redemptions, while private equity capital is committed for many years. Both are usually structured as limited partnerships, but a hedge fund charges its management fee on assets under management, while a private equity fund charges it on committed capital.

Strategy families

KEY CONCEPT

Family	Core idea	Sub-strategies	Typical net exposure
Equity hedge	Long and short positions in listed equities (or equity derivatives/indexes)	Fundamental long/short; fundamental growth; fundamental value; market neutral; short bias	Varies: long/short and growth usually net long; market neutral near zero; short bias net short
Event-driven	Profit from corporate events such as takeovers, distress, activism, spin-offs	Merger arbitrage; distressed/restructuring; activist shareholder; special situations	Usually net long
Relative value	Buy one security, short a related one, and profit when the price gap between them closes	Convertible arbitrage fixed income; specific fixed income (ABS, MBS, high yield); general fixed income; multistrategy	Paired long and short positions can reduce broad market exposure
Opportunistic	Top-down bets on macro trends and commodity/financial futures	Macro; managed futures (CTAs)	Can be long or short any asset class

Equity hedge and event-driven managers mostly build positions company by company (bottom-up analysis). Macro managers start from views on the economy as a whole (top-down analysis).

Equity hedge sub-strategies

- *Fundamental long/short:* long undervalued names picked by fundamental research, short a basket or index to trim market risk; most run a long bias. A manager with no negative view on particular stocks can take the hedging short in a market index.
- *Fundamental growth:* long companies expected to keep growing revenue and earnings quickly (seeking capital appreciation); short low- or no-growth firms; net long.
- *Fundamental value:* long shares judged cheap versus intrinsic value, short shares judged expensive; results track value versus growth.
- *Market neutral:* longs and shorts of roughly equal size so that broad market moves cancel; profits come from relative moves; leverage may be used.
- *Short bias:* mostly short positions in shares the manager believes are overvalued (e.g., weak business models, aggressive accounting), perhaps a few longs, with negative net market exposure; a contrarian style.

Event-driven sub-strategies

- *Merger arbitrage:* long the target, short the acquirer (typical for a stock deal). Not riskless: the deal can be re-priced or collapse.

- *Distressed/restructuring:* buy deeply discounted securities (often debt) of firms in or near bankruptcy, expecting value to be unlocked by the restructuring; may short overvalued claims too.
- *Activist shareholder:* take a stake large enough to push management toward value-raising changes.
- *Special situations:* trade around issuance, buybacks, spin-offs, asset sales, or capital distributions.

Relative value sub-strategies

- *Convertible arbitrage fixed income:* exploits mispricing between a company's convertible bonds, its common shares and options on those shares, for example by buying the convertible bond and selling the shares short.
- *Specific fixed income (ABS, MBS, high yield):* exploits pricing and quality discrepancies in asset-backed, mortgage-backed or high-yield securities.
- *General fixed income:* trades mispricing between fixed-income securities of different issuers and types.
- *Multistrategy:* looks for mispricing between related securities, both inside one asset class and across asset classes and markets.

Opportunistic sub-strategies

Opportunistic funds often trade ETFs and derivatives as well as individual securities. *Macro* funds base long or short positions in currencies, commodities, equities or fixed income on their view of global economic trends; they benefit from volatility around big economic events, and central-bank smoothing of shocks hurts them. *Managed futures* funds trade commodity or financial futures (commodity specialists are CTAs). Commodity prices tend to self-correct in a way financial asset prices do not: high prices cut demand, which pulls prices down, and low prices cut supply, which pushes prices up.

Liquidity, fees and transparency

Compared with mutual funds and ETFs, hedge funds cost more, because an incentive fee comes on top of a generally high management fee, and they are less liquid. The main terms:

KEY CONCEPT

Term	Meaning
Lockup period	Minimum period after the initial investment during which the investor cannot redeem (hard lockup) or can redeem only with a significant penalty (soft lockup).
Notice period	After the lockup has expired and a redemption request is made, the time the fund has to pay out (commonly 30–90 days).
Liquidity gate	A partial limit on redemptions, e.g., a cap on the fraction of the fund that may be withdrawn per period; less drastic than a full suspension.
Redemption fee	Charge paid by the redeeming investor to offset the trading costs of meeting the withdrawal.
Management fee	Annual fee on assets under management (historically ~2%).
Incentive (performance) fee	Share of profits (historically ~20%; some newer funds use ~1% management plus ~30% performance fee based on return relative to a benchmark rather than total return).

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Notice periods and gates let the manager unwind positions in an orderly way. Redemptions tend to spike after poor performance, and the cost of meeting them can further damage the remaining investors. Transparency is limited because managers protect proprietary strategies, which makes independent valuation of holdings difficult.

Example (high-water mark). Maple Ridge Fund's NAV per unit peaked at 110 two years ago, fell to 100, and ends this year at 121 before incentive fees. With a 20% incentive fee and a high-water mark, the fee applies only to the gain above 110: $0.20 \times (121 - 110) = 2.2$ per unit rather than $0.20 \times (121 - 100) = 4.2$.

COMMON EXAM TRAPS

- Swapping *lockup* (cannot redeem yet) and *notice period* (time the fund has to honor a request).
- Classifying market neutral as relative value (it is equity hedge) or merger arbitrage as relative value (it is event driven).
- Reversing the merger arbitrage trade: it is long the target and short the acquirer.
- Assuming hedge funds are low risk because they own liquid assets; leverage and derivatives change the risk profile.

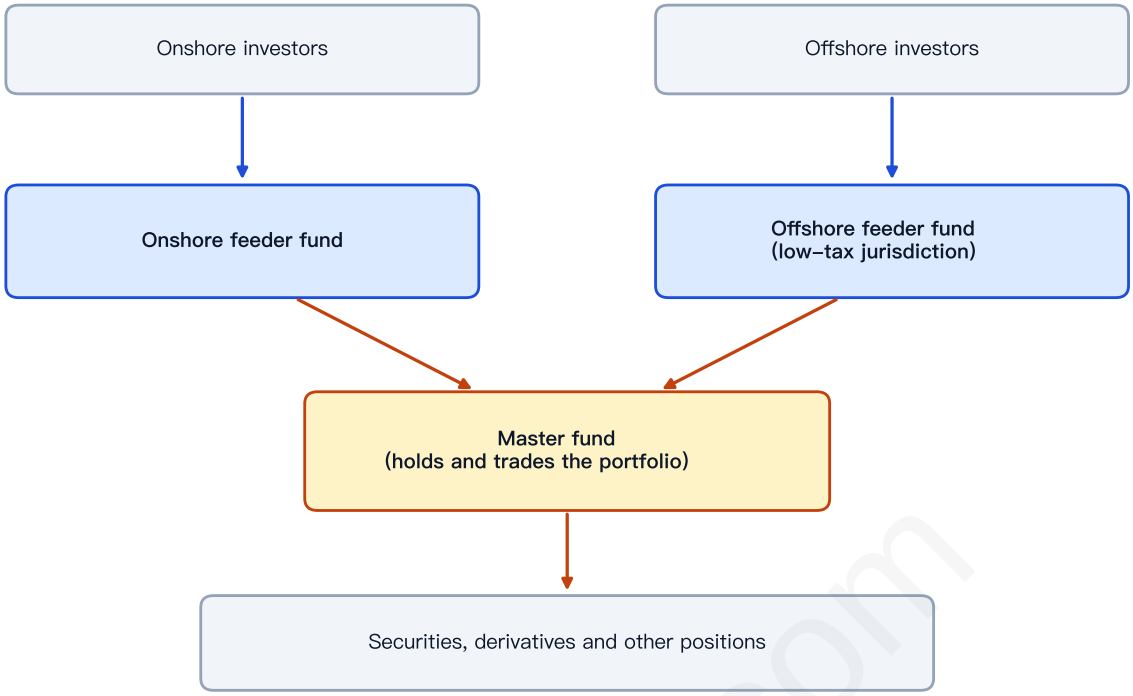
LOS 85.b — Legal forms, vehicles and indirect access

Legal form. Most hedge funds are **limited partnerships** (or limited liability companies). The **general partner** (or managing member) is the manager and is paid partly on performance; investors are **limited partners**. The rights and obligations of both sides are set out in the fund documents: the **partnership agreement**, the **private placement memorandum (PPM)**, or the articles of incorporation. A **side letter** is different: it is a bilateral agreement granting one investor special terms (e.g., lower fees, extra reporting, better liquidity) and can override the general documents for that investor only. Funds usually have an indefinite life.

Commingled fund vs. separately managed account (SMA)

Feature	Commingled fund	SMA
Investors	Many, capital pooled	One (usually a large institution)
Customization	Standard terms for all	Portfolio and mandate tailored to the investor
Fees	Standard	Often negotiated lower
Drawbacks	Terms not tailored to any one investor	More operational oversight, so suited to larger or institutional investors; the manager has no own capital at stake, so interests are less closely aligned; the account may receive allocations of only the manager's most liquid trades

A **master-feeder** structure is common for commingled funds: an onshore feeder and an offshore feeder (in a low-tax jurisdiction) both invest in a single master fund that holds the portfolio. Benefits: tax efficiency, economies of scale, and access to global capital. The structure also sidesteps regional regulatory requirements.



Master-feeder structure

A **fund of funds (FoF)** is a vehicle that invests in several hedge funds. Advantages: diversification across strategies and managers, the FoF manager's expertise in selecting and monitoring funds, access for smaller investors to funds they could not enter directly, and often shorter lockups/better exit liquidity. Disadvantage: a second layer of fees (historically about 1% management plus 10% incentive) on top of the underlying funds' fees, which can substantially cut net returns. A FoF does not promise higher returns.

COMMON EXAM TRAPS

- Treating a side letter as a fund-wide governing document.
- Listing "stronger manager motivation" as an SMA benefit; weaker incentive alignment is an SMA drawback.
- Saying a FoF lowers fees; it adds fees.

LOS 85.c — Where returns come from, index biases and diversification

Hedge fund returns come from three sources:

$$R_{\text{fund}} = \underbrace{\text{market beta}}_{\text{broad market}} + \underbrace{\text{strategy beta}}_{\text{sector/strategy exposure}} + \underbrace{\alpha}_{\text{manager skill}}$$

- **Market beta**: the part of return that comes from the broad market index.
- **Strategy beta**: the return attributable to the specific sectors or strategy the fund is exposed to.
- **Alpha**: the additional return delivered by the manager's security selection.

Market beta can be bought cheaply through index funds, so hedge funds often short it away. Managers use leverage to magnify strategy beta and alpha. High fees are a drag on net results.

Risk-adjusted comparison. Because hedge funds differ widely in risk, compare them per unit of risk. One simple measure is the **coefficient of variation**:

$$CV = \frac{\sigma}{\bar{R}}$$

A lower CV means less risk per unit of return, i.e., better risk-adjusted performance. A low standard deviation alone ignores return; a high return or alpha alone ignores risk.

Example. Fund P averages 10% with σ = 12% (CV = 1.2); Fund Q averages 6% with σ = 5% (CV = 0.83). Q is better risk-adjusted even though P earns more.

Index biases. Hedge fund indexes are built from data that managers report voluntarily. Poor performers tend not to report, so index returns are biased upward, and measured correlations with traditional assets may be distorted.

KEY CONCEPT

Bias	What happens	Effect on reported index performance
Survivorship	Funds that fail or stop reporting drop out (or never reach minimum size/age to enter), so their poor results vanish	Overstates returns
Backfill	When a fund joins, its earlier (typically strong) history is added to the index	Overstates returns
Selection	Index providers assign funds to strategy categories inconsistently or use differing inclusion rules	Distorts peer-group (category) comparisons; can also tilt reported performance upward when inclusion favors stronger funds

Diversification. Results depend heavily on the measurement period, but hedge funds as a group have added diversification to portfolios of traditional assets. Their returns are more correlated with equities than with bonds.

COMMON EXAM TRAPS

- Options that call the voluntary-reporting bias downward or negligible are wrong, because the funds that stop reporting are the weak ones.
- Survivorship and backfill bias do not offset each other; both push index returns up.
- Asked what leverage is meant to magnify, "market beta" is the trap answer.

BOTTOM LINE

- A hedge fund is a privately offered pool open to investors above a wealth or sophistication threshold, lightly regulated and free to use long and short positions, leverage and derivatives, and it is usually evaluated on an absolute or risk-adjusted basis instead of against a conventional benchmark.

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- Compared with private equity, hedge funds hold more liquid assets, have a shorter horizon and allow periodic redemptions, and they charge the management fee on assets under management where private equity charges it on committed capital.
- The four strategy families are equity hedge (including market neutral), event-driven (including merger arbitrage, long the target and short the acquirer), relative value (including convertible arbitrage) and opportunistic (macro and managed futures).
- During a lockup period the investor cannot redeem (hard lockup) or can redeem only with a significant penalty (soft lockup), a notice period is the time the fund has to pay out a redemption, a gate partially limits redemptions, and a high-water mark restricts the incentive fee to value above the fund's previous peak.
- A separately managed account is tailored to one investor and often has lower fees but needs more oversight and aligns interests less closely, while a fund of funds offers diversification and access for smaller investors at the cost of a second layer of fees.
- Hedge fund returns come from market beta, strategy beta and alpha; funds often short away market beta because it is cheap to buy through index funds, and they use leverage to magnify strategy beta and alpha.
- The coefficient of variation, $\sigma/\bar{R}$, measures risk per unit of return, and a lower CV means better risk-adjusted performance, whereas a low standard deviation or a high return alone ignores the other side.
- Because index data are reported voluntarily, survivorship and backfill bias both overstate hedge fund index returns, and selection bias distorts category comparisons.

QUICK CHECK

Question 1

An adviser makes three claims to a client about hedge funds as a vehicle. Which claim is *least* accurate?

A They are generally offered only to investors who meet accredited, qualified or institutional-investor thresholds.

B Their use of leverage and derivatives can give them a risk and return profile quite unlike that of the assets they hold.

C They are regulated about as heavily as mutual funds, which confines them to a narrow set of permitted strategies.

Answer: C

Hedge funds are private pooled vehicles that face light regulation and give managers wide latitude over strategy, leverage and instruments, so the claim that they are tightly regulated and strategy-constrained is false. The other statements are accurate: access is limited to investors meeting high suitability standards, and although most holdings are liquid stocks and bonds, leverage and derivatives change the fund's risk profile, which is why managing the fund's risk exposure matters.

Why the other options are wrong

- **A.** Accurate. Hedge funds are sold privately to investors who satisfy wealth or sophistication requirements.

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- **B.** Accurate. Leverage and derivatives can make the fund's risk and return behave very differently from the underlying debt and equity positions. For the same reason, owning mostly liquid assets does not make a hedge fund lower risk than, say, a private equity fund.

Key takeaway Hedge funds = private, lightly regulated, flexible. Liquid holdings do not equal low risk once leverage and derivatives are involved.

Practice questions

Question 2

A hedge fund that follows a macro strategy is *most likely* to make which of the following trades?

A Buying the shares of a company that has agreed to be acquired and selling short the acquirer's shares.

B Buying a company's convertible bonds and selling short the same company's common shares.

C Selling a country's currency and its government bond futures ahead of an expected rise in its inflation.

Question 3

The Northgate Teachers' Pension Plan is choosing between investing in a hedge fund's commingled vehicle and hiring the same manager through a separately managed account (SMA). Which of the following is a drawback of the SMA route?

A Fees that are often negotiated lower than the pooled vehicle charges.

B An investor-specific investment mandate tailored to the plan.

C Weaker alignment of interests, because the manager has no stake in the account.

Question 4

An analyst reviewing a hedge fund index database notes three practices. Each one illustrates a different hedge fund index bias: survivorship bias, backfill bias or selection bias. Which practice is the example of selection bias?

A Two funds running almost identical convertible arbitrage books are placed in different strategy categories by the index provider.

B A fund that suffered heavy losses and closed is removed from the index, and its record no longer appears in the index history.

C A fund joins the index after three strong years, and its returns for those earlier years are added to the index history.

Answers: 2. C · 3. C · 4. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/alternative-investments/high-water-mark/. The full bank has 21 questions on this reading.

ALTERNATIVE INVESTMENTS · READING 86

Introduction to Digital Assets

What you'll learn

LOS 86.a Describe how distributed ledger technology works (ledger, consensus protocols, network types) and its financial applications, including the main types of digital assets.

LOS 86.b Explain how digital assets differ from traditional asset classes in value, validation, use as a medium of exchange and regulation.

LOS 86.c Describe exchanges, direct and indirect investment vehicles, and asset-backed tokens and DeFi as forms of digital asset investment.

LOS 86.d Analyze the drivers of digital asset returns, their volatility and their potential diversification benefits.

MODULE 86.1

Distributed Ledger Technology

This reading explains distributed ledger technology, blockchains and their consensus protocols, permissionless and permissioned networks, and the main types of digital asset. It then compares digital assets with traditional assets, describes the ways to invest in them directly and indirectly, and covers their return drivers, risk and diversification potential.

LOS 86.a — Distributed ledger technology and the digital assets built on it

What a digital asset is

Digital assets are assets that are created, stored and transferred electronically. The class is young and includes **cryptocurrencies**, **tokens** and digital collectibles. Every digital asset is secured and validated with **distributed ledger technology (DLT)**, often called blockchain technology. A cryptocurrency runs on its own blockchain; a **crypto token** is built on top of a blockchain that already exists.

The distributed ledger

A **distributed ledger** is a database that is shared among market participants. Each participant keeps an identical copy, and the ledger holds a record of every transaction.

DLT benefits	DLT drawbacks
Accuracy, transparency and security of records	Data protection concerns
Rapid transfer of ownership	Possible violations of privacy
Peer-to-peer (P2P) creation, trading and monitoring of assets, with no intermediary	Very large computing power (and energy) needed to verify transactions

A DLT network has three parts:

1. a **digital ledger** (the shared record itself);
2. a **consensus mechanism**, which is how the network agrees on the state of the ledger by validating transactions and then updating the ledger; and
3. a **network of participants** (who may perform the network's functions).

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Every copy is identical; a validated transaction updates the shared ledger.

A DLT network: consensus validates transactions before the shared ledger is updated

Validation plus updating makes records **immutable**, meaning they cannot be changed afterwards, while keeping them visible to all participants. **Cryptography** encrypts the data so that unauthorized parties cannot read it.

Smart contracts are computer programs on a DLT network that execute by themselves when pre-agreed terms and conditions are met. Examples: automatically settling contingent claims, or moving collateral to a lender when a borrower defaults.

Blockchains and consensus protocols

A **blockchain** is a digital ledger that stores information in **blocks** recorded in sequence. Each block holds a group of transactions plus a cryptographic link (a **hash**) to the block before it, so the blocks form a chain. A new transaction joins the chain only after authorized participants validate it.

Consensus protocols decide how blocks are chained together and are designed to resist manipulation.

KEY CONCEPT

	Proof of work (PoW)	Proof of stake (PoS)
Who validates	**Miners**	**Validators**
How	Computers race to solve a cryptographic problem that verifies the transaction	Participants pledge collateral (**staking**) to vouch that a block is valid; other validators then confirm it
Reward	Newly earned cryptocurrency	A return on the staked collateral
Protection against attack	Rewriting history would require controlling most (51%) of the network's computing power, which is very costly	Exam convention: validators protect the network from malicious attacks by controlling most of its computational power. Current practice: PoS security rests on staked collateral, so an attacker would need to control most of the stake rather than most of the computing power
Status	Most widely used	Emerging

Because a PoW attack needs a majority of the network, a blockchain with many participants is harder to attack and more likely to succeed.

Permissionless versus permissioned networks

KEY CONCEPT

	Permissionless network (open)	Permissioned network
Who can transact	Any user; all network functions open to every participant	Some users restricted from some activities
Visibility	All transactions visible to all users	Access levels set by permissions, e.g., members add transactions, a regulator may only view history
Trust	Parties need not trust each other: consensus, not a central authority, confirms transactions, and the record is immutable	Controlled by whoever sets the permissions
Cost	Less cost effective	More cost effective (stronger restrictions)
Example	Bitcoin and many other cryptocurrencies	Private networks run by a group of institutions

Types of digital assets

Digital assets fall into two families, cryptocurrencies and tokens.

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Main types of digital assets

Family	Type	Example or note
Cryptocurrencies	Bitcoin	The most widely traded cryptocurrency
Cryptocurrencies	Altcoins: other cryptocurrencies	Ether, on the Ethereum network
Cryptocurrencies	Altcoins: stablecoins	Tether, pegged to the US dollar
Cryptocurrencies	Altcoins: meme coins	Dogecoin
Cryptocurrencies	Central bank digital currencies (CBDCs)	A central bank's equivalent of a cryptocurrency
Tokens	Nonfungible tokens (NFTs)	Each token is unique, e.g., a piece of digital artwork
Tokens	Security tokens	Initial coin offerings (ICOs)
Tokens	Utility tokens	Paying for network services and network fees
Tokens	Governance tokens	Voting on how a permissionless network operates

- **Cryptocurrency**: a digital currency issued privately, with no central bank backing. It allows near-real-time transactions without an intermediary. Many cryptocurrencies limit the number of units that can be issued to support their role as a store of value, yet prices are still very volatile. **Bitcoin** is the most widely traded. **Altcoins** are alternatives built on Bitcoin's technology. The best known is **Ether** on the **Ethereum** network, a programmable blockchain on which users build smart contracts, so its value is not limited to being a store of value. As of mid-2022, Bitcoin and Ether together made up more than 80% of the cryptocurrency market.
- **Stablecoin**: designed to hold a stable value by being **linked to the value of another asset** (such as the US dollar); Tether, pegged to the US dollar, is a widely used example. Reserve-backed stablecoins are secured by a **reserve basket of assets**. **Algorithmic (smart) stablecoins** instead use algorithms to mint additional units when demand increases; they may hold no actual reserves, and a peg that depends only on the algorithm can fail, as TerraUSD's did in 2022. Exam convention: a stablecoin cannot be converted into fiat currency and has no legal or regulatory backing. Current practice: some issuers do redeem their stablecoins for fiat currency, and some jurisdictions have begun to regulate stablecoin issuers. An **asset-backed token** is a special case: its value is pegged to a target asset such as gold.
- **Meme coin**: launched for entertainment. Prices can soar on social media attention and then collapse. The best known is Dogecoin.
- **Central bank digital currency (CBDC)**: a central bank's equivalent of a cryptocurrency, in effect a digital version of its banknotes and coins. Unlike cryptocurrencies, it is government issued.

- **Tokenization**: using DLT to record and track the ownership history of physical assets (e.g., real estate), which reduces the work needed to verify ownership when assets change hands. The token is the unit that represents the ownership right.
- **Nonfungible token (NFT)**: a form of tokenization that ties a digital asset to a certificate of authenticity. Each NFT represents a distinct item (e.g., a piece of digital art), unlike fungible tokens, which are interchangeable.
- **Security token**: tracks ownership rights in publicly traded securities digitally. A blockchain can handle custody, settlement, recordkeeping and post-trade processing on one ledger.
- **Initial coin offering (ICO)**: a way for a company to raise money by selling **crypto tokens** in exchange for money or other cryptocurrency. An ICO is classified as an example of a security token. The tokens are often usable later for the issuer's products or services.
- **Utility token**: used to pay for services on the network, such as service payments and network fees. It compensates holders only for network activity, while a security token may pay dividends.
- **Governance token**: issued on permissionless networks. It gives holders **voting rights** over how the network operates, e.g., voting on how to fix a technical problem.

ICO versus regulated IPO

KEY CONCEPT

	ICO	Regulated IPO
Regulation	Unregulated (many jurisdictions considering rules because of fraud)	Regulated
Issuance cost	Potentially lower	Higher
Time to raise capital	Potentially shorter	Longer
Voting rights	Usually none	Common shares normally carry votes
What investors get	Crypto tokens	Shares (registered securities)

Example

A cooperative owns a $12 million grain terminal. It records ownership on a distributed ledger and issues 120,000 tokens, each representing $1/120{,}000$ of the terminal. Each token then represents a $100 claim ($12{,}000{,}000/120{,}000 = 100$). A buyer with $5,000 can hold 50 tokens, and each transfer of tokens is logged on the immutable ledger. This is tokenization: ownership of a physical asset is tracked on DLT. Because each token is a digital ownership claim on a physical asset, the tokens are also asset-backed tokens (LOS 86.c).

COMMON EXAM TRAPS

- Proof of work and proof of stake are consensus protocols. They are not types of ledger or types of network of participants.
- Options often swap the roles: miners and computing power go with PoW; validators and staked collateral go with PoS.

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- A statement that parties on a permissionless network must trust each other is false, because consensus replaces trust. The cost advantage belongs to permissioned networks.
- Token, tokenization, smart contract and blockchain are often offered together. Match the noun the stem asks for: the ownership unit (token), the process of recording physical-asset ownership on DLT (tokenization), self-executing code (smart contract) or the ledger structure (blockchain).
- Stablecoins stay stable by being linked to other assets, through reserves or, for algorithmic stablecoins, through supply adjustments. Fiat convertibility and government backing are not the mechanism.
- ICO buyers usually receive crypto tokens with no voting rights. Governance tokens are the ones built around voting.
- A digital currency with government backing is a CBDC, not a stablecoin.

BOTTOM LINE

- Digital assets are created, stored and transferred electronically and secured with distributed ledger technology; a cryptocurrency runs on its own blockchain, while a crypto token is built on an existing one.
- The three parts of a DLT network are the digital ledger, the consensus mechanism and the network of participants, and validating and then updating the ledger makes its records immutable while keeping them visible to all participants.
- Under proof of work, miners solve a cryptographic problem and an attack would need most of the network's computing power; under proof of stake, validators pledge collateral, and the exam convention says validators guard the network by controlling most of its computational power, while in current practice an attacker would need most of the stake.
- A permissionless network is open to every user with all transactions visible and no need for parties to trust one another, while a permissioned network restricts some users' activities and is more cost effective.
- Stablecoins hold their value by being linked to another asset through reserves or, for algorithmic stablecoins, supply adjustments; exam convention: they cannot be converted into fiat currency and have no legal or regulatory backing; current practice: some issuers redeem them for fiat and some jurisdictions regulate issuers.
- Tokenization records ownership of physical assets on DLT, an NFT represents a distinct item, ICO buyers usually receive crypto tokens with no voting rights in an unregulated offering, utility tokens pay for network services, and governance tokens give voting rights over the network.

QUICK CHECK

Question 1

Two blockchains differ in how new blocks are accepted. On the first, miners compete to solve a cryptographic problem before a block is locked in. On the second, validators pledge some of their own holdings as collateral to vouch for a block, and further validators then confirm it. The two blockchains differ in their:

A network of participants.

B digital ledger.

C consensus protocol.

Answer: C

A DLT network has three components: the digital ledger, the consensus mechanism and the network of participants. Consensus protocols determine how blocks are validated and chained together. The first chain uses proof of work (miners solve cryptographic problems and earn cryptocurrency), and the second uses proof of stake (validators stake collateral and earn a return on it).

Why the other options are wrong

- **A.** The network of participants determines who may perform network functions. Those functions are open to all users in a permissionless network and restricted in a permissioned one. The network of participants does not set the rule used to validate and add blocks.
- **B.** The digital ledger is the shared, identical record of transactions itself. Proof of work and proof of stake are ways of agreeing on how to update that ledger. They are not ledgers in their own right.

Key takeaway Proof of work and proof of stake are the two main consensus protocols. Miners belong to PoW; validators and staking belong to PoS.

MODULE 86.2

Digital Asset Characteristics

LOS 86.b — Investment features of digital assets versus other asset classes

The number of cryptocurrencies has grown from dozens to thousands, and institutional investors are paying attention because of high expected returns and possible diversification benefits. Exchanges and custodians have built infrastructure to serve them. Digital assets nonetheless differ from traditional financial assets in four main ways:

KEY CONCEPT

Feature	Traditional financial assets	Digital assets
Inherent value	Backed by underlying assets and the **cash flows** they generate (interest, dividends)	Most have no backing and generate no cash flows, so they have **no fundamental value**. Prices reflect **expected price appreciation**, driven by **scarcity** (limited supply) and the ability to **transfer value** in the future
Transaction validation	Recorded on **private ledgers** kept by central intermediaries	Recorded on **decentralized digital ledgers**, often on **permissionless** blockchain networks

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Feature	Traditional financial assets	Digital assets
Medium of exchange	Traded and priced in globally accepted **fiat currencies**	Used as an alternative to fiat currency, mostly for online transactions. Mainstream finance is reluctant to accept them because of **high transaction costs**, the fact that they are **not legal tender** in many jurisdictions, and heavy restrictions in some countries. CBDCs are being evaluated as tokenized fiat currency
Regulation	Clear, established rules in most jurisdictions	**Lack well-developed standards**. Rules are still being written; US regulators treat some digital currencies as **commodities**, while others see them as nonfinancial assets. Crypto exchanges are often unregulated, so behavior such as market manipulation is not specifically prohibited

LOS 86.c — Investment forms and vehicles

Where cryptocurrencies trade

Direct owners hold a **cryptocurrency wallet**, which stores the digital encryption keys needed to access and trade the assets.

KEY CONCEPT

	Centralized exchange	Decentralized exchange
Structure	Privately held; trades run on the exchange's private servers	No central authority; runs on a distributed framework of many computers
Popularity	Most popular type (even though centralization conflicts with Bitcoin's design)	Less used
Security	More vulnerable: a compromised server can halt trading and leak data such as wallet keys	Harder to attack: if one computer is hit, the others keep the exchange running
Regulation	Some are regulated as financial exchanges, depending on jurisdiction	Difficult to regulate, because no person or group controls it
Price and volume information	Platforms provide price transparency and volume data	No central platform

On both types, weak oversight raises investor protection concerns. Fraud and manipulation are more likely, e.g., **pump-and-dump schemes** that hype a coin on social media to push its price up.

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Direct investment

A **direct investment** is a transaction recorded on the blockchain itself: buying tokens on a crypto exchange, trading an NFT, or taking part in an ICO. Most crypto exchanges trade continuously. Key risks:

- **Fraud**: theft, scam ICOs and attempts to obtain wallet credentials, on top of the manipulation risk that comes with weak exchange oversight.
- **Loss of access**: coins sit in wallets that require unique passkeys. If the passkey is lost, the coins cannot be retrieved and are effectively worthless. An estimated 20% of all Bitcoin sits in lost or abandoned wallets.
- **Whales**: a few large holders who own enough of a small cryptocurrency to manipulate its price. This is mainly a risk in thinly held coins; in the largest, widely held coins no single holder is likely to have that power.

Indirect investment

KEY CONCEPT

Vehicle	How it gives exposure	Points to remember
Cryptocurrency coin trusts	Shares in a trust whose assets are large holdings of the cryptocurrency itself	Trade over the counter, similar to a closed-end fund; more transparency; the investor needs no digital wallet or encryption keys; some charge high fees
Cryptocurrency futures contracts	Agreement to buy or sell a set quantity of a cryptocurrency at a future date, based on spot prices from crypto exchanges	Typically **cash settled** and leveraged; more volatile and less liquid than traditional futures
Cryptocurrency exchange-traded products (e.g., ETFs)	Mimic crypto returns using cash or cryptocurrency derivatives	Exchange traded rather than over the counter. Exam convention: they typically do not hold the coins directly. Current practice: some spot ETPs launched since 2024 do hold them
Cryptocurrency stocks	Shares of firms connected to digital assets: exchanges, payment providers, firms that accept, hold or mine crypto, blockchain service providers	Exposure through the business, not the coin
Cryptocurrency hedge funds	Discretionary long, long/short, quantitative and multistrategy approaches	Some also mine Bitcoin to add return

Digital investment in nondigital assets

Asset-backed tokens are digital claims on physical or financial (nondigital) assets such as oil, gold, real estate or equities, and they are collateralized by the underlying asset. They:

- can potentially make high-priced assets, such as real estate, more liquid through **fractional ownership**;

- create an **immutable ownership record**, which improves transparency and lowers transaction costs;
- are generally classified by regulators as securities.

They are often issued on Ethereum, where smart contracts allow P2P interaction. **Decentralized applications (dApps)** record transactions on the blockchain without a central system. **Decentralized finance (DeFi)** is the resulting marketplace of open-source dApps that can serve as a medium of exchange and a store of value, tokenize assets and keep immutable ownership records. Today most dApps focus on speculating in digital assets.

LOS 86.d — Risk, return and diversification

Return drivers. Returns on cryptocurrencies come mainly from **asset appreciation**. A major driver of prices is **limited supply**: Bitcoin is capped at 21 million units, and some call it "digital gold" for that reason. Prices are also affected by:

- market adoption and network impacts (network effects);
- technological innovation;
- regulation, including how new rules develop;
- speculation and investors' risk appetite.

Risk. Bitcoin and other cryptocurrencies have delivered high returns with high volatility compared with traditional asset classes, with very large rises followed by steep falls. Fraud, crime, uncertain regulation and outright bans in some countries (China banned cryptocurrencies in 2021) add to the uncertainty. Over January 2011 to January 2022, Bitcoin's average monthly log return was about 8.8%, against roughly 0.2% to 1.1% for US equities, global equities and global bonds, and its monthly standard deviation (about 0.32) was about eight times that of equities. Its coefficient of variation (standard deviation per unit of average return, about 3.7) was close to that of US equities and below those of global equities and global bonds.

Diversification. Historically, crypto returns have shown low correlations with traditional asset class returns (Bitcoin's monthly correlations with US equities, global equities and global bonds were only about 0.14 to 0.22 over the same period), so they may provide diversification benefits. The evidence is not settled, and correlations can rise during periods of extreme market stress, just when diversification is most needed.

Example

A coin's price goes from $8,000 to $20,000 over one year and then falls to $11,000 over the next. The first-year return is $20{,}000/8{,}000 - 1 = 150\%$, and the second-year return is $11{,}000/20{,}000 - 1 = -45\%$. The two-year holding period return is $11{,}000/8{,}000 - 1 = 37.5\%$. No dividend or interest was received, so the whole return comes from price change. Such large swings are typical of the asset class's high volatility.

COMMON EXAM TRAPS

- The value of an unbacked cryptocurrency does not come from cash flows. "Projected cash flows" or "dividends" is the wrong answer to a question about the value or price drivers of such a cryptocurrency. This does not extend to every digital asset: security tokens may pay dividends, and asset-backed tokens draw value from the underlying asset.
- Options often attach "more popular" or "more vulnerable to attack" to decentralized exchanges. Both describe centralized exchanges.
- A vehicle that holds the coins and trades over the counter is a coin trust. Under the exam convention an ETF does not hold the coins.

- Asset-backed tokens lower transaction costs and increase transparency; options that reverse either effect are wrong.

BOTTOM LINE

- Unlike traditional financial assets, most digital assets have no backing and generate no cash flows, so they have no fundamental value; they are recorded on decentralized ledgers, are not legal tender in many jurisdictions and lack well-developed regulatory standards.
- Centralized crypto exchanges are the most popular type but more vulnerable to attack, while decentralized exchanges keep running if one computer is hit and are difficult to regulate.
- Direct investment carries the risks of fraud, permanent loss of coins whose passkey is lost, and, in thinly held coins, price manipulation by whales.
- Indirect exposure comes from coin trusts, which hold the coins and trade over the counter, cash-settled and leveraged futures, exchange-traded products (exam convention: typically not holding the coins; current practice: some spot ETPs launched since 2024 do), crypto stocks and crypto hedge funds.
- Asset-backed tokens are digital claims collateralized by nondigital assets, which allow fractional ownership, create an immutable ownership record that improves transparency and lowers transaction costs, and are generally classified by regulators as securities.
- Cryptocurrency returns come mainly from price appreciation, with limited supply a major driver, and have been high but highly volatile; their historically low correlations with traditional assets may offer diversification, although correlations can rise in periods of extreme stress.

QUICK CHECK

Question 2

An investment committee is debating whether to add digital assets to a multi-asset portfolio. Regarding their role as an investment class, which statement is *most accurate*?

A Their returns have historically moved closely with stocks and bonds.

B Their low historical correlations with traditional assets may aid diversification.

C They have behaved as a relatively stable, low-volatility asset class.

Answer: B

Cryptocurrency returns have historically shown low correlations with traditional asset class returns, so they may provide diversification benefits. The evidence is not conclusive, and correlations can rise in periods of extreme market stress, so the benefit is potential rather than assured.

Why the other options are wrong

- **A.** Returns have not closely tracked traditional asset classes; the low correlations are what make diversification possible. Digital assets have also produced much higher (and more volatile) returns than traditional assets.
- **C.** Digital assets are a new, highly speculative asset class with no clear legal protections, marked by high returns and very high volatility, so they are not a stable asset class.

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Key takeaway Crypto offers potential diversification (low historical correlation) but with high volatility, and correlations may rise under stress.

Practice questions

Question 3

Kestrel Robotics is weighing two ways of raising new capital: a regulated initial public offering (IPO) or an initial coin offering (ICO). Compared with the IPO, the ICO would *most likely* involve:

A a longer capital-raising period.

B more voting rights for investors.

C lower issuance costs.

Answers: 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/alternative-investments/introduction-to-digital-assets/. The full bank has 16 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Portfolio Construction

Exam weight 8–12% · 6 readings

Portfolio risk and return and the benefit of diversification, the capital market line and the CAPM, the portfolio management process and the investment policy statement, behavioral biases and an introduction to risk management.

Reading 87	Modern Portfolio Theory
Reading 88	Systematic vs Unsystematic Risk
Reading 89	Defined Benefit vs Defined Contribution
Reading 90	Basics of Portfolio Planning and Construction
Reading 91	The Behavioral Biases of Individuals
Reading 92	Value At Risk

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PORTFOLIO CONSTRUCTION · READING 87

Modern Portfolio Theory

What you'll learn

LOS 87.a Describe the major asset classes (small- and large-cap stocks, corporate and government bonds, T-bills), their historical risk–return tradeoff, non-normal returns and liquidity.

LOS 87.b Explain risk aversion (versus risk neutrality and risk seeking) and what it implies for choosing among portfolios.

LOS 87.c Explain how an investor's utility function and indifference curves, combined with the capital allocation line, determine her optimal portfolio.

LOS 87.d Calculate and interpret the mean, variance, covariance and correlation of asset returns from historical data.

LOS 87.e Calculate and interpret the variance and standard deviation of a two-asset portfolio.

LOS 87.f Describe how combining assets whose returns are less than perfectly correlated reduces portfolio risk.

LOS 87.g Describe and interpret the minimum-variance frontier, the efficient frontier and the global minimum-variance portfolio.

MODULE 87.1

Historical Risk and Return

This reading describes the risk and return history of the major asset classes and explains risk aversion, utility, indifference curves, the capital allocation line and the choice of an optimal portfolio. It shows how to calculate mean, variance, covariance and correlation from historical returns and the standard deviation of a two-asset portfolio, how correlation drives diversification, and how the minimum-variance frontier, the global minimum-variance portfolio and the efficient frontier are defined.

LOS 87.a — Characteristics of the major asset classes

Portfolio construction starts with the **asset classes** an investor can combine. The classic building blocks are **small-capitalization stocks**, **large-capitalization stocks**, **long-term corporate bonds**, **long-term government bonds** (Treasury bonds) and **Treasury bills** (T-bills).

Risk and return go together

Long histories of market data (US data from 1926 onward, and similar data for other markets around the world) show a clear **risk–return tradeoff**: the asset classes with the highest **average annual returns** also have the highest **standard deviation of returns**.

KEY CONCEPT

US asset class, 1926–2017	Average annual return (geometric mean)	Standard deviation (annualized monthly)
Small-cap stocks	12.1%	31.7%
Large-cap stocks	10.2%	19.8%

US asset class, 1926–2017	Average annual return (geometric mean)	Standard deviation (annualized monthly)
Long-term corporate bonds	6.1%	8.3%
Long-term government bonds	5.5%	9.9%
Treasury bills	3.4%	3.1%
Inflation	2.9%	4.0%

The pattern holds broadly rather than pair by pair: over this period long-term government bonds were slightly more volatile than long-term corporate bonds while earning a little less.

Over long periods, equities have earned higher average returns than fixed-income securities, but with a higher standard deviation of returns. Small-cap stocks have had the highest average return and the highest standard deviation, and T-bills the lowest of both.

Why the pattern makes sense

Theory predicts this pattern. Investors are risk averse, so they hold a riskier asset class only if they expect a higher average return as compensation for the extra risk (LOS 87.b).

Nominal versus real returns

Year-by-year nominal returns on US equities have swung widely, from losses above 40% to gains above 50%. Subtracting inflation gives an approximate **real return**. On the 1926–2017 figures, T-bills earned a real return of only about $3.4\% - 2.9\% = 0.5\%$, while large-cap stocks earned about $10.2\% - 2.9\% = 7.3\%$. Because inflation itself moved around a lot over the period, real returns have been more stable than nominal returns.

Example. Suppose a fund's nominal return last year was 9.0% and inflation was 3.5%. The approximate real return is $9.0\% - 3.5\% = 5.5\%$. If the same fund had returned 1.0% in a year of 6.0% inflation, its approximate real return would be -5.0%. Nominal returns can hide the effect of inflation.

Beyond mean and variance

KEY CONCEPT

Describing an investment only by its **expected return** and **variance** (or standard deviation) is a simplification, because actual return distributions are not normal:

- **Negative skewness**: returns show a tendency toward large downside surprises.
- **Excess kurtosis** (fatter tails; kurtosis above 3): extreme outcomes, both up and down, happen more often than a normal distribution predicts.

These non-normal features (skewness $\neq 0$, kurtosis $\neq 3$) should be considered when analyzing investments.

Liquidity is another characteristic. Illiquidity can depress a security's price and so raise its expected return. It matters most in **emerging markets** and for thinly traded securities, for example **low-quality corporate bonds**; developed-market government securities are highly liquid.

COMMON EXAM TRAPS

- An option giving equities a higher average return *and* a lower standard deviation than bonds contradicts the historical record.
- Long-term government bonds are not the lowest-risk asset class: their prices move with interest rates, so they are more volatile than T-bills.
- Negative skewness describes the shape of the distribution (a long left tail), not a negative average return.

BOTTOM LINE

- Over long periods the asset classes with the highest average returns have also had the highest standard deviations, small-cap stocks being highest on both and T-bills lowest, although the pattern holds broadly and not for every pair.
- Over long periods equities have earned higher average returns than fixed-income securities with a higher standard deviation, which fits risk-averse investors holding riskier asset classes only for a higher expected return.
- The approximate real return is the nominal return minus inflation, and over 1926–2017 real returns were more stable than nominal returns because inflation itself varied a lot.
- Return distributions are not normal: negative skewness means a tendency toward large downside surprises, and excess kurtosis (kurtosis above 3) means extreme outcomes occur more often than a normal distribution predicts.
- Illiquidity can depress a security's price and so raise its expected return, which matters most in emerging markets and for thinly traded securities such as low-quality corporate bonds.

QUICK CHECK

Question 1

A pension trustee reviews close to a century of capital market history for the United States and other developed markets, comparing equity asset classes with fixed-income asset classes. Relative to fixed-income securities, equities have *most likely* shown:

A higher average annual returns and higher standard deviation of returns.

B lower average annual returns and higher standard deviation of returns.

C higher average annual returns and lower standard deviation of returns.

Answer: A

Long-run data show a positive risk–return tradeoff across asset classes. Both small-cap and large-cap stocks have earned higher average annual returns than long-term corporate and government bonds, and they have done so with a larger standard deviation of returns. Results in other developed markets are similar.

Why the other options are wrong

- **B.** Equities have earned higher average returns than bonds over long periods. The extra return is the reward for their extra volatility.

- **C.** Equities have been more volatile than bonds; higher return with lower risk would contradict the risk–return tradeoff.

Key takeaway Over long periods: more average return has come with more standard deviation (small caps > large caps > bonds > T-bills).

MODULE 87.2

Risk Aversion

LOS 87.b — Risk aversion and portfolio selection

A **risk-averse** investor dislikes risk: given two investments with the **same expected return**, she chooses the one with the **lower risk** (standard deviation). She will hold riskier assets if the extra expected return is adequate compensation for the extra risk. Financial models assume investors are risk averse, and the market prices of financial assets reflect the preferences of risk-averse investors, so **riskier assets must offer higher expected returns**.

KEY CONCEPT

Investor type	Choice between two investments with equal expected return
Risk averse	Picks the less risky one
Risk neutral	Indifferent (cares only about expected return)
Risk seeking (risk loving)	Picks the riskier one

Illustration. A game pays \$200 or \$0 with equal probability (expected payoff \$100). A risk-averse investor prefers a sure \$100 to the game; a risk-seeking investor prefers the game; a risk-neutral investor is indifferent.

Dominance. A risk-averse investor never picks a portfolio that is beaten on both counts, that is, when another available portfolio has a higher (or equal) expected return and lower (or equal) risk, with at least one strictly better. Between a low-return/low-risk portfolio and a high-return/high-risk one, either can be a rational choice; which one depends on the investor's degree of risk aversion.

LOS 87.c — Utility, indifference curves and the optimal portfolio

Utility function

An investor's **utility function** expresses her trade-off between risk and return:

KEY CONCEPT

$$U = E(r) - \frac{1}{2}A\sigma^2$$

where $E(r)$ is expected return and σ^2 the variance of returns (both in decimals), and A is the investor's **degree of risk aversion**, also called the **risk aversion coefficient**. The larger A, the more utility the investor gives up for each unit of variance. A risk-averse investor has $A > 0$, a risk-neutral investor $A = 0$ and a risk-seeking investor $A < 0$.

> *Example.* With $A = 2$, a portfolio with $E(r) = 8\%$ and $\sigma = 10\%$ gives $U = 0.08 - 0.5 \times 2 \times 0.10^2 = 0.08 - 0.01 = 0.07$.

Two results follow directly from the formula. A risk-free asset has $\sigma = 0$, so its utility equals its return for every investor, whatever the value of A. A risky portfolio, by contrast, is worth less to a more risk-averse investor: with the same 8% and 10%, an investor with $A = 6$ gets $U = 0.08 - 0.03 = 0.05$, below the 0.07 of the investor with $A = 2$.

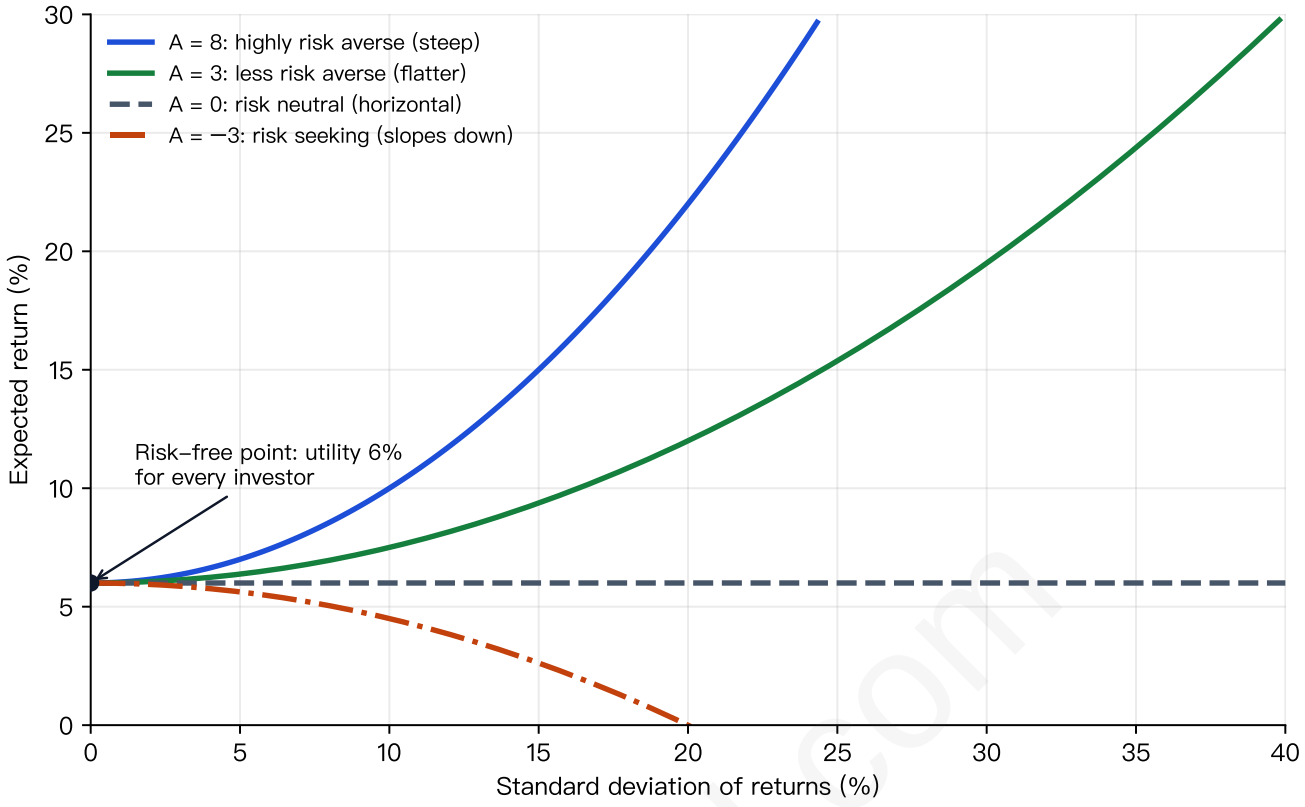
Indifference curves

An **indifference curve** plots the risk–return combinations that give an investor the **same expected utility**. Drawing the curves from expected return and standard deviation alone assumes that these two are the only portfolio characteristics the investor cares about. For a risk-averse investor the curves **slope upward** (more risk must be paid for with more expected return) and curves higher up and to the left give more utility.

- A more risk-averse investor needs more extra return per unit of extra risk, so her indifference curves are **steeper** (a higher A).
- A less risk-averse investor has **flatter** indifference curves.

Solving the utility function for expected return gives $E(r) = U + \frac{1}{2}A\sigma^2$, so the slope follows the sign of A: upward when $A > 0$, flat when $A = 0$ (risk neutral) and downward when $A < 0$ (risk seeking). The figure draws each kind with the same utility of 6%.

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Indifference curves with a utility of 6% for investors with different risk aversion coefficients (illustrative numbers)

Adding the risk-free asset: the capital allocation line

Combine a risky portfolio (weight w) with a **risk-free asset** (weight $1 - w$). The risk-free asset's standard deviation is zero, and so is its correlation with the risky portfolio. In the two-asset portfolio formulas (LOS 87.e), every term involving the risk-free asset's risk therefore drops out, leaving

KEY CONCEPT

$$E(R_p) = w\,E(R_{risky}) + (1 - w)R_f \qquad \sigma_p = w\,\sigma_{risky}$$

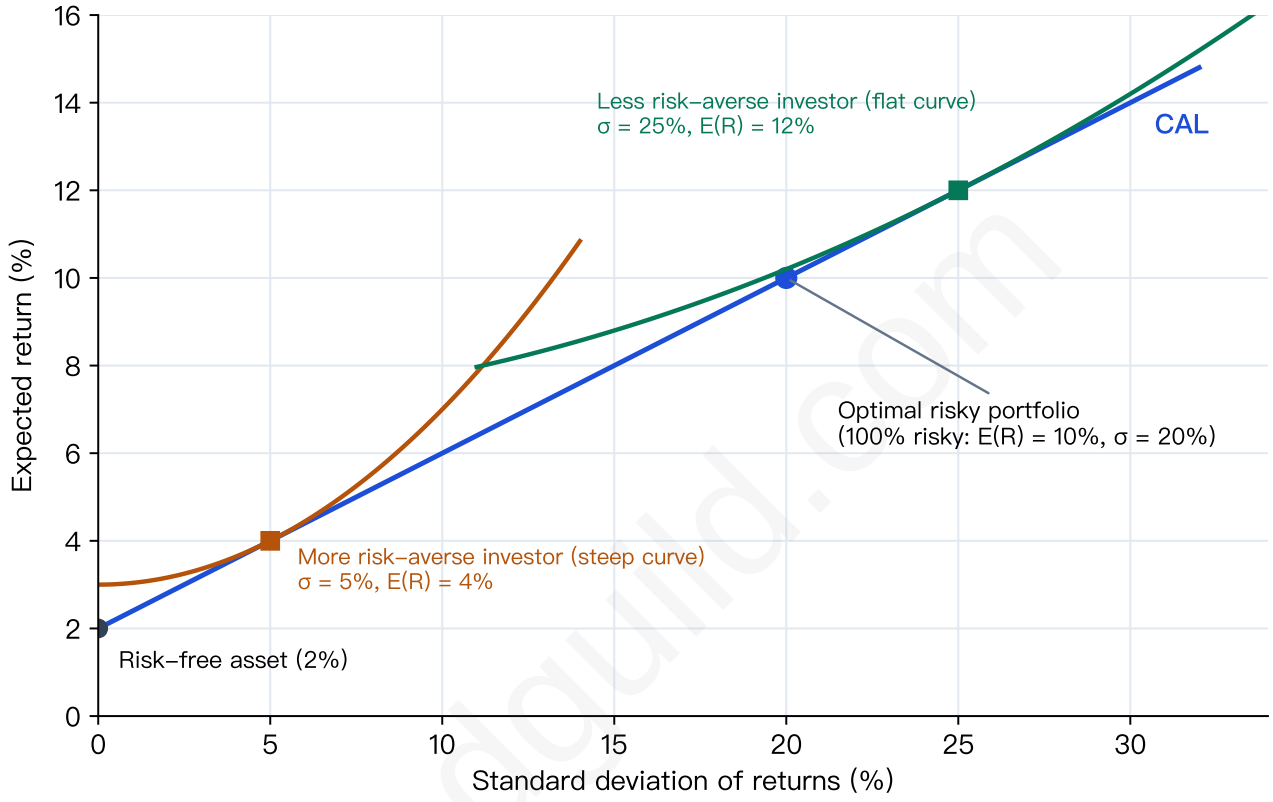
Portfolio risk is just the fraction w of the risky portfolio's risk, so all combinations lie on a **straight line**. The line from the risk-free rate through the **optimal risky portfolio** is the **capital allocation line** (CAL). $w > 1$ means borrowing at the risk-free rate to invest more than 100% of equity in the risky portfolio (a leveraged position, beyond the risky portfolio on the CAL).

The **two-fund separation theorem** says that every investor's optimal portfolio is some combination of the same two "funds": the **optimal risky portfolio** and the **risk-free asset**. Investors differ only in how much they put in each.

Example. $R_f = 2\%$; optimal risky portfolio $E(R) = 10\%$, $\sigma = 20\%$. With 30% in the risky portfolio: $E(R_p) = 0.30(10\%) + 0.70(2\%) = 4.4\%$ and $\sigma_p = 0.30 \times 20\% = 6\%$.

Choosing the optimal portfolio

The investor's **optimal portfolio** is the attainable portfolio that gives her the **highest utility**: the point where her **highest attainable indifference curve is tangent** to the CAL. With risky assets only, the tangency is with the **efficient frontier**, the set of risky portfolios with the highest expected return for each level of risk (LOS 87.g). Curves above it cannot be reached; curves below it are reachable but give less utility. The optimal portfolio is therefore not simply the one with the highest return, and it differs across investors because their indifference curves differ.



Capital allocation line and the optimal portfolios of two investors

The steep-curve (more risk-averse) investor is tangent near the risk-free end of the CAL: mostly T-bills, with low risk and low expected return. The flat-curve (less risk-averse) investor is tangent further out, with more in the risky portfolio (possibly borrowing) and higher risk and higher expected return.

COMMON EXAM TRAPS

- Risk aversion does not mean always choosing the least risky asset; it means choosing the less risky of two assets with the same expected return.
- Do not reverse the slopes: a steep curve belongs to the more risk-averse investor, a flat curve to the less risk-averse one.
- A leveraged investor (more than 100% in the risky portfolio) can still be risk averse, only less so, with flatter curves.
- The line joining the risk-free asset and a risky portfolio is the **capital allocation line**; the capital market line is the special case where the risky portfolio is the market portfolio (Reading 88).
- The optimum is the tangency with the highest attainable indifference curve. The lowest curve, or any point where a curve crosses the frontier, is wrong.

- In the utility formula, enter returns and standard deviations as decimals (0.08, not 8) and keep the $\frac{1}{2}$. With A = 2, $E(r) = 8\%$ and $\sigma = 10\%$, dropping the $\frac{1}{2}$ gives 0.06 and entering 8 and 10 gives −92, instead of 0.07.

EXAM SHORTCUTS

- A risk-free asset has $\sigma = 0$, so its utility equals its return for every investor whatever the value of A, and no utility calculation is needed to rank it.

BOTTOM LINE

- Given two investments with the same expected return, a risk-averse investor picks the less risky one, a risk-neutral investor is indifferent and a risk-seeking investor picks the riskier one.
- Utility is $U = E(r) - \frac{1}{2}A\sigma^2$ with returns in decimals, where A is positive for a risk-averse investor, zero for a risk-neutral one and negative for a risk seeker, and a risky portfolio is worth less to a more risk-averse investor.
- An indifference curve joins risk-return combinations with the same expected utility; for a risk-averse investor it slopes upward, a more risk-averse investor has steeper curves, and curves higher and to the left give more utility.
- Combining a risky portfolio with the risk-free asset gives $E(R_p) = wE(R_{risky}) + (1 - w)R_f$ and $\sigma_p = w\sigma_{risky}$, a straight capital allocation line, with $w > 1$ meaning borrowing at the risk-free rate.
- Under two-fund separation every investor's optimal portfolio combines the same optimal risky portfolio with the risk-free asset, and investors differ only in how much they put in each.
- The optimal portfolio is where the investor's highest attainable indifference curve is tangent to the CAL, or to the efficient frontier when only risky assets are available, so a more risk-averse investor sits nearer the risk-free end.

QUICK CHECK

Question 2

Priya Nair and Tomas Ekberg share the same capital market expectations. Ekberg's risk–return indifference curves are flatter than Nair's. Compared with Nair, Ekberg is *most likely*:

A less risk averse and will choose an optimal portfolio with a higher expected return.

B more risk averse and will choose an optimal portfolio with a higher expected return.

C less risk averse and will choose an optimal portfolio with a lower expected return.

Answer: A

Flatter indifference curves mean Ekberg requires less extra expected return for each additional unit of risk, so he is less risk averse than Nair. Facing the same set of portfolios, his flatter curve is tangent further out along the frontier (or CAL), so his optimal portfolio has more risk and a higher expected return.

Why the other options are wrong

- **B.** Flatter curves indicate lower risk aversion; a more risk-averse investor has steeper curves.

- **C.** The degree of risk aversion is right, but a less risk-averse investor's tangency point lies further out, at a higher expected return.

Key takeaway Flatter curves mean less risk aversion and an optimal portfolio with more risk and higher expected return.

MODULE 87.3

Portfolio Standard Deviation

LOS 87.d — Mean, variance, covariance and correlation from historical data

One asset

The **mean return** is the average of the period returns. **Variance** and **standard deviation** measure how widely returns spread around that mean. They are the usual measures of investment risk.

$$\sigma^2 = \frac{\sum_{t=1}^{T}(R_t - \mu)^2}{T} \quad \text{(population)} \qquad s^2 = \frac{\sum_{t=1}^{T}(R_t - \bar{R})^2}{T-1} \quad \text{(sample)}$$

Historical returns are a sample, so the **sample variance** (divide by $T-1$) is normally used. Standard deviation is the square root of variance.

Two assets: covariance

Covariance measures how two variables move together over time.

$$\text{Cov}_{1,2} = \frac{\sum_{t=1}^{T}(R_{1,t} - \bar{R}_1)(R_{2,t} - \bar{R}_2)}{T-1}$$

- **Positive covariance**: the returns tend to be above (or below) their means in the same periods, so they tend to move together.
- **Negative covariance**: when one return is above its mean, the other tends to be below its mean. It does not mean they always move in opposite directions.
- **Zero covariance**: no linear relationship, so knowing one return says nothing about the other in a linear sense.

Covariance is an absolute measure in squared return units. Its size depends on the assets' volatilities, so on its own it says little about the strength of the relationship.

Correlation

Standardizing covariance by the two standard deviations gives the **correlation coefficient**:

KEY CONCEPT

$$\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \sigma_2} \quad \Longleftrightarrow \quad \text{Cov}_{1,2} = \rho_{1,2}\,\sigma_1\,\sigma_2$$

Correlation has no units and always lies between −1 and +1.

KEY CONCEPT

Correlation	Meaning
$\rho = +1$	**Perfectly positively correlated**: deviations from the means are always proportional and in the same direction
$\rho = -1$	**Perfectly negatively correlated**: deviations always proportional and in opposite directions
$\rho = 0$	**Uncorrelated**: no linear relationship

Example. Three annual returns: Asset A 4%, 10%, 1%; Asset B 6%, 12%, 0%.

- Means: $\bar{R}_A = 15/3 = 5\%$, $\bar{R}_B = 18/3 = 6\%$.
- Deviations: A $-1, +5, -4$; B $0, +6, -6$.
- $s_A^2 = (1 + 25 + 16)/2 = 21$, so $s_A = 4.58\%$; $s_B^2 = (0 + 36 + 36)/2 = 36$, so $s_B = 6\%$.
- $\text{Cov}_{A,B} = [(-1)(0) + (5)(6) + (-4)(-6)]/2 = 54/2 = 27$ (in %²).
- $\rho_{A,B} = 27/(4.58 \times 6) = 0.98$.

LOS 87.e — Portfolio standard deviation

For a two-asset portfolio with weights w_1 and $w_2 = 1 - w_1$:

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

KEY CONCEPT

$$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\text{Cov}_{1,2} = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$$

$$\sigma_p = \sqrt{\sigma_p^2}$$

The inputs are the **weights**, each asset's **standard deviation** (or **variance**) and the **covariance** (or correlation). Expected returns are needed for $E(R_p)$ but not for σ_p. Beta, which measures an asset's systematic risk relative to the market (Reading 88), is not an input.

Example. 60% in an equity fund ($\sigma = 18\%$) and 40% in a bond fund ($\sigma = 10\%$), $\rho = 0.20$:

$$\sigma_p^2 = 0.36(0.0324) + 0.16(0.0100) + 2(0.6)(0.4)(0.2)(0.18)(0.10) = 0.011664 + 0.0016 + 0.001728 = 0.014992$$

$\sigma_p = \sqrt{0.014992} = 12.24\%$. With $\rho = +1$ the answer would be the weighted average $0.6(18\%) + 0.4(10\%) = 14.8\%$.

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COMMON EXAM TRAPS

- When variances are given, take square roots before dividing the covariance by $\sigma_1\sigma_2$.
- Correlation outside −1 to +1 means an arithmetic error (e.g., dividing the wrong way).
- Sample statistics divide by $T-1$, not T. Dividing by T = 3 in the three-year example gives a variance of 14 for Asset A and a covariance of 18 instead of 21 and 27.
- Portfolio variance or standard deviation: read which one is asked.
- Square the weights and the standard deviations (in decimals) in the first two terms, and keep the factor 2 on the covariance term. In the 60/40 example, dropping the 2 gives 11.89% and leaving the weights unsquared gives 15.86%, instead of 12.24%.

BOTTOM LINE

- Historical returns are a sample, so the sample variance divides the squared deviations by $T-1$, and the standard deviation is its square root.
- Covariance measures how two returns move together: positive when they tend to be above or below their means in the same periods, negative when one tends to be above its mean while the other is below, and zero when there is no linear relationship.
- Covariance is in squared return units and depends on the assets' volatilities, whereas the correlation $\rho_{1,2} = \mathrm{Cov}_{1,2}/(\sigma_1\sigma_2)$ has no units and lies between −1 and +1.
- Two-asset portfolio variance is $w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{1,2}\sigma_1\sigma_2$, so its inputs are the weights, the standard deviations and the covariance or correlation, with expected returns and beta not needed for σ_p.

QUICK CHECK

Question 3

An analyst collects four years of returns for two stocks:

Annual returns on two stocks (%)

Year	Stock P	Stock Q
1	9	14
2	3	6
3	12	10
4	8	10

Using the sample formula, the covariance between the returns of Stock P and Stock Q (in percent squared) is *closest* to:

A

B

C

Answer: A

Sample covariance is the sum of the products of the two stocks' deviations from their own means, divided by $T-1$. The products sum to 24 over four observations, so the covariance is $24/3 = 8$.

Means: $\bar{R}_P = (9+3+12+8)/4 = 8\%$; $\bar{R}_Q = (14+6+10+10)/4 = 10\%$

Year	$R_P - \bar{R}_P$	$R_Q - \bar{R}_Q$	Product
1	1	4	4
2	−5	−4	20
3	4	0	0
4	0	0	0

$$\text{Cov}_{P,Q} = \frac{4+20+0+0}{4-1} = \frac{24}{3} = 8$$

Why the other options are wrong

- **B.** 14 is the sample variance of Stock P $(1+25+16+0)/3$. The question asks for the covariance between P and Q.
- **C.** 24 is the sum of the cross-products before dividing by $T-1=3$.

Key takeaway Covariance: multiply paired deviations, sum, divide by $T-1$. Don't confuse it with one stock's variance.

MODULE 87.4

The Efficient Frontier

LOS 87.f — Investing in assets that are less than perfectly correlated

When $\rho_{1,2} = +1$, the portfolio variance formula collapses to a perfect square:

$$\sigma_p^2 = (w_1\sigma_1 + w_2\sigma_2)^2 \quad \Rightarrow \quad \sigma_p = w_1\sigma_1 + w_2\sigma_2$$

With **perfect positive correlation**, portfolio standard deviation is simply the **weighted average** of the assets' standard deviations: there is **no diversification benefit**. The lowest-risk (long-only) portfolio is then 100% in the asset with the lower standard deviation.

For any correlation below +1, the covariance term is smaller, so portfolio standard deviation is less than the weighted average of the individual standard deviations. That reduction is the **diversification benefit**, and the lower the correlation, the larger it is. At $\rho = 0$ the covariance term vanishes; for negative ρ it becomes negative and cuts risk further. At $\rho = -1$ the variance is again a perfect square, $(w_1\sigma_1 - w_2\sigma_2)^2$, so $\sigma_p = |w_1\sigma_1 - w_2\sigma_2|$. Choosing the weights so that $w_1\sigma_1 = (1-w_1)\sigma_2$ sets this to zero and gives a **zero-variance** portfolio:

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KEY CONCEPT

$$w_1 = \frac{\sigma_2}{\sigma_1 + \sigma_2}$$

With long positions only, a zero-variance portfolio of two risky assets is possible only when $\rho = -1$; at $\rho = 0$ risk falls but cannot reach zero.

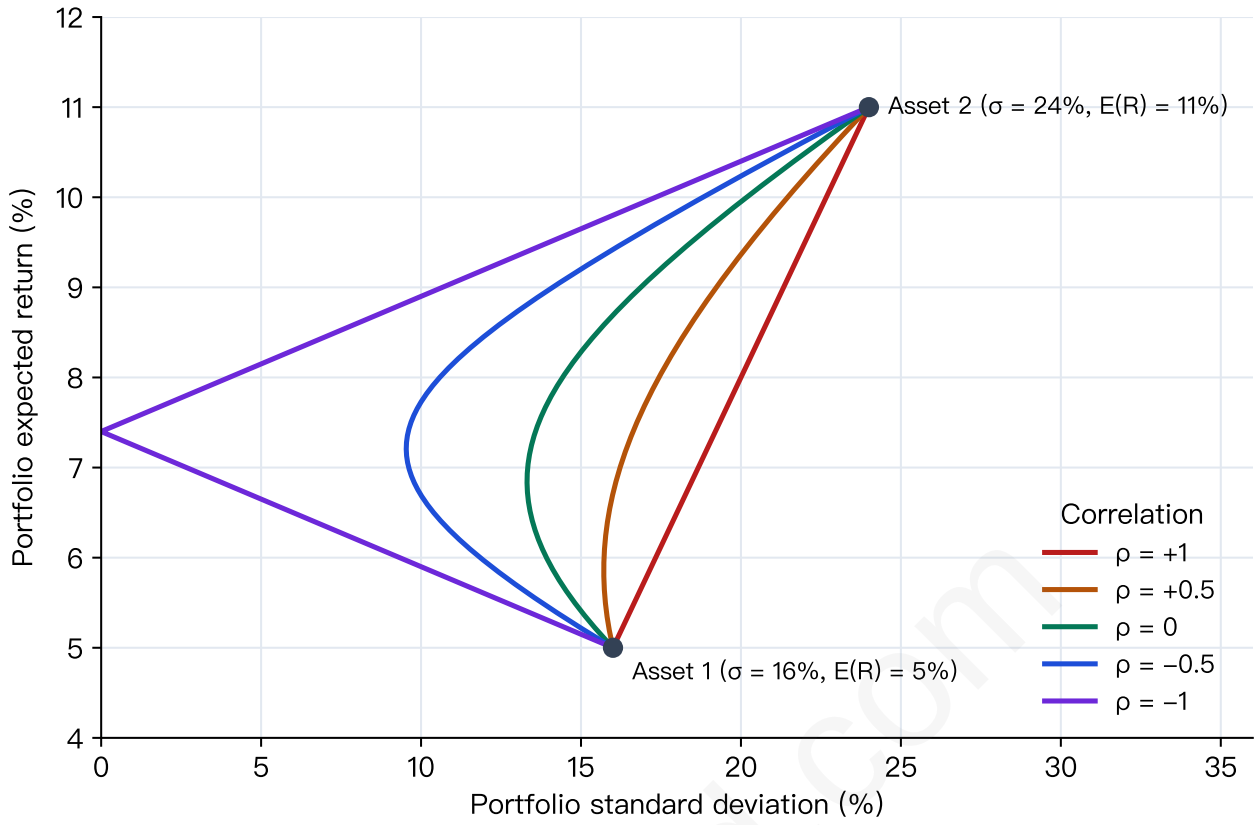
Example. Two assets with $\sigma_1 = 16\%$, $\sigma_2 = 24\%$, equally weighted:

KEY CONCEPT

ρ	Portfolio variance	Portfolio σ
+1	0.0400	20.00%
+0.5	0.0304	17.44%
0	0.0208	14.42%
−0.5	0.0112	10.58%
−1	0.0016	4.00%

At $\rho = -1$ the zero-variance mix is $w_1 = 24/40 = 60\%$; the equal-weighted mix still has 4.00% risk. The figure shows all long-only combinations of two assets ($E(R_1) = 5\%$, $E(R_2) = 11\%$) for several correlations. Correlation does not enter $E(R_p) = w_1 E(R_1) + w_2 E(R_2)$, so a given mix of weights has the same expected return on every curve. A lower correlation only cuts the risk of that mix, which is why the curve bulges further to the left. The $\rho = -1$ curve reaches the vertical axis at $0.60(5\%) + 0.40(11\%) = 7.4\%$, the expected return of the zero-variance mix.

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Two-asset portfolio risk and return for different correlations

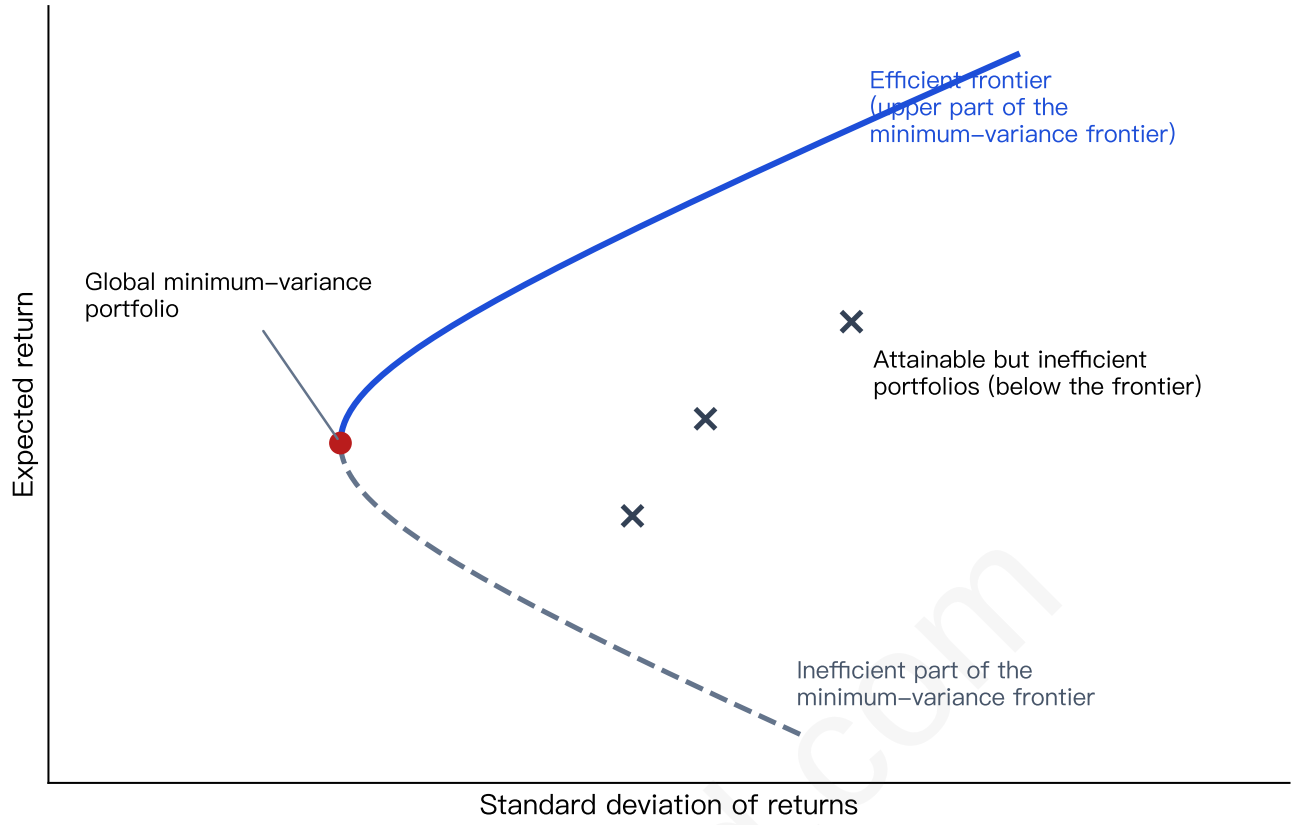
Adding an asset. If a new asset has the same standard deviation as the existing portfolio and a correlation with it below +1, adding it reduces portfolio standard deviation. With equal risk and return, the candidate with the lowest covariance (correlation) with the portfolio reduces risk the most. For the same reason investors add other asset classes, such as bonds, real estate and foreign stocks, to their domestic stocks.

LOS 87.g — Minimum-variance frontier, efficient frontier and the global minimum-variance portfolio

For each level of expected return, the portfolio with the lowest standard deviation is a **minimum-variance portfolio**; together they form the **minimum-variance frontier**. The minimum-variance portfolio farthest to the left, which is the lowest-risk risky portfolio of all, is the **global minimum-variance portfolio**.

KEY CONCEPT

Risk-averse investors want the highest expected return for each level of risk. The portfolios that offer it form the **efficient frontier**: the **upper part of the minimum-variance frontier**, starting at the global minimum-variance portfolio. Equivalently, an efficient portfolio is **not dominated** by any other attainable portfolio: no attainable portfolio offers a higher expected return with the same or lower risk. Efficient portfolios do not dominate one another, because moving along the frontier trades more risk for more expected return.



Minimum-variance frontier, efficient frontier and global minimum-variance portfolio

- A portfolio below the efficient frontier is **inefficient**: another attainable portfolio offers a higher expected return for the same risk (or the same return with lower risk). Risk-averse investors will not choose it.
- A portfolio above the efficient frontier is **not attainable**.
- The frontier is **concave**: its slope decreases as risk increases (each extra unit of risk buys less extra return).
- Lower correlations among assets move the frontier to the left (up and to the left, the "northwest"): the same returns with less risk.

Inputs (Markowitz). Risk is measured by the **variance (standard deviation) of returns**. Building the efficient frontier requires each security's **expected return**, **variance** and the **covariances** between all pairs of securities. Investors' risk aversion is not an input; it matters only when choosing a portfolio on the frontier (Module 87.2).

Spotting an inefficient portfolio. Portfolio K: $E(R) = 9\%$, $\sigma = 17\%$; Portfolio L: $E(R) = 10\%$, $\sigma = 15\%$. L has the higher return and the lower risk, so K cannot be on the efficient frontier.

When investors can lend and borrow at a risk-free rate below the expected returns on the efficient frontier, they combine the risk-free asset with the optimal risky portfolio (two-fund separation). The capital allocation line touches the concave efficient frontier only at the optimal risky portfolio, so every other frontier portfolio, including the global minimum-variance portfolio, lies below the line and is not an optimal holding for a rational investor. That includes the frontier portfolios beyond the tangency point. For an investor who wants more expected return than the optimal risky portfolio offers, borrowing at the risk-free rate to move further out along the line gives a higher expected return for the same standard deviation.

COMMON EXAM TRAPS

- Efficient frontier = highest return for each level of **risk**. "Each level of risk tolerance" is a distractor.

- A covariance equal to $\sigma_1\sigma_2$ signals $\rho = +1$, even when no correlation is quoted.
- Diversification helps whenever $\rho < +1$, including positive correlations below +1.

EXAM SHORTCUTS

- With $\rho = +1$, portfolio standard deviation is the weighted average of the assets' standard deviations, so the full variance formula is not needed.
- A portfolio with a lower expected return and a higher standard deviation than another attainable portfolio is dominated, so it can be ruled out as efficient without plotting the frontier.

BOTTOM LINE

- For any correlation below +1, portfolio standard deviation is less than the weighted average of the assets' standard deviations, and the lower the correlation, the larger this diversification benefit.
- With long positions only, a zero-variance portfolio of two risky assets exists only when $\rho = -1$, with $w_1 = \sigma_2/(\sigma_1 + \sigma_2)$.
- Correlation does not affect a portfolio's expected return, only its risk, so a lower correlation moves each mix of weights to the left at the same expected return.
- Adding an asset with the same standard deviation as the portfolio and a correlation with it below +1 reduces portfolio risk, and among otherwise equal candidates the one with the lowest correlation reduces it most.
- The efficient frontier is the upper part of the minimum-variance frontier starting at the global minimum-variance portfolio; portfolios below it are inefficient, portfolios above it are not attainable, and it is concave.
- The Markowitz inputs are each security's expected return and variance and the covariances between all pairs, investors' risk aversion is not an input, and when investors can lend and borrow at a risk-free rate below the frontier's expected returns, the CAL touches the frontier only at the optimal risky portfolio.

QUICK CHECK

Question 4

Fund Lark has a standard deviation of returns of 7.4%, and Fund Wren has a standard deviation of returns of 5.2%. The two funds' returns are perfectly positively correlated. If short sales are not allowed, which allocation minimizes the standard deviation of a portfolio of the two funds?

A Lark 0%, Wren 100%.

B Lark 100%, Wren 0%.

C Lark 41%, Wren 59%.

Answer: A

With a correlation of +1 there is no diversification benefit: portfolio standard deviation is just the weighted average of the two funds' standard deviations. The weighted average is lowest when everything is in the fund with the lower standard deviation, Wren (5.2%).

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With $\rho = +1$: $\sigma_p = w_L(7.4\%) + w_W(5.2\%)$, a weighted average that is minimized at $w_W = 100\%$, giving $\sigma_p = 5.2\%$.

Why the other options are wrong

- **B.** Putting everything in Lark gives the highest possible portfolio risk (7.4%), because Lark is the more volatile fund.
- **C.** These weights ($w_{Lark} = 5.2/12.6 = 41\%$) would create a zero-risk portfolio only if the correlation were −1. With $\rho = +1$, this mix has $\sigma_p = 0.41(7.4\%) + 0.59(5.2\%) = 6.1\%$, more than Wren alone.

Key takeaway With perfect positive correlation, hold only the lower-σ asset to minimize risk.

Practice questions

Question 5

Investors can borrow and lend at the risk-free rate, which is below the expected return of every portfolio on the efficient frontier of risky assets, and all investors hold the same expectations about risky assets. A rational, risk-averse investor is *least likely* to choose which of the following as her entire portfolio?

A 100% in the risk-free asset.

B The global minimum-variance portfolio.

C 140% in the optimal risky portfolio, financed by borrowing at the risk-free rate.

Answers: 5. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/portfolio-construction/modern-portfolio-theory/. The full bank has 75 questions on this reading.

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PORTFOLIO CONSTRUCTION · READING 88

Systematic vs Unsystematic Risk

What you'll learn

LOS 88.a Describe what happens to risk and expected return when a risk-free asset is combined with a portfolio of risky assets.

LOS 88.b Explain the capital allocation line (CAL) and the capital market line (CML), including lending and borrowing portfolios and the market portfolio.

LOS 88.c Explain systematic and nonsystematic risk and why nonsystematic risk earns no additional expected return.

LOS 88.d Explain return generating models, including multifactor models and the market model, and their uses.

LOS 88.e Calculate and interpret beta from covariance, correlation and standard deviations, or from the CAPM relation.

LOS 88.f Explain the capital asset pricing model (CAPM), its assumptions, and the security market line (SML), and contrast the SML with the CML.

LOS 88.g Calculate and interpret an asset's expected (required) return using the CAPM.

LOS 88.h Describe and demonstrate applications of the CAPM and the SML, such as identifying undervalued and overvalued securities.

LOS 88.i Calculate and interpret the Sharpe ratio, Treynor measure, M-squared and Jensen's alpha.

MODULE 88.1

Systematic Risk and Beta

This reading adds a risk-free asset to a risky portfolio to derive the capital allocation line and the capital market line, separates systematic from unsystematic risk, and covers return generating models, the market model and the calculation of beta. It then derives the CAPM and the security market line, shows how to calculate required returns and identify mispriced securities, and how to compute and interpret the Sharpe ratio, M-squared, the Treynor measure and Jensen's alpha.

LOS 88.a — Adding a risk-free asset to a risky portfolio

A **risk-free asset** has a standard deviation of returns of zero, and its returns have **zero correlation** with the returns of any risky asset. When an investor puts a weight w in a risky portfolio P and $1 - w$ in the risk-free asset, the two-asset portfolio formulas reduce to two simple expressions:

$$E(R_C) = w\,E(R_P) + (1 - w)\,R_f$$

$$\sigma_C = w\,\sigma_P$$

- The **expected return** is a weighted average of the **risk-free rate** and the risky portfolio's expected return.
- The **standard deviation** is the risky weight times the risky portfolio's standard deviation, because the risk-free asset contributes no variance and no covariance.
- Both $E(R_C)$ and σ_C are linear in w, so every combination plots on a **straight line** that starts at R_f on the vertical axis and passes through P.

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A single risky portfolio plus the risk-free asset therefore lets investors with very different degrees of risk aversion reach the level of risk they want. The resulting combinations can dominate risky portfolios that lie below the line.

Example. A risky fund has $E(R) = 9\%$ and $\sigma = 15\%$; $R_f = 3\%$.

Weight in risky fund	Expected return	Standard deviation
0.70 (lending 30%)	$0.70(9) + 0.30(3) = 7.2\%$	$0.70(15) = 10.5\%$
1.00	9.0%	15.0%
1.25 (borrowing 25%)	$1.25(9) - 0.25(3) = 10.5\%$	$1.25(15) = 18.75\%$

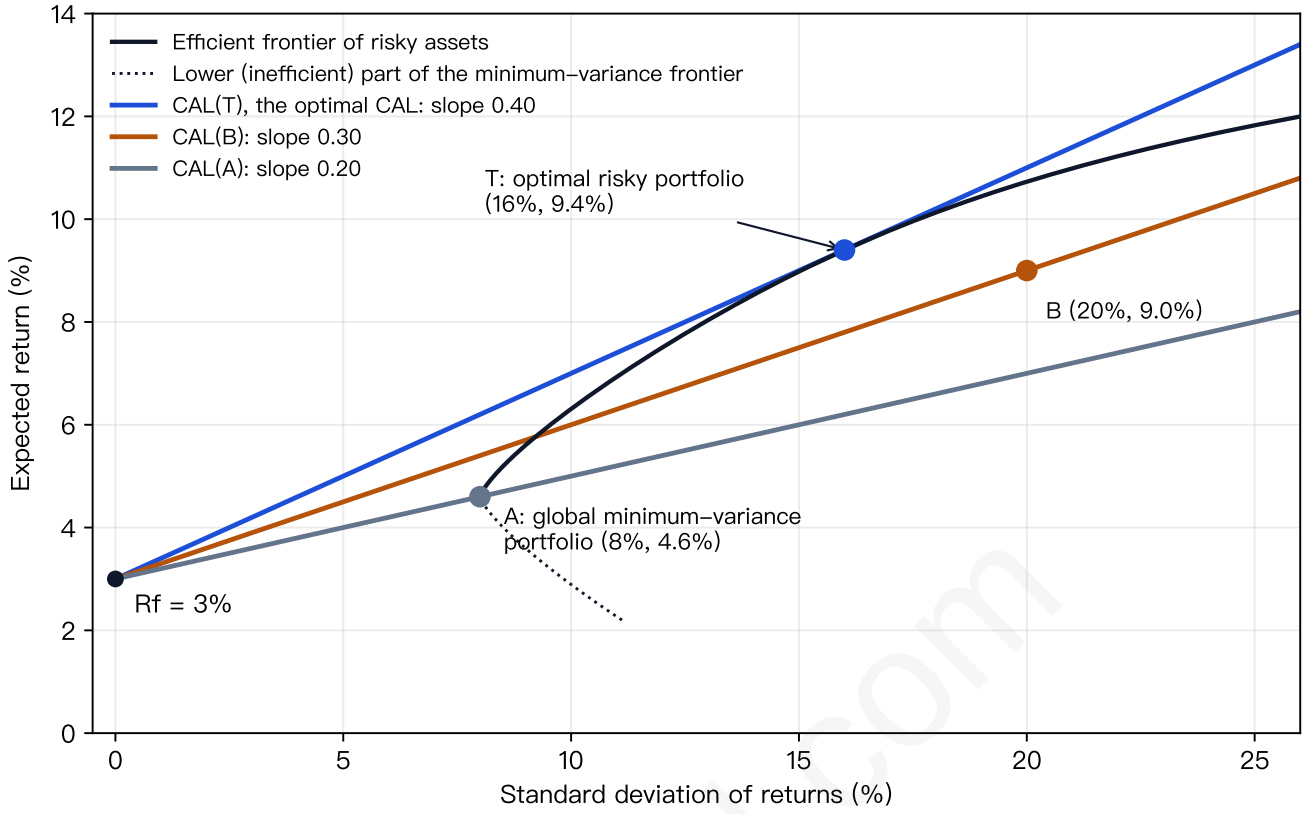
Common exam traps

- A 50/50 mix has exactly half the risky portfolio's standard deviation. Expecting diversification to push it lower is the mistake: the risk-free return never varies, so it cannot offset any of P's swings.
- The variance is not a weighted average of the variances. The standard deviation is a weighted average of the standard deviations (one of which is zero). For the 0.70 weight in the example, averaging the variances gives a standard deviation of 12.55% instead of 10.5%.

LOS 88.b — The capital allocation line (CAL) and the capital market line (CML)

The **capital allocation line (CAL)** is the line of risk–return combinations available from the risk-free asset and one particular risky portfolio. Each risky portfolio produces its own CAL. Of all the CALs, the investor should use the one whose set of portfolios she prefers most. That is the steepest line from R_f, the one **tangent** to the efficient frontier, called the **optimal CAL**. The tangency point is the **optimal risky portfolio**: of all risky portfolios, it has the highest excess return per unit of standard deviation, $[E(R_P) - R_f]/\sigma_P$, which is the Sharpe ratio of LOS 88.i. Because every CAL starts at R_f, the optimal CAL offers a higher expected return than any other CAL at every level of risk above zero.

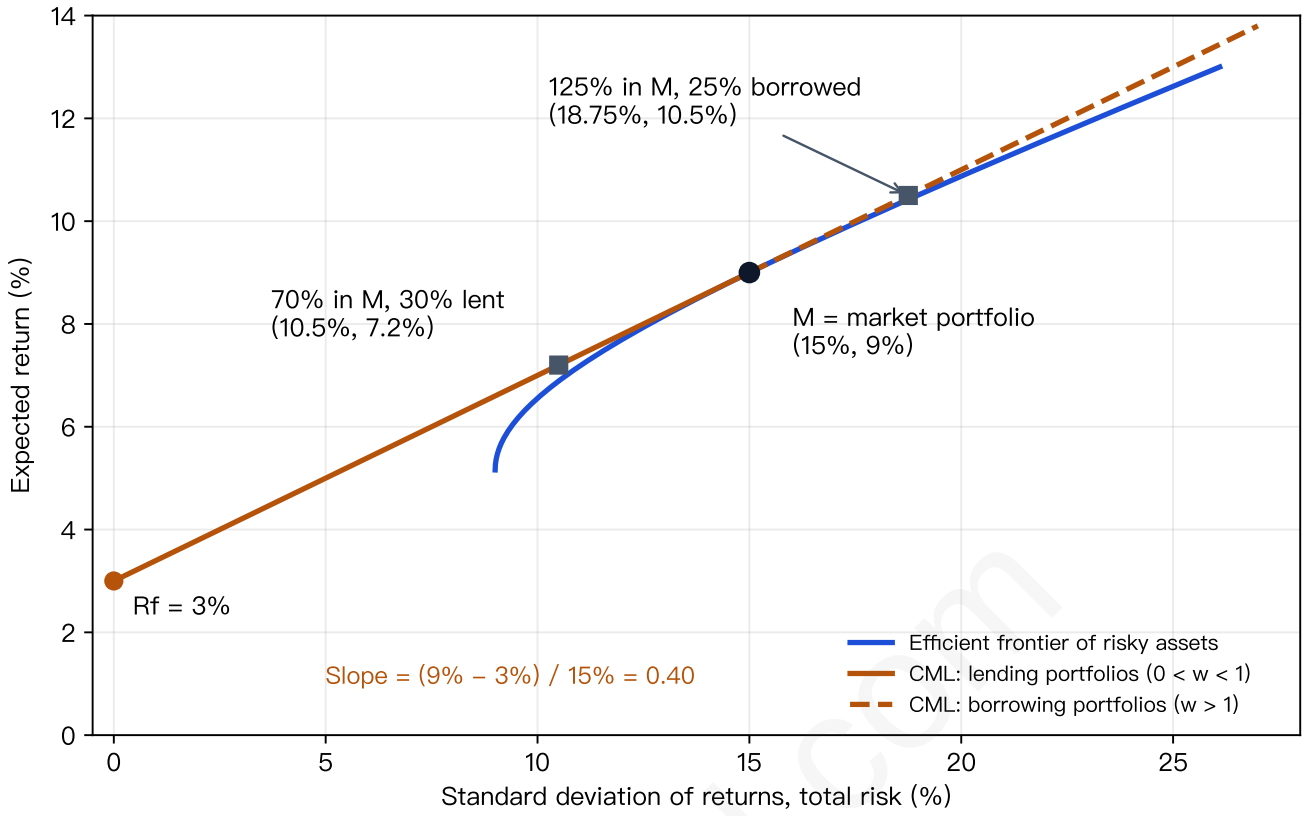
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Three capital allocation lines from the same risk-free rate: the steepest one is tangent to the efficient frontier

In the figure, $R_f = 3\%$. B has a higher expected return than the global minimum-variance portfolio A and a steeper CAL: $(9.0 - 3)/20 = 0.30$ against $(4.6 - 3)/8 = 0.20$. T's CAL is steeper still, $(9.4 - 3)/16 = 0.40$, so T is the optimal risky portfolio. B lies inside the frontier, and T offers both less risk and more expected return than B. The investor then picks the point on the optimal CAL that maximizes her **expected utility**, where her highest indifference curve touches the line.

If investors had different forecasts, each would find a different tangency portfolio and a different CAL. Capital market theory assumes **homogeneous expectations**: everyone has the same estimates of expected returns, standard deviations and correlations. Everyone then faces the same efficient frontier and the same tangency portfolio, and that portfolio must be the **market portfolio**. The market portfolio is the portfolio of **all risky assets** in existence, held in market-value weights; it contains no risk-free asset. The common optimal CAL is called the **capital market line (CML)**.



The capital market line: lending and borrowing portfolios built from the risk-free asset and the market portfolio

KEY CONCEPT

$$E(R_P) = R_f + \left[\frac{E(R_M) - R_f}{\sigma_M}\right]\sigma_P$$

- The intercept is R_f. The slope, $\frac{E(R_M) - R_f}{\sigma_M}$, is the excess return per unit of total risk (standard deviation). It equals the Sharpe ratio of the market portfolio.
- $E(R_M) - R_f$ is the **market risk premium**. Writing the CML as $E(R_P) = R_f + [E(R_M) - R_f](\sigma_P/\sigma_M)$ shows that each unit of market risk σ_M an investor accepts adds one market risk premium to her expected return.
- Risk on the horizontal axis is **standard deviation (total risk)**. Beta is the risk measure of the security market line (LOS 88.f).
- Points between R_f and M are **lending portfolios**, with part of the money invested at R_f. Points beyond M, further up the line, are **borrowing portfolios**: the investor borrows at R_f and invests more than 100% of her equity in M (buying on margin). All portfolios on the CML hold the same risky portfolio (M), so their returns are **perfectly positively correlated** with each other and they carry no unsystematic risk.

Example. $R_f = 3\%$, $E(R_M) = 9\%$, $\sigma_M = 15\%$. Slope $= 6/15 = 0.40$. A CML portfolio with $\sigma_P = 12\%$ has $E(R_P) = 3 + 0.40(12) = 7.8\%$; its weight in M is $12/15 = 0.80$. The risk-free asset has a beta of 0 and M has a beta of 1, so the beta of any CML portfolio equals its weight in M: 0.80 here, and 1.25 for an

investor who borrows a quarter of her equity and puts 125% of it in M.

Two-fund separation. Combining the CML with an investor's indifference curves splits portfolio choice into two separate decisions. Which risky assets to hold is the same decision for everyone: hold the market portfolio. How much risk to take, meaning how much to lend or borrow at R_f, depends on the investor's own risk aversion. This is the **two-fund separation theorem** (the **separation theorem**): every investor's optimal portfolio combines just two funds, the risk-free asset and the optimal risky portfolio. Security selection thereby reduces to building one well-diversified portfolio.

Passive vs. active. Investors who believe market prices are informationally efficient follow a **passive investment strategy**. They hold an index fund as a proxy for M and split their money between it and the risk-free asset. Investors who think they can identify mispriced securities use **active portfolio management**, overweighting securities they consider undervalued and underweighting those they consider overvalued.

Common exam traps

- Calling the market portfolio "all stocks", or a mix of risky assets and the risk-free asset, is wrong; a broad equity index is only a proxy for it.
- The market portfolio is found by drawing a line from the risk-free rate tangent to the efficient frontier. A line from some other point on the return axis, or the point where an investor's indifference curve touches the frontier, does not locate it.
- Reading capital market theory as "everyone holds exactly M" is a mistake; only the risky part of each investor's portfolio is the same.

LOS 88.c — Systematic and unsystematic risk

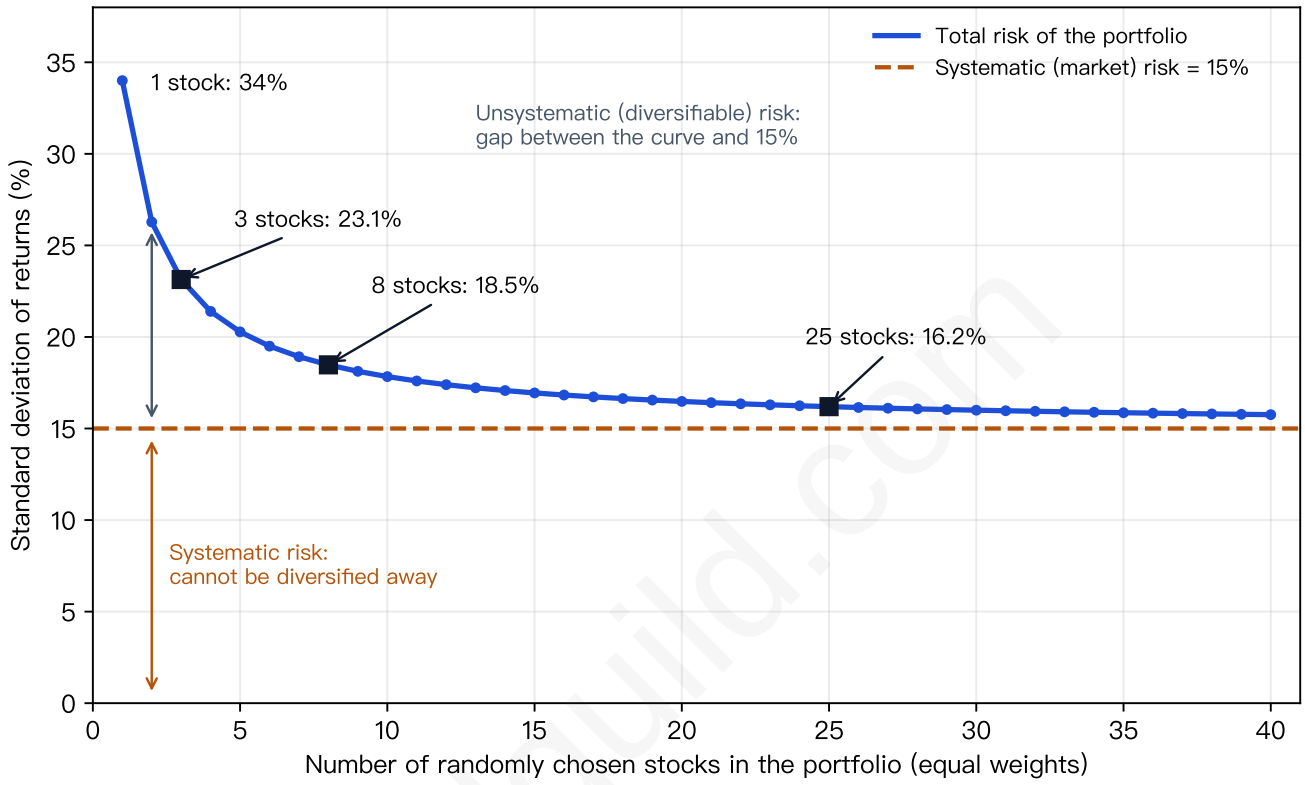
$$\text{total risk} = \text{systematic risk} + \text{unsystematic risk}$$

KEY CONCEPT

Systematic risk	Unsystematic risk
Also called **market risk** or **nondiversifiable risk**	Also called **unique risk**, **firm-specific risk** or **diversifiable risk**
Caused by economy-wide factors (GDP growth, interest rates, inflation)	Caused by events specific to one company or industry
Cannot be removed by diversification	Falls as more (imperfectly correlated) securities are added
Priced: compensated with higher expected return	**Not priced** in equilibrium

Total risk is measured by the standard deviation (or variance) of returns. Exam convention: total risk = systematic risk + unsystematic risk is stated as a simple sum, with total risk measured by standard deviation. Current practice: the sum is exact for variances, as in the market model $\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_e^2$; the standard deviations of the two parts do not add up to the total standard deviation.

Systematic risk applies to single securities as well as to portfolios. Makers of luxury goods, such as sports cars and premium motorcycles, respond strongly to the business cycle and have high systematic risk. Utilities respond little to market-wide factors and have low systematic risk. As the number of randomly chosen stocks in a portfolio grows, total risk declines and approaches market risk. According to one study, about 12 to 18 stocks capture roughly 90% of the maximum diversification benefit; another put the number at about 30 stocks. Either figure is far smaller than the number of securities in the market.



Portfolio risk falls toward systematic risk as randomly chosen stocks are added (illustrative numbers)

As unsystematic risk is diversified away, the portfolio behaves more and more like the market, so its correlation with the market moves toward +1. The market portfolio is perfectly diversified, and all of its risk is systematic.

Bonds. The same reasoning applies to a bond portfolio. A surprise that hits one issuer, such as a downgrade, can be diversified away by holding bonds of many issuers, while economy-wide changes, such as a recession or a rise in interest rates or inflation, affect all bonds and cannot be diversified away.

Why unsystematic risk earns no premium. Capital market theory assumes diversification is costless; a low-cost index fund comes close. Investors will not be paid for bearing a risk they could remove for free, so in equilibrium the expected (required) return depends only on systematic risk. A single-drug biotech stock can have very high total risk yet low systematic risk, and therefore a low equilibrium return. A steady machine-tool maker with modest total risk but high sensitivity to the business cycle requires a higher return. A portfolio of 50 or 100 such biotech stocks would carry little of that firm-specific uncertainty, because the successes and failures offset one another. A security's systematic risk is measured by what it contributes to the risk of a well-diversified portfolio.

Common exam traps

- Individual securities carry both systematic and unsystematic risk. Only well-diversified portfolios are (approximately) free of unsystematic risk.
- Adding randomly chosen stocks reduces unsystematic risk and leaves the expected level of systematic risk unchanged. Deliberately adding high-beta (low-beta) stocks does raise (lower) the portfolio's systematic risk.
- Rejecting total risk = systematic risk + unsystematic risk because standard deviations do not add is the mistake; under the exam convention above, that identity is the expected answer.

LOS 88.d — Return generating models and the market model

A **return generating model** estimates an asset's expected return from its exposure to one or more factors. Factors may be **macroeconomic** (GDP growth, inflation, consumer confidence), **fundamental** (earnings, earnings growth, firm size, research spending) or **statistical**. In practice, most models rely mainly on macroeconomic and fundamental factors. Statistical factors frequently lack any grounding in finance theory. They may capture a pattern specific to one sample period that was found by **data mining** (testing one data set over and over).

The **multifactor model** in excess-return form is:

$$E(R_i) - R_f = \beta_{i,1}E(F_1) + \beta_{i,2}E(F_2) + \cdots + \beta_{i,k}E(F_k)$$

Each β is a **factor sensitivity** (**factor loading**). The first factor is often the market's excess return.

- The **Fama and French** model uses three factors: the market's excess return, **firm size**, and the **book-to-market** (book value to market value) ratio.
- **Carhart** adds a fourth factor, **price momentum**, based on prior-period returns. The four factors explain a good share of the differences in US equity returns over the period in which the model was estimated.

A **single-factor model** uses just one factor, whatever that factor is (for example, only the book-to-market ratio, or only the earnings growth rate). The single-factor model whose factor is the market's excess return is the **single-index model**: $E(R_i) - R_f = \beta_i[E(R_m) - R_f]$.

The **market model** is a simplified single-index model estimated by regressing the asset's return on the market's **rate of return**:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

- α_i and β_i are estimated from historical returns. The expected return is $\alpha_i + \beta_i E(R_m)$.
- It is used to estimate β_i and to measure **abnormal return**, $e_i = R_i - (\alpha_i + \beta_i R_m)$: the actual return minus the return expected given the actual market return.
- For consistency with the single-index model, α_i can be set to $R_f(1 - \beta_i)$.

Example. A stock's market model is $R_i = 0.4\% + 1.1R_m$. The market returns 6%, so the expected return is $0.4 + 1.1(6) = 7.0\%$. If the stock actually returned 8.5%, its abnormal return is $8.5 - 7.0 = 1.5\%$.

Common exam traps

- The market model regresses on the market's rate of return (R_m), not on its excess return or a risk-adjusted return.

- The number of factors decides "single-factor" versus "multifactor"; the type of factor does not.

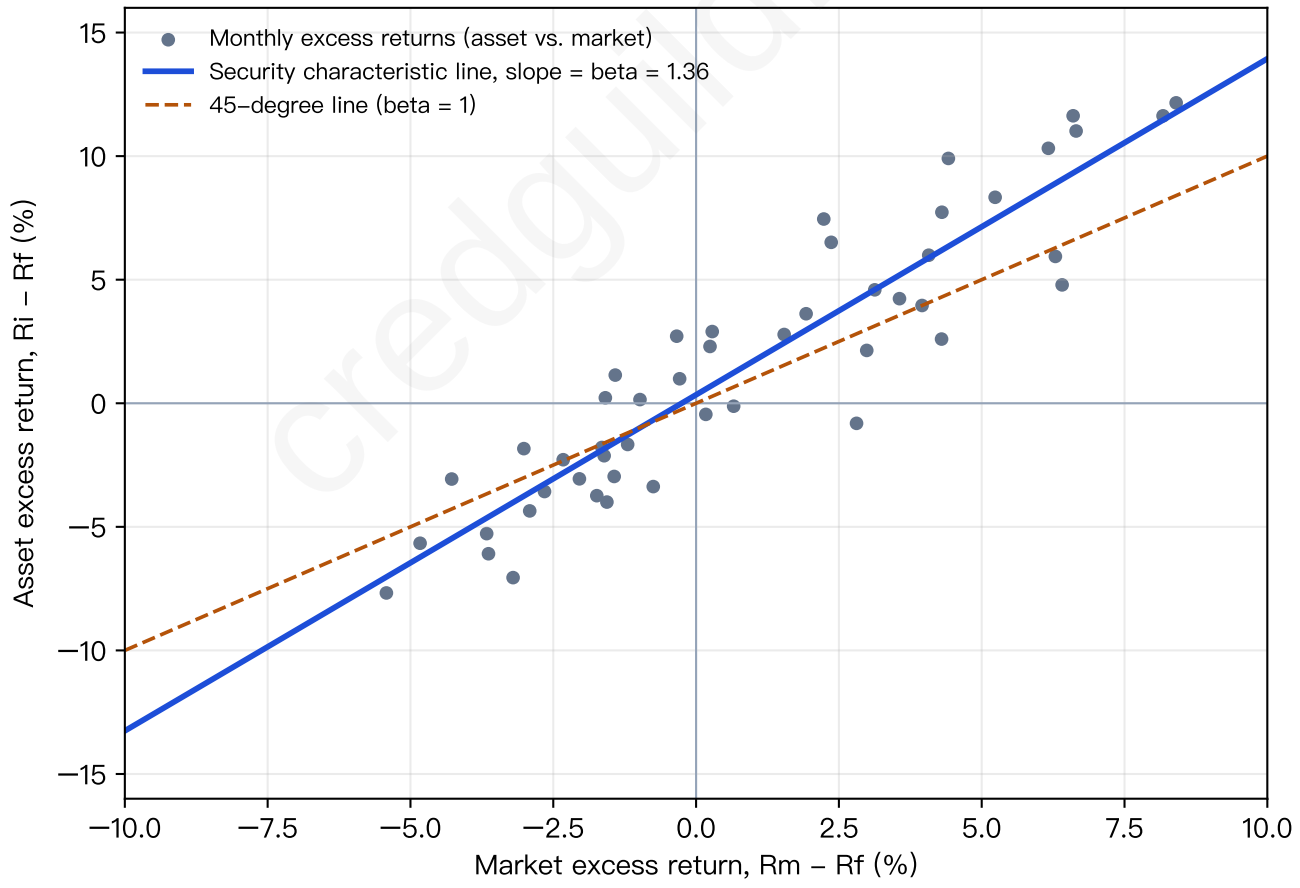
LOS 88.e — Calculating and interpreting beta

Beta is a standardized measure of the covariance of an asset's return with the market's return. It measures **systematic risk** only:

KEY CONCEPT

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma_m^2} = \frac{\rho_{i,m}\,\sigma_i\,\sigma_m}{\sigma_m^2} = \rho_{i,m}\frac{\sigma_i}{\sigma_m}$$

- The market's beta is 1, because the covariance of the market's return with itself is its variance: $Cov(R_m, R_m)/\sigma_m^2 = 1$. The (value-weighted) average beta of all stocks is also 1. A beta of 0 means the asset's return is uncorrelated with the market. Negative betas are possible.
- In practice beta is estimated as the slope of a regression of the asset's excess returns on the market's excess returns. The least squares line, which minimizes the sum of squared vertical distances of the points from the line, is the **security characteristic line**. Its slope is $Cov_{i,m}/\sigma_m^2$. A slope steeper than 45° means $\beta > 1$: the asset's return reacts more strongly to systematic factors than the market's does.



Security characteristic line: asset excess returns regressed on market excess returns

Example. $\sigma_i = 28\%$, $\sigma_m = 14\%$, $\rho = 0.45$.
Covariance $= 0.45 \times 0.28 \times 0.14 = 0.01764$; $\beta = 0.01764/0.14^2 = 0.90$. Equivalently, $\beta = 0.45 \times (28/14) = 0.90$.

Backing beta out of the CAPM. The CAPM (LOS 88.f) states $E(R_i) = R_f + \beta_i[E(R_m) - R_f]$. Rearranging it gives $\beta_i = [E(R_i) - R_f]/[E(R_m) - R_f]$. If a stock is expected to earn 12% with $R_f = 2\%$ and $E(R_m) = 7\%$, $\beta = 10/5 = 2.0$.

Common exam traps

- Divide the covariance by the market variance. Dividing by the market standard deviation is a common mistake. In the example, dividing 0.01764 by 0.14 gives 0.126 instead of 0.90.
- When given variances, take square roots before using $\rho\,\sigma_i/\sigma_m$.
- The ratio is σ_i/σ_m, the asset's standard deviation over the market's. Do not invert it, and do not forget to multiply by the correlation. In the example, inverting the ratio gives 0.225 and leaving out the correlation gives 2.0, instead of 0.90.
- When backing out beta, subtract R_f from both the asset's and the market's expected return. In the example, 12/7 gives 1.71 and 10/7 gives 1.43 instead of 2.0.

EXAM SHORTCUTS

- The risk-free asset has a beta of 0 and the market portfolio a beta of 1, so the beta of any portfolio on the CML equals its weight in M, with no covariance calculation needed.

BOTTOM LINE

- Combining a risky portfolio P with the risk-free asset gives $E(R_C) = wE(R_P) + (1-w)R_f$ and $\sigma_C = w\sigma_P$, so every combination lies on a straight line from R_f through P.
- The optimal CAL is the steepest line from R_f, tangent to the efficient frontier at the optimal risky portfolio, which has the highest excess return per unit of standard deviation of all risky portfolios.
- Under homogeneous expectations every investor's tangency portfolio is the market portfolio of all risky assets in market-value weights, and the CML, $E(R_P) = R_f + [(E(R_M) - R_f)/\sigma_M]\sigma_P$, measures risk by standard deviation, with investors differing only in how much they lend or borrow at R_f.
- Exam convention: total risk = systematic risk + unsystematic risk, with total risk measured by standard deviation; current practice: the sum is exact for variances only; in either case only systematic risk is priced, because capital market theory assumes unsystematic risk can be diversified away at no cost.
- The market model $R_i = \alpha_i + \beta_i R_m + e_i$ regresses an asset's return on the market's rate of return and gives the abnormal return $R_i - (\alpha_i + \beta_i R_m)$, while multifactor models such as Fama and French (market, size, book-to-market) and Carhart (adding momentum) use several factors.
- Beta is $Cov(R_i, R_m)/\sigma_m^2 = \rho_{i,m}\sigma_i/\sigma_m$, measures systematic risk only, and equals 1 for the market.

QUICK CHECK

Question 1

An investor combines a risk-free asset with a portfolio of risky assets. Which statement about the new portfolio is *least accurate*?

A Its variance equals the weighted average of the variances of the risk-free asset and the risky portfolio.

B Its expected return is a weighted average of the risk-free rate and the risky portfolio's expected return.

C Its standard deviation equals the risky portfolio's weight multiplied by the risky portfolio's standard deviation.

Answer: A

The variance of the new portfolio is $w^2\sigma_P^2$, not $w\sigma_P^2 + (1-w)(0)$. The standard deviation, rather than the variance, is the weighted average of the two assets' standard deviations (zero for the risk-free asset and σ_P for the risky portfolio).

Example with $w = 0.5$, $\sigma_P = 20\%$: variance $= 0.5^2 \times 0.04 = 0.01$ (standard deviation 10%), whereas a weighted average of variances would give $0.5 \times 0.04 = 0.02$ (standard deviation 14.1%).

Why the other options are wrong

- **B.** Accurate: $E(R_C) = wE(R_P) + (1-w)R_f$.
- **C.** Accurate: because the risk-free asset has zero standard deviation and zero correlation, $\sigma_C = w\,\sigma_P$.

Key takeaway Standard deviation is linear in the risky weight; variance is not.

MODULE 88.2

The CAPM and the SML

LOS 88.f — The CAPM, its assumptions, and the security market line (SML)

Only systematic risk is priced, so the relevant risk of any asset is its covariance with the market. Plotting expected return against $Cov(R_i, R_m)$, or against **beta** after standardizing by the market variance, gives the **security market line (SML)**. The equation of the SML is the **capital asset pricing model (CAPM)**:

$$E(R_i) = R_f + \beta_i\,[E(R_m) - R_f], \qquad \beta_i = \frac{Cov(R_i, R_m)}{\sigma_m^2}$$

The intercept is R_f (beta 0), the market plots at $\beta = 1$, and the slope is the **market risk premium** $E(R_m) - R_f$. With a positive market risk premium, expected return rises **linearly** with systematic risk.

Assumptions of the CAPM

Assumption	Meaning
Risk aversion	Investors demand a higher expected return to accept more risk

Assumption	Meaning
Utility maximizing investors	Rational investors each choose the risk–return combination that maximizes their own expected utility
Frictionless markets	No taxes, no transaction costs and no other impediments to trading
One-period horizon	All investors share the same single-period time horizon
Homogeneous expectations	All investors have the same estimates of expected returns, standard deviations and correlations
Divisible assets	All investments are infinitely divisible
Competitive markets	Investors are **price takers**; no single investor's trades can influence prices

Comparing the CML and the SML

KEY CONCEPT

	Capital market line (CML)	Security market line (SML)
Risk measure (x-axis)	Standard deviation (total risk)	Beta (systematic risk)
What plots on it in equilibrium	Only **efficient** portfolios (R_f combined with M)	**All** properly priced securities and portfolios, diversified or not
Slope	$[E(R_m) - R_f]/\sigma_m$	$E(R_m) - R_f$
Diversification of points on it	Always well diversified; only systematic risk	Can be single stocks or concentrated portfolios; any unsystematic risk they carry is not priced

The two lines are drawn against different risk measures, so there is no point at which they "converge". In equilibrium an inefficient portfolio or a single stock plots below the CML but on the SML. Every asset with a beta of 1 plots at the same point on the SML as the market, whatever its total risk.

Common exam traps

- Picking the stock with the highest standard deviation as the one with the highest equilibrium return is the classic mistake; the CAPM prices beta only.
- Risk aversion does not require holding the risk-free asset. A risk-averse investor may still borrow to buy more of M.
- The security characteristic line (LOS 88.e) is not the SML. Its slope is one asset's beta, while the slope of the SML is the market risk premium.

LOS 88.g — Expected (required) return with the CAPM

Substitute the inputs into $E(R_i) = R_f + \beta_i[E(R_m) - R_f]$. Check whether the question gives the **expected market return** $E(R_m)$ or the market risk premium $E(R_m) - R_f$.

Example. $R_f = 2.5\%$, $E(R_m) = 7.5\%$, $\beta = 1.4$: $E(R_i) = 2.5 + 1.4(7.5 - 2.5) = 9.5\%$. Because $\beta > 1$, the stock's expected return exceeds the market's.

Useful rearrangements and relationships:

- Market risk premium: $E(R_m) - R_f = [E(R_i) - R_f]/\beta_i$. The same equation solved for beta is shown under LOS 88.e.
- Nominal risk-free rate ≈ real risk-free rate + expected inflation.
- With a **positive market risk premium** ($E(R_m) > R_f$), a **negative beta** gives a required return below R_f. If $E(R_m) < R_f$, a negative beta gives a required return above R_f. For example, with $\beta = -0.5$, $R_f = 4\%$ and $E(R_m) = 2\%$, the required return is $4 - 0.5(-2) = 5\%$.
- Holding everything else constant and assuming a positive market risk premium, a higher beta raises the required return, so the price investors will pay today falls. With a negative market risk premium the beta ranking reverses.

Common exam traps

- If the premium is given, do not subtract R_f again.
- If the market return is given, subtract R_f before multiplying by beta. In the example, $2.5 + 1.4(7.5)$ gives 13.0% instead of 9.5%.
- Do not use the real risk-free rate when a nominal required return is asked for.

LOS 88.h — Applying the CAPM and the SML: finding mispriced securities

In equilibrium, a security's expected return equals its **required return** from the SML. An analyst compares her own forecast with the CAPM required return:

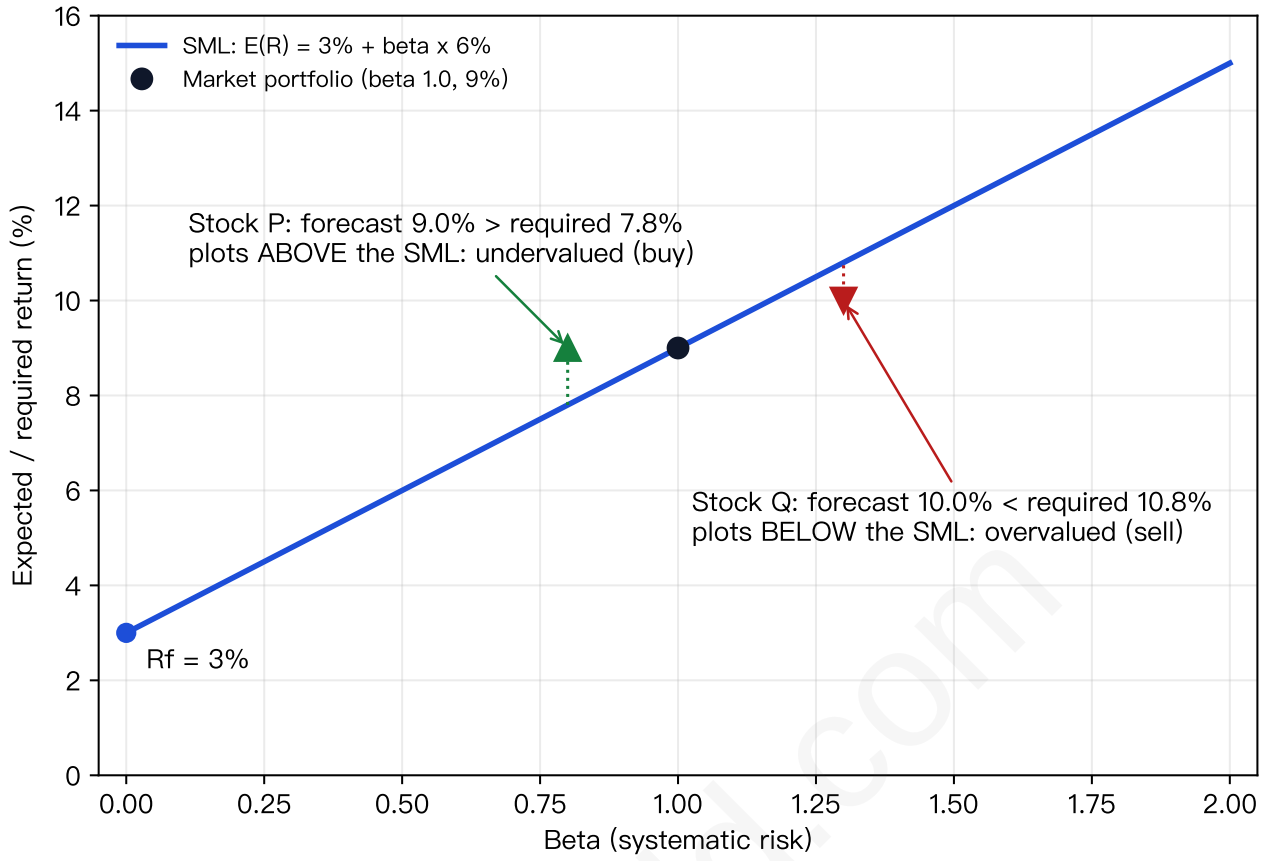
$$\text{forecast return} = \frac{P_1 - P_0 + D_1}{P_0} \qquad \text{required return} = R_f + \beta[E(R_m) - R_f]$$

KEY CONCEPT

Comparison	Position vs. SML	Valuation	Action
Forecast > required	Plots above the SML	**Undervalued**	Buy
Forecast < required	Plots below the SML	**Overvalued**	Sell or sell short
Forecast = required	Plots on the SML	**Properly valued**	Indifferent

The forecast minus the required return is the security's expected **alpha**, the size of the mispricing. When forced to sell one of two undervalued holdings, sell the one with the smaller alpha.

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Using the SML to identify undervalued and overvalued stocks

Example ($R_f = 3\%$, $E(R_m) = 9\%$).

- Stock P: $\beta = 0.8$, bought at 50, forecast price 53 plus a 1.50 dividend. Forecast $= (53 - 50 + 1.50)/50 = 9.0\%$; required $= 3 + 0.8(6) = 7.8\%$. P is undervalued by 1.2% and plots above the SML.
- Stock Q: $\beta = 1.3$, forecast 10.0%; required $= 3 + 1.3(6) = 10.8\%$. Q is overvalued by 0.8% and plots below the SML.

The graph and the algebra always give the same conclusion, because the SML is the graph of the CAPM equation.

Common exam traps

- Include the dividend in the forecast holding period return. For Stock P, leaving out the 1.50 dividend gives a forecast of 6.0%, below the 7.8% required return, instead of 9.0%.
- Judging a stock against R_f instead of against its own required return is a mistake. With a positive market risk premium, a negative-beta stock can be undervalued even when its forecast return is below R_f.

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LOS 88.i — Sharpe ratio, Treynor measure, M-squared and Jensen's alpha

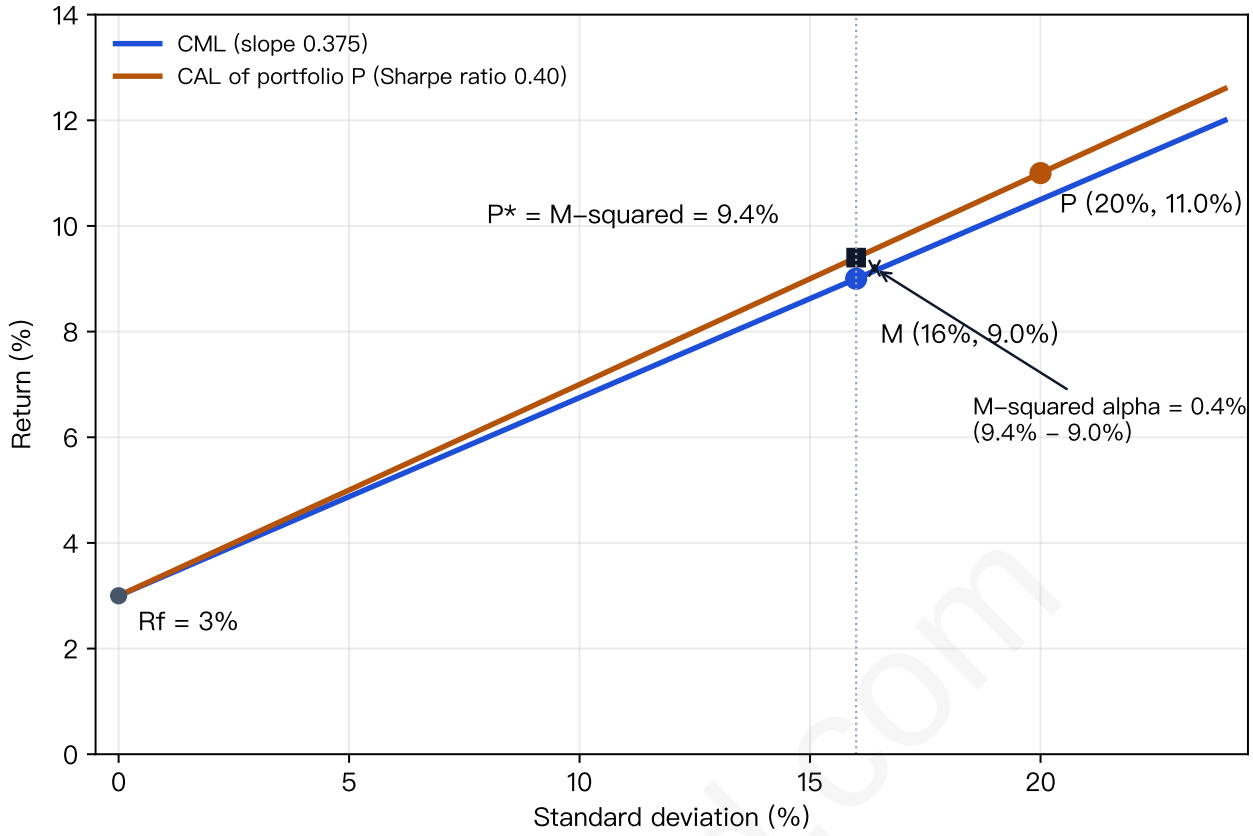
Performance evaluation analyzes the risk and return of an active manager's portfolio. **Attribution analysis**, part of performance evaluation, looks for the sources of the difference between the active portfolio's return and a passive benchmark's return. A higher return than the benchmark proves little on its own: a portfolio with more risk (especially more beta) than the benchmark should earn more over time, so returns must be adjusted for risk.

KEY CONCEPT

Measure	Formula	Risk used	Units / reading
Sharpe ratio	$\frac{R_P - R_f}{\sigma_P}$	Total risk	Slope of the portfolio's CAL; compare with other portfolios or the CML slope
M-squared (M²)	$R_f + \frac{R_P - R_f}{\sigma_P}\sigma_M$	Total risk	Percentage return of P leveraged/deleveraged to the market's σ
M² alpha	$M^2 - R_M$	Total risk	Positive if P plots above the CML
Treynor measure	$\frac{R_P - R_f}{\beta_P}$	Systematic risk	Excess return per unit of beta (a slope)
Jensen's alpha	$R_P - [R_f + \beta_P(R_M - R_f)]$	Systematic risk	Percentage return above the SML

- The Sharpe ratio can be used **ex ante** (expected values) or **ex post** (realized means and sample standard deviation). Its value is meaningful only in comparison with another portfolio's Sharpe ratio.
- M² produces the **same rankings as the Sharpe ratio** but is stated in percentage terms. Because $M^2 = SR_P \times \sigma_M + R_f$, M² exceeds R_M (M² alpha > 0) exactly when P's Sharpe ratio exceeds the CML slope. The Treynor measure and Jensen's alpha are the beta-based analogues: Treynor is a slope, and Jensen's alpha is a percentage.
- The choice of risk measure depends on whether the portfolio carries unsystematic risk. A single manager's portfolio or a concentrated holding should be judged with total-risk measures (Sharpe or M²). A well-diversified fund, such as one spread over many managers, can be judged with beta-based measures (Treynor or Jensen's alpha).
- Portfolios above the CML have higher Sharpe ratios than any CML portfolio and positive M² alphas. Portfolios above the SML have higher Treynor measures than any asset on the SML and positive Jensen's alphas.

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M-squared: leveraging or deleveraging portfolio P to the market's standard deviation

> *Example.* $R_P = 11\%$, $\sigma_P = 20\%$, $\beta_P = 1.1$; $R_f = 3\%$, $R_M = 9\%$, $\sigma_M = 16\%$.

- Sharpe $= 8/20 = 0.40$, above the CML slope of $6/16 = 0.375$, so P beat the market on a total-risk basis.
- M² $= 3 + 0.40(16) = 9.4\%$; M² alpha $= 9.4 - 9.0 = 0.4\%$.
- Treynor $= 8/1.1 = 7.27\%$, against a market Treynor measure of 6%.
- Jensen's alpha $= 11 - [3 + 1.1(6)] = 11 - 9.6 = 1.4\%$.

> *Example (the two families can rank funds differently).* $R_f = 3\%$, $R_M = 9\%$, $\sigma_M = 16\%$.

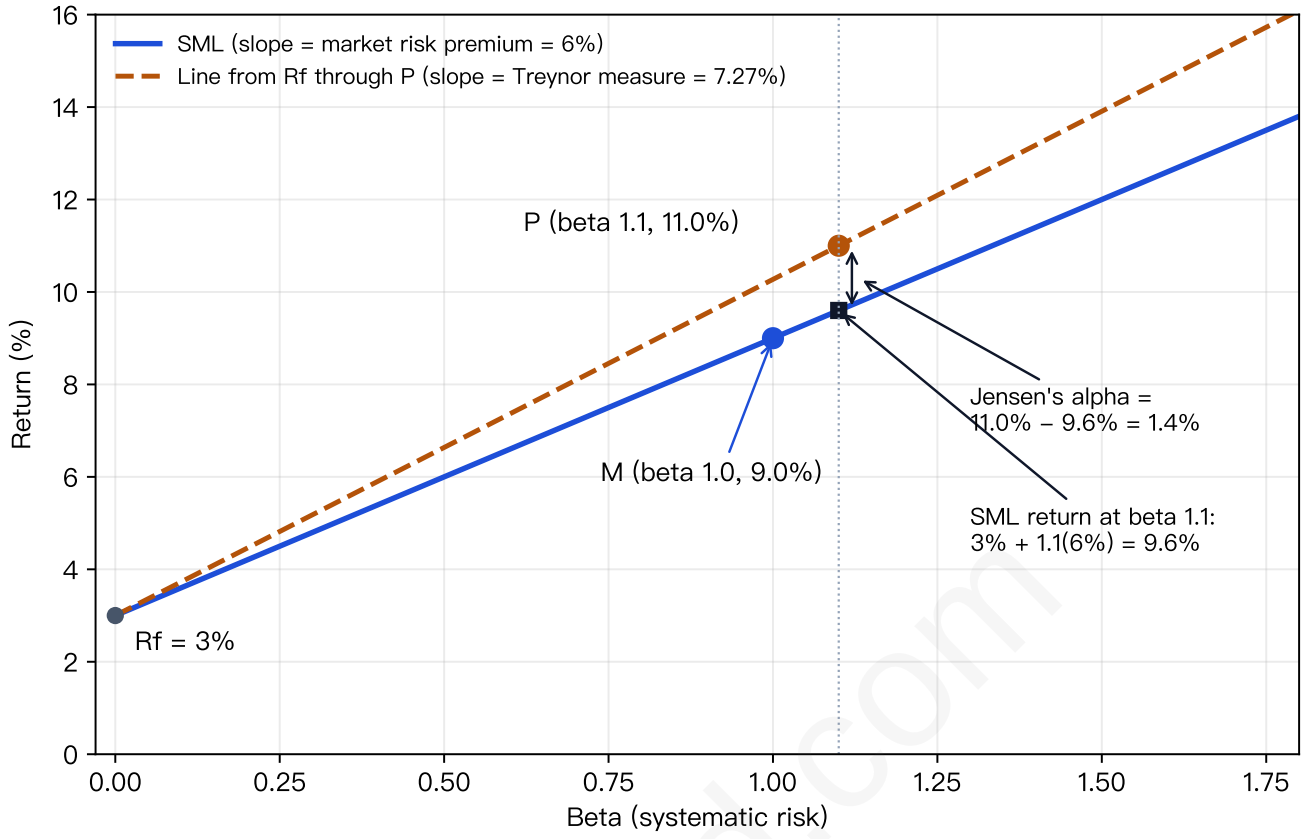
Risk-adjusted performance of two funds and the market (illustrative numbers)

	Return	σ	β	Sharpe ratio	M² alpha	Treynor measure	Jensen's alpha
Fund X	12%	24%	0.9	9/24 = 0.375	3 + 0.375(16) − 9 = 0.0%	9/0.9 = 10.0%	12 − [3 + 0.9(6)] = +3.6%
Fund Y	10%	14%	1.2	7/14 = 0.50	3 + 0.50(16) − 9 = +2.0%	7/1.2 = 5.8%	10 − [3 + 1.2(6)] = −0.2%
Market	9%	16%	1.0	6/16 = 0.375	0.0%	6/1.0 = 6.0%	0.0%

Fund Y wins on the total-risk measures (Sharpe 0.50 against 0.375; M² alpha +2.0% against 0.0%). Fund X wins on the beta-based measures (Treynor 10.0% against 5.8%; Jensen's alpha +3.6% against −0.2%). X has a low beta but a high standard deviation, so much of its risk is unsystematic. If X is the investor's whole portfolio, that risk matters and Y is better. If X is one small part of a well-diversified fund, its unsystematic risk is diversified away and X is better. Sharpe and Treynor values are read by comparing them with another portfolio or with the market; M² alpha and Jensen's alpha are read directly, where a value above zero means the fund beat the market after adjusting for risk.

On a graph of return against beta, P's Treynor measure is the slope of the line from R_f through P, and its Jensen's alpha is the vertical gap between P and the SML at $\beta = 1.1$.

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Treynor measure and Jensen's alpha for portfolio P, shown against the SML

The inputs to these measures, including the expected market return (and hence the market risk premium) and betas, are estimated with error. The expected market return need not equal its historical average. Results from the models are only as good as those estimates.

Common exam traps

- Sharpe and M² use standard deviation; Treynor and Jensen use beta. Distractors often use the wrong risk measure.
- When M² alpha is asked, subtract the market return from M²; reporting M² itself is a common slip. In the example, reporting M² gives 9.4% instead of an M² alpha of 0.4%.
- Forgetting to subtract R_f in the numerator of the Sharpe or Treynor ratio. In the example, the Sharpe ratio becomes 0.55 instead of 0.40 and the Treynor measure 10.0% instead of 7.27%.

EXAM SHORTCUTS

- M² is above R_M exactly when the portfolio's Sharpe ratio is above the CML slope, so the sign of the M² alpha can be read from the Sharpe ratio comparison.
- Any asset with a beta of 1 has the market's required return under the CAPM whatever its total risk, so no calculation is needed for it.

BOTTOM LINE

- The CAPM, $E(R_i) = R_f + \beta_i[E(R_m) - R_f]$, is the equation of the security market line, whose intercept is R_f and whose slope is the market risk premium.
- The CAPM assumes risk-averse, utility-maximizing investors, frictionless markets, a single common period, homogeneous expectations, infinitely divisible assets and competitive markets in which investors are price takers.
- The CML measures risk by standard deviation and in equilibrium holds only efficient portfolios, while the SML measures risk by beta and holds all properly priced securities and portfolios, so a single stock plots below the CML but on the SML.
- A security whose forecast return exceeds its CAPM required return plots above the SML and is undervalued, one whose forecast is below plots below the SML and is overvalued, and the difference is its expected alpha.
- With a positive market risk premium a negative beta gives a required return below R_f, and when $E(R_m)$ is below R_f a negative beta gives a required return above R_f.
- The Sharpe ratio and M² use total risk and suit a single manager's or concentrated portfolio, M² ranking portfolios as the Sharpe ratio does, while the Treynor measure and Jensen's alpha use beta and suit a well-diversified fund.

QUICK CHECK

Question 2

Denholm Freight shares have a beta of 1.15. The risk-free rate is 2.5% and the expected return on the market is 8.5%. According to the capital asset pricing model, the required return on Denholm shares is *closest* to:

A 12.3%.

B 9.8%.

C 9.4%.

Answer: C

The CAPM adds a beta-adjusted market risk premium to the risk-free rate. The market risk premium is the expected market return minus the risk-free rate, 6.0%.

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] = 2.5\% + 1.15(8.5\% - 2.5\%) = 2.5\% + 6.9\% = 9.4\%$$

Why the other options are wrong

- **A.** 12.3% multiplies beta by the full market return instead of the market risk premium: $2.5\% + 1.15 \times 8.5\% = 12.3\%$.
- **B.** 9.8% is simply beta times the market return ($1.15 \times 8.5\%$), ignoring the risk-free rate and the premium structure of the CAPM.

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Key takeaway CAPM: $E(R_i) = R_f + \beta_i[E(R_m) - R_f]$. Subtract R_f from the market return before multiplying by beta.

Practice questions

Question 3

A fixed-income manager holds bonds from many different issuers. After this diversification, the risk from which of the following events is *most likely* to remain in the portfolio?

A A downgrade of one issuer after an accounting scandal.

B A recession that lowers revenues at most companies.

C A fire that shuts the main plant of one issuer.

Question 4

An analyst's CAPM-based expected return for Harwell Mining shares is 2.2 times the 10% return expected on the market portfolio. With a risk-free rate of 4%, the beta of Harwell Mining shares is *closest* to:

A 3.0.

B 2.2.

C 1.8.

Question 5

About 60% of Mariela Cortez's wealth is in shares of the software company she founded; the rest is in a broad mix of bonds and equity index funds. Which of the following risk-adjusted performance measures is *least appropriate* for evaluating her overall portfolio?

A Sharpe ratio.

B Jensen's alpha.

C M-squared.

Answers: 3. B · 4. A · 5. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/portfolio-construction/systematic-vs-unsystematic-risk/. The full bank has 105 questions on this reading.

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PORTFOLIO CONSTRUCTION · READING 89

Defined Benefit vs Defined Contribution

What you'll learn

LOS 89.a Describe the portfolio approach: judge investments by their effect on total portfolio risk and return; diversification and the diversification ratio.

LOS 89.b Describe the planning, execution and feedback steps of the portfolio management process, including top-down and bottom-up analysis.

LOS 89.c Describe the main types of investors and their risk tolerance, horizon, liquidity and income needs.

LOS 89.d Describe and contrast defined contribution and defined benefit pension plans, including who bears investment risk.

LOS 89.e Describe the asset management industry: firm types, active vs passive management and industry trends.

LOS 89.f Describe mutual funds and compare them with ETFs, separately managed accounts, hedge funds, and private equity and venture capital funds.

MODULE 89.1

Portfolio Management Process

This reading introduces the portfolio approach, the three steps of the portfolio management process, the needs of different types of investors, the difference between defined contribution and defined benefit pension plans, and the asset management industry with its pooled investment vehicles.

LOS 89.a — The portfolio approach to investing

Under the **portfolio approach** (also called the **portfolio perspective**), every candidate investment is judged by what it adds to, or takes away from, the risk and return of the investor's whole portfolio. The alternative is to study the risk and return of each security on its own and to ignore how it fits with everything else the investor owns. An investor who puts all of her savings in the one stock she believes is "the best" is not using the portfolio perspective. She carries far more risk than someone holding a diversified mix of stocks.

- **Diversification** lets an investor cut portfolio risk without necessarily giving up expected return. It mainly narrows the spread of outcomes, making very high and very low returns both less likely. It is not a tool for raising expected return.
- **Modern portfolio theory (MPT)** concludes that the extra risk of holding a single security, which could have been diversified away, is not rewarded with a higher expected return.
- **Harry Markowitz** (early 1950s) used the **standard deviation of returns** as the measure of risk and showed how combining risky assets changes portfolio risk and expected return. His key result: unless the assets' returns are **perfectly positively correlated**, combining them reduces risk.
- In the 1960s, Treynor, Sharpe, Mossin and Lintner extended this work into MPT, in which equilibrium expected returns are a linear function of **market risk**, the part of risk that diversification cannot remove.

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Measuring the benefit: the diversification ratio

KEY CONCEPT

$$\text{Diversification ratio} = \frac{\sigma \text{ of an equally weighted portfolio of the } n \text{ securities}}{\text{average } \sigma \text{ of the } n \text{ individual securities}}$$

The denominator is the average standalone risk of the securities in the group.

Ratio	Interpretation
= 1	No risk reduction at all from combining the securities
below 1	Some risk reduction; the lower the ratio, the greater the benefit

Example. A group of 12 stocks has an average standard deviation of 40%. An equally weighted portfolio of the 12 stocks has a standard deviation of 26%. The diversification ratio is $26/40 = 0.65$: the portfolio carries 65% of the risk of a typical single holding, a 35% reduction.

Two caveats apply. First, an equally weighted portfolio is only a convenient yardstick; optimization software can find the weights that give the lowest standard deviation for the group. Second, diversification works best in normal markets. In a financial crisis, such as the credit contagion of 2008, correlations tend to rise, so diversification protects less at the very time it is most wanted.

Common exam traps

- Judging a security by its own expected return, its excess-return potential, its fundamentals or its intrinsic value versus price is standalone analysis, not the portfolio approach.
- Do not invert the ratio. Portfolio risk goes in the numerator, and an equally weighted portfolio's standard deviation can never exceed the average standard deviation of its holdings, so a value above 1 means the ratio was turned upside down. In the 12-stock example, the inverted ratio is $40/26 = 1.54$ instead of 0.65.
- Low correlations measured in calm markets overstate the protection a portfolio will get in a crisis.

LOS 89.b — The three steps of the portfolio management process

KEY CONCEPT

Step	What happens	Main output
Planning step	Analyze the investor's **risk tolerance**, **return objectives**, **time horizon**, tax exposure, **liquidity needs**, income needs and any unique circumstances or preferences	An **investment policy statement (IPS)** setting out objectives and constraints, plus an objective **benchmark** against which results will later be judged

Step	What happens	Main output
Execution step	**Asset allocation** (decide how funds are split among asset classes, often using **top-down analysis**), **security analysis** (often **bottom-up analysis**) and **portfolio construction**	The portfolio itself
Feedback step	**Portfolio monitoring and rebalancing** (restore target weights as prices, markets and client circumstances change); **performance measurement and reporting** against the IPS benchmark	Rebalanced portfolio; performance reports

The IPS is the foundation of the whole process, so writing the IPS comes first. It should be reviewed at least every few years and whenever the client's objectives or constraints change significantly. Recent portfolio performance is not a reason to revise it.

Top-down versus bottom-up analysis (both belong to execution)

- **Top-down analysis** works from the broad to the specific. It starts with the macroeconomic picture: forecasts of GDP growth, inflation and interest rates, and the stance of government policy such as **fiscal policy** (spending and taxes) and monetary policy. From this the manager decides which asset classes look most attractive. The resulting portfolio is usually spread across publicly traded equities, fixed-income securities, cash, private equity, hedge funds, real estate, and commodities and other real assets. Within an asset class the analysis then narrows to **industry analysis** and only after that to **company analysis**. Industry analysis comes first because an industry's outlook shapes the results of the firms in it.
- **Bottom-up analysis** starts with individual securities: analysts use valuation models to find securities that look undervalued.

Example (sorting activities). Over two months an adviser (1) interviews a new client about goals and constraints and drafts an IPS naming a 70/30 global equity/bond index as the benchmark; (2) uses macro forecasts to pick a 60% equity / 30% bond / 10% infrastructure mix and selects funds and securities; (3) after an equity rally, trims equities back to 60% and reports the quarter's return against the benchmark. Item (1) is planning, (2) is execution and (3) is feedback.

Common exam traps

- Choosing the benchmark is part of planning, because it is written into the IPS. Using it to judge results is feedback.
- Top-down analysis starts from the economy, yet it is an execution activity, not part of planning. Industry figures such as industry return on equity come after macro factors such as fiscal policy.
- Rebalancing involves trading, but it belongs to feedback, not execution.

LOS 89.c — Types of investors and their needs

- **Individual investors** save for goals such as a home, children's education or retirement. Many countries offer tax-deferred retirement accounts, and **defined contribution pension plans** are a popular vehicle. Their needs depend entirely on the individual.
- An **endowment** is a fund dedicated to supporting a specific purpose on an ongoing basis, such as a university's programs. A **foundation** is a fund set up for charitable purposes, such as supporting research on a disease. A typical aim is to fund the activity indefinitely without eroding the real (inflation-adjusted) value of its assets.
- A **bank** aims to earn more on its loans and investments than it pays on deposits. It keeps risk low and needs liquidity to meet depositor withdrawals.
- **Insurance companies** invest premiums to pay claims as they arise. **Life insurance companies** have a relatively long horizon. **Property and casualty (P&C) insurance companies** have a shorter one because their claims tend to come sooner.
- **Investment companies**, such as **mutual funds**, manage pooled money in a particular style or segment of the market.
- **Sovereign wealth funds** are pools of assets owned by a government.

Investor	Risk tolerance	Investment horizon	Liquidity needs	Income needs
Individuals	Depends on the individual	Depends on the individual	Depends on the individual	Depends on the individual
Banks	Low	Short	High	Pay interest
Endowments and foundations	High	Long	Low	Spending level
Insurance companies	Low	Long (life), short (P&C)	High	Low
Mutual funds	Depends on the fund	Depends on the fund	High	Depends on the fund
Defined benefit pension plans	High	Long	Low	Depends on participants' ages

A defined benefit plan's income need depends on its participants' ages: a plan whose members are mostly retired and drawing benefits needs more current income than one whose members are mostly young workers.

Common exam traps

- When a life insurer and a P&C insurer are compared, time horizon is the only difference in the profile.
- A long horizon alone does not identify the investor. A life insurer also has a long horizon, but unlike an endowment, a foundation or a DB plan it has low risk tolerance and high liquidity needs.

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LOS 89.d — Defined contribution versus defined benefit pension plans

Feature	Defined contribution pension plan	Defined benefit pension plan
What the employer promises	A contribution each period into the employee's own account (based on, e.g., salary, service, age, profits or a match of the employee's contribution); no promise about the account's future value	**Periodic payments after retirement**, usually based on years of service and salary at or near retirement
Who makes the investment decisions	The employee	The employer / plan sponsor (through the plan's fund)
Who bears the **investment risk**	The employee	The employer (plan sponsor)
Effect of poor investment returns	Smaller retirement account for the employee	Employer must contribute more to the fund

Example (defined benefit). A plan pays 1.5% of final salary for each year of service. An employee retiring after 25 years with a final salary of $80,000 receives $0.015 \times 25 \times \$80{,}000 = \$30{,}000$ per year for life. Because that amount is fixed by formula, weak fund returns are the employer's problem rather than the retiree's.

Common exam traps

- The fund manager bears the investment risk in neither plan. In a DC plan the employer's obligation ends once it has made its contribution, so a fall in the account's value is the employee's loss.
- A promised contribution is not a promised benefit. A stem that describes a promised retirement income describes a DB plan.

EXAM SHORTCUTS

- Sort a portfolio activity by its output: anything written into the IPS, including the benchmark, is planning; allocating, selecting and building is execution; monitoring, rebalancing and measuring performance is feedback.

BOTTOM LINE

- The portfolio approach judges each investment by its effect on the risk and return of the whole portfolio.
- Combining assets whose returns are not perfectly positively correlated reduces risk without necessarily giving up expected return.
- The diversification ratio is the standard deviation of an equally weighted portfolio ÷ the average standard deviation of its securities; the lower the ratio, the greater the benefit.
- Planning produces the IPS and its benchmark; execution covers asset allocation, security analysis and portfolio construction; feedback covers monitoring, rebalancing and performance measurement.

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- In a DC plan the employee makes the investment decisions and bears the investment risk; in a DB plan the employer promises the benefit and bears the risk.

QUICK CHECK

Question 1

An investor who applies modern portfolio theory in the tradition of Markowitz is deciding whether to add a utility stock to her holdings. She should *most appropriately* judge the stock by:

A its expected return viewed on its own.

B how it would change her portfolio's overall risk and return.

C whether its intrinsic value exceeds its market price.

Answer: B

Markowitz's framework, the basis of modern portfolio theory, evaluates an investment by its effect on the risk and expected return of the portfolio as a whole. Because risk that can be diversified away is not rewarded, how the asset interacts with existing holdings matters more than its standalone characteristics.

Why the other options are wrong

- **A.** Looking only at expected return ignores risk and ignores how the stock combines with the rest of the portfolio.
- **C.** Comparing intrinsic value with market price is standalone security valuation (bottom-up analysis). The Markowitz framework calls for a portfolio-level evaluation.

Key takeaway Modern portfolio theory: no asset is judged in isolation.

MODULE 89.2

Asset Management and Pooled Investments

LOS 89.e — The asset management industry

The **asset management industry** consists of firms that manage investments for clients. They are **buy-side** firms. Broker-dealers and investment banks, by contrast, are **sell-side** firms. Asset managers may be independent or divisions of larger financial groups.

Type of firm	What it offers
Full-service asset manager	Many investment styles and asset classes under one firm
Specialist asset manager	Focus on one investment style or one asset class
Multi-boutique firm	A **holding company** that owns several different specialist asset managers
Traditional asset manager	Focus on equities and fixed-income securities

Type of firm	What it offers
Alternative asset manager	Focus on private equity, hedge funds, real estate, commodities and similar assets

Active versus passive management

- **Active management** tries to beat a chosen benchmark by relying on the manager's skill, using, for example, fundamental or technical analysis.
- **Passive management** tries to replicate a benchmark index, either by tracking a broad market index or through a **smart beta** approach that targets exposure to a particular risk factor.
- Passive management accounts for roughly one-fifth of **assets under management (AUM)**. Passive fees are lower, so passive management's share of industry revenue is smaller still than its share of AUM.

Industry trends

- The market share of passive management has been growing. Lower fees help, as do doubts about whether active managers add value after risk adjustment, especially in developed markets thought to be fairly efficient. Most AUM is still actively managed.
- Profit margins are higher in alternative asset classes, so many traditional managers have moved into alternatives, blurring the line between the two types.
- The volume of available data has grown enormously, pushing managers to spend on information technology and on data services bought from third parties.
- **Robo-advisors** use computer algorithms to give advice based on an investor's requirements and constraints. They appeal to younger investors and smaller accounts, and they have lowered barriers to entry into asset management, for example for insurance companies.

Common exam traps

- A holding company whose separate firms each specialize is a multi-boutique firm, even though the group as a whole covers many styles. Do not call it a full-service manager.
- Smart beta is passive management even though it tilts toward a risk factor. Reliance on fundamental or technical analysis signals active management.

LOS 89.f — Mutual funds and other pooled investments

A **pooled investment** is a single portfolio holding money from many investors; each investor owns shares in the pool. The **net asset value (NAV)** per share is

KEY CONCEPT

$$NAV = \frac{\text{fund assets} - \text{fund liabilities}}{\text{shares outstanding}}$$

Example. A fund holds assets worth \$412 million, owes \$12 million and has 16 million shares outstanding: $NAV = (412 - 12)/16 = \$25.00$ per share.

Open-end versus closed-end funds

- **Open-end fund**: investors buy newly issued shares at NAV and can **redeem** shares at NAV by selling them back to the fund, so the number of shares changes with investor demand. New cash is invested in more portfolio securities.
- **Closed-end fund**: a fixed number of shares. The fund neither takes new money nor redeems shares. The shares trade like stocks, on an exchange or over the counter. Because the price depends on supply and demand for the shares, the market price can differ significantly from NAV (a premium or discount).
- All mutual funds charge an ongoing **management fee** expressed as a percentage of NAV. **No-load funds** charge no purchase or redemption fees; **load funds** charge up-front fees, redemption fees or both.

Types of mutual funds

- **Money market funds** hold short-term debt securities. They pay interest income, and the chance that the share value changes is very small. NAV is typically held at one unit of currency per share, although some funds' NAVs have fallen in stressed markets.
- **Bond mutual funds** hold fixed-income securities and differ by maturity, credit rating, issuer and type (government, tax-exempt, high-yield, global).
- **Stock mutual funds** include **index funds**, which are passively managed to match an index, and **actively managed funds**, which select securities to beat their benchmarks. Actively managed funds charge higher fees and have higher **turnover**, which creates higher tax liabilities than index funds.
- **Balanced (hybrid) funds** hold both debt and equity securities.

Other pooled vehicles

Feature	Open-end fund	Closed-end fund	Exchange-traded fund (ETF)
Where investors trade	With the fund	In the market	In the market
Price versus NAV	Trades at NAV	Can differ significantly from NAV	Kept very close to NAV by special redemption provisions (arbitrage)
Typical management style	Active or index	Often active	Mostly passive (index); management fees generally low
Intraday trading, short sales, margin purchases	No (priced once a day at closing NAV)	Yes	Yes
Brokerage commission / bid–ask spread	No	Yes	Yes
Dividends	Can be reinvested in fund shares	Depends on the fund	Usually paid in cash

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Feature	Open-end fund	Closed-end fund	Exchange-traded fund (ETF)
Capital gains tax from other investors' exits	Can arise when redemptions force sales of appreciated securities	No forced sales, because shares are never redeemed	May be lower: investor sales of ETF shares do not force the fund to sell securities

- A **separately managed account (SMA)** is a portfolio owned by a single investor and managed to that investor's needs; no shares are issued. It suits investors with substantial assets and carries an annual fee based on assets.
- **Hedge funds** are far less regulated than mutual funds. They limit the number of investors, often to **qualified (accredited) investors**, set high minimums (commonly $250,000 to $1 million) and are not offered to the general public. Typical fees are 2% of assets plus 20% of excess performance. They follow many strategies, often with **leverage**, short positions and derivatives. Examples are **global macro** funds and **event-driven** funds, which trade around corporate events such as mergers and acquisitions.
- **Private equity (buyout) funds** buy all the shares of a company, often a public one, and take it private, usually financing a large percentage of the purchase price with debt. They restructure the company to raise cash flow and typically exit within three to five years, for example through an initial public offering (IPO) or a sale.
- **Venture capital funds** resemble buyout funds but invest in start-up companies, usually taking equity stakes with little borrowed money and providing advice and expertise. Both buyout and venture capital managers take an active role in managing the companies they own.

Common exam traps

- ETFs and closed-end funds both trade in the market, but only a closed-end fund can sit at a large discount or premium to NAV for months. A persistent wide gap points to a closed-end fund, not an ETF.
- An exit through an IPO does not make a fund a venture capital fund. A fund that takes an established listed company private and later floats it again is a buyout fund; an event-driven hedge fund trades around mergers without running the companies.

BOTTOM LINE

- Asset managers are buy-side firms; a multi-boutique firm owns several specialist managers through one holding company.
- Active management tries to beat a benchmark; passive management, including smart beta, tracks one, and its share of AUM has been growing.
- NAV per share = (fund assets − fund liabilities) ÷ shares outstanding.
- Open-end funds issue and redeem shares at NAV; closed-end funds keep a fixed share count, and their market prices can differ significantly from NAV; ETFs trade in the market at prices kept close to NAV.
- Hedge funds are far less regulated than mutual funds, are often limited to qualified investors and typically charge 2% of assets plus 20% of excess performance.

QUICK CHECK

Question 2

Shares of the Norcastle Frontier Fund trade on a stock exchange and have been priced at a discount of about 14% to the fund's net asset value per share for several months. Norcastle is *most likely* a(n):

A closed-end fund.

B open-end fund.

C exchange-traded fund.

Answer: A

Closed-end fund shares trade in the market at prices set by supply and demand for the shares, so they can sit at a large and persistent premium or discount to NAV.

Why the other options are wrong

- **B.** Open-end fund shares are bought from and redeemed by the fund at NAV, so they cannot trade at a discount to NAV (and they do not trade on an exchange).
- **C.** An ETF's special redemption provisions let arbitrageurs profit from any gap between price and NAV, which keeps ETF prices very close to NAV; a 14% gap lasting months is inconsistent with an ETF.

Key takeaway A large, lasting premium or discount to NAV points to a closed-end fund.

Practice questions

Question 3

An analyst using a top-down approach is evaluating the outlook for chemical company stocks. Which of the following factors should the analyst examine *first*?

A Return on equity for the chemicals industry.

B The outlook for fiscal policy.

C Competitive risks within the chemicals industry.

Question 4

A pooled investment fund buys all of the outstanding shares of a listed industrial manufacturer, removing it from the stock exchange. The fund installs a new executive team and restructures the company's operations, and four years later it sells the company back to the public through an initial public offering. The fund is *most likely* a(n):

A venture capital fund.

B event-driven fund.

C private equity fund.

Answers: 3. B · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/portfolio-construction/defined-benefit-vs-defined-contribution/. The full bank has 34 questions on this reading.

PORTFOLIO CONSTRUCTION · READING 90

Basics of Portfolio Planning and Construction

What you'll learn

LOS 90.a Explain why a written investment policy statement is the starting point of the portfolio management process.

LOS 90.b Identify the main sections of an IPS and its minimum required content.

LOS 90.c Describe absolute and relative risk and return objectives and why they must be compatible.

LOS 90.d Distinguish willingness from ability (capacity) to take risk and combine them into overall risk tolerance.

LOS 90.e Classify liquidity, time horizon, tax, legal and regulatory, and unique-circumstance constraints and their portfolio implications.

LOS 90.f Specify asset classes for asset allocation (homogeneous within, low correlation between, covering the permitted universe).

LOS 90.g Describe portfolio construction: strategic vs tactical asset allocation, security selection, risk budgeting and core-satellite.

LOS 90.h Describe ESG approaches and how they affect benchmarks, voting and portfolio performance.

MODULE 90.1

Portfolio Planning and Construction

This reading explains why a written investment policy statement matters, what it contains, and how to set risk and return objectives and judge willingness and ability to take risk. It then covers the five investment constraints, the specification of asset classes, the roles of strategic and tactical asset allocation, security selection and risk budgeting, and ESG approaches in portfolio planning.

LOS 90.a — Why a written investment policy statement matters

The **investment policy statement (IPS)** is the written plan that starts the **portfolio management process**. Before any security is bought, the manager and client agree in writing on what the client needs, what the client can and will tolerate, and how results will be judged.

A written IPS is valuable for three main reasons:

- It forces discipline. The client must articulate circumstances, objectives and constraints instead of reacting to the market mood of the day.
- It keeps goals realistic. High return and low risk are popular wishes but usually incompatible, so the IPS makes the client reconcile return expectations with **risk tolerance**.
- It sets a yardstick. Performance is judged against a stated **benchmark** rather than with hindsight.

An IPS does not pick individual stocks or time the market. It works at the level of objectives, constraints, asset classes and policy.

Common exam traps

- The IPS comes first in the process; it is not a report card produced at the end.

LOS 90.b — Major components of an IPS

A typical IPS contains the following sections:

KEY CONCEPT

Section	What it covers
Client description / introduction	The client's circumstances and situation
Statement of purpose	What the IPS is meant to achieve
Statement of duties and responsibilities	Roles of the client, the **custodian** and the **investment manager**
Procedures	How the IPS is updated and how unforeseen events are handled
Investment objectives	**Return objectives** and **risk objectives** (risk tolerance)
Investment constraints	Liquidity, time horizon, taxes, legal and regulatory, unique circumstances
Investment guidelines	How the policy is executed: permitted asset types, use of **leverage** and **derivatives**
Evaluation and review	The **benchmark** and how results are assessed
Appendices	The **strategic asset allocation** (policy portfolio), permitted ranges and the **rebalancing** policy

The bare minimum. Even a short IPS must contain (1) a clear statement of the client's circumstances and constraints, (2) an investment strategy based on them and (3) a benchmark for evaluating performance. Detailed update procedures are common but are not part of that minimum. Nor is a precise annual target return, although return objectives may still be stated in the IPS, for example as a percentage.

The manager's fee schedule is not one of the IPS components listed above.

Common exam traps

- Permitted assets and leverage limits are investment guidelines. They are neither objectives nor constraints.

LOS 90.c — Risk and return objectives

Risk objectives translate the client's risk tolerance into specific limits on portfolio risk. **Return objectives** usually follow from a future financial goal, such as a target level of retirement income. Both can be stated in **absolute** or **relative** terms, and a return objective may be set as a percentage or as a currency amount.

KEY CONCEPT

	Absolute	Relative
Risk objective	"Portfolio value must not fall more than 8% in any 12-month period" or, in probability form, "no more than a 10% probability of losing more than 6% in a year"	"Return must not trail the benchmark index by more than 3% in any year", or a probability of underperforming a benchmark by a given margin
Return objective	A nominal figure ("at least 6.5% a year") or a real figure ("inflation plus 2.5%")	"Beat a stated equity index by 1% a year"; for a bank, a spread over its **cost of funds**

- Stating a risk objective as the **probability** of a given loss does not make it relative. It is relative only if it refers to a benchmark.
- A return of "inflation plus x%" is still an absolute (real) objective. A limit measured against a market interest rate, such as never earning less than the three-month Treasury bill rate over a calendar year, is a relative risk objective, because that rate serves as the benchmark.
- Very strict absolute risk objectives, such as no loss in any year, push the portfolio toward guaranteed instruments such as Treasury bills.
- **Peer-group benchmarks** (for example, "top quartile of endowments") are not **investable**: nobody can build that portfolio in advance.
- Risk and return objectives must be set together and must be compatible. The client's risk tolerance determines which returns are realistically attainable, and a return-only objective can push the client into far too much risk.

Example. A client wants 11% a year but says she cannot accept any calendar-year loss. Portfolios with an 11% expected return in her market carry a standard deviation near 16%. If returns are roughly normal, $P(R < 0) = N(-11/16) \approx 25\%$, so a losing year can be expected about one year in four. The objectives are inconsistent, and the adviser must either lower the return target or relax the risk limit.

LOS 90.d — Willingness versus ability (capacity) to take risk

An investor's overall risk tolerance combines two separate assessments: how much risk the investor wants to take and how much risk the investor's finances can bear.

KEY CONCEPT

	Willingness to take risk	Ability (capacity) to take risk
Driven by	Psychology: attitudes, beliefs, personality, financial knowledge	Financial circumstances
Measured by	Questionnaires, interviews (subjective)	Analysis of wealth, liabilities, income, horizon

	Willingness to take risk	Ability (capacity) to take risk
Higher when	Investor is comfortable with volatility	Long **time horizon**, large wealth relative to liabilities and spending needs, stable job and income, ample insurance, few dependents

Large near-term **spending needs** reduce the ability to bear risk, because the portfolio must reliably fund a fixed obligation. The expected rate of return, the level of inflation and the client's tax bracket are not inputs to risk tolerance. Expected return follows from risk tolerance, inflation matters for the return objective, and taxes are a constraint.

The table shows how the two assessments combine into overall risk tolerance.

Combining willingness and ability to take risk

Willingness \ Ability	Below-average ability	Above-average ability
Below-average willingness	Below-average risk tolerance (no conflict)	Conflict: educate the client; usually conform to the lower (below average)
Above-average willingness	Conflict: ability prevails (below average)	Above-average risk tolerance (no conflict)

When the two conflict, the lower assessment governs. With high willingness and low ability, ability prevails, and a portfolio tilted toward bonds rather than equities is appropriate. With low willingness and high ability, the adviser may educate the client and correct misconceptions but should not try to change the client's personality; a portfolio the client is clearly uncomfortable with is unlikely to end well.

Example. A 36-year-old client has a secure salaried job, no debt, savings well above her planned spending and 25 years until retirement. She also says she would sell all her equities if her account fell 10% in a year.

1. Sort the facts. Job security, wealth relative to needs and the long horizon are financial circumstances, so they bear on ability. Her reaction to a 10% fall is an attitude, so it bears on willingness.
2. Rate each dimension. Ability is high; willingness is low.
3. Combine them. The two conflict and willingness is the lower one. The adviser explains how a 25-year horizon can absorb short-term losses, but if her attitude does not change, the portfolio conforms to her low willingness and overall risk tolerance is below average.

Common exam traps

- Averaging a high willingness and a low ability to reach "moderate" or "average" risk tolerance is a mistake; the lower assessment governs.
- Re-administering the questionnaire does not fix a low ability to take risk.

LOS 90.e — Investment constraints

The five constraint categories are:

- **Liquidity**: the need to turn assets into spendable cash quickly without large price concessions, for example for regular withdrawals, tuition, a planned purchase or insurance claims. High liquidity needs call for cash and short-term bonds; illiquid hedge funds or private equity are unsuitable.
- **Time horizon**: often the period until withdrawals begin. Longer horizons allow more risk and less liquidity. A short horizon points to low-risk, liquid assets such as government securities or bank deposits. Short horizons and liquidity needs usually go together.
- **Tax concerns**: the investor's marginal tax rate, the different treatment of income and capital gains, and whether accounts are taxable, **tax-deferred** or **tax-exempt**. In a fully taxable account, an investor in a high tax bracket may prefer tax-exempt bonds (in the US) or equities whose return is expected to come mainly from capital gains, because gains are commonly taxed more lightly than income. Fully taxed income (corporate bond interest, for example) is best placed in tax-deferred accounts, while long-term capital gains, tax-exempt interest and (where they are taxed favorably) dividends suit accounts without tax deferral. Portfolio choices should compare expected after-tax returns with risk. Selling holdings to diversify can trigger capital gains tax.
- **Legal and regulatory factors**: constraints imposed by law or regulators, such as trust and pension rules, minimum payout requirements imposed by statute, limits on asset types or percentages, and restrictions on trading by corporate insiders.
- **Unique circumstances**: client-specific preferences or restrictions, often non-financial. Examples are **responsible investing** exclusions (tobacco, weapons, gambling), religious restrictions (for example, no interest-bearing securities), avoiding a competitor's shares, or diversifying away from an employer's industry.

A handy checklist for a whole IPS is **R-R-T-T-L-L-U**: Risk, Return, Time horizon, Tax, Liquidity, Legal, Unique. The first two are **objectives**; the last five are **constraints**.

Common exam traps

- A rule the investor chooses is a unique circumstance. A rule imposed by law is legal and regulatory, even if it concerns payouts.
- A required minimum return is a return objective, even though it reads like a limit on the portfolio.
- The amount and timing of withdrawals are a liquidity matter. What the client spends the money on once it has left the portfolio is not a constraint at all.

LOS 90.f — Specifying asset classes

An **asset class** is a group of assets with similar risk and return behavior. For **strategic asset allocation** the classes should be defined so that:

- returns are **highly correlated within** each class, because its members behave alike;
- returns have **low correlation between** classes, which is what produces diversification benefits;
- taken together, the classes approximate the **universe of permissible investments** in the IPS.

Specify the type of security first, then subdivide. Equities can be split into domestic and foreign, large-cap and small-cap, developed and emerging; bonds into government and corporate, investment grade and high yield, or by maturity. A label such as "emerging markets" or "technology sector" alone is incomplete because it does not say whether the assets are equities or bonds. The traditional classes are equities, bonds, cash and real estate. **Alternative investments** include hedge funds, private equity, commodities, artwork and intellectual property.

The manager then gathers expected returns, standard deviations and correlations for each class.

LOS 90.g — Portfolio construction and the role of asset allocation

1. Estimate return, risk and correlation for each asset class and build an **efficient frontier** of asset-class portfolios.
2. Choose the efficient portfolio that best matches the objectives and constraints in the IPS. Its weights are the **strategic asset allocation** (the policy portfolio), the long-term baseline. **Rebalancing** later moves drifted weights back to these targets; it is not an active strategy and not a tactical decision.
3. Decide how much **active management** to add:
   - **Tactical asset allocation** is a temporary deviation from strategic weights to exploit perceived short-term mispricing between asset classes. For example, a 10% real estate target might currently be held at 13% because property looks cheap.
   - **Security selection** is a deviation from the index weights of individual securities within a class. For classes without investable indexes (hedge funds, individual properties, art), selection is unavoidable.
4. Each active strategy may add return, but it also adds risk compared with a passive portfolio of asset-class indexes. **Risk budgeting** sets a total risk limit and allocates it among strategic allocation risk, tactical allocation risk and security selection risk.

Using several active managers against the same benchmark creates two problems:

- Their positions may offset, with one overweighting what another underweights. **Net active risk** is then below **gross active risk**, and the risk budget is underused.
- Combined trading may be excessive and tax-inefficient.

A **core-satellite approach** addresses both. Most of the portfolio (the core) is held passively in index funds, and a smaller satellite portion is actively managed. The success of tactical allocation and security selection depends on real opportunities and on manager skill.

Example (illustrative). A pension fund's strategic asset allocation, grouped by the role each asset class plays in the portfolio:

Role	Asset class	Target weight
Growth	Global equities	44%
	Private equity	10%
	High-yield credit	10%
Downturn hedging	Government bonds	18%
	Cash	6%
Inflation hedging	Inflation-linked bonds	7%
	Real estate	5%
Total		**100%** (growth 64%, downturn hedging 24%, inflation hedging 12%)

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Common exam traps

- Setting target weights from the client's objectives is strategic asset allocation, even if the weights are aggressive.
- Changing a weight because the client's tax rate changed is an IPS (strategic) change. Tactical allocation responds to short-term valuation views.
- Market forecasts do matter, because the efficient frontier is built from expected returns and risks.

LOS 90.h — ESG considerations in portfolio planning

Environmental, social and governance (ESG) approaches:

Approach	What the investor does
Negative screening	Excludes specific companies or industries on ESG grounds
Positive screening	Invests in companies with good ESG practices
Thematic investing	Picks sectors or companies that advance a specific ESG goal
Impact investing	Seeks a financial return together with a positive ESG outcome
Engagement/active ownership	Uses shareholdings (votes, dialogue with management) to push for better ESG practices
ESG integration	Considers ESG factors throughout asset allocation and security selection

- A negatively screened portfolio should be judged against an index that excludes the same kinds of companies. A broad market index is not an appropriate benchmark for it.
- Positive screening, thematic and impact approaches need customized portfolios and benchmarks, and often specialist managers.
- Under active ownership, clarify whether clients will vote their own shares or have the manager vote according to specified ESG factors. Either arrangement is acceptable.
- The effect on returns is uncertain. A smaller universe and research costs may reduce returns, while good governance and lower ESG risks may raise them.

BOTTOM LINE

- The investment policy statement is the written plan that starts the portfolio management process: it forces the client to articulate circumstances, objectives and constraints, reconciles return expectations with risk tolerance, and sets a benchmark for judging results.
- At a minimum an IPS describes the client's circumstances and constraints, sets out a strategy built on them and names a benchmark for evaluating performance; permitted assets and leverage limits belong in the investment guidelines.
- Risk and return objectives can be absolute or relative, a risk objective being relative only when it refers to a benchmark, and the two objectives must be set together so that they are compatible.
- Willingness to take risk reflects the investor's psychology and ability to take risk reflects financial circumstances such as horizon, wealth relative to needs and income stability; when the two conflict the lower assessment governs, although with high ability and low willingness the adviser may educate the client.

- The constraints are liquidity, time horizon, taxes, legal and regulatory factors and unique circumstances, a rule imposed by law being a legal constraint and a rule the investor chooses a unique circumstance.
- Asset classes for strategic allocation should have highly correlated returns within each class, low correlations between classes, and together approximate the universe of permissible investments.
- The strategic asset allocation is the efficient asset-class portfolio that best fits the IPS, tactical asset allocation is a temporary deviation from it to exploit perceived short-term mispricing, and risk budgeting allocates a total risk limit among strategic, tactical and security selection risk.
- ESG approaches include negative and positive screening, thematic and impact investing, engagement and ESG integration, and a negatively screened portfolio should be judged against an index that excludes the same kinds of companies.

QUICK CHECK

Question 1

An adviser explains to a new client why they should agree on a written investment policy statement (IPS) before any money is invested. Which of the following is *least likely* to be one of the reasons she gives? A written IPS:

A names the individual securities the portfolio will buy first.

B makes the client spell out objectives, circumstances and constraints.

C helps keep return goals realistic given the client's risk tolerance.

Answer: A

An IPS works at the level of objectives, constraints, strategy and benchmark. It does not go down to the choice of particular securities, which is a later, implementation decision within the policy the IPS sets.

Why the other options are wrong

- **B.** Forcing the client to articulate needs, circumstances and constraints is a central purpose of a written IPS; it imposes discipline on both parties.
- **C.** The IPS makes the client reconcile return expectations with risk tolerance, so it does help keep goals realistic and makes the risks of investing explicit.

Key takeaway The IPS is the starting point of the portfolio management process. It sets policy; security selection comes later.

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Practice questions

Question 2

A client's IPS lists several investment objectives. Which one is a relative return objective?

A Achieve a nominal return of at least 7% per year.

B Keep to 5% the probability of losing more than 7% in any 12-month period.

C Outperform a global equity index by 1.5% per year.

Question 3

Holding everything else constant, which of the following changes would *most likely* reduce a client's ability to take investment risk?

A A sharp rise in near-term spending needs.

B A rise in the client's net worth.

C The purchase of additional insurance coverage.

Question 4

Which of the following is a characteristic of a well-specified set of asset classes for strategic asset allocation?

A Each class groups assets with widely differing risk and expected return.

B Taken together, the classes cover roughly the whole universe the IPS permits.

C Returns of each class are highly correlated with those of the other classes.

Question 5

Which of the following statements about integrating environmental, social and governance (ESG) considerations into portfolio construction is *most* accurate?

A Adding ESG constraints to the investment process will most likely reduce portfolio returns.

B Clients pursuing active ownership must vote their shares themselves instead of through the manager.

C A broad market index is unlikely to be an appropriate benchmark for a negatively screened portfolio.

Answers: 2. C · 3. A · 4. B · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/portfolio-construction/basics-of-portfolio-planning-and-construction/. The full bank has 38 questions on this reading.

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PORTFOLIO CONSTRUCTION · READING 91

The Behavioral Biases of Individuals

What you'll learn

LOS 91.a Compare cognitive errors (faulty reasoning, can be mitigated) with emotional biases (feelings-based, often accommodated).

LOS 91.b Describe the six emotional biases and their implications for financial decision making.

LOS 91.c Explain how behavioral biases may contribute to bubbles, the value/growth anomaly and home bias.

MODULE 91.1

Cognitive Errors vs. Emotional Biases

This reading separates cognitive errors from emotional biases and explains why the first are usually mitigated and the second accommodated. It describes the belief perseverance and information-processing biases, the six emotional biases, and how behavioral biases may help explain bubbles and crashes, the value premium and home bias.

LOS 91.a — Cognitive errors versus emotional biases

Traditional finance assumes investors are fully rational and process all relevant information objectively. **Behavioral finance**, pioneered by Kahneman and Tversky, shows that people depart from rationality in **systematic** ways. Because these departures are widespread rather than random, they are predictable. Kahneman, Tversky and others suggest that if clients and their advisers understood these biases better, prices and returns would move closer to the efficient-market outcome of traditional theory.

Biases fall into two families:

KEY CONCEPT

	Cognitive errors	Emotional biases
Source	Faulty reasoning: misunderstanding statistics, information-processing mistakes, illogical reasoning, memory errors	Feelings, impulses or intuition; not the product of conscious thought
Nature	An attempt to reason that goes wrong	A spontaneous reaction
Fix	Can often be **mitigated** (reduced) through awareness, education, training or better information	Difficult to overcome; may have to be **accommodated** (the portfolio is adjusted to live with the bias)
Sub-groups	**Belief perseverance biases** and **information-processing biases**	Six biases (see Module 91.2)

Many biases have both cognitive and emotional elements. When they do, the adviser is more likely to succeed by working on the cognitive part. A useful test: if a simple change in thinking or better information would make the investor drop the bias, treat it as **cognitive**. If the view rests on emotion the investor is unwilling or unable to change, treat it as **emotional**.

Example. A client keeps too much money in low-yield deposits because he misreads a volatility chart. Once the adviser shows him the long-run return data, he rebalances. Better information changed his decision, so the problem was a cognitive error. Another client refuses to sell a family-business stake despite a clear case for diversifying, saying that selling "just feels wrong". Her view rests on feelings she will not change, so it is an emotional bias to be accommodated.

LOS 91.b — Cognitive errors: belief perseverance biases

Belief perseverance is the tendency to cling to prior beliefs. It is rooted in **cognitive dissonance**: holding conflicting beliefs, or receiving information that challenges a current belief, causes stress. A person can relieve it by dropping the old belief or by discounting the new information (questioning its truth, source, relevance or importance). When the second route is easier, the result is a bias toward existing beliefs.

Bias	Description	Typical consequence
Conservatism bias	Forms a reasonable initial view but fails to update it (or updates too slowly) when new information arrives; overweights prior probabilities	Holding positions too long; ignoring complex new data
Confirmation bias	Seeks and notices information that supports existing beliefs; avoids or discounts conflicting evidence, or distorts it to fit the prior view	Screens set up to confirm a view; overconfidence in a held belief. Remedy: deliberately seek contrary views
Representativeness bias	Classifies something by a few features and assumes it will behave like the category; treats two things that are alike in some respects as more alike in other respects than they are	Two forms: **base-rate neglect** (ignoring how common a trait is in the population) and **sample-size neglect** (generalizing from a small sample, e.g. three good years of a fund manager). Leads to overweighting a few characteristics and relying on simple rules and classifications instead of thorough analysis
Illusion of control bias	Believes one can control or influence outcomes that one cannot. Often linked with emotional biases: illusion of knowledge, self-attribution and overconfidence	Overweighting the employer's stock or other "controllable" holdings; poor diversification

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Bias	Description	Typical consequence
Hindsight bias	Selective memory: sees past outcomes as predictable or even inevitable ("I knew it all along"), remembers correct forecasts and forgets wrong ones, overestimates what could have been known	Overconfidence; discarding sound methods that happened not to work

Example (representativeness). An analyst meets the founder of a young biotech company. The firm has a charismatic leader, a strong patent and fast hiring, the traits she associates with the best-known biotech successes, so she projects rapid growth. She ignores the base rate: only a small fraction of early-stage biotech firms ever bring a product to market. Judging the firm by how closely it resembles a few famous winners, rather than by how often firms like it succeed, is base-rate neglect.

LOS 91.b — Cognitive errors: information-processing biases

These biases concern how information is processed rather than the decision itself.

Bias	Description	Typical consequence
Anchoring and adjustment bias	Starts from an initial number (current price, last EPS, prior estimate) and adjusts too little as new data arrive	Underreacting to new information. Remedy: judge new data on its own merits rather than as an adjustment to the starting number
Mental accounting bias	Treats money differently depending on its source or the "account" it is in, instead of viewing the portfolio as a whole	Speculating with "found money" (bonuses, refunds), keeping an inheritance in low-risk bonds, treating income and capital gains differently, holding offsetting positions instead of considering correlations. New York taxi drivers who treat each day as a separate account and stop once a daily income target is met are a well-known case
Framing bias	Makes different choices depending on how the same facts are presented (as a gain or as a loss; over a short or a long horizon)	Overreacting to short-term volatility; mis-measured risk tolerance if questionnaire wording is biased
Availability bias	Judges probability by how easily examples come to mind; overweights recent, vivid, familiar or heavily publicized events	Choosing managers by advertising; limiting the universe to familiar firms; overreacting to recent news and ignoring long-run history

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Example (framing). An investor is offered a choice for a $30,000 position. Framed as gains, the choice is "keep $20,000 for certain" or "a one-third chance of keeping all $30,000 and a two-thirds chance of keeping $15,000". Both have an expected value of $20,000: one-third of 30,000 is 10,000, and two-thirds of 15,000 is 10,000. Most people take the sure amount. Framed as losses ("lose $10,000 for certain" versus the equivalent gamble), many switch to the gamble, even though the outcomes are identical.

Classification summary

KEY CONCEPT

Belief perseverance (cognitive)	Information processing (cognitive)	Emotional
Conservatism	Anchoring and adjustment	**Loss-aversion bias**
Confirmation	Mental accounting	**Overconfidence bias**
Representativeness	Framing	**Self-control bias**
Illusion of control	Availability	**Status quo bias**
Hindsight		**Endowment bias**
		Regret-aversion bias

Common exam traps

- Conservatism and confirmation differ. Conservatism is failing to update a view; confirmation is selectively gathering or interpreting evidence to fit it.
- Conservatism and anchoring also differ, although both lead to underreaction. Anchoring starts from a specific number, such as a price or a prior EPS figure, and adjusts too little; conservatism is a belief perseverance bias in which a prior view or forecast is kept, or revised too slowly, as new information arrives.
- Availability is about ease of recall (recent, vivid, publicized events). Confirmation is about agreement with a prior view, and framing is about presentation.
- Mental accounting and framing are cognitive (information-processing) biases, even though they feel intuitive.

EXAM SHORTCUTS

- To classify a bias, ask whether better information or a simple change in thinking would make the investor drop it: if so, treat it as cognitive; if the view rests on emotion the investor will not or cannot change, treat it as emotional.

BOTTOM LINE

- Cognitive errors come from faulty reasoning and can often be mitigated through awareness, education, training or better information, while emotional biases arise from feelings or impulses and are hard to overcome, so the portfolio may have to accommodate them.

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- Belief perseverance biases, rooted in cognitive dissonance, are conservatism, confirmation, representativeness, illusion of control and hindsight bias.
- Information-processing biases are anchoring and adjustment, mental accounting, framing and availability.
- Conservatism is failing to update a reasonable view, or updating it too slowly, confirmation bias is seeking and interpreting evidence to fit an existing view, and anchoring is adjusting too little from a specific starting number.
- Representativeness takes two forms, base-rate neglect (ignoring how common a trait is in the population) and sample-size neglect (generalizing from a small sample).
- Availability bias judges probability by how easily examples come to mind, and framing bias produces different choices from the same facts depending on how they are presented.

QUICK CHECK

Question 1

An adviser notes several tendencies in a client's behavior. Which one is *most likely* a cognitive error rather than an emotional bias? The client:

A takes on more risk rather than selling a stock at a loss.

B avoids buying stocks for fear of the regret she would feel if they fell.

C assumes a new company will behave like others it seems to resemble.

Answer: C

Classifying something by a few features and assuming it will share the characteristics of that category is representativeness bias, a belief perseverance bias and therefore a cognitive error.

Why the other options are wrong

- **A.** Taking extra risk to avoid realizing a loss is loss-aversion bias, an emotional bias.
- **B.** Inaction driven by fear of regretting a wrong decision is regret-aversion bias, an emotional bias.

Key takeaway Representativeness (categorizing) is cognitive; loss aversion and regret aversion are emotional.

MODULE 91.2

Emotional Biases

LOS 91.b — Emotional biases

Six biases are generally classified as **emotional biases**. LOS 91.a compares them with cognitive errors and explains why they are usually accommodated rather than corrected.

KEY CONCEPT

Bias	Description	Typical consequence
Loss-aversion bias	Losses hurt more than equal gains please; decisions are judged against a **reference point**	Trading too much by selling winners for small gains (higher costs, lower returns); holding deteriorating losers and taking excess risk after a decline in the hope of getting back to breakeven; a position may be seen as a gain or a loss depending on how the reference point is framed
Overconfidence bias	Overestimating one's own knowledge, reasoning or forecasting ability; includes **illusion of knowledge**; **self-attribution bias**, which is **self-enhancing** when it takes credit for successes and **self-protecting** when it blames others or circumstances for failures; **prediction overconfidence** (ranges too narrow); and **certainty overconfidence** (probability of being right overstated)	Underestimating risk, overestimating return, overtrading, too little diversification
Self-control bias	Lack of discipline: short-term satisfaction wins over long-term goals; **hyperbolic discounting** (preferring small payoffs now to larger ones later)	Undersaving for retirement, then taking excessive risk to catch up; overemphasis on income assets. Remedy: a formal plan and budget, reviewed regularly
Status quo bias	Comfort with the current situation; doing nothing is the default	Keeping an inappropriate allocation for years; ignoring better alternatives. Default options (e.g. automatic enrollment with opt-out) exploit it
Endowment bias	An asset feels more valuable simply because one owns it; owners tend to ask a higher price to sell it than they would pay to buy it	Holding inherited or familiar assets that no longer fit. Test: "Would you buy this today with new money?"
Regret-aversion bias	Fear that an action will turn out wrong leads to inaction; errors of **commission** weigh more than errors of **omission**. **Herding** is a form of regret aversion	Excess conservatism; following the crowd

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Loss aversion and risk aversion. A **risk-averse** investor, faced with two choices with the same expected return, picks the less risky one. A **loss-averse** investor measures outcomes against a reference point and may take on more risk to avoid realizing a loss than to secure an equal gain.

Example (loss aversion). An investor holds a stock bought at €80 that now trades at €56, a 30% loss. His research no longer supports the stock, but he says he will sell "once it gets back to €80". He is using the purchase price as the reference point and refusing to realize the loss, which is typical loss-aversion behavior. The only relevant question is whether the stock is worth holding at €56.

Common exam traps

- Status quo bias, regret aversion and endowment bias can all produce inaction. Status quo bias is comfort with the default; regret aversion is fear that acting will be a mistake; endowment bias makes an asset seem worth more because it is owned.
- Self-control bias concerns saving and spending discipline rather than trading decisions.
- Overconfidence has cognitive elements but is classified as an emotional bias, because it is hard to correct and rooted in the wish to feel good.

LOS 91.c — Behavioral biases and market characteristics

A **market anomaly** is a result that does not fit the prevailing model of risk and return. A **market inefficiency** is an anomaly that offers positive risk-adjusted returns. Many apparent anomalies disappear once small samples, time-period bias or a **misspecified risk model** are accounted for.

KEY CONCEPT

Bubbles and crashes. Behavioral finance does not offer an overall explanation of bubbles, but several biases may contribute:

- **Overconfidence** causes investors to trade too much, underestimate risk and diversify too little.
- **Self-attribution** and **hindsight bias** (taking credit for gains in a bull market) reinforce overconfidence.
- **Confirmation bias** leads investors to dismiss evidence that prices cannot keep rising and to treat early declines as buying opportunities.
- **Anchoring** leads investors to treat recent highs as rational prices even as prices fall.
- **Fear of regret** keeps even skeptical investors in the market.

Exploiting a bubble is hard. Leaving too early and staying too long are both costly, and short selling a rising market is risky and restricted.

Value vs growth. Value stocks (low market-to-book, low P/E, high dividend yield) have outperformed growth stocks over long periods. In Fama and French's (1992) tests, the apparent value premium disappeared once size and book-to-market factors were included in the return model, which suggests the extra return compensates for risk. A behavioral explanation is the **halo effect**, a form of representativeness in which a firm's good features (fast growth, a rising share price) are extended to the conclusion that it is a good stock. The result is overvaluation of growth stocks.

Home bias. Investors overweight domestic or local companies despite the benefits of global diversification, perhaps believing they have better information or simply feeling more comfortable. Investors also tend to overrate firms whose products they use or whose marketing they see often.

Common exam traps

- Under-diversification is linked to overconfidence, illusion of control and availability (a preference for familiar firms), not to anchoring.
- Anchoring and regret aversion tend to keep investors in a bubble; they do not deflate it.
- The halo effect overvalues growth stocks, not local or familiar ones. Overweighting companies close to home is home bias.

BOTTOM LINE

- The six emotional biases are loss aversion, overconfidence, self-control, status quo, endowment and regret aversion.
- A loss-averse investor feels losses more than equal gains and judges outcomes against a reference point, so may sell winners too early, hold losers and take on more risk to avoid realizing a loss, unlike a risk-averse investor, who picks the less risky of two choices with the same expected return.
- Overconfidence, which includes illusion of knowledge, self-attribution, prediction overconfidence and certainty overconfidence, leads investors to underestimate risk, overtrade and diversify too little, and it is classified as emotional although it has cognitive elements.
- Self-control bias puts short-term satisfaction ahead of long-term goals, status quo bias makes doing nothing the default, endowment bias values an asset more because it is owned, and regret aversion makes errors of commission weigh more than errors of omission.
- A market anomaly does not fit the prevailing risk-return model, and it is a market inefficiency only if it offers positive risk-adjusted returns; many apparent anomalies disappear once small samples, time-period bias or a misspecified risk model are accounted for.
- Biases such as overconfidence, confirmation, anchoring and fear of regret may contribute to bubbles, the halo effect may explain overvaluation of growth stocks, and home bias leads investors to overweight domestic or local companies.

QUICK CHECK

Question 2

Studies find that many investors hold far fewer securities than traditional portfolio theory would recommend. Which behavioral bias *best* explains this under-diversification?

A Overconfidence.

B Anchoring.

C Fear of regret.

Answer: A

Overconfidence leads investors to overestimate their knowledge and underestimate risk, which results in overtrading and insufficient diversification.

Why the other options are wrong

- **B.** Anchoring may cause investors to treat recent highs as rational prices even as prices fall; it is not the usual explanation for under-diversification.
- **C.** Fear of regret (of missing gains) may keep skeptical investors in an overvalued market; it does not explain holding too few securities.

Key takeaway Under-diversification, overtrading and underestimating risk are hallmarks of overconfidence.

Practice questions

Question 3

Which of the following is classified as an information-processing bias?

A Regret aversion.

B Anchoring and adjustment.

C Status quo.

Question 4

Which of the following groups consists *only* of cognitive errors?

A Representativeness, framing, and overconfidence biases.

B Confirmation, anchoring and adjustment, and availability biases.

C Endowment, regret-aversion, and self-control biases.

Answers: 3. B · 4. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/portfolio-construction/the-behavioral-biases-of-individuals/. The full bank has 24 questions on this reading.

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PORTFOLIO CONSTRUCTION · READING 92

Value At Risk

What you'll learn

LOS 92.a Define risk management as choosing and maintaining the optimal bundle of risks, not minimizing risk.

LOS 92.b Describe the features of a risk management framework.

LOS 92.c Define risk governance and describe what makes it effective.

LOS 92.d Explain how risk tolerance is set and how it shapes risk management.

LOS 92.e Describe risk budgeting and its role within risk governance.

LOS 92.f Identify financial and non-financial risks and describe how they interact.

LOS 92.g Describe methods for measuring (e.g., duration, beta, Greeks, VaR, CVaR, stress tests, scenarios) and modifying (avoid, prevent, accept, transfer, shift) risk exposures.

MODULE 92.1

Introduction to Risk Management

This reading defines risk management and the features of a risk management framework, and explains risk governance, risk tolerance and risk budgeting. It then classifies financial and non-financial risks and their interactions, and describes how risk exposures are measured (including VaR and CVaR) and modified.

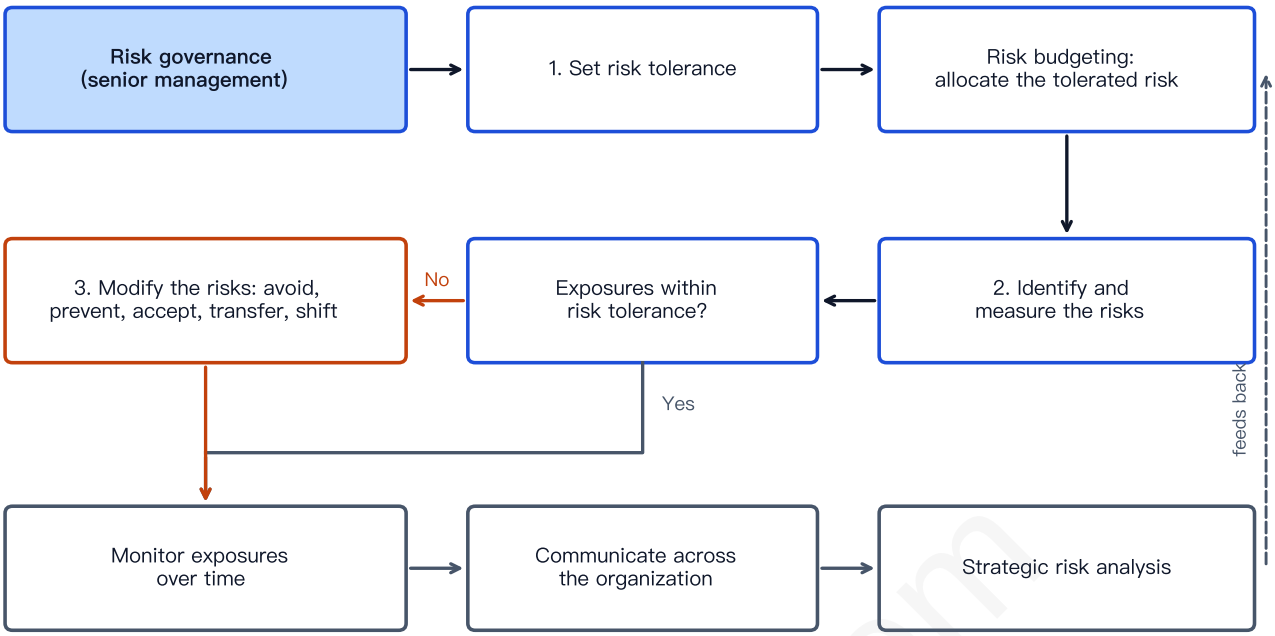
LOS 92.a — What risk management is

Risk management is the process by which an organization, a portfolio manager or an individual (1) identifies its **risk tolerance**, (2) identifies and measures the risks it faces, and (3) modifies and monitors those risks, so that the bundle of risks it keeps is the one best suited to its goals.

- The goal is to hold the right risks. It is not to minimize or eliminate risk: returns above the risk-free rate come from bearing risk. An organization may deliberately increase its exposure to risks it is well placed to manage and reduce its exposure (through organizational changes, insurance or hedging) to risks it handles poorly.
- Managers cannot control the returns of any single period. They can control which risks they take and how much total risk they take. Risk management therefore means choosing the **optimal bundle of risks** and putting in place the strategies that achieve it.
- The same logic applies to individuals, who choose the bundle of risks that maximizes their expected utility.

The figure puts the steps in order. Risk governance sets the tolerance; the risks are then identified and measured; exposures are compared with the tolerance and modified when they are out of line; and the results are monitored, communicated and fed into strategic risk analysis.

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The risk management process as a loop

LOS 92.b — Features of a risk management framework

A comprehensive **risk management framework** addresses all of the following:

1. Establishing processes and policies for risk governance.
2. Determining the organization's risk tolerance.
3. Identifying and measuring existing risks.
4. Managing and mitigating risks so that the organization ends up with its optimal bundle of risks.
5. Monitoring risk exposures over time.
6. Communicating across the organization.
7. Performing strategic risk analysis.

Disciplinary action against individual employees (for example, penalizing a trader who breached a limit) is not one of these features.

LOS 92.c — Risk governance

Risk governance is senior management's determination of the organization's risk tolerance, of the elements of its optimal risk exposure strategy, and of the framework for oversight of the risk management function. It is set at the enterprise level and aims to manage risk in a way that supports the organization's overall goals. It gives organization-wide guidance on which risks to pursue efficiently, which to limit, and which to reduce or avoid. A **risk management committee** gives different parts of the organization a forum to raise issues of risk measurement, integration of risks and mitigation.

Term	Meaning	Relationship
Risk governance	Senior management's overall oversight of risk management	The umbrella concept
Risk tolerance	How much overall risk the organization will take, and which risks	One element decided under risk governance
Risk budgeting	Allocating the acceptable risk across assets/investments	One element carried out within risk governance

LOS 92.d — How risk tolerance affects risk management

Risk tolerance is the overall amount of risk an organization will take in pursuing its goals. Top management sets it by identifying the risks the firm can take effectively and those it should reduce or avoid. The factors that determine it include:

- expertise in its lines of business;
- skill at responding to negative outside events;
- the regulatory environment;
- financial strength and the ability to withstand losses.

Management must examine risks that arise both inside and outside the organization and weigh each one against the expected benefit of bearing it. A low risk tolerance does not require cutting every identified risk. Some risks are worth keeping because they are consistent with the organization's objectives.

LOS 92.e — Risk budgeting

Risk budgeting allocates the organization's total acceptable risk (its risk tolerance) to assets or investments by considering their risk characteristics and how they combine. The aim is to place that risk where expected returns over time are greatest.

- The budget can be a single metric: return variance, portfolio beta, portfolio duration or value at risk.
- It can be split by investment category (e.g., domestic equities, domestic debt, international equities, international debt).
- It can be built from risk factors (interest rate risk, equity market risk, foreign exchange risk), which are estimated and aggregated to check that the total matches the risk tolerance.

LOS 92.f — Financial and non-financial risks and how they interact

Financial risks arise from exposure to financial markets. **Non-financial risks** arise from the organization's operations and from sources outside it.

KEY CONCEPT

Financial risks	Non-financial risks
Credit risk: the counterparty may not meet its contractual obligations	**Operational risk**: losses from human error, faulty processes, inadequate security or business interruption (includes **cyber risk**)

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Financial risks	Non-financial risks
Liquidity risk: having to sell an asset below its fair value because of market conditions	**Solvency risk**: running out of cash and being unable to continue operating
Market risk: uncertainty about the prices of stocks, commodities and currencies, and about interest rates	**Regulatory risk**: the regulatory environment changes, adding costs or restricting activities
	Governmental or political risk (includes **tax risk**): political action outside the regulatory framework, such as a tax increase, imposes costs
	Legal risk: uncertainty about exposure to future legal action
	Model risk: valuations from the organization's models are wrong
	Tail risk: extreme outcomes are more likely than the organization's analysis indicates (e.g., from wrongly assuming normality)
	Accounting risk: accounting policies or estimates are judged incorrect

Individuals also face **mortality risk** (dying before providing for dependents, usually addressed with life insurance) and **longevity risk** (outliving one's assets, which a lifetime annuity reduces), as well as the risk of health care expenses (health insurance). The approach is the same as for an organization: decide which risks to bear (self-insure), which to prevent or avoid, and which to take in order to maximize expected utility.

Interactions. Risks are not independent. Suppose a commodity producer hedges price risk with a swap arranged with a dealer. If commodity prices fall sharply, the dealer owes the producer a large payment under the swap, so the producer's credit risk rises. If the dealer disputes the contract, legal risk appears. Credit and legal losses may force the producer to sell assets in a weak market (liquidity risk), and those losses drain cash (solvency risk). Interactions like these are frequent and matter most in periods of market stress, so risks must be assessed together as well as one at a time.

LOS 92.g — Measuring and modifying risk exposures

Measures

KEY CONCEPT

Measure	What it measures
Standard deviation	Volatility of asset prices or interest rates (stand-alone risk); can mislead for non-normal distributions (negative skew, fat tails)
Beta	Market risk of equities/equity portfolios; appropriate for securities held in a well-diversified portfolio

Measure	What it measures
Duration	Price sensitivity of debt securities to interest rate changes
Delta	Sensitivity of a derivative's value to the underlying's price
Gamma	Sensitivity of delta to the underlying's price
Vega	Sensitivity of a derivative's value to the underlying's volatility
Rho	Sensitivity of a derivative's value to the risk-free rate

Common measures of tail risk (also called downside risk) are:

- **Value at risk (VaR)** is the minimum loss over a given period that will occur with a stated probability, so a VaR figure always states an amount, a period and a probability. Equivalently, it is the maximum potential loss at the matching confidence level (95% for a 5% VaR), but larger losses still occur in the tail, so VaR is not the worst possible loss. Results depend heavily on the inputs and the model chosen, so VaR should be used together with other measures. Banks use it widely, including to set minimum capital requirements.
- **Conditional VaR (CVaR)** is the expected loss given that the loss exceeds the VaR threshold (the probability-weighted average of those losses). It is similar to the loss given default used for debt securities.

Example. A fund reports a one-week VaR of $600,000 at 5%. In about 5% of weeks the fund should expect to lose at least $600,000. If the losses in those worst weeks average $850,000, the CVaR is $850,000.

Two methods supplement VaR. **Stress testing** examines the effect of a specific (usually extreme) change in one key variable. **Scenario analysis** combines changes in several inputs at once. Rare events, operational risks and tax or regulatory changes often need subjective estimates. The market prices of insurance and derivatives also reveal market participants' estimates of expected losses, and operational risk can be estimated from large samples of firms (the overall probability of a large operational loss and the average size of such losses). A subjective estimate of a risk's probability and size is still better than ignoring the risk.

Modifying risk exposures

KEY CONCEPT

Method	How it works	Example
Avoid a risk	Do not engage in the activity	Not investing in a politically unstable country. Usually a top-management decision, made when the risk outweighs the activity's benefits
Prevent a risk	Take steps to prevent the loss or make it less likely	Stronger data security against breaches, when the benefit exceeds the cost
Accept (bear) a risk / **self-insurance**	Keep the risk, ideally efficiently (e.g., via diversification or a loss reserve)	Reserve account for small claims. Self-insurance only means bearing the losses; it can reflect inaction rather than a deliberate decision

Method	How it works	Example
Risk transfer	Another party takes on the risk	**Insurance**, **surety bonds** (pay if a third party fails to perform), **fidelity bonds** (employee theft or misconduct). Insurers diversify across risks that ideally are not highly correlated, and may buy **reinsurance** for concentrated risks
Risk shifting	Change the distribution of possible outcomes, mainly with **derivatives**	Forwards, futures or swaps to hedge currency risk; buying puts to set a floor; writing calls (gives up upside, and the premium reduces the downside)

The choice is always a cost–benefit comparison, and several methods may be combined for a single risk. An insurance policy with a deductible is an example: the insurer takes on losses above the deductible (risk transfer), while the insured bears the losses up to it (self-insurance). The aim is a risk profile consistent with the organization's risk tolerance.

Common exam traps

- A what-if run that moves interest rates and oil prices together is scenario analysis. A stress test looks at one key variable.
- Do not read VaR as the worst possible loss or as the average loss in the tail. The average of the losses beyond VaR is CVaR.
- Buying puts or writing calls reduces a loss by changing the distribution of outcomes, so it is risk shifting. Risk transfer means another party, such as an insurer, takes the risk on.
- Beta does not measure interest rate sensitivity. Duration and rho do.
- Solvency, model, tax and tail risk sound financial, yet they are classified as non-financial risks.

BOTTOM LINE

- Risk management identifies the risk tolerance, identifies and measures the risks faced, and modifies and monitors them, with the aim of holding the optimal bundle of risks rather than minimizing or eliminating risk.
- Risk governance is senior management's determination of risk tolerance, of the elements of the optimal risk exposure strategy and of the framework for overseeing risk management, set at the enterprise level.
- Risk tolerance depends on expertise in the lines of business, skill at responding to negative outside events, the regulatory environment and financial strength, and a low risk tolerance does not require cutting every identified risk.
- Risk budgeting allocates the total acceptable risk to assets or investments, using a single metric such as variance, beta, duration or VaR, a split by investment category, or aggregated risk factors.
- Financial risks are credit, liquidity and market risk, while operational, solvency, regulatory, political and tax, legal, model, tail and accounting risks are non-financial, and because risks interact, especially in periods of market stress, they must be assessed together as well as one at a time.
- VaR is the minimum loss over a stated period that will occur with a stated probability, so it is not the worst possible loss, and CVaR is the expected loss given that the loss exceeds the VaR threshold.

- A stress test examines a specific, usually extreme, change in one key variable, while scenario analysis changes several inputs at once.
- Risk can be avoided, prevented, accepted through self-insurance, transferred to another party such as an insurer, or shifted by changing the distribution of outcomes with derivatives, the choice being a cost–benefit comparison.

QUICK CHECK

Question 1

Which of the following is *most likely* one of the goals of an organization's risk management process?

A Identifying the risks the organization faces.

B Minimizing the risks the organization faces.

C Eliminating the risks the organization faces.

Answer: A

The risk management process identifies the organization's risk tolerance, identifies and measures the risks it faces, and then modifies and monitors those risks. Its aim is the optimal bundle of risks rather than the smallest possible amount of risk.

Why the other options are wrong

- **B.** Minimizing risk is not the goal. Returns above the risk-free rate require bearing risk, and an organization may deliberately increase risks it manages well.
- **C.** Eliminating all risk would also eliminate the returns that come from bearing it; risk management keeps the risks that fit the organization's tolerance and goals.

Key takeaway Risk management chooses which risks to bear; it does not try to avoid all risk.

Practice questions

Question 2

Which statement about how an organization sets its risk tolerance is *most accurate*?

A The organization's financial strength is one of the factors it should weigh.

B An organization with low risk tolerance should cut every risk it identifies.

C Only risks that come from outside the organization need to be considered.

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Question 3

To protect its equity portfolio, the Ashcombe Teachers' Pension Fund buys over-the-counter put options from a single dealer. Over the next quarter, equity prices fall 25% and the puts move deep in the money. Which risk has *most likely* increased the most for the fund as a result of this move?

A Market risk on the equity portfolio.

B Credit risk on the put options.

C Operational risk in the fund's trading.

Question 4

Westbury College buys a flood insurance policy for its campus buildings. For each flood claim, the college pays the first $400,000 of damage itself, and the insurer pays the rest. Which description of this arrangement is *most accurate*?

A Losses of any size are transferred to the insurer, so the college bears no flood risk.

B The policy is a form of risk shifting, because it changes the distribution of the college's outcomes.

C The first $400,000 of each loss is self-insured, and the part above it is transferred.

Answers: 2. A · 3. B · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/portfolio-construction/value-at-risk/. The full bank has 20 questions on this reading.

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CFA® PROGRAM LEVEL I · TOPIC

Ethical and Professional Standards

Exam weight 10–15% · 10 readings

Why trust matters in investment work, the CFA Institute Code of Ethics and the seven Standards of Professional Conduct with guidance for each Standard, and how to apply them to the kind of cases set at Level I.

Reading 93	Ethics and Trust in the Investment Profession
Reading 94	Code of Ethics
Reading 95	Knowledge of the Law
Reading 96	Material Nonpublic Information
Reading 97	Duties to Clients
Reading 98	Duties to Employers
Reading 99	Investment Analysis Recommendations and Actions
Reading 100	Conflicts of Interest
Reading 101	Responsibilities as a CFA Institute Member
Reading 102	Application of the Code and Standards: Level I

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ETHICAL AND PROFESSIONAL STANDARDS · READING 93

Ethics and Trust in the Investment Profession

What you'll learn

LOS 93.a Explain what ethics means: shared beliefs about good (acceptable) and bad (unacceptable) behavior, balancing self-interest against effects on stakeholders.

LOS 93.b Describe how a code of ethics communicates a group's values and how it can help define a profession; distinguish a code of ethics from standards of conduct.

LOS 93.c Describe what makes an occupation a profession and the ways professions build and keep public trust.

LOS 93.d Explain why investment management needs especially high ethical standards and how unethical conduct harms clients, firms and the wider economy.

LOS 93.e Explain professionalism in investment management, including the difference between a suitability standard and a fiduciary standard.

LOS 93.f Identify the main challenges to ethical behavior, especially situational influences and overconfidence in one's own ethics.

LOS 93.g Compare ethical standards with legal standards and explain why neither is a subset of the other.

LOS 93.h Describe the Identify / Consider / Decide and act / Reflect framework for ethical decision making.

MODULE 93.1

Ethics and Trust

This reading defines ethics, codes of ethics and standards of conduct, explains how professions establish trust and why investment management needs high ethical standards, and contrasts the suitability and fiduciary standards. It then covers the challenges to ethical behavior, the difference between ethical and legal standards, and a four-step framework for ethical decision making that a candidate must be able to apply.

LOS 93.a — What ethics means

Ethics is the body of shared beliefs about which behavior is good (acceptable) and which is bad (unacceptable). **Ethical conduct** is behavior that conforms to moral principles and meets the ethical expectations of society.

A second, practical description: ethical conduct is behavior that improves outcomes for **stakeholders**, meaning everyone directly or indirectly affected by a decision. For an investment professional the stakeholders include clients, coworkers, the employer and the investment profession as a whole. Ethical behavior therefore balances self-interest against the impact on others. A decision can bring good results for the person who makes it and still have negative consequences for a stakeholder such as a coworker.

LOS 93.b — Codes of ethics and standards of conduct

A **code of ethics** sets down in writing the moral principles that tell members of a group what behavior is acceptable. It communicates the group's values, principles and general expectations to its own members and to the public. A code may be **principles-based** (broad statements), **rules-based** (specific requirements) or a mix of the two.

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Standards of conduct go further. They set out the minimum level of acceptable behavior in particular situations. A code of ethics may include standards of conduct, but it does not have to: a code can consist of general principles alone.

KEY CONCEPT

	Code of ethics	Standards of conduct
Purpose	States values and general expectations	Sets a minimum acceptable level of behavior
Level of detail	Usually general principles	Specific guidance for situations
Relationship	The overall statement	Optional part of a code

A **profession** is made up of people who use specialized knowledge and skills in the service of others and who commit to a code of ethics. A professional code tells the public that members will apply their expertise for clients' benefit in an honest and ethical way, which can increase public trust. It cannot guarantee that every member will behave ethically. A profession may properly point to its code when it presents itself to the public.

LOS 93.c — Professions and how they establish trust

A profession is an occupational group (doctors, lawyers, engineers) whose core requirement is **specialized expert knowledge**, usually combined with a focus on ethical behavior and on **service to society**. A profession may also have:

- a code and standards of professional behavior, with a **regulatory body** that enforces them and monitors members' conduct;
- attention to clients' needs, with client interests placed first;
- a focus on, or a requirement for, continuing education.

Professions establish trust by:

1. demanding a high level of expertise, knowledge and skill;
2. setting **standards of ethical behavior**;
3. **monitoring professional conduct**;
4. encouraging **continuing education**;
5. focusing on clients' needs;
6. mentoring and inspiring others in the profession.

> *Example.* An actuarial society admits only members who pass a series of exams, publishes a conduct code, investigates complaints and requires 30 hours of study a year. Each of these requirements is a way to establish trust.

LOS 93.d — Why investment management needs high ethical standards

Investment professionals are entrusted with client wealth and must use their specialized skills both to protect it and to grow it. Investment advice and management are **intangible products**. Their quality and value are harder for a client to judge than those of a meal or a laptop, so trust matters more than in most businesses.

Unethical behavior has consequences at three levels:

- Clients lose wealth.
- Firms and professionals find it harder to succeed, because potential investors are less likely to use their services.
- Society suffers. The financial industry links savers with businesses that need financing. Less trust means less money entrusted to the industry and a higher perceived risk of investing, so investors require higher returns and the **cost of capital** rises. Misleading or false information also misallocates capital away from the most productive firms, which slows economic growth and harms everyone in the economy.

KEY CONCEPT

Effect of unethical conduct on	Direction
Perceived investment risk	Increases
Cost of capital for businesses	Increases
Amount of capital invested	Decreases
Efficiency of capital allocation	Decreases
Rate of economic growth	Decreases
Use of investment firms' and professionals' services	Decreases

LOS 93.e — Professionalism: suitability vs fiduciary standards

Clients often know little about securities, fee structures and possible sources of bias in recommendations, so they rely on the adviser's expertise, judgment and ethics. Some financial professionals must meet a suitability standard; others must meet a fiduciary standard:

	Suitability standard	Fiduciary standard
Requirement	The characteristics of recommended securities must match the client's return requirements and risk tolerance	Professional must use knowledge and expertise to act in the best interests of the client
Strength	Weaker	Stronger

LOS 93.f — Challenges to ethical behavior

1. **Overconfidence in one's own ethics.** People tend to rate their own behavior as more ethical than other people's and to overstate how much their personal traits determine what they do.
2. **Situational influences.** External pressures are thought to shape the ethical quality of behavior more strongly than a person's internal traits do. Examples are social pressure from colleagues, loyalty to a supervisor, employer or coworkers, and the prospect of more money or prestige. These pressures push people to put self-interest and short-term results ahead of long-term consequences.
3. **Rules-focused compliance cultures.** A firm that concentrates on strict, rules-based compliance can breed a culture in which staff ask only what they can do (what is allowed). The question of what they should do, based on ethical principles and long-term effects, often goes unasked.

A lack of ethical principles is a personal trait of the decision maker, so it is not a situational influence.

LOS 93.g — Ethical standards vs legal standards

Law and ethics overlap, but neither is a subset of the other:

- Some legal acts are unethical. Recommending a relative's company without disclosing the relationship may break no law.
- Some illegal acts are considered ethical by many. Examples are **whistle-blowing** that breaks a confidentiality law to expose harm to clients, and **civil disobedience**.

Legal and ethical are separate tests (examples)

	Legal	Illegal
Ethical	Most everyday professional conduct, such as disclosing a conflict to clients	Breaking a curfew to drive an injured neighbor to hospital
Unethical	Keeping a referral fee secret from clients where no law requires it to be disclosed	Fraud, such as sending clients falsified account statements

Ethical principles often set a higher standard than law and require more judgment about the impact on many stakeholders. Laws are more specific and often respond to past misconduct: the 1929 crash led to the Glass-Steagall Act, the Securities Act of 1933 and the Securities Exchange Act of 1934; the Enron and WorldCom scandals led to Sarbanes-Oxley; and the 2008 crisis led to the Dodd-Frank Act. New laws can in turn create opportunities for new kinds of unethical behavior.

LOS 93.h — A framework for ethical decision making

Building ethics into a firm's decision process is an important part of developing an ethical culture. Adopting a code of ethics is the first requirement, and backing from senior management matters a great deal. It gives the firm a routine for teaching ethical decision making and reinforcing it through practice, and it lets teams weigh **alternative actions** and their short- and long-term consequences for different stakeholders. A framework helps people identify the issues, examine them from several perspectives, develop judgment and avoid unanticipated (unintended) ethical consequences.

The **ethical decision-making framework** has four steps:

KEY CONCEPT

Step	What to do
1. Identify	Relevant facts (those known and those wanted), stakeholders and the duties owed to each, ethical principles, conflicts of interest
2. Consider	Situational influences and personal biases; additional guidance (mentor, colleagues, legal/compliance); alternative actions and their short- and long-term effects
3. Decide and act	Choose and carry out the action

Step	What to do
4. Reflect	Was the outcome as anticipated? Why or why not? Were duties and principles properly weighed?

Example. A fund manager is offered a paid trip by a broker. Identify: the facts, her duties to clients and employer, and the conflict. Consider: pressure from the broker relationship, advice from compliance, and alternatives such as declining or paying her own way. Decide and act: she pays her own way. Reflect: did the decision work out as expected?

Common exam traps

- One person's private beliefs about right and wrong are not ethics in this sense, and a written code or firm rulebook is only one way of expressing ethics.
- Standards of conduct are not a statement of best practice.
- Fair dealing is Standard III(B), a duty under the CFA Institute Standards. It is not another name for the suitability standard or the fiduciary standard.
- Higher pay, or seeing work as a calling, does not make an occupation a profession. Putting clients first is a common characteristic of professions but is not the definition.
- The framework's steps differ from the qualitative characteristics of financial reports (relevance, faithful representation) and from the steps of a financial analysis project.

BOTTOM LINE

- Ethics is the body of shared beliefs about which behavior is good and which is bad, and ethical conduct can also be described as behavior that improves outcomes for stakeholders, balancing self-interest against the impact on others.
- A code of ethics states a group's values, principles and general expectations, while standards of conduct set the minimum acceptable behavior in particular situations and are an optional part of a code.
- Professions establish trust through demanding requirements for expertise, knowledge and skill, standards of ethical behavior, monitoring of professional conduct, continuing education, a focus on clients' needs, and mentoring others in the profession.
- Investment advice and management are intangible products whose quality clients find hard to judge, and unethical conduct raises perceived investment risk and the cost of capital while reducing capital invested, the efficiency of capital allocation and economic growth.
- The suitability standard requires recommended securities to match the client's return requirements and risk tolerance, while the stronger fiduciary standard requires acting in the client's best interests.
- The challenges to ethical behavior are overconfidence in one's own ethics, situational influences and rules-focused compliance cultures; a lack of ethical principles is a personal trait, not a situational influence.
- Law and ethics overlap but neither is a subset of the other: some legal acts are unethical, and some illegal acts, such as whistle-blowing or civil disobedience, are considered ethical by many.

- The ethical decision-making framework has four steps: identify the facts, stakeholders, duties, principles and conflicts; consider influences, guidance and alternatives; decide and act; and reflect on the outcome.

QUICK CHECK

Question 1

Ethics is *best* described as:

A a written set of rules adopted by a firm.

B commonly shared beliefs about good and bad behavior.

C one individual's belief about proper conduct.

Answer: B

Ethics is a set of shared beliefs about what behavior is good or acceptable and what behavior is bad or unacceptable. It is broader than any particular written rulebook and broader than one person's view.

Why the other options are wrong

- **A.** A written code of standards is one way a group expresses its ethics; it is narrower than ethics itself.
- **C.** A single person's belief about proper conduct is a personal view. Ethics rests on beliefs that are commonly shared.

Key takeaway Ethics means commonly shared beliefs about good and bad behavior. It is broader than one person's view or a single written code.

Practice questions

Question 2

Which of the following *best* distinguishes the members of a profession from people working in an ordinary occupation? Members of a profession:

A earn higher compensation for their services.

B regard their work as a vocation or calling.

C agree to comply with a common code of ethics.

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Question 3

Anika Dorsey, a junior trader, goes along with her desk's habit of delaying the reporting of losing trades. She does so because her supervisor and colleagues expect it and her bonus depends on the desk's reported profits. Lapses like Dorsey's are *most likely* to be explained by:

A situational influences.

B a lack of technical training.

C her personal character traits.

Question 4

Which of the following lists the steps of the ethical decision-making framework presented in the Level I CFA curriculum?

A Relevance; faithful representation; comparability; verifiability; timeliness; understandability.

B Define the objective; collect and analyze data; report findings; update the analysis.

C Identify facts and duties; consider influences and alternatives; decide and act; reflect.

Answers: 2. C · 3. A · 4. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/ethics-and-trust-in-the-investment-profession/. The full bank has 24 questions on this reading.

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ETHICAL AND PROFESSIONAL STANDARDS · READING 94

Code of Ethics

What you'll learn

LOS 94.a Describe how the CFA Institute Professional Conduct Program is organized, how inquiries start and proceed, and which sanctions can result.

LOS 94.b Identify the six components of the Code of Ethics and the seven Standards of Professional Conduct, and tell Code components apart from Standards.

LOS 94.c Explain the ethical responsibilities set out in the Code and Standards, including every sub-section of each Standard.

MODULE 94.1

Code and Standards

This reading describes how the CFA Institute Professional Conduct Program enforces the Code and Standards, sets out the Code of Ethics with its six components, and lists the seven Standards of Professional Conduct with their sub-sections. A candidate must be able to explain how an inquiry starts and how it can end, and to match a duty to the Code or to the Standard it comes from.

LOS 94.a — The Professional Conduct Program and enforcement

The Professional Conduct Program is how CFA Institute enforces the Code and Standards: it looks into possible violations and, where needed, imposes sanctions. The **Professional Conduct Program** (PCP) is governed by the CFA Institute Bylaws and by its Rules of Procedure for Proceedings Related to Professional Conduct. Two principles run through it: **fairness** toward the members and candidates involved, and **confidentiality** of the proceedings.

- The **CFA Institute Board of Governors** has overall responsibility for the program.
- Its **Disciplinary Review Committee** is responsible for enforcing the Code and Standards.
- The **Professional Conduct staff** carry out inquiries.

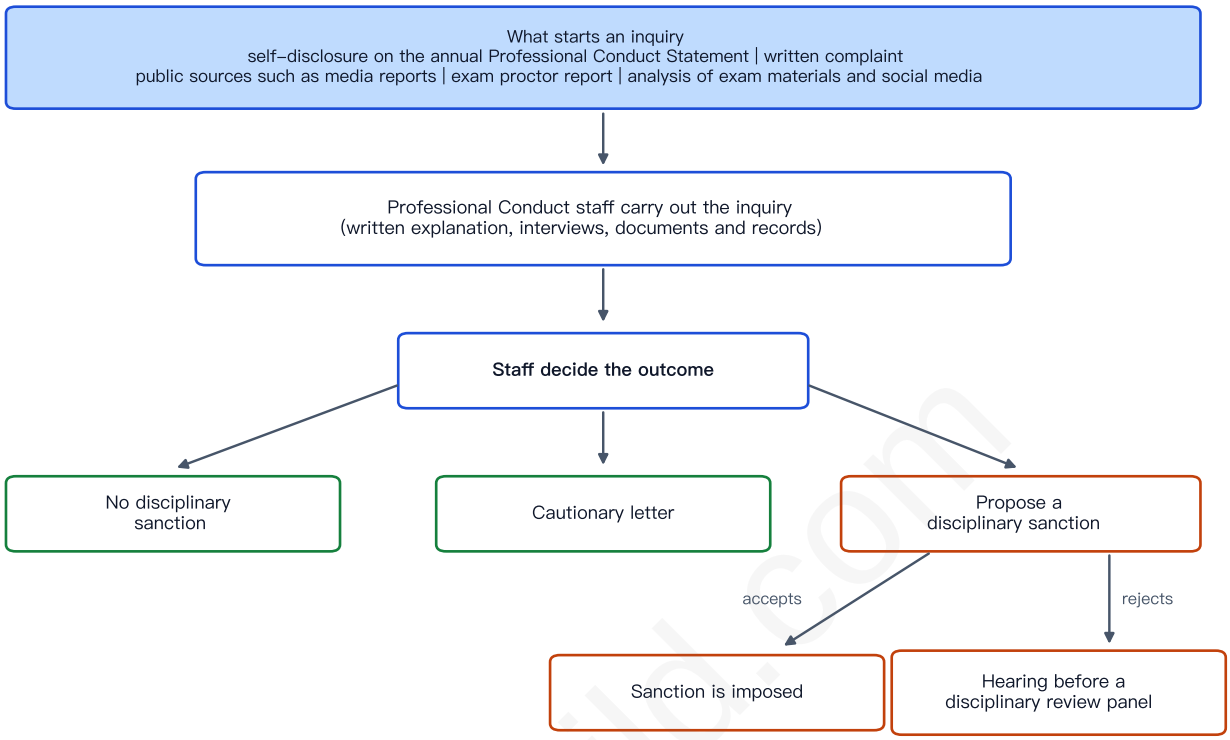
An inquiry can start in five ways. Complaints are only one of them.

1. **Self-disclosure** on the annual **Professional Conduct Statement** (being party to civil litigation, being under criminal investigation, or being named in a written complaint), for example a statement that reports a pending lawsuit;
2. **written complaints** that reach the Professional Conduct staff about how a member or candidate has acted professionally, such as a client's letter to CFA Institute;
3. evidence of misconduct from **public sources**, such as a newspaper article on a fraud;
4. a report by a **CFA exam proctor** of a possible violation during the exam;
5. CFA Institute's own **analysis of exam materials and monitoring of social media**, which can reveal a candidate posting exam content online.

During an inquiry the staff may ask the member or candidate (in writing) for an explanation, interview him or her, interview the complainant or third parties, and collect relevant documents and records.

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The staff may conclude with (1) no disciplinary sanction, (2) a **cautionary letter**, or (3) disciplinary sanction. A member or candidate offered a proposed sanction can either accept it or reject it. If it is rejected, the matter goes to a **disciplinary review panel** of CFA Institute members for a hearing.



Oversight: CFA Institute Board of Governors. Enforcement of the Code and Standards: Disciplinary Review Committee.
Sanctions can include public censure (condemnation by peers) and suspension of a candidate's participation in the CFA Program.

How a Professional Conduct inquiry proceeds

Sanctions may include **condemnation by the member's peers** (public censure) and **suspension of a candidate's continued participation in the CFA Program**.

Exam convention: the curriculum lists these two sanctions. Current practice: the preamble to the Code and Standards also names **revocation** of membership, of candidacy in the CFA Program and of the right to use the CFA designation.

Fines, repayment of profits and bans from working in the industry are not on the list. The program enforces the Code and Standards, which bind members of CFA Institute (including charterholders) and candidates for the CFA designation.

Common exam traps

- Staff do not suspend a member simply because an inquiry is in progress.
- A member who does not yet hold the charter is still bound by the Code and Standards and can be sanctioned; the Code covers all members of CFA Institute, not only charterholders.
- Misconduct under Standard I(D) concerns professional activities. Acts outside a member's professional capacity, such as a minor offense unrelated to work, are not addressed by it, while disclosing exam content violates Standard VII(A).

WORKED EXAMPLE: AN INQUIRY UNDER THE PROFESSIONAL CONDUCT PROGRAM

A Level II candidate posts a description of several questions from her exam on a social media forum the day after the exam. CFA Institute's monitoring of social media finds the post. Explain how the matter reaches the Professional Conduct staff, what the staff may do, which Standard is involved and how the matter can end.

Step 1. CFA Institute's own monitoring of social media and review of exam materials is one of the five ways an inquiry can start, so no complaint is needed.

Step 2. The staff may ask the candidate in writing for an explanation, interview her and any third parties, and collect relevant documents and records.

Step 3. Disclosing exam content violates Standard VII(A), which covers conduct as a participant in CFA Institute programs.

Step 4. The staff may conclude with no disciplinary sanction, a cautionary letter or a disciplinary sanction. If the staff propose a sanction, the candidate can accept it or reject it; if she rejects it, the matter goes to a hearing before a disciplinary review panel made up of CFA Institute members.

Result. Under the exam convention, the sanctions that apply to a candidate include suspension of her continued participation in the CFA Program; current practice also names revocation of candidacy in the CFA Program.

LOS 94.b — The Code of Ethics and the seven Standards

KEY CONCEPT

Code of Ethics. Members of CFA Institute and candidates for the CFA designation (**Members and Candidates**) must (the Code's text is in italics):

1. *Act with **integrity, competence, diligence, respect** and in an ethical manner with the public, clients, prospective clients, employers, employees, colleagues in the investment profession, and other participants in the **global capital markets**.*
2. *Place the **integrity of the investment profession** and the interests of clients above their own personal interests.*
3. *Use **reasonable care** and exercise **independent professional judgment** when conducting investment analysis, making investment recommendations, taking investment actions, and engaging in other professional activities.*
4. ***Practice and encourage others to practice** in a professional and ethical manner that will **reflect credit** on themselves and the profession.*
5. ***Promote the integrity and viability** of the global capital markets for the ultimate benefit of society.*
6. ***Maintain and improve their professional competence** and strive to maintain and improve the competence of other investment professionals.*

Exam convention: the curriculum prints component 1 with a comma after "respect". Current practice: the official Code text has no comma there, as shown above.

The seven Standards of Professional Conduct:
I. Professionalism · II. Integrity of Capital Markets · III. Duties to Clients · IV. Duties to Employers · V. Investment Analysis, Recommendations, and Actions · VI. Conflicts of Interest · VII. Responsibilities as a CFA Institute Member or CFA Candidate.

The Code is short and general. Specific duties, such as keeping records or distinguishing fact from opinion, come from the Standards. The table maps phrases that often appear in questions to their source.

Sounds like…	Where it lives
"integrity, competence, diligence, respect"	Code, component 1
"use reasonable care and exercise independent professional judgment"	Code, component 3
"reflect credit on the profession"	Code, component 4
"professional competence" of self and others	Code, component 6
act with and maintain the competence one's own responsibilities require	Standard I(E)
comply with laws; stricter law applies	Standard I(A)
diligence, independence and thoroughness in analysis	Standard V(A)
suitability of investments	Standard III(C)
priority of client transactions	Standard VI(B)
integrity of the CFA designation and exams	Standard VII(A)

Several plausible-sounding duties are not components of the Code: complying with the **Global Investment Performance Standards (GIPS)**; advocating new laws; educating the general public; honoring contractual provisions; maximizing return per unit of risk.

Common exam traps

- Component 1 names specific groups (public, clients, prospective clients, employers, employees, colleagues, other market participants). It does not name "CFA Program candidates" as a group.
- The four qualities in component 1 are integrity, competence, diligence and respect. Honesty, candor, skill and humility are not on the list.

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LOS 94.c — The Standards and their sub-sections

KEY CONCEPT

Standard	Sub-sections	Core requirement in brief
I Professionalism	I(A) Knowledge of the Law; I(B) Independence and Objectivity; I(C) Misrepresentation; I(D) Misconduct; I(E) Competence	Know and follow the stricter of applicable law and the Code and Standards and dissociate from violations; maintain independence and objectivity, with no gift or benefit that could compromise it; no knowing misstatements; no professional conduct involving dishonesty, fraud or deceit, and no act that reflects badly on professional reputation, integrity or competence; stay competent
II Integrity of Capital Markets	II(A) Material Nonpublic Information; II(B) Market Manipulation	Don't act or cause others to act on material nonpublic information; don't distort prices or inflate volume to mislead
III Duties to Clients	III(A) Loyalty, Prudence, and Care; III(B) Fair Dealing; III(C) Suitability; III(D) Performance Presentation; III(E) Preservation of Confidentiality	Duty of loyalty, reasonable care and prudent judgment, with clients' interests ahead of the employer's and one's own; treat all clients fairly; in an advisory relationship, inquire into the client's experience, objectives and constraints before acting and reassess regularly, confirm that each investment suits the client's financial situation and written objectives, mandates and constraints, and judge suitability in the total-portfolio context; when managing to a stated mandate, act only within its objectives and constraints; fair, accurate, complete performance; keep information about current, former and prospective clients confidential unless it concerns illegal activities on the part of the client, disclosure is required by law, or the client or prospective client permits it
IV Duties to Employers	IV(A) Loyalty; IV(B) Additional Compensation Arrangements; IV(C) Responsibilities of Supervisors	Act for the employer's benefit: don't withhold skills, divulge confidential information or otherwise harm the employer; get written consent from all parties before accepting compensation or benefits that compete or could conflict with the employer's interest; make reasonable efforts to make supervisees comply

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Standard	Sub-sections	Core requirement in brief
V Investment Analysis, Recommendations, and Actions	V(A) Diligence and Reasonable Basis; V(B) Communication with Clients and Prospective Clients; V(C) Record Retention	Diligence, independence and thoroughness, with a reasonable and adequate basis; disclose the nature and cost of services, the investment process (and material changes to it promptly), and significant risks and limitations; include the important factors in client communications; separate fact from opinion; keep records supporting analysis, recommendations and client communications
VI Conflicts of Interest	VI(A) Avoid or Disclose Conflicts; VI(B) Priority of Transactions; VI(C) Referral Fees	Avoid, or fully and fairly disclose, matters that could impair independence and objectivity or interfere with duties, in prominent, plain-language disclosures; client and employer trades before personal trades; disclose referral compensation
VII Responsibilities as a CFA Institute Member or CFA Candidate	VII(A) Conduct as Participants in CFA Institute Programs; VII(B) Reference to CFA Institute, the CFA Designation, and the CFA Program	Don't compromise the reputation or integrity of CFA Institute or the CFA designation, or the integrity, validity or security of CFA Institute programs; don't misrepresent or exaggerate what membership, the charter or candidacy means

Exam convention: the curriculum's list of the Standards extends the first III(E) exception to illegal activities of a prospective client as well as of the client. Current practice: the official Standard III(E) names illegal activities on the part of the client only, so the exception is not extended to a prospective client.

Mini-cases

- *Standard I(C) Misrepresentation.* A salesperson tells a prospect that her firm has an emerging-markets desk, not knowing whether that is true. Presenting a claim as fact without knowing whether it is true is misleading. Once she learns it is false she must correct it, and staying silent after learning the truth is a further failure.
- *Standard V(C) Record Retention.* A manager recommends a trade to a client over dinner. Back at the office he should document the recommendation, its basis and the client's response. Suitability should already have been assessed before the recommendation was made, and nothing should be executed until the client gives instructions.
- *Standard III(B) Fair Dealing.* Before changing a research rating, the firm should limit the number of people who know the change is coming and shorten the time between the decision and its release. Its personnel guidelines for pre-dissemination should forbid anyone with prior knowledge from discussing or acting on the pending change, and the change should go to all interested clients at the same time.
- *Standard I(A) Knowledge of the Law.* Where local law is less demanding than the Code and Standards, the member follows the Code and Standards because they are more strict. Where local law is stricter, the member follows local law.
- *Standard III(E) Preservation of Confidentiality.* Information about a client's illegal activity is an exception to the duty of confidentiality, so disclosing it to the authorities is permitted without the client's consent.

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Common exam traps

- A misrepresentation need not be a deliberate lie. Presenting as fact a claim one knows is unverified, or leaving a statement uncorrected after learning it is false, is enough.
- Additional compensation arrangements belong to Standard IV Duties to Employers (IV(B)), even when a client offers the compensation.

EXAM SHORTCUTS

- Specific duties such as keeping records or distinguishing fact from opinion come from the Standards, so an answer that places them in the Code of Ethics can be ruled out.

BOTTOM LINE

- The Professional Conduct Program enforces the Code and Standards on the principles of fairness and confidentiality: the Disciplinary Review Committee is responsible for enforcement and the Professional Conduct staff carry out inquiries.
- An inquiry can start from self-disclosure on the annual Professional Conduct Statement, a written complaint, evidence from public sources, a report by an exam proctor, or CFA Institute's own monitoring of social media and review of exam materials.
- The Professional Conduct staff may conclude with no disciplinary sanction, a cautionary letter or a disciplinary sanction, and a member or candidate who rejects a proposed sanction is referred to a disciplinary review panel for a hearing.
- Exam convention: the sanctions listed are condemnation by the member's peers (public censure) and suspension of a candidate's continued participation in the CFA Program. Current practice: the preamble to the Code and Standards also names revocation of membership, of candidacy and of the right to use the CFA designation.
- Component 1 of the Code names four qualities, integrity, competence, diligence and respect, while complying with GIPS, advocating new laws, educating the general public, honoring contractual provisions and maximizing return per unit of risk are not components of the Code.
- Standards I to VII cover, in order: professionalism; integrity of capital markets; duties to clients; duties to employers; investment analysis, recommendations, and actions; conflicts of interest; and responsibilities as a CFA Institute member or candidate.
- Under Standard I(A), a member follows the Code and Standards where local law is less demanding and follows local law where it is stricter.
- The Code and Standards bind members of CFA Institute, including charterholders, and candidates for the CFA designation, so a member who does not yet hold the charter can still be sanctioned.

QUICK CHECK

Question 1

Moses Adeyemi, CFA, is told that the Professional Conduct staff have concluded that he violated the Code and Standards, and they propose a disciplinary sanction. If Adeyemi rejects the proposed sanction:

A he is automatically suspended from membership.

B the matter is referred to a disciplinary review panel for a hearing.

C the matter is closed unless a new inquiry is opened.

Answer: B

A member or candidate may accept or reject a sanction proposed by the Professional Conduct staff. If it is rejected, the matter is referred to a disciplinary review panel of CFA Institute members, which holds a hearing.

Why the other options are wrong

- **A.** Rejecting a proposed sanction does not trigger an automatic suspension; it leads to a hearing.
- **C.** Rejection does not end the matter; the existing case is referred to a disciplinary review panel of CFA Institute members for a hearing.

Key takeaway A rejected sanction goes to a hearing before a disciplinary review panel.

Practice questions

Question 2

Sven Adler, a CFA candidate, is making flash cards. He marks each duty "Code" if it is one of the six components of the Code of Ethics and "Standard" if it appears only in the Standards of Professional Conduct. Which of the following duties should Adler mark "Standard"? Members and candidates must:

A not engage in conduct that compromises the integrity or reputation of CFA Institute, the CFA designation, or its programs.

B place the integrity of the investment profession and the interests of clients above their own personal interests.

C promote the integrity and viability of the global capital markets for the ultimate benefit of society.

Question 3

Elena Varga, CFA, is a relationship manager at Northgate Wealth Partners. A prospective client asks whether Northgate can manage a portfolio of Indian equities. Varga has heard that her colleague Tomas Reyes might have managed Indian equities at a previous employer, and without checking she tells the prospect that Northgate has that expertise. Two days later she learns that Reyes has never managed Indian equities. She does not contact the prospect, who later opens an account with Northgate. Varga has:

A violated the Code and Standards both when she made the unverified claim and when she failed to correct it.

B violated the Code and Standards only by failing to correct her statement once she learned it was false.

C not violated the Code and Standards, because she did not intend to mislead the prospect.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/code-of-ethics/. The full bank has 34 questions on this reading.

ETHICAL AND PROFESSIONAL STANDARDS · READING 95

Knowledge of the Law

What you'll learn

LOS 95.a Apply the Code and Standards, in particular Standards I(A) Knowledge of the Law, I(B) Independence and Objectivity, I(C) Misrepresentation, I(D) Misconduct, and I(E) Competence, to situations involving professional integrity.

LOS 95.b Recommend practices and procedures (e.g., consulting counsel and dissociating, restricted lists, gift limits, background checks, continuing education) that prevent violations of the Code and Standards.

LOS 95.c Identify conduct that conforms to the Code and Standards and conduct that violates them.

MODULE 95.1

Guidance for Standards I(A), I(B), I(C), I(D), and I(E)

This reading covers Standard I, Professionalism: Knowledge of the Law, Independence and Objectivity, Misrepresentation, Misconduct and Competence. For each part a candidate must be able to explain what it requires, apply it to a fact pattern, and recommend practices and procedures that help prevent violations.

Standard I **Professionalism** has five parts: Standard I(A) **Knowledge of the Law**, Standard I(B) **Independence and Objectivity**, Standard I(C) **Misrepresentation**, Standard I(D) **Misconduct**, and Standard I(E) **Competence**. Questions on Standard I often bring in other Standards too, such as a trustee who trades on a tip or an adviser who never updates client data. Those related Standards are summarized in a table at the end.

LOS 95.a / 95.c — Standard I(A) Knowledge of the Law

KEY CONCEPT

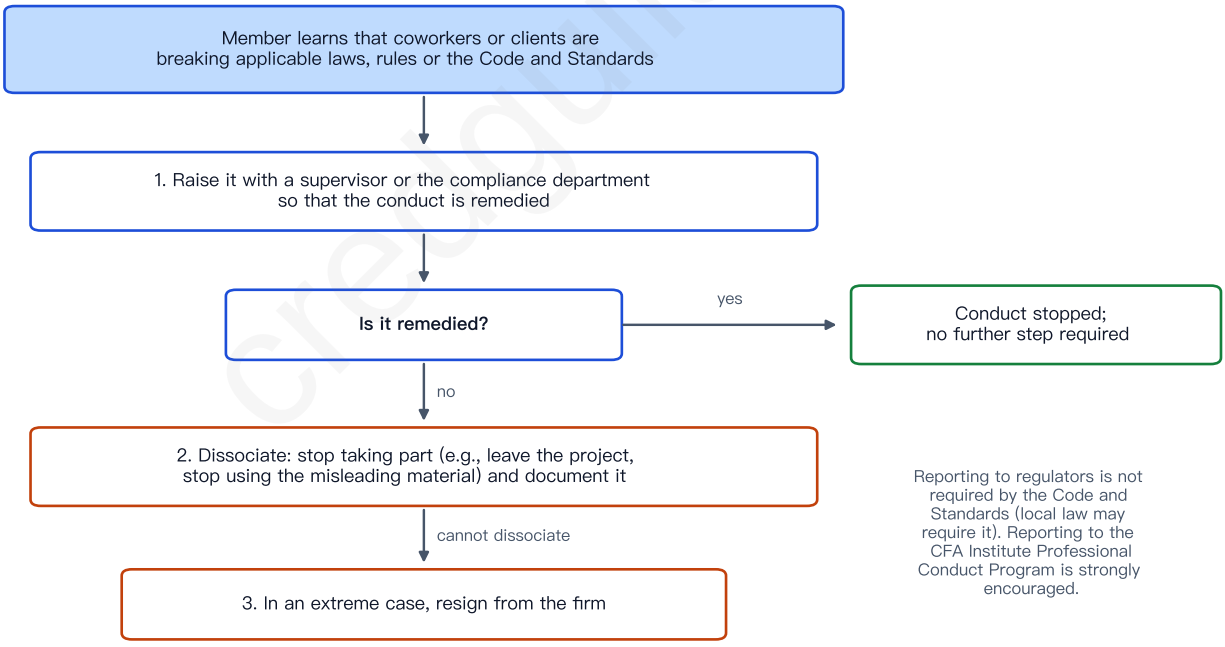
Members and candidates must understand and comply with all applicable laws, rules, and regulations (including the CFA Institute Code of Ethics and Standards of Professional Conduct) of any government, regulatory organization, licensing agency, or professional association governing their professional activities. In the event of conflict, Members and Candidates must comply with the more strict law, rule, or regulation. Members and Candidates must not knowingly participate or assist in and must dissociate from any violation of such laws, rules, or regulations.

Guidance

- *Strictest rule wins.* The Code and Standards set a minimum level of professional conduct. A member compares the laws that apply to the activity (those of the country where he or she resides and of the country where the business is done, to the extent each governs it) with the **Code and Standards**, and follows whichever is strictest on each point. Where applicable law is weaker or silent, the Code and Standards govern, so a country with no securities law relaxes nothing.
- A member cannot use a permissive local rule to do something the Code and Standards forbid. If a market legally allows trading on **material nonpublic information**, a member still may not trade on it, either for clients or for his or her own account.
- **Knowledge of the law** means keeping current. Members need not be legal experts, but they must know the rules that govern their jobs, including new rules. Breaking an applicable rule the member did not know about is still a violation, even if a regulator's grace period allows the breach to be corrected without penalty. Relying

solely on the firm's compliance department is not enough.

- **Knowingly participating or assisting** includes looking the other way: a member who knows (or who is dealing with conduct whose illegality would be evident to anyone knowing the law) and takes no action to stop it or to dissociate can be treated as a participant. Senior management approval does not excuse a member. Passing the task to someone else, such as asking a supervisor to sign a document the member has refused to sign, is still assisting.
- *Reporting.* The Code and Standards do not require members to report wrongdoers to governmental or regulatory authorities, although local law may require it and reporting may be prudent. CFA Institute strongly encourages members to report potential violations of the Code and Standards by other members and candidates to the CFA Institute **Professional Conduct Program**.
- *Escalation path.* A member who learns that coworkers or clients are breaking applicable rules first goes to a supervisor or the compliance department to have the conduct remedied. If they will not or cannot fix it, the member must **dissociate** (e.g., stop working with a trading desk that allocates client trades unfairly, stop using marketing material the member knows or should know is misleading). In an extreme case, dissociation may mean resigning from the firm.
- Because the Code and Standards are themselves among the rules a member must obey, any violation of the Code and Standards is also a violation of Standard I(A).
- Standard I(A) covers the rules that govern **professional activities**. A scuffle at a recreational sports match or a traffic offense is personal conduct. It falls outside Standard I(A) and, for the same reason, outside Standard I(D) Misconduct (see below).



Staying silent while continuing to work with the people involved can itself be treated as participating or assisting.

What a member does on learning of a violation (Standard I(A))

> *Example: which rule applies?*

Situation	Law where member resides	Law where business is done	Member must follow
Home law weaker than the Code; client country has no securities law	Weaker than Code	None	Code and Standards in both countries
Client country's disclosure rule is stricter than the Code	Weaker than Code	Stricter than Code	Client country's law for that business
Home country forbids gifts from issuers, and the ban covers its residents' business abroad; business country allows them	Stricter than Code	Weaker than Code	Home law (no gifts)
Home law is stricter than the Code but tells residents to follow local law for business done abroad; client country's law is weaker than the Code	Stricter than Code, but defers to local law abroad	Weaker than Code	Code and Standards

Recommended procedures (LOS 95.b)

- *When in doubt about legality:* consult a supervisor, compliance personnel or a lawyer (legal counsel).
- *When dissociating:* keep **records documenting the violation**, encourage the employer to bring it to an end, and remove one's name from documents one cannot stand behind.
- *Members (or their employers) should:* set up procedures that keep employees **informed of changes** in relevant laws, rules, and regulations; **review the written compliance procedures** regularly; and **maintain copies of current laws**, rules, and regulations.
- *Firms should:* have a **code of ethics**, give employees information on the laws, rules, and regulations that govern their professional activities, and have **procedures for reporting suspected violations**.

WORKED EXAMPLE: DISSOCIATING FROM A VIOLATION

A portfolio manager learns that her firm's trading desk regularly gives its best fills on block trades to a few large accounts, which breaks the firm's allocation rules and applicable regulation. She is not involved in the allocations, but her own clients' orders go through the desk. Explain what Standard I(A) requires of her and what she should do.

Step 1. Standard I(A) forbids knowingly participating or assisting in a violation of applicable laws, rules or regulations and requires dissociation from it. Looking the other way after learning of the practice could make her a participant.

Step 2. She takes the matter to her supervisor or the compliance department to have the conduct remedied.

Step 3. If they will not or cannot fix it, she must dissociate, for example by not working with the desk for her clients' trades. In an extreme case, dissociation may mean resigning from the firm.

Step 4. While dissociating, she keeps records documenting the violation and encourages the firm to bring it to an end.

Result. The Code and Standards do not require her to report the desk to a regulator, although local law may require it and reporting may be prudent; CFA Institute strongly encourages members to report potential violations by other members and candidates to the Professional Conduct Program.

LOS 95.a / 95.c — Standard I(B) Independence and Objectivity

KEY CONCEPT

Members and candidates must use reasonable care and judgment to achieve and maintain independence and objectivity in their professional activities. Members and Candidates must not offer, solicit, or accept any gift, benefit, compensation, or consideration that reasonably could be expected to compromise their own or another's independence and objectivity.

Guidance

- *Gifts and entertainment.* Modest gifts and ordinary business entertainment are acceptable. The test is objective: whether the item could reasonably be expected to compromise judgment, whatever the member believes about her own resolve. The Standard does not require token items from non-clients to be reported to the employer, although firm policy may. Distinguish the source:
  - a gift from a client usually rewards past service and is considered less likely to impair objectivity. It may be accepted, but it must be disclosed to the employer, before acceptance if possible and otherwise afterward. Standard IV(B) Additional Compensation Arrangements requires written consent when the benefit is compensation tied to future performance;
  - a gift from a party seeking to influence the member (an issuer, a broker, a fund sponsor) calls for more caution.
- *Offering benefits.* The Standard covers giving as well as receiving. A manager competing for a university endowment's mandate who treats the chair of its investment committee to a weekend at a luxury resort violates Standard I(B), even if his own firm approved the cost as a marketing expense.
- **Pressure and inducements** to give a security a particular rating, to **select certain outside managers or vendors**, or to produce favorable (or unfavorable) research all threaten objectivity. A member who lets them shape the analysis violates the Standard. Allocating shares of an **oversubscribed IPO** to one's personal account violates this Standard and Standard III(B) Fair Dealing.
- *Sell-side and investment banking pressure.* Research must reflect the analyst's own view even when the firm's investment banking department has a relationship with the issuer. Firms separate research from banking with **firewalls**. When a firm does not want to issue a negative opinion on a banking client, it places the company on a **restricted list** and publishes only factual information, instead of issuing favorable opinions it does not hold. A banker may tell a prospective corporate finance client that the firm will cover the company, but may not promise favorable research: the analysts must stay free to reach their own conclusion.
- *Buy-side and issuer pressure.* Portfolio managers who hold a stock may lean on analysts not to downgrade; companies may threaten to cut off access. Neither is a reason to change a rating.
- **Issuer-paid research** is permitted if compensation is a **flat fee** (best practice: agreed before the report is written) that is not tied to the report's conclusions, recommendations, or market impact, and if the fact that the research is issuer-paid is disclosed.

- *Credit ratings.* Ratings from recognized agencies are not automatically objective; members should consider the conflicts of interest in the rating process and may need independent research to judge a rating's soundness.
- *Using outside research.* A member may rely on third-party research (another firm's reports, a data vendor, a model licensed from outside) after a **reasonable and diligent review** of its assumptions and soundness. Nothing requires a member to use only his own or his firm's research.
- **Other pressures** include fund manager and custodial relationships, credit rating agency work, performance measurement, and the member's own holdings in a covered stock (disclose them).

Recommended procedures (LOS 95.b)

- *Members:* pay (or have the firm pay) for travel to company events when practicable; limit use of corporate aircraft to trips where commercial travel is not an alternative.
- *Firms:* **restrict employee participation in IPOs and private placements** and require **pre-approval** for participation; appoint a **compliance officer**, have **written policies on independence and objectivity**, and establish clear procedures for reporting violations; allow only **token items** as gifts from anyone other than clients.

LOS 95.a / 95.c — Standard I(C) Misrepresentation

Members and candidates must not knowingly make any misrepresentations relating to investment analysis, recommendations, actions, or other professional activities.

Guidance

- **Misrepresentation** includes knowingly misleading investors, **omitting relevant information**, **presenting selective data** to mislead, and **plagiarism**.
- Typical violations: presenting third-party research as one's own; guaranteeing a specific return on securities not explicitly guaranteed by a government body or financial institution; picking the **valuation service** that assigns the highest values to untraded holdings; choosing a **benchmark** that is not comparable to the strategy; performance or attribution data that **omit accounts** or relevant variables; false or misleading claims about the experience, expertise or capabilities of the analyst or the firm; and using misleading **third-party marketing materials**.
- *Services and qualifications.* Members must not overstate their own experience, credentials, or their firm's capabilities. Telling a client "our firm can handle all your needs" is acceptable if it is true, based on the firm's actual capabilities and the client's facts. A trainee claiming years of experience in areas she does not cover is a violation. Claiming a degree before it has been awarded (for example, by handing out business cards that show it) is a violation. Ordering cards to use once the degree is awarded is not.
- *Guarantees.* Accurately describing an instrument that really is guaranteed by a government body or financial institution (e.g., a government-backed bond) is not a misrepresentation. An opinion ("I think this stock looks attractive") is not a guarantee.
- *Keeping information current.* If a member is responsible for a firm website or marketing material, outdated or inaccurate content that clients rely on is a misrepresentation. An honest slip, such as a misplaced decimal point in a published report, is not a knowing misstatement when it is made. Once the member discovers it, she must stop its circulation and correct it promptly, because continuing to circulate it after that point would be a knowing misrepresentation.

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- **Plagiarism** means taking charts, graphs, spreadsheets, models, forecasts, reports, ideas or quotations that others created and using them without **crediting the source**. Quotations attributed to "leading analysts" without naming them, and slightly reworded excerpts from another analyst's report used without credit, are plagiarism. Exception: projections, statistics, and tables from **recognized financial and statistical reporting services** (factual data such as government bond yields or published index levels, as distinct from someone's analysis or opinion) may be used without citation. Models or research developed by colleagues at the member's own firm may be used, and the names of colleagues who have left may be omitted, as long as the member does not present that earlier work as solely his own.
- Forecasts or statistics by others used with the source named but with their original qualifications stripped out still mislead.

Recommended procedures (LOS 95.b)

- *Members:* prepare a summary of their own experience, qualifications, and the services they can perform; cite the source of any summaries of others' material; **keep copies of all reports, articles, and other materials** used in preparing research; encourage employers to **verify marketing materials provided by third parties**.
- *Firms:* provide a written list of the firm's available services and qualifications, and **periodically review** employees' documents and communications (including websites) to catch any misstatement of the qualifications and capabilities of employees or the firm.

LOS 95.a / 95.c — Standard I(D) Misconduct

Members and candidates must not engage in any professional conduct involving dishonesty, fraud, or deceit or commit any act that reflects adversely on their professional reputation, integrity, or competence.

Guidance

- Any professional conduct involving **dishonesty, fraud, or deceit** is a violation: lying under oath (perjury) in a case involving the firm, padding expense reports, falsifying a résumé, making promises the member knows cannot be kept.
- The Standard applies only to conduct in **professional activities**; acts outside a member's professional capacity are not addressed by it. Professional activities include dealings with clients, employees, and coworkers; activities as a member, charterholder, or candidate; volunteer work for a CFA society or CFA Institute; attending industry events, meetings, or courses; writing about investments (articles, reports, message boards) even outside the workplace; and communications with regulators and professional organizations.
- Misconduct can involve legal as well as illegal acts. Behavior that impairs the ability to perform professional duties, such as **excessive drinking during business hours** or coming to work intoxicated, violates the Standard. So does failing to act when professional obligations require it, such as **neglecting due diligence** on a recommendation.
- Not every illegal act is misconduct: **civil disobedience** (such as being arrested at a peaceful protest) is generally not a violation because it does not reflect on honesty, integrity, or competence. **Personal bankruptcy** is not a violation unless fraudulent or deceitful professional conduct was part of the circumstances.
- Members must not use enforcement of this Standard to settle personal, political, or other disputes unrelated to professional ethics or competence.

Recommended procedures (LOS 95.b)

Firms should adopt a **code of ethics** that states plainly that unethical behavior will not be tolerated, provide employees with a **list of potential violations and sanctions** (up to dismissal), and **check references** of potential employees (background checks on character and eligibility to work in the industry). Such checks are a recommended practice and do not breach any privacy obligation under the Code.

LOS 95.a / 95.c — Standard I(E) Competence

Members and candidates must act with and maintain the competence necessary to fulfill their professional responsibilities.

Guidance

- **Competence** means having, and keeping up, the knowledge, skills and abilities that the member's professional responsibilities require.
- What counts as competent depends on the role the member takes on.
- When a member accepts a new or unfamiliar role, it is the member's own responsibility, not the employer's, to develop the abilities, skills, and knowledge the role calls for.
- **Continuing education** is recommended. The Standard does not require it.
- A poor outcome does not by itself show a lack of competence. A recommendation that turns out badly, made with reasonable analysis, violates neither I(E) nor V(A) Diligence and Reasonable Basis.

Recommended procedures (LOS 95.b)

Ways to maintain competence include continuing education, professional development and employer-provided training; earning professional designations; going to relevant seminars and conferences; joining professional organizations; and informal self-study.

Related Standards that appear in the same fact patterns

Each of these Standards has its own module with the full guidance. The table gives only the points that the questions in this module rely on.

Standard	Key point when it appears alongside Standard I
II(A) **Material Nonpublic Information**	Do not act or cause others to act until the information is public (a magazine article out "tomorrow" is still nonpublic; news given on a conference call to analysts is not public until the announced press release). The recommended response is to make reasonable efforts to achieve **public dissemination** of the information by the company. The **mosaic theory** permits conclusions built from public information plus **nonmaterial nonpublic information** (e.g., executives' travel). The Standard applies to anyone who possesses such information, however it was obtained (overheard, read in a spouse's papers). Firms prevent misuse with **firewalls** (control of interdepartmental information), **watch or restricted lists** and review of trading.

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Standard	Key point when it appears alongside Standard I
III(A) **Loyalty, Prudence, and Care**	Duty is to the ultimate beneficiaries: for a pension plan, the **plan participants and beneficiaries**, not the sponsor or the company paying the adviser. **Proxy voting** must be informed and in clients' interests (not rubber-stamped with management, and not passed to a sponsor when the manager has been given voting authority). **Soft dollars**: client brokerage must be used to benefit the client, so it may pay for research that benefits the client but never for the manager's own operating expenses. Governance and soft-dollar issues fall under this Standard. Because of cost–benefit considerations, it may not be necessary to vote every proxy.
III(B) **Fair Dealing**	Do not use hot-issue allocations to reward, placate or retain a client; offer a new issue to every client for whom it is suitable and **prorate** it if it is oversubscribed. Disseminate recommendation changes through a system designed to treat all clients **fairly** (fair, not necessarily simultaneous); never tip selected colleagues, analysts or journalists first. An issuer should release material news by **press release**, not to favored analysts. A client's order that goes against the current recommendation may be executed after the client is told of the recommendation. A family member's fee-paying account in which the member has no beneficial interest is treated like any other client account, neither favored nor excluded, including in the order of trades under VI(B).
III(C) **Suitability**	Reasonable inquiry into the client's investment experience, objectives (risk *and* return) and constraints before acting, **reassessed and updated regularly** (not only when something changes). A process that looks only at the probability of reaching a return target ignores risk tolerance. Suitability is judged for the **portfolio as a whole**, not security by security; update the policy statement when circumstances change (e.g., the client sells the concentrated holding that a hedge was built around).
III(D) **Performance Presentation**	Performance must be fair, accurate and complete: presenting one account's results as representative of all accounts is a violation. Recommended practice: present a weighted composite of similar portfolios, include terminated accounts and state when they were terminated, and disclose whether results are gross or net of fees.
III(E) **Preservation of Confidentiality**	Keep information about current, former and prospective clients confidential; for example, never pass client names to a charity, however worthy. Cooperating with an investigation by the CFA Institute **Professional Conduct Program** is permitted.
IV(A) **Loyalty**	Employer's models and records are employer property; copying them without permission is a violation, even if the employer no longer uses them. Honor legal and confidentiality agreements with former employers. While still employed, a member may prepare to compete (register a firm, sign a lease) but must not solicit clients or take records; after leaving, and absent a non-compete agreement, former clients may be contacted using memory and public sources. Giving the employer a copy of the Code and Standards is recommended, not required.
IV(B) **Additional Compensation Arrangements**	Written consent from all parties involved before accepting compensation, such as board fees and options, that competes with or could create a conflict with the employer's interest.

Standard	Key point when it appears alongside Standard I
IV(C) **Responsibilities of Supervisors**	A supervisor needs adequate compliance procedures; if the firm will not adopt them, the member should **decline in writing** to accept supervisory responsibility until it does. Missing procedures (e.g., none for allocating block trades) are a supervisory failure.
V(A) **Diligence and Reasonable Basis**	Recommendations need a reasonable and adequate basis; extrapolating a manager's offhand conjecture into a published claim fails this test, and so does changing a rating on a colleague's unfinished notes. A thoroughly researched quantitative model that the manager understands is a reasonable basis; applying a model one does not understand is a diligence failure. A poor outcome, or a later restatement by the issuer, does not by itself show a lack of diligence.
V(B) **Communication with Clients and Prospective Clients**	Disclose the basic format and general principles of the investment process and **promptly disclose changes** that could materially affect it (e.g., a new valuation method, or managers overriding a quantitative model with their own research). Routine refinements of a model need not be disclosed one by one. Distinguish fact from opinion.
VI(A) **Avoid or Disclose Conflicts**	Disclose board seats, stock options and similar interests to the employer and clients. The firm's **market-making** activity in a security and board seats held by the firm's principals must be disclosed in research and to clients. Personal matters such as a bankruptcy need disclosure only if they create a conflict.
VI(B) **Priority of Transactions**	Client and employer trades come before trades in which the member has beneficial ownership. Firms may require duplicate confirmations and pre-clearance; they need not collect the holdings of employees' friends. Order: clients first, then the employer's own account, then the member's personal account. A family member's fee-paying client account ranks with other clients (see III(B)). The member may own the same securities as clients, as long as client trades come first.
VI(C) **Referral Fees**	Disclose referral fees to the employer, clients and prospects; where local law bans them outright, the stricter rule (the ban) governs.
VII(B) **Reference to CFA Institute, the CFA Designation, and the CFA Program**	Members must sign the Professional Conduct Statement and pay dues each year; a member who does not is no longer an active member, so presenting the CFA designation as current would misrepresent membership. A true statement, such as having completed the CFA Program in three years, is acceptable; claiming superior ability because of it is not. No form of the designation (e.g., "CFA in waiting") before the charter is awarded — state facts such as the levels passed. Refer to someone as a CFA candidate only while he or she is registered for a CFA exam or awaiting its results. Statements that the charter reflects commitment to ethics are fine; implying superior performance is not.

Common exam traps across the reading

- "The Code and Standards apply only where the law is silent" is wrong.

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- Most items turn on a single fact in the stem, such as commercial travel being unavailable, information already being public, or a statement being based on fact.

BOTTOM LINE

- Under Standard I(A), a member compares the applicable laws with the Code and Standards and follows whichever is strictest on each point, so where applicable law is weaker or silent the Code and Standards govern.
- A member who learns that coworkers or clients are breaking applicable rules goes to a supervisor or the compliance department to have the conduct remedied and, if it is not remedied, must dissociate, which in an extreme case may mean resigning.
- The Code and Standards do not require a member to report wrongdoers to governmental or regulatory authorities, although local law may require it.
- Under Standard I(B), modest gifts and ordinary business entertainment are acceptable, the test being whether an item could reasonably be expected to compromise independence and objectivity, and a gift from a client may be accepted but must be disclosed to the employer.
- Issuer-paid research is permitted if the compensation is a flat fee not tied to the report's conclusions, recommendations or market impact and the issuer payment is disclosed.
- Under Standard I(C), misrepresentation covers plagiarism, selective data presented in order to mislead, the omission of relevant information and any knowing attempt to mislead investors, but factual data from recognized financial and statistical reporting services may be used without citation.
- Standard I(D) covers only professional conduct: dishonesty, fraud or deceit violates it, while civil disobedience is generally not a violation and personal bankruptcy is not one unless fraudulent or deceitful professional conduct was involved.
- Under Standard I(E), the member, not the employer, is responsible for developing the competence a new role requires, and continuing education is recommended but not required.

QUICK CHECK

Question 1

After 18 years with a large Toronto asset manager, Lionel Batiste, CFA, sets up his own advisory firm in Port Aurelia, a jurisdiction whose securities rules are far weaker than both Canadian law and the CFA Institute Standards of Professional Conduct. Most of his clients are Canadian residents who moved their accounts to the new firm, and Canadian law continues to apply to his dealings with them. Batiste finds that Port Aurelia rules would let him skip several client disclosures that he always had to make in Canada. According to the Code and Standards, Batiste must follow:

A Port Aurelia rules, provided he tells his Canadian clients where they differ from Canadian law.

B the Code and Standards in every case, because as a charterholder he answers only to them.

C whichever is stricter of the Code and Standards and Canadian law.

Answer: C

Under Standard I(A) Knowledge of the Law, when applicable laws and the Code and Standards differ, members must follow the strictest requirement: the law where they live, the law where they do business (including the law governing their clients' accounts), or the Code and Standards. Because Canadian law still governs his dealings with Canadian clients, Batiste must meet the stricter of Canadian law and the Code and Standards.

Why the other options are wrong

- **A.** Moving to a less strict jurisdiction does not lower the standard. Disclosing the differences to clients does not allow a member to follow a weaker rule.
- **B.** The Code and Standards are a minimum, not the only rule. If applicable law (here, Canadian law) is stricter on a point, the member must comply with that law as well.

Key takeaway When several rule sets apply, follow the strictest one. Moving offshore never lowers the bar.

Practice questions

Question 2

Irene Vautrin, CFA, has managed domestic large-cap equity portfolios for ten years. Her firm asks her to take over its frontier-market bond fund next month, an asset class she has never analyzed. Which response is *most consistent* with Standard I(E) Competence?

A Take responsibility for building the knowledge and skills the new role requires.

B Accept the role and rely on the firm to provide any training it considers necessary.

C Decline the role, because the Standard does not allow members to change asset classes.

Question 3

Hana Petrov runs an independent advisory practice with 140 individual clients. When she takes on a new client, she collects the information she needs to make recommendations, such as the client's tax situation, income, spending needs, and willingness to take risk. With respect to this client information, the Standards require Petrov to:

A update it only when she makes a material change to the client's portfolio.

B reassess and update it on a regular basis.

C update it only after learning of a material change in the client's circumstances.

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Question 4

While looking for a file on her firm's shared drive, Nadia Petrova, an equity analyst, opens a colleague's half-finished draft notes on a retailer that Petrova covers. After reading them, and without doing any further work, Petrova immediately changes her own recommendation on the retailer from sell to buy. Which Standard has Petrova violated?

A Standard V(A) Diligence and Reasonable Basis.

B Standard I(B) Independence and Objectivity.

C Standard III(A) Loyalty, Prudence, and Care.

Question 5

Marguerite Olsen has worked as an independent financial adviser for 20 years. She was awarded the CFA charter in 2011 but has not paid this year's CFA Institute dues because she feels the designation brings her little business, so her membership has lapsed. She still has several boxes of business cards printed with "CFA" after her name, and her brochures state that she completed the CFA Program in three years, which is true. Olsen:

A must stop handing out the cards that show the CFA designation, but may keep using the existing brochures.

B must stop handing out both the cards that show the CFA designation and the existing brochures.

C may keep using the existing brochures, and may hand out the cards until they run out, provided new cards omit the designation.

Answers: 2. A · 3. B · 4. A · 5. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/knowledge-of-the-law/. The full bank has 141 questions on this reading.

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ETHICAL AND PROFESSIONAL STANDARDS · READING 96

Material Nonpublic Information

What you'll learn

LOS 96.a Apply Standard II(A) Material Nonpublic Information and Standard II(B) Market Manipulation to situations involving professional integrity.

LOS 96.b Recommend practices and procedures (public dissemination, firewalls, restricted and watch lists, trading review) that prevent violations of Standard II.

LOS 96.c Identify conduct that conforms to, and conduct that violates, Standards II(A) and II(B).

MODULE 96.1

Guidance for Standards II(A) and II(B)

This reading covers Standard II, Integrity of Capital Markets, with its two parts, Material Nonpublic Information and Market Manipulation. A candidate must be able to apply both to a fact pattern, including the tests of materiality and of public dissemination and the mosaic theory, recommend procedures such as firewalls that prevent violations, and tell conduct that conforms from conduct that violates.

Standard II protects the **integrity of capital markets**: prices should reflect information that all investors can obtain, and trading activity should be genuine. It has two parts, **Standard II(A) Material Nonpublic Information** and **Standard II(B) Market Manipulation**. The three LOS cover applying the two Standards to a fact pattern (96.a), the procedures that prevent violations (96.b), and sorting conduct into "conforms" and "violates" (96.c).

LOS 96.a — Applying Standards II(A) and II(B) to fact patterns

Standard II(A) Material Nonpublic Information

Members and candidates who possess material nonpublic information that could affect the value of an investment must not act or cause others to act on the information.

The rule exists because investors take part in markets only if they believe everyone trades on the same public information. If insiders could profit from what others cannot know, markets would look rigged, investors would commit less capital and prices would reflect information less well.

The prohibition applies only when the information passes both tests:

KEY CONCEPT

Test	Question to ask	Passes when…	Fails when…
Material	Would disclosure move the price, or would a **reasonable investor** want to know it before investing?	Earnings results or guidance, mergers and **tender offers**, big contracts won or lost, significant litigation, regulatory decisions, dividend changes, a sudden change in a widely followed analyst's recommendation	The information is vague or **ambiguous** as to its price effect (e.g., only that two people met), so it may not be material; the less reliable the source, the less likely the information is material

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Test	Question to ask	Passes when…	Fails when…
Nonpublic	Has it been disseminated to the **marketplace in general**?	Only insiders, a few analysts, or a closed group know it	It has been released broadly (press release, regulatory filing, general newswire)

Points to apply:

- **Selective disclosure** is not public disclosure. When a company releases information to a chosen few, it creates the risk that those who receive it will trade on it. Telling a group of analysts on a conference call, a roadshow audience or a small investor group leaves the information nonpublic for everyone who heard it.
- *The source does not matter.* Information overheard at a restaurant, passed on by a relative or a housekeeper, or supplied by an outside party such as a supplier or plaintiff is still covered. The Standard applies whether or not the insider breached a duty and whether or not the information was **misappropriated**. Once a member knows the information is material and nonpublic, the member may not act on it.
- **"Act or cause others to act"** covers trading for oneself, for clients, for the employer and for family members. It also covers accepting and executing an order from someone the member knows is trading on the information, and recommending or tipping.
- *Indirect trades are still trades.* The prohibition covers the security itself, **mutual funds** and exchange-traded funds that hold it, and related **swaps** and **options** contracts. Buying funds that hold the affected stock (and shorting similar funds that do not) is still acting on the information.
- *Check before acting.* The member is responsible for finding out whether information has been publicly disseminated before acting on it or causing others to act on it.
- *Information received for a deal.* Members involved in transactions such as investment banking assignments may receive material nonpublic information from the firms involved. They may use it for the purpose for which it was provided (working on the deal) and for no other purpose until it becomes public.
- *Wait for public dissemination.* The prohibition lasts until the information has been made public. Until then, trading on it, publishing a report on it and telling clients are all prohibited.
- *Possession alone is not a violation.* Analysts often receive nonpublic information by accident or in the course of their work. The violation is acting on it or causing others to act.
- *Knowing that an item is false can itself be material nonpublic information.* If only one person knows that a market-moving statement is fake, that knowledge is material and nonpublic.

The mosaic theory

Under the **mosaic theory**, an analyst may reach a conclusion by combining **public information** with **nonmaterial nonpublic information** and may act on that conclusion even if the conclusion itself would be material had the company disclosed it. Channel checks with suppliers and customers, counting inventory in a parking lot, reading trade publications and noticing where executives travel are all legitimate research. The mosaic theory never permits using an item that is itself material and nonpublic.

Which information may a member act on?

	Public	Nonpublic
Material	May act (e.g., earnings in a press release)	May not act or cause others to act until it is public
Nonmaterial	May act	May act; a usable input to a mosaic conclusion (e.g., a count of trucks at a plant)

Analysts should keep records of the research behind a mosaic conclusion.

Standard II(B) Market Manipulation

Members and candidates must not engage in practices that distort prices or artificially inflate trading volume with the intent to mislead market participants.

The Standard is broken in two ways: by spreading false information to affect prices or volume, and by making trades intended to mislead market participants.

KEY CONCEPT

Conduct	Examples
Spreading false or misleading information to affect prices or volume	False rumors or fabricated posts on social media; overstating an earnings projection to lift a price; promoting a stock one holds with false claims; misrepresenting a product or technology
Trades intended to mislead market participants	Matched buy and sell orders between related accounts to inflate volume; buying a dominant position in an asset to move the price of a related **derivative** (or the reverse)

The deciding element is the **intent to mislead**; whether the member profits does not matter. A member's trades can legitimately move prices and volume, and that alone is not a violation. The Standard is violated when the trading or the information is intended to mislead. Conduct with no such intent includes trading strategies that exploit differences in market power, information or other **market inefficiencies**, transactions made for tax purposes (e.g., selling for a tax loss while holding a synthetic long position), and **block trades** used to limit the price impact of a large order in a thinly traded security.

Example. A trader holds a large short position in a small biotech stock and then posts invented reports of a failed clinical trial. The short position alone is legitimate. The false posts violate Standard II(B), because they spread false information to move the price. The trade itself was not intended to mislead anyone, so it is not a violation.

LOS 96.b — Procedures that prevent violations

Recommendations for members and candidates who hold material nonpublic information:

- Make reasonable efforts to achieve **public dissemination** of the information by the company (for example, encourage the issuer to announce it).

- Encourage their firms to adopt procedures that prevent the misuse of material nonpublic information.

Recommendations for firms. The core control is a **firewall** (information barrier) between departments that receive inside information (investment banking, corporate finance) and sales, trading and research. Its elements:

KEY CONCEPT

Firewall element	What it does
Clearance area (compliance or legal department)	Exercises substantial control over communications between departments; any necessary contact across the wall is cleared and documented there
Review of **employee trades**	Pre-clearance and reporting of personal trades, so trading on inside information is detected
Watch, **restricted** and **rumor** lists	A watch list lets compliance quietly monitor trading in securities the firm has sensitive information about; a restricted list prohibits or limits trading in them; a rumor list tracks securities that are the subject of market rumors
Monitoring and restricting **proprietary trading**	Limits the firm's own trading while it holds material nonpublic information

A blanket ban on all proprietary trading in a security whenever the firm holds inside information may itself be inappropriate, because the sudden absence of the firm's usual trading can signal the information to the market. In such cases the firm should only take the opposite side of **unsolicited customer trades**.

Other good practice includes written procedures and training on what counts as material and nonpublic information.

LOS 96.c — Conduct that conforms vs. conduct that violates

The points under LOS 96.a already sort most conduct into the two groups. Two further contrasts:

Conforms	Violates
Declining an order from an insider who reveals an unannounced takeover	Executing that order because it is "for the insider's children"
Exploiting genuine pricing inefficiencies	Inventing a warning to move futures prices and then trading against it

COMMON EXAM TRAPS

- Standard IV(A) Loyalty concerns duties to the employer (independent practice, leaving an employer, whistleblowing). Trading on an inside tip because a supervisor ordered it violates II(A); it is not a loyalty violation.

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EXAM SHORTCUTS

- Under Standard II(B), whether the member profited does not matter; the deciding element is the intent to mislead, so a trade that merely moves the price is not a violation without it.

BOTTOM LINE

- Standard II(A) applies only when information is both material, meaning disclosure would move the price or a reasonable investor would want to know it, and nonpublic, meaning it has not been disseminated to the marketplace in general.
- Selective disclosure, such as telling a group of analysts on a conference call or a roadshow audience, leaves the information nonpublic for everyone who heard it.
- Acting or causing others to act on material nonpublic information includes trading for oneself, clients, the employer or family members, executing an order from someone known to be trading on it, recommending or tipping, and trading related funds, swaps or options, while possession alone is not a violation.
- Under the mosaic theory, an analyst may act on a conclusion drawn from public information and nonmaterial nonpublic information even if the conclusion would be material had the company disclosed it, but never on an item that is itself material and nonpublic.
- Standard II(B) is violated by spreading false or misleading information to affect prices or volume, or by trades intended to mislead market participants.
- Strategies that exploit market inefficiencies, transactions made for tax purposes and block trades used to limit price impact are not manipulation when they carry no intent to mislead.
- A member who holds material nonpublic information should make reasonable efforts to achieve public dissemination of it by the company.
- The firm's core control is a firewall, with a clearance area, review of employee trades, watch, restricted and rumor lists, and monitoring and restriction of proprietary trading.

QUICK CHECK

Question 1

Adaeze Okafor, CFA, is a broker. The chief executive of Stellan Pharmaceuticals, a listed company, telephones her and asks her to buy Stellan shares for trust accounts he set up for his grandchildren, remarking that the purchase will look very smart "once the merger we're negotiating is announced." Okafor searches news services and finds no public mention of a merger. Okafor should:

A execute the order, because Standard III(B) Fair Dealing requires her to treat all client orders alike.

B execute the order, because the shares are for the grandchildren's accounts rather than the executive's own.

C decline the order, because executing it would mean acting on material nonpublic information.

Answer: C

A pending merger is material, and Okafor has confirmed it has not been made public. Executing the trade would mean acting (and helping the insider act) on material nonpublic information, which Standard II(A) prohibits: Members and Candidates must not act or cause others to act on such information.

Why the other options are wrong

- **A.** Fair Dealing is about treating clients fairly when disseminating recommendations and executing trades; it never requires a member to execute a trade that violates Standard II(A).
- **B.** Whose account benefits is irrelevant: buying for trusts set up for family members on the strength of inside information is still acting on material nonpublic information, and no duty to a client overrides Standard II(A).

Key takeaway Once a client reveals material nonpublic information behind an order, the member may not execute it. The identity of the account holder does not change that.

Practice questions

Question 2

Colette Brenner is convinced that Lunaris Robotics is overvalued, and three weeks ago she sold short 60,000 Lunaris shares. This week she has been posting false, damaging claims about Lunaris's management on several investment message boards. Under Standard II(B) Market Manipulation, Brenner's conduct is:

A a violation through the false claims, but not through her trade.

B a violation through both her trade and the false claims.

C a violation through her trade, but not through the false claims.

Question 3

Tobias Engel, an investment analyst, has gathered several items of nonpublic information through his contacts at medical device makers. None of the items is material on its own, but by putting them together he correctly concludes that one of the companies will report much higher earnings than expected next year. According to the CFA Institute Standards of Professional Conduct, Engel:

A cannot legally trade or make recommendations based on this conclusion.

B may use the conclusion in his investment recommendations and decisions.

C may use the conclusion only after a compliance officer has approved it.

Answers: 2. A · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/material-nonpublic-information/. The full bank has 32 questions on this reading.

ETHICAL AND PROFESSIONAL STANDARDS · READING 97

Duties to Clients

What you'll learn

LOS 97.a Apply Standard III (loyalty, prudence and care; fair dealing; suitability; performance presentation; confidentiality) to situations involving professional integrity.

LOS 97.b Recommend practices and procedures that prevent violations of Standard III.

LOS 97.c Identify conduct that conforms to Standard III and conduct that violates it.

MODULE 97.1

Guidance for Standards III(A), III(B), III(C), III(D), and III(E)

This reading covers Standard III, Duties to Clients: Loyalty, Prudence, and Care; Fair Dealing; Suitability; Performance Presentation; and Preservation of Confidentiality. A candidate must be able to apply each duty to a fact pattern, recommend procedures for compliance, and tell conduct that conforms from conduct that violates.

Standard III sets out what members and candidates owe the people whose money or information they handle. Five Standards sit under it: Standard III(A) **Loyalty, Prudence, and Care**, Standard III(B) **Fair Dealing**, Standard III(C) **Suitability**, Standard III(D) **Performance Presentation**, and Standard III(E) **Preservation of Confidentiality**. The three LOS cover applying them to fact patterns (97.a), recommending compliance procedures (97.b) and separating conduct that conforms from conduct that violates (97.c).

LOS 97.a — Applying the five duties to clients

III(A) Loyalty, Prudence, and Care

Members and candidates have a duty of loyalty to their clients and must act with reasonable care and exercise prudent judgment. Members and candidates must act for the benefit of their clients and place their clients' interests before their employer's or their own interests.

The Standard does not create a **fiduciary duty** where none already exists. It does require acting in the client's best interest and recommending products that suit the client's investment objectives and risk tolerance, with the prudence, care, skill and diligence that a person acting in a similar role and familiar with such matters would use under the circumstances.

- **Governing documents and total portfolio.** Manage pooled client assets according to the governing documents (trust documents, investment management agreements), and make decisions in the context of the **total portfolio**.
- **Disclose limitations.** Tell clients about any limits on the advisory relationship (e.g., an adviser who may recommend only her own firm's products).
- **Identify the client.** For a pension plan, the duty runs to the **plan participants and beneficiaries**. The sponsoring company, its management and its shareholders are not the client. In some roles the "client" is the **investing public as a whole** rather than a specific person. For a pooled fund, decisions serve the fund's investors as a whole, in line with the fund's mandate, rather than each investor's individual needs.
- **Diversification.** A prudent manager of retirement assets keeps the portfolio properly diversified. Holding some of the sponsor's own stock is not banned; concentrating the plan in it at management's request is imprudent.

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- **Client brokerage** (the commissions generated by client trades, also called **soft dollars** or **soft commissions**) is an **asset of the client**, because the client's trades pay for it. It must be used to benefit the client, e.g., to buy research that assists the manager in managing client accounts, and the arrangement should be disclosed to the client. Using it for research relevant only to the manager's own holdings, or for personal benefits the member could not otherwise obtain, breaches the duty.
- **Best execution**: seek the most favorable terms reasonably available for each trade. A trader who only executes client orders may owe no fiduciary duty, but the Standard still applies: in that role, loyalty and care mean seeking best execution on each order.
- Trading a client's account more often than its objectives require, in order to generate commissions for the member or the firm, puts their interests ahead of the client's and violates the Standard.
- **Directed brokerage**: when a *client* instructs that its trades go to a particular broker, following the instruction is acceptable because the brokerage belongs to the client. The member should disclose that best execution may not be achieved.
- **Proxy voting**: proxies have economic value, so they must be voted in an informed, responsible way in the clients' interest. Management's wishes do not decide the vote. A **cost-benefit analysis** may show that voting a particular proxy is not worthwhile, so voting every proxy is not required.
- **Trade errors**: a broker that absorbs the cost of its own trading error, with no implied promise of future business, raises no issue.

III(B) Fair Dealing

*Members and candidates must deal **fairly and objectively** with all clients when providing investment analysis, making investment recommendations, taking investment action, or engaging in other professional activities.*

Fair treatment is not the same as equal treatment. Firms may offer different **levels of service** (e.g., a premium research tier for a fee) as long as the tiers are disclosed to all clients and prospects, are available to anyone willing to pay for them, and do not disadvantage any client. Differences in when e-mails or other messages happen to arrive are a normal part of business.

- Do not discriminate among clients when disseminating recommendations. Delivery channels may differ: e-mail to clients who supplied an address and same-day post to the rest is fair. The duty is owed to clients, so a firm may keep its research for its clients and need not release it to the public.
- Give every client a fair opportunity to act on each recommendation. A client unaware of a changed recommendation should be told of the change before an order for that security is accepted.
- Treat individual and institutional clients fairly and impartially. A member must not exploit his or her position in the industry at clients' expense (e.g., by taking shares of an oversubscribed IPO for personal accounts).
- Clients may not be ranked by account size, fees or trading volume for the order in which they hear of recommendation changes or receive fills.
- **Trade allocation**: when an order is only partly filled or filled in blocks at different prices, allocate **pro rata** (in proportion to each account's order size) across all accounts for which the trade is suitable, at the same average price. Allocation by alphabet, fee level or other arbitrary rule is unfair. So is waiting to see which fills have gained before deciding which accounts get them.
- Tipping portfolio managers of discretionary accounts before the report reaches other clients favors some clients over others. Acting on the tip is also a violation.

Example. A manager's block order for a stock fills in three lots of equal size at $41.10, $41.30 and $41.50. Fair Dealing requires every participating account to receive its pro rata share of all three lots at the average price of $41.30. Giving the cheapest lot to favored clients would be a violation.

Example. A downgrade goes to every client at the same time. That afternoon the analyst who wrote it walks the firm's largest institutional clients through the report by phone, a service they pay for under the firm's disclosed premium tier. This is fair dealing, because everyone received the recommendation first. If she had told those clients that her private view was less negative than the published one, she would violate Standard III(B): they would be acting on a recommendation that other clients never received.

III(C) Suitability

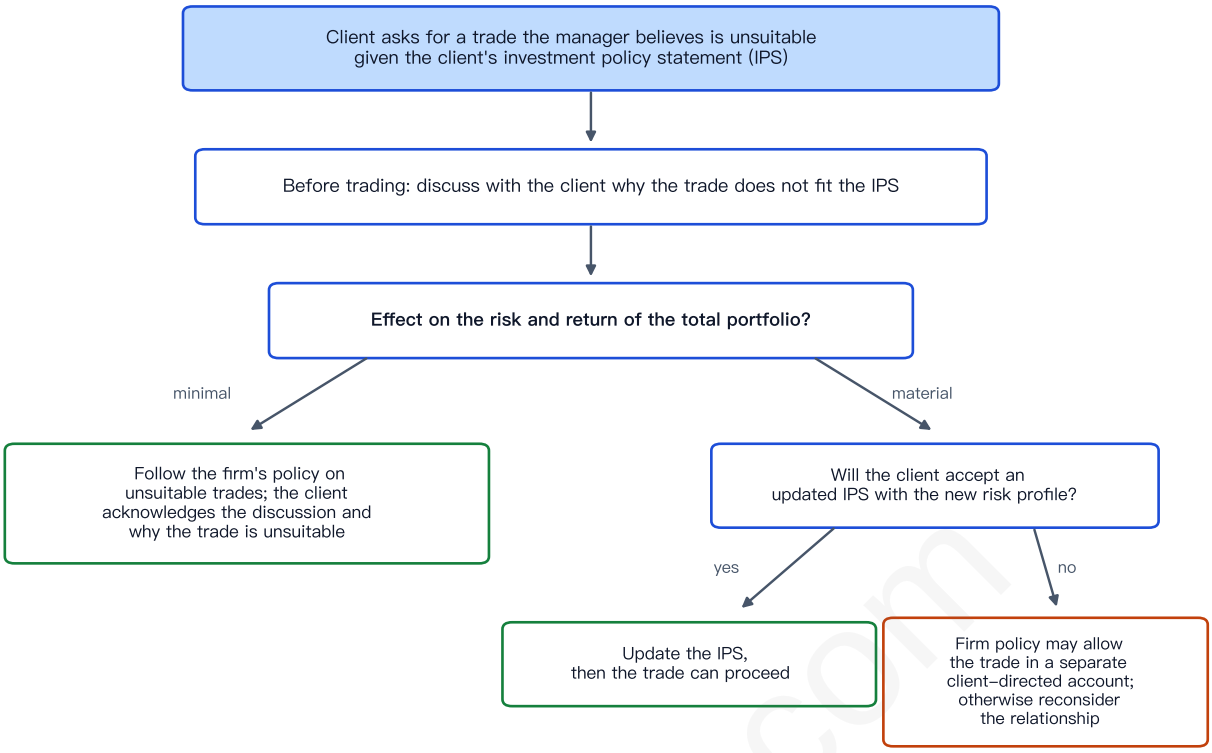
KEY CONCEPT

*1. When members and candidates are in an **advisory relationship** with a client, they must: a) Make a **reasonable inquiry** into a client's or prospective client's investment experience, **risk and return objectives**, and **financial constraints** prior to making any investment recommendation or taking investment action and must reassess and update this information regularly. b) Determine that an investment is suitable to the client's financial situation and consistent with the client's written objectives, **mandates**, and constraints before making an investment recommendation or taking investment action. c) Judge the suitability of investments in the context of the client's **total portfolio**. 2. When members and candidates are responsible for managing a portfolio to a specific **mandate, strategy, or style**, they must make only investment recommendations or take only investment actions that are consistent with the stated objectives and constraints of the portfolio.*

- The financial constraints cover liquidity needs, expected cash flows, time horizon, tax, and legal and regulatory circumstances. The information is usually documented in an **investment policy statement** (IPS) gathered at the start of the relationship, and it is reassessed regularly and whenever circumstances change.
- Consider whether the use of **leverage** is suitable for the client.
- Because suitability is judged for the portfolio as a whole, a volatile holding can suit a moderate-risk client if it lowers the risk of the total portfolio.

Under part 2, a fund whose mandate admits only dividend-paying stocks may not buy a non-dividend growth stock, however attractive. The manager of such a fund judges investments against the fund's mandate. Whether the fund suits an individual investor is the concern of the adviser who recommends it.

Unsolicited trade requests that look unsuitable: the member does not execute the trade until he or she has discussed with the client why it does not fit the IPS. If the effect on the total portfolio's risk/return profile is minimal, the member may then follow the firm's policy on unsuitable trades; the client must acknowledge the discussion and why the trade is unsuitable. If the effect is material, the client may agree to revise the IPS and accept the changed risk profile. If the client will not, firm policy may allow the trade in a separate **client-directed account**. Failing other options, the member may need to reconsider the relationship.



Handling an unsolicited trade request that looks unsuitable (Standard III(C))

III(D) Performance Presentation

*When communicating investment performance information, members and candidates must make reasonable efforts to ensure that it is **fair, accurate, and complete**.*

- The Standard covers oral as well as written performance communications.
- The same facts often breach Standard I(C) Misrepresentation too, which counts selective performance data and results that omit accounts among its violations. Standard III(D) is the Standard aimed specifically at performance communications.
- Do not present a new fund's "history" built from the past returns of assets it now holds, or from other accounts, as if the fund itself had earned it.
- Do not cherry-pick. A list of winning picks misrepresents results unless it is the complete list, however accurate each return on it is.
- A firm's stated rules on which accounts enter a composite, and from what date, apply to every account alike; bringing an account in early or leaving one out because of its results misstates performance.
- Adhering to the Global Investment Performance Standards (GIPS) is voluntary, but a claim of GIPS compliance that the firm's calculations do not support misleads clients and prospects about its performance and violates the Standard.
- Do not turn a single fund's or single period's result into a claim about the firm's typical results. Never guarantee future returns, and never suggest, directly or by implication, that past returns can be repeated.
- Stating accurate past results is acceptable. Brief presentations are allowed if **detailed information is available on request** and the presentation indicates that it offers only limited information.

III(E) Preservation of Confidentiality

*Members and candidates must keep information about **current, former, and prospective clients** confidential unless: 1. The information concerns illegal activities on the part of the client, 2. Disclosure is required by law, or 3. The client or prospective client permits disclosure of the information.*

Information may be shared with the CFA Institute **Professional Conduct Program** in an investigation, because the program keeps it confidential. No family tie or private agreement lets a member withhold it. Inside the firm, share client information only with colleagues who work on the client's account. Illegal activity by a client may create an obligation to report it to the authorities.

When unsure whether an exception applies, the member consults compliance or legal counsel. Client data must also be protected: storing identifiable client records on unsecured personal devices or networks is a failure to preserve confidentiality. When a client reveals her own information in an online group or chat the member runs, the disclosure is the client's, not the member's; the member's duty is to take reasonable precautions, such as warning users that posts are visible to all and removing such a post promptly.

LOS 97.b — Recommended procedures for compliance

Standard	Recommended practices
III(A)	Send clients, at least quarterly, **itemized statements** of all securities in custody and all debits, credits and transactions; encourage firm policies that: follow applicable rules and laws, establish each client's investment objectives, consider suitability, diversify, deal fairly with all clients, disclose conflicts and compensation arrangements, vote proxies in the best interest of clients and ultimate beneficiaries, maintain confidentiality, seek best execution
III(B)	Encourage the firm to set compliance procedures for proper dissemination of recommendations and fair treatment of all clients; limit the number of people who know a recommendation is about to be released; shorten the time between the decision and its dissemination; publish guidelines that bar staff who know of a pending recommendation from discussing it or acting on it before release; send new or changed recommendations at the same time to every client who has expressed interest in the security or for whom it is suitable (and, where possible, at the same time inside and outside the firm); keep a list of clients and their holdings; develop and disclose written **trade allocation** procedures (pro rata, same price); establish systematic account review; disclose available levels of service
III(C)	Written IPS for each client covering type of client and any separate beneficiaries, return and risk objectives, constraints (liquidity, cash flows, time, tax, legal and regulatory) and performance benchmarks; review objectives and constraints periodically
III(D)	Encourage the firm to adhere to the **Global Investment Performance Standards (GIPS)**; consider the sophistication of the audience; present a weighted composite of similar portfolios rather than a single account; include terminated accounts (and state when they ended); disclose gross/net of fees and any model results; keep the data and records behind the figures
III(E)	Share client information only with authorized colleagues who also work for that client; follow firm procedures for storing electronic data, and recommend such procedures if the firm has none

LOS 97.c — Violation or not? Quick reference

KEY CONCEPT

Situation	Conclusion
Soft dollars buy research that helps manage client accounts	Conforms (disclose)
Soft dollars buy research relevant only to the manager's personal holdings	Violates III(A)
Client directs its brokerage to a named broker	Conforms
Manager skips a proxy vote after a cost-benefit analysis	Conforms
Clients told of a recommendation in order of account size, with signed consent	Violates III(B)
Pro rata allocation across all suitable accounts	Conforms
Premium service tier, disclosed and open to all	Conforms
New ideas pitched to a prospect before existing clients get them, with no fact-finding	Violates III(B) and III(C)
No client update for several years despite regular mailings	Violates III(C)
New fund advertises returns of its current holdings as "historical"	Violates III(D)
Confidential data shared with the Professional Conduct Program	Conforms
Client's plans mentioned to a colleague not working on the account	Violates III(E)

COMMON EXAM TRAPS

- Client **consent** or **disclosure** does not make an unfair allocation or dissemination practice acceptable.
- Suitability requires *active* updating. A newsletter asking clients to call in is not an update.
- Pro rata allocation is in proportion to each account's order size, never to the size of the account.
- Relabeling a client's objectives (say, from conservative to growth) so that a favored product fits is not an IPS update. The IPS is revised with the client when the client's circumstances or wishes change, and before any trade that relies on the new profile.

EXAM SHORTCUTS

- Client consent or disclosure does not make an unfair allocation or dissemination practice acceptable, so an answer that relies on consent to justify one can be ruled out.

BOTTOM LINE

- Standard III(A) requires loyalty to clients, reasonable care and prudent judgment, with clients' interests placed before the employer's and the member's own, but it does not create a fiduciary duty where none already exists.
- For a pension plan, the duty of loyalty runs to the plan participants and beneficiaries, not to the sponsoring company, its management or its shareholders.

- Client brokerage (soft dollars) is an asset of the client and must be used for the client's benefit, such as research that assists in managing client accounts, while a client's instruction to direct its trades to a named broker may be followed with disclosure that best execution may not be achieved.
- Proxies must be voted in an informed and responsible way in clients' interests, although a cost-benefit analysis may show that voting a particular proxy is not worthwhile.
- Fair dealing is not equal treatment: disclosed service tiers open to anyone willing to pay are allowed, but a partly filled order is allocated pro rata to each account's order size, across all accounts for which the trade is suitable, at the same average price.
- In an advisory relationship, Standard III(C) requires a reasonable inquiry into the client's experience, objectives and constraints before acting, regular reassessment, and a judgment of suitability in the context of the total portfolio; a manager running a portfolio to a mandate acts only within its stated objectives and constraints.
- Standard III(D) requires reasonable efforts to make performance information fair, accurate and complete, and a brief presentation is allowed if detailed information is available on request and the presentation says it gives only limited information.
- Under Standard III(E), confidential information about current, former and prospective clients may be disclosed only if it concerns illegal activities by the client, if the law requires disclosure, or if the client or prospective client permits it, and giving it to the Professional Conduct Program in an investigation is permitted.

QUICK CHECK

Question 1

Lucia Ferraz, CFA, manages the Tessaro Emerging Markets Equity Fund and routes the fund's trades to Blakemore Securities, which gives her soft dollar credits for buying research. In her own personal account, Ferraz holds several high-yield corporate bonds; the fund holds no bonds. She spends some of the soft dollars on reports about emerging-market companies and some on a credit newsletter that covers the high-yield bonds she owns personally. Her use of the soft dollars is:

A a violation for both the equity research and the credit newsletter.

B not a violation of the Code and Standards.

C a violation for the credit newsletter but not for the equity research.

Answer: C

Soft dollars come from client brokerage, which is an asset of the client. Under Standard III(A) Loyalty, Prudence, and Care, they must be used to benefit the client, for example by paying for research that assists in managing client accounts. Emerging-market equity research helps manage the fund and is acceptable; a credit newsletter on bonds that only Ferraz owns benefits her personally and is a violation.

Why the other options are wrong

- **A.** Research on emerging-market companies is directly relevant to the fund's holdings, so buying it with the fund's soft dollars is appropriate.
- **B.** The credit newsletter serves only Ferraz's personal bond holdings; paying for it with client brokerage uses client assets for her own benefit.

Key takeaway Ask whether the soft-dollar purchase benefits the client. Research that helps manage client accounts does; research on the manager's personal holdings does not.

Practice questions

Question 2

Dmitri Olsen, CFA, finishes a research report with a buy recommendation on Harrow Grain Logistics. Some weeks earlier he asked all of his clients for their e-mail addresses. At 11 a.m. he e-mails the report to the clients who supplied an address, and that afternoon he posts printed copies to all clients, who receive them two to four days later. Olsen has:

A violated the Standards because only some clients received the e-mail.

B not violated the Standards, because he disseminated the report fairly.

C violated the Standards because the e-mail went out before the printed report arrived.

Question 3

Gloria Ansel, a retired bus driver who lives on the income from a small annuity, asks Viktor Lindqvist, CFA, to buy 8,000 shares of Quarrystone Genetics, a newly formed biotechnology start-up. The purchase would materially change the risk of her portfolio, and Lindqvist believes it is unsuitable given her investment policy statement. Consistent with the Standards, Lindqvist should *most appropriately*:

A refuse the order because the investment is unsuitable for her.

B follow his firm's procedure for obtaining her approval of an unsolicited trade.

C talk with her about whether she wants to revise her investment policy statement.

Question 4

Carden Asset Management launches a new emerging-market debt fund. It correctly calculates the returns over the past four years of each bond now in the fund and, using the fund's current weights, arrives at an average annual return of 14%. Carden's launch advertising describes the fund as having produced 14% a year historically. Under the Standards, Carden's advertising is:

A in compliance with the Standards.

B a violation, because the advertisement implies that the firm earned this return.

C a violation, because the Standards prohibit advertising historical returns on high-risk assets.

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Question 5

Tobias Ferrante, CFA, has gathered evidence suggesting that one of his clients is likely laundering money. According to Standard III(E) Preservation of Confidentiality, Ferrante is prohibited from which of the following actions?

A Report his conclusion to CFA Institute.

B Report his conclusion to the appropriate government authorities.

C Neither; both actions are permissible.

Answers: 2. B · 3. C · 4. B · 5. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/duties-to-clients/. The full bank has 57 questions on this reading.

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ETHICAL AND PROFESSIONAL STANDARDS · READING 98

Duties to Employers

What you'll learn

LOS 98.a Apply Standard IV(A) Loyalty, IV(B) Additional Compensation Arrangements and IV(C) Responsibilities of Supervisors to situations involving professional integrity.

LOS 98.b Recommend practices and procedures that help members, employers and supervisors prevent violations of Standard IV.

LOS 98.c Identify conduct that conforms to Standard IV and conduct that violates it.

MODULE 98.1

Guidance for Standards IV(A), IV(B), and IV(C)

This reading covers Standard IV, Duties to Employers: Loyalty, Additional Compensation Arrangements, and Responsibilities of Supervisors. A candidate must be able to apply each part to a fact pattern, recommend practices and procedures that prevent violations, and tell conduct that conforms from conduct that violates.

Overview — what Standard IV protects

Standard IV Duties to Employers has three parts. **Standard IV(A) Loyalty** protects the employer's business, clients, information and property. **Standard IV(B) Additional Compensation Arrangements** keeps outside benefits from drawing an employee's loyalty or objectivity away from the employer. **Standard IV(C) Responsibilities of Supervisors** makes anyone with authority over others answerable for trying to prevent their violations. The three LOS cover applying the Standards to a fact pattern (LOS 98.a), recommending procedures that prevent violations (LOS 98.b) and sorting conduct into "conforms" and "violates" (LOS 98.c).

Loyalty to an employer never requires a member to break the law, to harm clients or to breach the Code and Standards. Clients' interests rank above the employer's, but members should still weigh how their actions affect the firm's integrity and long-term sustainability.

LOS 98.a — Standard IV(A) Loyalty

*In matters related to their employment, members and candidates must act for the benefit of their employer and not deprive their employer of the advantage of their skills and abilities, divulge **confidential information**, or otherwise cause harm to their employer.*

Harm to the employer includes any activity that injures the firm or deprives it of profit.

Who is an employer? An **employer–employee relationship** exists whenever a person works in the service of another. It does not require a written contract, and it does not require **monetary compensation**. Services exchanged for a benefit (for example, managing a club's reserves instead of paying membership dues) create a relationship in which the member should treat the organization as an employer. A member who is an **independent contractor** rather than an employee must abide by the terms of the agreement with the hiring firm; the contract defines the duties.

Personal life. Loyalty does not mean the employer comes before family or other personal obligations. Employer and employee are expected to discuss such conflicts and balance them with work duties.

Independent practice. Doing the same kind of work outside the job, for **compensation or other benefit**, in competition with the employer is **independent practice**.

- It is not banned. The member must **notify the employer** and obtain the employer's **consent** to all of its terms before the practice begins.
- The notification should fully describe the services: the **types of services** (nature of the activities), the **expected duration** and the **compensation** to be received. It need not disclose confidential information about outside clients. The employer may set lawful terms for the practice.
- Help given with no compensation or other benefit (reviewing a relative's portfolio for free, on one's own time) is not independent practice, so no consent is required. A non-cash benefit, such as free use of a property, counts as compensation.
- A member who already runs an independent practice and then takes a job must obtain the new employer's permission to continue the practice if it competes with the employer.

Leaving an employer. Until the resignation is **effective**, the member must keep acting in the employer's interest. Conduct that can breach the Standard around a departure includes **misappropriation of trade secrets**, **misuse of confidential information**, **soliciting the employer's clients** before leaving, **self-dealing** and **misappropriation of client lists**.

KEY CONCEPT

Allowed before leaving (preparations)	Not allowed before leaving
Interviewing, signing a contract with a new employer	Soliciting the employer's current clients or prospective clients
Leasing office space, buying equipment, arranging a business loan	Telling current clients about the new firm's lower fees
Registering a new firm with regulators	Organizing a mass resignation of the employer's staff (inducing colleagues to leave with you)
Not telling the employer that one is preparing to leave	Copying client lists, models, research notes or other files

After leaving. A former employee may contact former clients, unless a valid **non-compete agreement** prevents it, provided the contact details come from memory or public sources (a directory, a website) rather than from records taken from the old firm. Soliciting former clients is not itself a violation, and it needs no permission from the new or old employer. Merely knowing who the former clients are is generally not confidential information, unless an agreement or the law makes it so.

Industry agreements. Where employers have signed an agreement that lets departing brokers take specified client information with them (for example, the US **Protocol for Broker Recruiting**), a member who stays within the terms of that agreement does not violate the Standard.

Social media. Members must follow the employer's **social media** policies. A member who plans to leave must make sure that any social media use (for example, announcing the move on a professional networking profile) complies with the employer's policy on notifying clients about departures.

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Employer records. Records created or kept for the employer—such as client lists, models, spreadsheets and research notes—are the employer's property, whatever the medium, including a home computer, tablet or mobile phone. Taking or using them after leaving requires the former employer's permission. A member's **skills, knowledge and experience** are not employer records and may be used elsewhere.

Departure procedures. A member who ignores the employer's legitimate procedures for leaving (for example, the policy to tell clients and introduce the successor) violates Standard IV(A) Loyalty.

Whistleblowing. In isolated cases, a member may act against the employer's interest to protect clients or market integrity, for example by reporting illegal or unethical conduct through an appropriate channel. This is not a breach of loyalty when the action is not for **personal gain** and complies with applicable law. It does not automatically permit disclosure of confidential client information: that information may be disclosed only when it concerns illegal client activity, disclosure is required by law, or the client permits it. Some firms are required to have **whistleblowing** procedures that allow anonymous reporting. Members should know their firm's policy and encourage the firm to adopt best practices.

Common exam traps

- Preparing to compete is permitted. Competing (soliciting clients, taking records) before the resignation takes effect is not.
- Consent is needed for competing independent practice. Preparing for it needs no consent or notice.
- Copying client records for the member's own use violates IV(A), even for a harmless purpose such as sending clients thank-you notes after leaving. Providing records to regulators as part of whistleblowing may be consistent with IV(A) only when the reporting and any disclosure comply with applicable law and the duty to preserve client confidentiality.

LOS 98.a — Standard IV(B) Additional Compensation Arrangements

*Members and candidates must not accept gifts, benefits, compensation, or consideration that competes with or might reasonably be expected to create a conflict of interest with their employer's interest unless they obtain **written consent from all parties involved**.*

The term covers more than pay: it includes anything of value a client provides, directly or indirectly, and any benefit a third party provides.

- The consent that matters most is the employer's. When a client or company offers the benefit (for example, a board seat), that party is also involved in the arrangement.
- **Written consent** includes e-mail. Telling a supervisor orally is not enough.
- Typical cases: a client promises a bonus, a holiday home or free club membership if the portfolio beats a benchmark (**performance-linked benefits**); a client offers free use of its facilities for as long as the member covers the account; a broker offers the member discounts on personal trades if the firm keeps its business (even when the firm already pays the lowest available commissions); a company pays a fee for serving on its board.
- The Standard does not require the member to refuse the benefit. It requires written consent before accepting it.
- It does not reach ordinary gifts that are unrelated to the member's services. A relative's long-standing birthday present that predates the client relationship, or taking turns paying for lunch with a client, is not an additional compensation arrangement.

- **Future versus past performance.** A client bonus that depends on how the account performs in the future is an **additional compensation arrangement**, and written consent is needed in advance. A client bonus that rewards performance already achieved is a **gift**; it must be disclosed to the employer under **Standard I(B) Independence and Objectivity**.
- A member hired **part time** should raise any arrangements that may compete with the employer when hired and respect any limits the employer sets.

LOS 98.a — Standard IV(C) Responsibilities of Supervisors

*Members and candidates must make **reasonable efforts** to ensure that anyone subject to their supervision or authority complies with applicable laws, rules, regulations, and the Code and Standards.*

The duty covers everyone under the member's supervision or authority, whether or not those people are CFA Institute members or candidates.

- Reasonable efforts cover three stages: preventing violations, detecting the ones that occur, and acting on any that are found. A supervisor who makes reasonable efforts to prevent and detect violations has not violated the Standard just because a subordinate commits one. A supervisor who makes no such effort violates the Standard even if no subordinate ever breaks a rule.
- Supervisors are expected to know the Code and Standards well and to apply them.
- Supervisory duties may be **delegated**, including to people who are not subject to the Standards, but the supervisor **retains supervisory responsibility** for the delegated duties and should instruct the delegates on how to prevent and detect violations.
- **Adequate compliance system.** An adequate **compliance system** satisfies three sets of requirements: those of the Code and Standards, those of regulators, and industry standards. A supervisor who finds the system inadequate must bring it to management's attention and recommend corrective action. If there are no compliance procedures, or the supervisor believes they are inadequate, the member must **decline in writing** to accept supervisory responsibility until the firm adopts adequate procedures. Urging management is necessary but does not replace declining. Resigning, relying on the Code alone or complaining to CFA Institute are not what the Standard asks for. The same applies when a supervisory role comes without the authority to act, for example a compliance head who may not enforce policies or see employees' communications with clients. Without that authority the member cannot make reasonable efforts to prevent and detect violations, so the role should be declined.
- **When a violation is suspected or found**, the supervisor should respond promptly and investigate thoroughly, while increasing supervision of the employee or limiting the employee's activities during the review, and involve compliance. Reporting the matter and warning the employee (a reprimand plus the employee's promise not to repeat the conduct) is not enough on its own.
- Supervisors should apply the same rigor to non-investment policies as to investment policies.

LOS 98.b — Recommended practices and procedures

Loyalty. A member is encouraged to hand the employer a copy of the Code and Standards. Keeping separate social media accounts for personal and professional use is best practice. Firms should avoid incentive and compensation systems that encourage unethical behavior.

Additional compensation. The member should make an immediate **written report** to the employer describing the benefit, including the terms of any oral or written agreement, the nature of the compensation, its amount and its duration. The Standard itself requires only the written consent; describing the nature, amount and duration is

a recommended procedure. Firms should have the offering party confirm the details of the additional compensation, including any performance incentives.

Supervisors. A member should recommend that the employer adopt a **code of ethics** and encourage the employer to give it to clients. The code of ethics should be kept separate from the detailed compliance procedures, because mixing the two can dilute the ethical message.

KEY CONCEPT

Adequate **compliance procedures** should:

- be clearly written and easy to understand;
- designate a **compliance officer** whose authority is clearly defined;
- include a system of checks and balances;
- define the scope of the procedures and the conduct they allow;
- explain how violations are reported and what sanctions apply;
- structure incentives so that unethical behavior is not rewarded (for example, not paying supervisors on the firm's total trading volume).

Once the program exists, the supervisor should distribute it to the relevant staff, update it as needed, keep educating staff and issue reminders, require **professional conduct evaluations** (for example, as part of each performance review) and review employee actions to monitor compliance and spot violations. The Standard does not require formal hearings after a violation.

Example. Daria Vint, CFA, becomes head of credit research at a firm with no written compliance procedures and no professional conduct evaluations. She asks management to adopt clearly written procedures that name a compliance officer and set out how violations are reported. Management refuses. She should put in writing that she declines supervisory responsibility until the firm adopts adequate procedures, while continuing to press for the procedures and documenting her requests.

LOS 98.c — Quick guide: conforms or violates?

KEY CONCEPT

Conduct	Conforms / Violates
Leasing office space for a future firm before resigning	Conforms
Offering current clients a fee discount to follow the member to a new firm before leaving	Violates IV(A)
Contacting former clients after leaving, using public directories	Conforms
Using research notes written for the former employer, without permission	Violates IV(A)
Paid evening consulting after getting the employer's written permission	Conforms
Accepting a client's performance bonus after only oral disclosure	Violates IV(B)

Conduct	Conforms / Violates
Board fee accepted after e-mail approval from compliance	Conforms
Delegating supervision and assuming the responsibility has passed	Violates IV(C)
Supervisor who restricts a suspect trader and refers the matter to compliance	Conforms
Supervisor who only warns an employee after finding a violation	Violates IV(C)
Announcing a move to a new firm on social media against the employer's client-notification policy	Violates IV(A)

EXAM SHORTCUTS

- Ask when a client bonus is earned: one that depends on future performance is an additional compensation arrangement needing written consent in advance under Standard IV(B), while one that rewards performance already achieved is a gift to be disclosed under Standard I(B).

BOTTOM LINE

- Under Standard IV(A), an employer–employee relationship exists whenever a person works in the service of another, with no written contract or monetary compensation required.
- Independent practice that competes with the employer for compensation or other benefit is allowed only after the member notifies the employer of the types of services, expected duration and compensation and obtains the employer's consent before the practice begins.
- Until a resignation is effective, a member may prepare to compete, for example by interviewing, leasing office space or registering a new firm, but may not solicit the employer's current or prospective clients, induce colleagues to leave, or copy client lists, models or other files.
- After leaving, and absent a valid non-compete agreement, a member may contact former clients using memory or public sources, while records created or kept for the employer remain its property in any medium.
- Whistleblowing against the employer's interest to protect clients or market integrity is not a breach of loyalty when it is not for personal gain and complies with applicable law.
- Under Standard IV(B), a benefit that competes with the employer's interest, or that might reasonably be expected to create a conflict of interest with it, may be accepted only with written consent, e-mail included, from all parties involved.
- Under Standard IV(C), a supervisor must make reasonable efforts to prevent, detect and act on violations by anyone under his or her supervision or authority, and delegating supervisory duties does not transfer the supervisory responsibility.
- A member asked to supervise where compliance procedures are absent or inadequate must decline in writing to accept supervisory responsibility until the firm adopts adequate procedures.

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QUICK CHECK

Question 1

Oscar Brennan, CFA, is a credit analyst at Halcyon Bank. A friend who runs an independent investment advisory practice asks Brennan to help review her clients' portfolios on evenings and weekends, for a fee. Under the Standard concerning loyalty, under what conditions may Brennan accept?

A Brennan must tell Halcyon the terms of the arrangement and obtain Halcyon's consent.

B Brennan must disclose the extra pay to Halcyon but does not need Halcyon's consent.

C Under none, because the arrangement would put Brennan in competition with Halcyon.

Answer: A

Under Standard IV(A) Loyalty, before engaging in independent practice for compensation, members must notify the employer of the services to be provided, their expected duration and the compensation, and must obtain the employer's consent before starting.

Why the other options are wrong

- **B.** Disclosure alone is not enough; the employer's consent is required before the paid work begins.
- **C.** Independent practice is not prohibited outright; it is allowed once the employer consents.

Key takeaway Paid outside advisory work: notify the employer (services, duration, pay) and obtain consent.

Practice questions

Question 2

Amelia Ross, CFA, has just joined Harrowby Trust as a trust officer. She learns that Harrowby routes all its securities trades through her cousin, a registered broker, whose commissions match the lowest rate available from other brokers. Her cousin tells her that if Harrowby keeps sending him its business, he will give her a large discount on all of her personal trades. Ross:

A must refuse the arrangement.

B must obtain written consent from all parties involved before accepting the arrangement.

C need not inform her employer, because Harrowby is paying the lowest available commissions.

Question 3

Arjun Mehta, CFA, leads a team of credit analysts at a regional bank; none of the analysts is a CFA charterholder or candidate. Mehta has several times urged the bank's senior partners to adopt a compliance system to make sure that applicable laws and regulations are followed, but they have rejected each proposal. Mehta should *most appropriately*:

A take no further action, because by urging the bank to adopt a compliance system he has met his obligations.

B decline in writing to accept supervisory responsibility until an adequate compliance system is adopted.

C report the bank's inadequate system in a written complaint to the CFA Institute Professional Conduct Program.

Answers: 2. B · 3. B

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/duties-to-employers/. The full bank has 61 questions on this reading.

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ETHICAL AND PROFESSIONAL STANDARDS · READING 99

Investment Analysis Recommendations and Actions

What you'll learn

LOS 99.a Apply Standard V (Diligence and Reasonable Basis; Communication with Clients and Prospective Clients; Record Retention) to situations involving investment analysis, recommendations and actions.

LOS 99.b Recommend practices and procedures that help prevent violations of Standards V(A), V(B) and V(C).

LOS 99.c Identify conduct that conforms to Standard V and conduct that violates it.

MODULE 99.1

Guidance for Standards V(A), V(B), and V(C)

This reading covers Standard V, Investment Analysis, Recommendations, and Actions: Diligence and Reasonable Basis, Communication with Clients and Prospective Clients, and Record Retention. A candidate must be able to apply each part to a fact pattern, recommend practices that help prevent violations, and tell conduct that conforms from conduct that violates.

LOS 99.a — Applying Standard V to investment analysis, recommendations and actions

Standard V has three parts, each governing a different stage of the research-and-advice process: doing the work (V(A)), telling clients about it (V(B)) and keeping the evidence (V(C)).

Standard V(A) Diligence and Reasonable Basis

*Members and candidates must: 1. Exercise **diligence, independence, and thoroughness** in analyzing investments, making investment recommendations, and taking investment actions. 2. Have a **reasonable and adequate basis**, supported by **appropriate research and investigation**, for any investment analysis, recommendation, or action.*

How much work is "enough" depends on the member's **investment philosophy**, his or her **role in the investment decision-making process** and the **resources and support** the employer provides; the level of research also differs with the product or service offered. Before recommending or acting, a diligent analyst typically weighs global and national economic conditions, the company's financial results and operating history, the stage of the business cycle, a fund's fees and historical results, the limitations of any quantitative model used, and whether peer-group comparisons are appropriate for valuation.

Relying on others' research. A member may base a recommendation on **third-party research** or on the firm's own research department, but V(A) still requires **reasonable and diligent efforts** to judge whether that research is **sound**: review its assumptions, how rigorous the analysis was, how timely it is, and whether its recommendations are objective and independent. A member who has made that review and found no reason to doubt the research may use it. A rumor, a tip overheard at a lunch, or a single article in the popular press is not a reasonable and adequate basis.

Quantitative research. The **limitations of any quantitative model** used are one of the factors to consider before making a recommendation. Scenario testing is a recommended firm procedure: written procedures should set a minimum level of scenario testing for computer-based models, with standards for the range of scenarios, model accuracy over time and the sensitivity of cash flows to assumptions and inputs. A conclusion drawn from too short a data period, or a blanket instruction to swap one style of stock for another across all accounts regardless of client needs, lacks a reasonable basis.

Pressure to issue a favorable view. If an analyst's own diligent research does not support a positive rating, issuing one anyway (for example, to help the firm's investment banking business) violates V(A), because the published view has no reasonable basis. It also violates Standard I(B) **Independence and Objectivity**, because the analyst let others' interests override his or her judgment.

> *Example.* An analyst has lost all her working papers for a report due tomorrow. She takes the company's own projections, trims them by a figure she "remembers", and pastes in charts from another firm's report without credit. She (1) has no reasonable basis (V(A)), (2) presents others' work as her own (Standard I(C) **Misrepresentation**: **plagiarism**), and (3) issues a report with no supporting records (V(C)).

Standard V(B) Communication with Clients and Prospective Clients

> **KEY CONCEPT**
>
> *Members and candidates must: 1. Disclose to clients and prospective clients the **nature of the services** provided, along with information about the **costs** to the client associated with those services. 2. Disclose to clients and prospective clients the **basic format and general principles of the investment processes** they use to analyze investments, select securities, and construct portfolios and must **promptly disclose any changes** that might materially affect those processes. 3. Disclose to clients and prospective clients **significant limitations and risks** associated with the investment process. 4. Use **reasonable judgment** in identifying which factors are important to their investment analyses, recommendations, or actions and include those factors in communications with clients and prospective clients. 5. **Distinguish between fact and opinion** in the presentation of investment analysis and recommendations.*

Key applications:

- **Reasonable judgment about what to include.** An analyst may emphasize some matters, touch briefly on others and omit some altogether, provided there is a reasonable basis for those choices. A temporary event years ago that was quickly reversed may reasonably be left out; heavy recent selling by insiders is a factor a reasonable investor would want and should be included.
- **Tailoring to the audience.** A report written for a specific group of clients with known needs can generally leave out more basic facts than a report for a wide audience. This flexibility covers background facts only: every research report, whatever its audience, must still describe the **basic characteristics** of the security analyzed. Sending a report only to the clients for whom the idea is relevant is allowed.
- **Fact versus opinion.** Historical data ("earnings have grown about 8% a year for five years") is fact. A projection must be presented as an opinion ("we expect", "could", "our estimate is"). Writing that a future event "will" happen turns an opinion into an apparent fact and violates V(B); so does turning management's optimism into a certainty.
- **Form of communication.** A recommendation may be short (a buy list, an e-mail, a brief note). V(B) does not require a detailed report with every recommendation, but clients must be informed that the full analysis supporting it is available. V(B) does not require analysts to issue explicit buy or sell ratings on every security they cover.
- **Changes in the process.** A material change in strategy, models or style must be communicated promptly, together with any newly identified risks and limitations. The same applies when the risk characteristics of an investment or strategy change significantly.
- **All forms of communication.** V(B) covers every means of communicating with clients (meetings, calls, e-mails, messages), not only written research reports.

- **Costs.** Client-facing members describe the services and their costs when the relationship begins and again whenever the services or costs change. Specific money amounts are not required (a percentage fee may make that impossible), but the cost information must be reasonably detailed, including costs from third parties used to deliver the services.
- **Risks and limitations.** For nontraditional products (such as structured securities or allocation strategies), members must explain the investment-specific risk factors. In all cases, members should clearly communicate potential gains and losses in terms of total returns. Projections from quantitative models must come with the model's limitations and key assumptions; expectations produced by statistical models are not facts. When projected results are shown, they should cover a range of scenarios, including ones that run against the expected trend: a strategy built on a currency's appreciation should also show what happens if that currency depreciates. Two examples of inherent limitations are **liquidity** (the ability to exit an investment without significant extra cost) and **capacity** (the ability of a strategy or vehicle to absorb more money without lowering the returns it can earn).

> *Example.* Brackenhill Partners markets a small-cap fund whose stocks have always been picked by its founding manager. When she retires, stock selection passes to a three-person committee, which also starts adding large-cap shares to the fund. Marketing updates the website and the pitch book for prospects the same week. Existing clients hear about the changes only at their annual reviews, several months later. Both changes can materially affect the investment process, so V(B) required prompt notice to current clients as well as prospects, together with any new risks, such as returns that may now depart further from the fund's small-cap benchmark.

Standard V(C) Record Retention

*Members and candidates must develop and maintain appropriate **records** to support their investment analyses, recommendations, actions, and other investment-related communications with clients and prospective clients.*

Records are the evidence behind V(A). When a client, the firm or a regulator later questions a recommendation or a trade, the file shows what analysis supported it and that the action followed the client's mandate. Without the file, even sound work cannot be demonstrated.

- Records can be in **any medium**: paper, electronic files, e-mails, messages and posts. The Standard does not say where they must be stored, so scanned copies kept off-site are acceptable.
- Records **belong to the firm**. A member who changes employers may not take them without permission. To reissue research at a new firm, the member must **re-create the supporting records** from public sources or information obtained from the covered company. Memory of files left behind is not a record.
- Where a regulator or firm policy sets a retention period, follow it. Where neither applies, the Standard recommends keeping records for **at least seven years**.
- Keeping records is generally the **firm's responsibility**, but a member must make sure the required records exist.

LOS 99.b — Practices and procedures that help prevent violations

KEY CONCEPT

Standard	Recommended practices
V(A)	A policy that research and recommendations must have a basis that can be substantiated as reasonable and adequate; detailed written guidance on research and due diligence; periodic review of third-party research (assumptions, rigor, timeliness, objectivity and independence); measurable criteria for research quality, with analyst pay linked to them; minimum scenario testing for computer-based models (range of scenarios, accuracy over time, sensitivity to inputs); policies for evaluating outside information providers and external advisers, including how often reviews are repeated
V(B)	Because choosing the relevant factors is a judgment call, keep records of the nature of the research and be ready to supply more information if a client or other user of the report asks
V(C)	If no regulation or firm policy sets a period, keep records for at least seven years; record keeping is generally the firm's responsibility

If a member thinks the firm's procedures fall short (for example, records may not be kept long enough), the right step is to **raise the issue with a supervisor or the compliance department** and suggest improvements. Refusing to work and keeping personal copies of firm records at home are not appropriate responses. Taking copies home can itself conflict with Standard IV(A) **Loyalty**.

The Standards call for diligence, attribution of others' work and supporting records. Legal review of each report is not among their requirements or recommended procedures.

LOS 99.c — Conduct that conforms versus conduct that violates

KEY CONCEPT

Situation	Conclusion
Recommendation based on the firm's research department's work, after reviewing it and finding no reason to doubt it	Conforms to V(A)
Recommendation based on an outside broker's report for general distribution, after reviewing it and finding no reason to doubt it	Conforms to V(A)
Changing a rating because of a colleague's offhand comment	Violates V(A)
Trading all discretionary accounts on a vague, unverified overheard rumor after a quick web search	Violates V(A)

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Situation	Conclusion
Research analyst e-mails a buy recommendation to all clients after careful work	Conforms
Using factual data from a recognized statistical service	No attribution needed, but data copied from a colleague's table should be checked for accuracy
"Company X will cut its dividend"	Violates V(B): a forecast stated as fact
A fee not disclosed in the client agreement	Violates V(B): cost of services not disclosed
Reissuing reports from memory at a new employer	Violates V(C)
Warning a client about the price impact of a quick liquidation, then executing	No violation

COMMON EXAM TRAPS

- **Suitability** (Standard III(C)) applies when advising a particular client. A research analyst publishing a general recommendation is judged under V(A); suitability for each recipient is not required.
- A vague, unverified rumor is generally not established **material nonpublic information** (MNPI). Materiality depends on specificity, reliability and likely price impact, so overheard information *can* be MNPI when it is specific and reliable.
- V(A) judges the work done before a recommendation, not how the investment turns out. A loss after careful, well-documented analysis is not a violation, and a gain does not cure a recommendation that rested on a tip or a hunch.

EXAM SHORTCUTS

- Standard V(A) judges the work done before a recommendation, so the investment's later loss or gain does not by itself decide whether the Standard was violated.

BOTTOM LINE

- Standard V(A) requires diligence, independence and thoroughness, and every analysis, recommendation or action must rest on a reasonable and adequate basis backed by appropriate research and investigation.
- How much research is enough depends on the member's investment philosophy, role in the decision-making process and the employer's resources and support.
- A member may rely on third-party or the firm's own research after a reasonable and diligent review of its assumptions, rigor, timeliness, objectivity and independence, but a rumor, an overheard tip or a single popular-press article is not a reasonable basis.
- Standard V(B) requires members to disclose the nature and costs of their services, the general principles and basic format of their investment process with prompt disclosure of material changes, and significant limitations and risks.

- A projection must be presented as an opinion, so writing that a future event will happen violates Standard V(B), while a short recommendation is allowed if clients are told that the full analysis is available.
- Under Standard V(C), records in any medium belong to the firm, and a member who changes employers must re-create supporting records from public sources or the covered company rather than rely on memory.
- Where no regulation or firm policy sets a retention period, records should be kept for at least seven years.
- Suitability under Standard III(C) applies when advising a particular client, while a research analyst publishing a general recommendation is judged under Standard V(A).

QUICK CHECK

Question 1

Marisol Vega, CFA, runs a one-person advisory practice. She frequently bases client recommendations on research bought from several outside providers. The Code and Standards *most likely* require Vega to:

A carry out her own research instead of depending on outside providers.

B name each outside provider whose research supports a recommendation.

C make reasonable efforts to confirm that the outside research has a sound basis.

Answer: C

Under Standard V(A) Diligence and Reasonable Basis, members who rely on third-party research must make reasonable and diligent efforts to determine that the research is sound, for example by reviewing its assumptions, rigor, timeliness and independence. Standard V(A) is about checking that the research is sound; it does not require her to name each provider in her recommendations.

Why the other options are wrong

- **A.** Relying on third-party research is allowed; the Standards do not require members to produce all research themselves.
- **B.** Naming each outside provider is not a requirement of Standard V(A); what the Standard requires is diligence about whether the research is sound.

Key takeaway A member who uses third-party research must make reasonable and diligent efforts to check that it is sound (Standard V(A)).

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Practice questions

Question 2

Ines Moreau, CFA, is writing a report on Aldercrest Power. Over the past year, four of the six utilities in its region cut their dividends roughly in half to fund capital spending, and Moreau reasons that Aldercrest will face similar pressure. Her report states: "Aldercrest will reduce its dividend from $3.20 to $1.60 a share by the third quarter to pay for its capital program. Investors who buy at today's price of about $64 could earn a total return above 18%." With respect to Standard V(B) Communication with Clients and Prospective Clients, Moreau:

A violated the Standard by promising a specific investment return.

B violated the Standard by failing to distinguish opinion from fact.

C did not violate the Standard.

Question 3

Dominic Ashe, CFA, recently left a large bank's research department to open his own equity research boutique. All of the files supporting his earlier reports stayed with the bank. Relying on his detailed memory of those files, Ashe rewrites several of his former reports, publishes them under his new firm's name and signs up several clients as a result. Ashe has not reconstructed any supporting records. Ashe is *most likely*:

A not in violation of any Standard.

B in violation of Standard V(A) Diligence and Reasonable Basis.

C in violation of Standard V(C) Record Retention.

Answers: 2. B · 3. C

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/investment-analysis-recommendations-and-actions/. The full bank has 33 questions on this reading.

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ETHICAL AND PROFESSIONAL STANDARDS · READING 100

Conflicts of Interest

What you'll learn

LOS 100.a Apply Standard VI (Avoid or Disclose Conflicts, Priority of Transactions, Referral Fees) to situations involving professional integrity.

LOS 100.b Recommend practices and procedures that prevent violations of Standard VI, such as disclosure policies, blackout periods and pre-clearance.

LOS 100.c Identify conduct that conforms to Standard VI and conduct that violates it.

MODULE 100.1

Guidance for Standards VI(A), VI(B), and VI(C)

This reading covers Standard VI, Conflicts of Interest: Avoid or Disclose Conflicts, Priority of Transactions, and Referral Fees. A candidate must be able to apply each part to a fact pattern, recommend procedures that prevent violations, and tell conduct that conforms from conduct that violates.

LOS 100.a — Applying Standard VI to real situations

Standard VI groups three rules about **conflicts of interest**: situations in which a member's own interests, or the interests of the member's firm, could pull against the duty owed to clients, prospective clients or the employer.

Standard VI(A) Avoid or Disclose Conflicts

*Members and candidates must **avoid** or make **full and fair disclosure** of all matters that could reasonably be expected to impair their **independence and objectivity** or interfere with respective duties to their clients, prospective clients, and employer. Members and candidates must ensure that such disclosures are **prominent**, are delivered in **plain language**, and communicate the relevant information effectively.*

Actual and potential conflicts should be avoided where that is reasonably possible; a conflict that cannot reasonably be avoided must be disclosed in full. A single line in fine print buried in a long legal appendix does not meet the Standard.

Disclosure lets clients and prospects judge motives and potential biases for themselves. The **most common conflict** requiring disclosure is actual **ownership of stock** in companies the member recommends or that clients hold. Toward the employer, members must give the employer enough information to assess how a conflict affects it, must take reasonable steps to avoid conflicts, and must report any conflict promptly once it arises.

The test is *reasonableness*: would a client or employer reasonably want to know this fact to judge whether the member's advice might be biased? Typical matters that must be disclosed:

KEY CONCEPT

Matter	Why it must be disclosed
Ownership of stock in a company the member recommends or that clients hold, including shares held in a trust in which the member has a financial interest and the power to vote (**beneficial ownership**)	The most common conflict: gains or losses on the position could bias the recommendation

Matter	Why it must be disclosed
Firm's material ownership of the covered issuer's securities	The firm benefits if the price rises
Market maker role, **investment banking / corporate finance relationship**, underwriting, M&A advisory work	The firm has a business interest in the issuer
Serving on the issuer's **board of directors** (member or senior firm officers)	Board members owe duties to the company and its shareholders
Paid consulting work for the covered company	Direct financial link to the issuer
Compensation arrangements that could create incentives at odds with clients' objectives: special compensation arrangements, bonus programs, commissions and incentives (e.g., a bonus tied to short-term trading profits in a long-term fund)	Clients should know how the professional is paid. Disclose when acting in an advisory capacity, and update the disclosure after a significant change in compensation structure

The Standard reaches only matters that could reasonably be expected to impair independence and objectivity or interfere with duties. A fact that gives neither the member nor the firm any financial interest in the issuer or any duty to it does not meet that test. Examples are a neighbor's holding in which the member has no interest, a relative's past job unrelated to the analysis, and firm property unrelated to the issuer.

Disclosure rather than refusal. VI(A) does not require an analyst to decline an assignment or to sell a position in the covered stock; full and fair disclosure of the holding meets the Standard. Inherited shares are treated like purchased shares: the analyst must tell the employer and disclose the holding in subsequent research.

Who needs to hear it. Conflicts that affect duties to the employer (e.g., a board seat or an unpaid committee role at an institution whose assets the firm manages) must be disclosed to the employer; conflicts that could bias advice go to clients and prospects as well. The disclosure goes in every communication the conflict could color: a member paid in a company's shares for promotional work discloses the holding in the promotional material as well as in any research report.

Standard VI(B) Priority of Transactions

*Investment transactions for **clients and employers** must have **priority** over investment transactions in which a Member or Candidate is the **beneficial owner**.*

Client transactions also take priority over transactions made on behalf of the **member's firm** (e.g., proprietary trading). Personal transactions include any transaction in which the member is a **beneficial owner**: the member's own account and any account in which he or she has a financial interest, such as shares held in a trust for the member.

- A member may trade for his or her own account only once clients and the employer have been given adequate opportunity to act on a recommendation (e.g., after client orders have been executed).
- **Front-running** is trading ahead of client orders, or ahead of a recommendation's release, to profit from them. It violates VI(B).
- **Family accounts that are client accounts** (family members who are fee-paying clients of the firm) are treated exactly like other client accounts: they are neither favored nor disadvantaged, and they may trade as soon as the recommendation has been disseminated to clients. If the member also has a beneficial interest in such an

account, the firm's pre-clearance and reporting requirements for personal holdings apply to it as well, and the account still may not be favored over other clients.
- If a security is not suitable for any client (e.g., a speculative stock for a manager whose only responsibility is a high-grade bond fund, or futures that none of the clients may hold), the member may trade it personally without first offering it to clients.
- Members must not act on information about **pending trades** for personal gain; the overriding test is that personal trades **do not disadvantage any client**. By that test, a personal trade is not prohibited merely because it runs against the firm's recommendation, such as selling shares the firm rates "buy".
- If investors ask, members must give them full disclosure of the firm's **personal trading policies**.
- **IPO / hot issues**: members can avoid IPO conflicts by not participating. Members should not take shares of an oversubscribed new issue for their own accounts when that could reduce client allocations, even if clients are told and do not object.

Standard VI(C) Referral Fees

*Members and candidates must disclose to their **employer, clients, and prospective clients**, as appropriate, any compensation, consideration, or benefit **received from or paid to others** for the recommendation of products or services.*

- Both directions count: fees the member receives for referring clients out *and* fees the member pays to others who refer clients in. The payer can be the member's own employer: a bonus for sending clients to another unit of the same firm must still be disclosed to those clients.
- **All types of consideration** must be disclosed; **non-cash benefits** count (free services, gifts, fee discounts); disclose their **nature and estimated value**.
- **Timing**: disclosure must be made **before** the client or prospect enters into an agreement for services, so the client can judge the full cost of the service and any partiality in the recommendation. A packet delivered only after someone has become a client comes too late.
- The Standard does not require disclosing what share of the member's business comes from referrals.

LOS 100.b — Procedures that prevent Standard VI violations

Firms and members should build controls so that conflicts are identified, avoided where possible, and disclosed when not.

Recommended procedures for VI(B):

- If the firm has not yet adopted the procedures below, members should urge it to do so.
- **Limitations on employee participation in equity IPOs.**
- **Restrictions on participation in private placements**: strict limits on employee purchases and proper supervisory procedures, because private placements raise the same kind of conflict as IPOs.
- **Blackout periods** (restricted periods) before client trades for employees involved in investment decision-making, so that no one front-runs client orders. How strict the blackout should be depends on the **size of the firm and the type of security**.
- **Reporting procedures**: **pre-clearance** procedures, duplicate trade confirmations, and disclosure of beneficial ownership positions and personal holdings.

Recommended procedures for VI(C):

- Members should urge their firms to set clear procedures for compensation received for referrals, and should give their employers **updates at least quarterly** on referral arrangements.

- Where a firm allows referral fees, it should have clear **approval procedures** and policies on the **nature and value** of referral compensation received.

COMMON EXAM TRAPS

- Allowing "small" or "limited" front-running is never an acceptable control.
- VI(B) ranks client and employer trades ahead of the member's own; III(B) Fair Dealing requires fair treatment among clients. A manager who allocates profitable block trades to his own account violates VI(B), and pushing the losing trades onto the largest client accounts is likely a III(B) violation as well.

LOS 100.c — Conduct that conforms and conduct that violates

KEY CONCEPT

Conduct	Verdict
Analyst owns the covered stock and discloses the holding in the report	Conforms to VI(A)
Analyst joins an unpaid investment committee of an institution whose assets are managed by her firm and tells no one	Violates VI(A)
Analyst buys, for an account she funds and controls for her child, a stock she has just upgraded, before the report reaches clients	Violates VI(B): she is the beneficial owner of the account, so the trade is hers, and it comes ahead of the clients; small size is irrelevant
Manager advises clients to sell, lets their orders execute, then sells her own position	Conforms to VI(B)
Manager buys for himself just before large client purchases	Violates VI(B) (front-running)
Planner mentions referral payments only in a welcome kit sent after the client signs	Violates VI(C): disclosure must come before the agreement
Member takes an equal share of a hot IPO alongside clients who agree	May violate VI(B) despite consent

Example. Anneliese is a fixed income analyst whose only duty is a municipal bond fund. She finds a small biotech stock she likes for herself. Because the stock could never be held by the fund, VI(B) does not require her to offer it to anyone first; she may buy it (subject to firm pre-clearance rules). If she later started covering biotech stocks, a personal holding would then have to be disclosed under VI(A) and client trades would take priority under VI(B).

Four Standards deal with benefits and conflicts that look alike. The trigger decides which one applies:

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KEY CONCEPT

Trigger	Standard	Required step	Who is told
Client's gift for results already achieved	I(B) Independence and Objectivity	Disclose, before accepting if possible	Employer
Benefit tied to future performance, or outside pay that competes or may conflict with the employer's interest	IV(B) Additional Compensation Arrangements	Written consent before accepting	All parties involved, above all the employer
Holding, relationship or pay scheme that could reasonably be expected to impair independence and objectivity or interfere with duties	VI(A) Avoid or Disclose Conflicts	Avoid where reasonably possible; otherwise disclose	Clients, prospects and employer, as relevant
Compensation received from or paid to others for recommending products or services	VI(C) Referral Fees	Disclose before the client or prospect agrees to the service	Employer, clients and prospects, as appropriate

EXAM SHORTCUTS

- To tell look-alike benefits apart, start from the trigger: a client's gift for results already achieved falls under I(B), a benefit tied to future performance or competing outside pay under IV(B), a holding, relationship or pay scheme that could impair independence and objectivity under VI(A), and pay for recommending products or services under VI(C).

BOTTOM LINE

- Standard VI(A) requires members to avoid conflicts where reasonably possible and otherwise to make full and fair disclosure in prominent, plain-language form, so a line of fine print buried in a legal appendix does not meet it.
- The conflict that most often requires disclosure is the member's own stock ownership in a company he or she recommends or that clients hold, including beneficial ownership through a trust, and disclosing the holding meets the Standard without declining the assignment or selling the stock.
- Market making, investment banking relationships, board seats held by the member or senior firm officers, paid consulting work for the issuer, and compensation arrangements that could create incentives at odds with clients' objectives must be disclosed.
- Under Standard VI(B), transactions for clients and employers have priority over those in which the member is the beneficial owner, client transactions also come before the firm's proprietary trading, and front-running violates the Standard.
- Fee-paying client accounts of family members are treated exactly like other client accounts, neither favored nor disadvantaged.
- A member may trade personally in a security that is not suitable for any client without first offering it to clients, the overriding test being that personal trades do not disadvantage any client.

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- Standard VI(C) requires disclosure of any compensation, consideration or benefit, cash or non-cash, received from or paid to others for recommending products or services, with its nature and estimated value, before the client or prospect enters into an agreement for services.
- Recommended procedures for Standard VI(B) include limits on employee participation in equity IPOs, restrictions on private placements, blackout periods before client trades, and reporting procedures such as pre-clearance and duplicate trade confirmations.

QUICK CHECK

Question 1

Rosalind Mercer, CFA, is an analyst at the investment bank Carrow & Pike. Her manager asks her to initiate coverage of Summit Ridge Ales. Carrow & Pike's corporate finance group has advised Summit Ridge on every acquisition it has made over the last nine years, and two of Carrow & Pike's senior partners serve as Summit Ridge directors. According to the Standards of Professional Conduct, may Mercer write the report?

A Yes, if she keeps her analysis independent and objective.

B No, because the conflicts are too significant.

C Yes, if she discloses the board seats and the advisory work.

Answer: C

Standard VI(A) requires members to disclose to clients and prospects matters that could reasonably be expected to impair objectivity. It does not prohibit an analyst with potential conflicts from writing the report; the investment banking relationship and the directorships must be disclosed.

Why the other options are wrong

- **A.** Independence and objectivity are required, but they do not replace disclosure: clients must be told about the relationships so they can judge the report.
- **B.** The Standard does not bar the analyst from writing the report; it requires the conflicts to be disclosed.

Key takeaway Conflicts rarely forbid the work. They require disclosure.

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Practice questions

Question 2

A brokerage firm is adding blackout periods to its personal-trading procedures for employees involved in investment decision-making. Under the recommended procedures for Standard VI(B) Priority of Transactions, how strict the blackout requirement should be depends *most* on:

A the size of the firm and the type of security.

B each employee's seniority and length of service.

C the number of clients and the average account size.

Question 3

Priya Natarajan is an estate-planning attorney. Colin Ashby, CFA, a wealth manager, regularly sends his clients to her for estate work. In exchange, Natarajan prepares the wills and trust documents for Ashby's own family without charging him. Natarajan has just been awarded the CFA charter and become a member of CFA Institute. Under the Standards, Natarajan must:

A tell prospective clients referred by Ashby about the free legal work and its estimated value.

B tell prospective clients referred by Ashby about the free legal work, but not its value.

C stop doing legal work for Ashby's family without charge.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/conflicts-of-interest/. The full bank has 42 questions on this reading.

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ETHICAL AND PROFESSIONAL STANDARDS · READING 101

Responsibilities as a CFA Institute Member

What you'll learn

LOS 101.a Apply Standard VII (Conduct as Participants in CFA Institute Programs; Reference to CFA Institute, the CFA Designation, and the CFA Program) to situations involving members and candidates.

LOS 101.b Recommend practices that help prevent violations of Standards VII(A) and VII(B).

LOS 101.c Identify conduct and statements that conform to Standard VII and those that violate it.

MODULE 101.1

Guidance for Standards VII(A) and VII(B)

This reading covers Standard VII, the responsibilities of CFA Institute members and CFA candidates: VII(A) governs conduct as participants in CFA Institute programs, and VII(B) governs how members and candidates refer to CFA Institute, the designation and the CFA Program. A candidate must be able to apply both parts to a fact pattern, recommend practices that prevent violations, and tell conduct that conforms from conduct that violates.

LOS 101.a — Applying Standard VII to members and candidates

Standard VII protects the value of CFA Institute programs and of the CFA designation. It has two parts.

Standard VII(A) Conduct as Participants in CFA Institute Programs

*Members and candidates must not engage in any conduct that compromises the **reputation or integrity of CFA Institute or the CFA designation** or the **integrity, validity, or security of CFA Institute programs**.*

Conduct covered by VII(A) includes:

- **Giving or receiving assistance (cheating)** on any CFA Institute exam, or offering to do so (for example, offering someone an advance look at exam questions, even if the offer is refused). Exam convention: cheating on any exam, not only a CFA Institute exam, is treated as VII(A) conduct. Current practice: Standard VII(A) covers conduct that affects CFA Institute, the CFA designation and CFA Institute programs, so cheating on an unrelated exam falls outside it. Such cheating is a Standard I(D) Misconduct matter only when that exam is part of the member's or candidate's professional activities.
- **Violating the rules and policies** of the CFA Program. A candidate who breaks any exam policy (for example, the rules on **calculators**, **personal belongings** or the **Candidate Pledge**) has violated VII(A).
- **Disclosing confidential exam information** to candidates or to the public. This covers specific questions, and also the **broad topic areas or formulas that were tested or not tested**. The obligation continues after the exam and after the testing window closes. Discussing the curriculum itself, as opposed to the exam, in study groups or online forums is allowed.
- **Improperly using the CFA designation** to further personal or professional goals.
- **Misrepresenting information** on the **Professional Conduct Statement** (PCS) or in connection with the CFA Institute Professional Development Program.

VII(A) does not restrict **expressing opinions**. Members and candidates may criticize CFA Institute, the CFA Program, pass rates, exam difficulty or CFA Institute's advocacy positions. An honestly held opinion, even a harsh or mistaken one, is neither dishonesty, fraud or deceit nor an act that reflects adversely on professional

reputation, integrity or competence, so it does not violate Standard I(D) **Misconduct** either.

Members who **volunteer** in the CFA Program (for example, as question writers, graders or proctors) must not ask for, or disclose, information about exam questions (whether under consideration or actually used), the grading process or the way questions are scored.

Standard VII(B) Reference to CFA Institute, the CFA Designation, and the CFA Program

*When referring to CFA Institute, CFA Institute membership, the CFA designation, or candidacy in the CFA Program, Members and Candidates must not **misrepresent or exaggerate** the meaning or implications of membership in CFA Institute, holding the CFA designation, or candidacy in the CFA Program.*

Membership and the right to use the designation. To remain an **active member** and keep the right to use the designation, a charterholder must (1) **pay annual membership dues** and (2) **file the annual Professional Conduct Statement**. A member who does not meet both is not active and may not use the CFA designation: unlike a university degree, the right to use it is not permanent. Nothing prevents a former charterholder from stating the facts of past status (e.g., "I was awarded the CFA charter in 2011"), as long as he or she does not imply the designation is currently held. How many attempts it took to pass the exams has no bearing on membership status.

Candidacy. References to **candidacy in the CFA Program** must not misrepresent or exaggerate what candidacy means. A candidate may describe it accurately, for example by naming the level for which he or she is registered ("Level II candidate in the CFA Program"), and may state facts such as the levels passed, the years in which they were passed, that each level was passed on the first attempt, or that the whole program was completed in three years. Claiming superior ability because of such facts is not permitted.

What is prohibited:

- Implying a **partial designation**: there is no partial CFA designation, so "CFA Level II" or "Level I CFA" after a name is not allowed.
- Making **promotional promises or guarantees tied to the designation**: over-promising individual competence or future investment results (higher returns, lower risk, better service), or claiming superior ability because the exams were passed quickly or on the first attempt. A guarantee of returns is also a **Misrepresentation** regardless of any reference to the designation.

What is allowed: stating that one is a charterholder, describing accurately what the designation requires (three levels of exams, work experience, commitment to the Code and Standards), and stating that holding the charter reflects a commitment to high ethical standards.

Using the designation after a name

KEY CONCEPT

Acceptable	Not acceptable	Why
Kim Ortega, CFA (dues paid and Professional Conduct Statement filed this year)	Kim Ortega, CFA (dues unpaid)	Only an **active member** may use the designation
Kim Ortega, Chartered Financial Analyst (active member)	Kim Ortega, CFA Level III	There is no **partial designation**

LOS 101.b — Practices that help prevent violations

- Errors in references to the designation or to candidacy are common, so members should make sure their **firms are aware of the proper references** to a member's CFA designation or candidacy, and firms should review résumés, business cards, websites and marketing materials before use.

WORKED EXAMPLE: REVIEWING A CANDIDATE'S BIOGRAPHY

Tomas Reyes passed Levels I and II of the CFA Program on his first attempts and is registered for the Level III exam. His firm's draft website biography reads: "Tomas Reyes, CFA Level II. Tomas is a Level III candidate in the CFA Program. Having passed each level on the first attempt, he can deliver higher returns than his peers." Decide which statements conform to Standard VII(B) and how the biography should be corrected.

Step 1. "CFA Level II" after his name implies a partial designation. No partial CFA designation exists, so this violates Standard VII(B).

Step 2. "Level III candidate in the CFA Program" describes his candidacy accurately by naming the level for which he is registered, so it conforms.

Step 3. Stating that he passed each level on the first attempt is a fact and is allowed. Promising higher returns because of it ties a promise of future investment results to his exam record and violates Standard VII(B).

Step 4. Members should make sure their firms know the proper references to the designation and to candidacy, and the firm should review the biography before it is used.

Result. A corrected biography reads: "Tomas Reyes is a Level III candidate in the CFA Program. He passed Levels I and II on his first attempts."

LOS 101.c — Conduct that conforms versus conduct that violates

KEY CONCEPT

Statement or act	Conclusion
"I passed Level II of the CFA Program in 2016."	Conforms (fact, no partial designation)
"I passed all three levels on my first attempt." (true)	Conforms
"Kim Ortega, CFA Level I"	Violates VII(B)
Business card shows "CFA" although annual dues have not been paid	Violates VII(B)
"Earning the charter proves my superior ability."	Violates VII(B)
All analysts are charterholders, "so we give better service than our rivals"	Violates VII(B) only because of the promise of better service; stating the fact is fine
Posting remembered exam questions after the window closes	Violates VII(A)

Statement or act	Conclusion
Blog listing topics that were left off the exam	Violates VII(A)
Public criticism of pass rates or advocacy positions	Conforms
Dues unpaid, Professional Conduct Statement filed	Not active; may not use the designation
False answers on the annual Professional Conduct Statement	Violates VII(A)
Bringing a prohibited item into the exam room	Violates VII(A)

COMMON EXAM TRAPS

- Misstating other résumé facts, such as a college major, is a Standard I(C) Misrepresentation issue; only statements about CFA Institute, the designation or candidacy fall under VII(B). One résumé can breach both.

BOTTOM LINE

- Conduct that breaches Standard VII(A) includes cheating, or offering to help someone cheat, on a CFA Institute exam, breaking CFA Program rules and policies, disclosing confidential exam information, improperly using the CFA designation and misrepresenting information on the Professional Conduct Statement.
- Confidential exam information includes specific questions and the broad topic areas or formulas that were or were not tested, the obligation continues after the testing window closes, and discussing the curriculum itself is allowed.
- Standard VII(A) does not restrict expressing opinions, so criticizing CFA Institute, the CFA Program, pass rates or CFA Institute's advocacy positions is allowed.
- To remain an active member and keep the right to use the CFA designation, a charterholder must pay annual membership dues and file the annual Professional Conduct Statement.
- No partial CFA designation exists, so a title such as "CFA Level II" is not allowed, but a candidate may state facts such as the levels passed, the years they were passed and that each was passed on the first attempt.
- Claiming superior ability, or promising higher returns, lower risk or better service, because of the designation or exam results violates Standard VII(B).
- Stating that one is a charterholder, describing accurately what the designation requires, and stating that holding the charter reflects a commitment to high ethical standards are allowed.
- Members should check that their firms know how to refer correctly to a member's CFA designation or candidacy.

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QUICK CHECK

Question 1

Tariq Haddad, a Level III candidate, takes part in an online forum where candidates discuss curriculum readings in depth during the study season. Four days after his testing window closes, frustrated by difficult questions, he posts several exam questions he remembers and asks other members how they answered them and why. Several other candidates respond by posting questions they remember. Haddad and the other candidates are *most likely*:

A not in violation, because the posts appeared only after the testing window had closed.

B in violation, because they disclosed confidential information about the exam.

C in violation, because they discussed curriculum topics in a public forum ahead of the exam.

Answer: B

Under Standard VII(A) Conduct as Participants in CFA Institute Programs, confidential information about the exam may not be disclosed, and that obligation continues after the exam and after the testing window has closed. Posting remembered exam questions is therefore a violation; discussing the curriculum itself in the forum beforehand was not.

Why the other options are wrong

- **A.** Confidentiality of exam content does not expire when the testing window closes.
- **C.** Discussing the curriculum itself before the exam is permitted; the violation is disclosing exam content.

Key takeaway Curriculum discussion is allowed. Exam content stays confidential with no time limit.

Practice questions

Question 2

A regional broker has just hired Mei Tanaka and Callum Reid. Each cleared both Level I and Level II at the first try, and neither business card refers to the CFA Program. Tanaka's marketing materials accurately describe the requirements of the CFA Program and add: "Because I cleared the first two levels at the first try, my clients can expect better results than other advisers deliver." Reid's materials say only, and truthfully, that he cleared the first two levels at the first try. Has either new hire violated the CFA Institute Standards of Professional Conduct?

A Only one of them has violated the Standards.

B Both of them have violated the Standards.

C Neither of them has violated the Standards.

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Question 3

Anika Sorensen, CFA, learned in March that a national securities regulator had opened an investigation into her trading practices. When she completes her annual Professional Conduct Statement in May, she answers "no" to the question asking whether she is the subject of any regulatory investigation. Sorensen has *most likely* violated:

A Standard VII(A) Conduct as Participants in CFA Institute Programs.

B Standard V(C) Record Retention.

C no Standard, because the investigation has not yet reached a conclusion.

Answers: 2. A · 3. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/responsibilities-as-a-cfa-institute-member/. The full bank has 25 questions on this reading.

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ETHICAL AND PROFESSIONAL STANDARDS · READING 102

Application of the Code and Standards: Level I

What you'll learn

LOS 102.a Evaluate practices, policies and conduct against the CFA Institute Code of Ethics and Standards of Professional Conduct.

LOS 102.b Explain why particular practices, policies and conduct do or do not violate the Code and Standards.

MODULE 102.1

Ethics Application

This reading tests two skills: evaluating practices, policies and conduct against the Code and Standards, and explaining why a given act does or does not violate them.

This reading applies the whole Code and Standards to short cases. It introduces no new rules; the skill tested is recognizing which Standard a fact pattern touches and whether the conduct crosses the line. Three questions help with every case: (1) whose interests are at stake (market, clients, employer, CFA Institute)? (2) which fact decides the outcome? (3) what should the member have done instead?

LOS 102.a — Evaluating practices, policies and conduct against the Code and Standards

Standard I: Professionalism

KEY CONCEPT

Standard	Key judgment in application cases
I(A) **Knowledge of the Law**	On learning of illegal or unethical conduct, the member first tries to stop it through supervisors or compliance. If that fails, the member must **dissociate** (e.g., ask to be reassigned, refuse to work on the affected clients, resign from a board). A partial fix is not enough: if some clients are still affected, the member must not keep working on those accounts. The Code and Standards do not require reporting to regulators unless the law requires it, although it may be advisable. Ignoring red flags (e.g., money-laundering indicators because the client is well connected) or forging signatures for convenience is a violation.
I(B) **Independence and Objectivity**	Gifts, political contributions or benefits intended to win business or influence judgment violate the Standard.
I(C) **Misrepresentation**	No guaranteeing returns on risky investments; no omission of material facts (e.g., key staff named in a proposal who have since left must be disclosed); no false statements such as claiming financing that does not exist. **Plagiarism**: charts, text or models copied from others must be attributed. Two exceptions: projections, statistics and tables from **recognized financial and statistical reporting services** may be used without acknowledgment, and models or research by colleagues at the member's own firm may be used without naming colleagues who have left, as long as the member does not present that work as solely his or her own.

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Standard	Key judgment in application cases
I(D) **Misconduct**	Has two parts: no professional conduct involving **dishonesty, fraud, or deceit**, and no act that reflects adversely on professional reputation, integrity, or competence. The second part reaches only work-related professional responsibilities and professional activities; acts outside the member's professional capacity are not covered. Minor offenses committed during civil disobedience to express personal beliefs are not necessarily a violation. Using an error-correction process to put personal money into a client account to flatter performance is deceit.
I(E) **Competence**	Accepting a new role (e.g., supervisory duties) requires taking steps to gain the needed competence.

Standard II: Integrity of Capital Markets

- II(A) **Material Nonpublic Information.** Overheard takeover news, or a regulator's reaction shared with a small group of analysts (**selective disclosure**), may not be acted on. The **mosaic theory** permits acting on a conclusion built from public information and nonmaterial nonpublic observations.
- II(B) **Market Manipulation.** Violations include misleading the market about liquidity (e.g., listing fake shareholders to meet exchange requirements), spreading false information, and trading to create artificial prices or volume. **Block trades** to limit price impact in a thinly traded stock are legitimate.

Standard III: Duties to Clients

- III(A) **Loyalty, Prudence, and Care.** Members cannot contract out of their duties to clients: client agreements may not excuse the firm from acting in clients' best interests or restrict clients' legal claims. For a self-directed, execution-only account the duties are narrower than in an advisory relationship, but the member must still act in good faith and describe the services honestly. How the firm will act if the account has a margin shortfall is a term the client can negotiate when the account is opened. A client may be charged expenses incurred to lower that client's total costs, but never expenses that benefit other clients or the member personally. **Soft dollars** (client brokerage) may pay only for research and services that benefit clients. Office furniture and personal expenses do not qualify.
- III(B) **Fair Dealing.** Recommendation changes must be disseminated fairly to all clients for whom they are relevant, at the same time or as close to it as practicable; answering follow-up questions afterward is acceptable. A fee-based premium service is allowed if all clients know it exists and it never delivers recommendation changes ahead of other clients; disclosing the service does not make early delivery acceptable.
- III(C) **Suitability.** Investigate a client's circumstances before acting, even on a client-requested change. Tax advantages do not excuse excessive risk.
- III(D) **Performance Presentation.** Do not present separate-account composites as a new fund's long track record. A **brief presentation** must make detailed information available on request and say that it offers only limited information.
- III(E) **Preservation of Confidentiality.** Protect client data; downloading client files to unsecured personal systems is a breach, and a compliance head whose controls allowed such a download may also be in breach. *Exception:* confidentiality does not prevent a member from cooperating with a CFA Institute **Professional Conduct Program (PCP)** investigation, because the PCP keeps the information confidential.

Standard IV: Duties to Employers

- IV(A) **Loyalty.** While still employed, a member may not disparage the current employer or solicit for a future employer, and may not take client lists or other employer property (even to send thank-you notes). Before performing outside work that could compete with the employer, including advisory work for which any benefit is received, the member must obtain the employer's permission. Whistleblowing to regulators to protect clients is not a loyalty violation.
- IV(B) **Additional Compensation Arrangements.** Any benefit that competes with or could create a conflict with the employer's interest (a bonus from a company seeking coverage; perks from a broker tied to the volume of employer business) requires **written consent** from all parties involved. When the outside party has already made its offer in writing, the employer's written consent is what remains.
- IV(C) **Responsibilities of Supervisors.** Make reasonable efforts to prevent and detect violations by all persons supervised, including those who are not CFA charterholders: distribute the compliance program, send periodic reminders, and include professional conduct in performance reviews. Without adequate procedures or authority, **decline supervisory responsibility** in writing.

Standard V: Investment Analysis, Recommendations, and Actions

- V(A) **Diligence and Reasonable Basis.** Analysis must be independent and thorough; relying on another analyst's unsupported work is also a violation. Numbers supplied by company management must be reviewed critically. Research from third parties may be used after a reasonable and diligent review of whether it is sound: its assumptions, the rigor of the analysis, how timely it is, and the objectivity and independence of its recommendations. Firms should set criteria for evaluating external advisers and outside information providers and say how often the reviews are repeated.
- V(B) **Communication with Clients and Prospective Clients.** Disclose methodology changes and distinguish fact from opinion. Reports need not give every area equal emphasis.
- V(C) **Record Retention.** Keep written client records up to date.

Standard VI: Conflicts of Interest

- VI(A) **Avoid or Disclose Conflicts.** Disclose personal holdings in stocks one covers (to the employer, and in research sent to clients) and time-consuming outside roles such as a trusteeship of a large endowment (to the employer); disclose payments from subadvisers to clients.
- VI(B) **Priority of Transactions.** Clients and the employer come first. Front-running client orders, tipping friends and family ahead of client trades, allocating winning block trades to personal accounts after the close (losers to large clients), or giving family members priority are violations. Personal transactions, those in which the member is a beneficial owner, follow client and employer transactions. Family accounts that are fee-paying client accounts are treated like any other client account, neither favored nor disadvantaged.
- VI(C) **Referral Fees.** Disclose to clients and prospects any benefit received for referrals and any benefit given to others for referring clients (including a share of fees paid to a bank or landlord that refers clients, and rewards such as fee discounts, gifts or hospitality for clients who refer others).

Standard VII: Responsibilities as a CFA Institute Member or CFA Candidate

- VII(A) Conduct as Participants in **CFA Institute Programs.** Candidates may share general impressions of difficulty, but not specific questions or **which topics were or were not tested**. Soliciting such information is also a violation.

- VII(B) Reference to CFA Institute, the **CFA Designation**, and the **CFA Program.** There is no "CFA Level I" or "CFA Level II" designation. A candidate may state that he or she is a "Level II Candidate in the CFA Program" or has passed a given level. A member whose dues are unpaid may not use the designation, and a firm may not describe all its senior staff as charterholders if any of them is not an active member.

LOS 102.b — Explaining why conduct does or does not violate the Standards

Most application questions offer three outcomes: *no violation*, *violation of the right Standard*, and *violation of a plausible but wrong Standard* (or the right Standard for the wrong reason). The explanation must name the deciding fact:

Some Standards cover neighboring ground. Ask what the conduct is about: a misstatement of any kind, performance figures, or what clients are told about the process; fairness among clients, or the member's own trades against clients'. (Gifts, outside pay, conflicts and referral fees are compared in the Standard VI notes.)

Standards that are often confused

Standard	What it governs	Typical breach
I(C) Misrepresentation	Any knowing misstatement or omission in professional activities, including plagiarism and false guarantees	Promising a fixed return on a stock fund; copying another analyst's chart without credit
III(D) Performance Presentation	Performance information for clients and prospects: fair, accurate and complete	Showing only the best accounts as the firm's record; implying past returns will be repeated
V(B) Communication with Clients and Prospective Clients	Services and costs, the investment process and changes to it, risks and limitations; fact versus opinion	Not telling clients about a switch to a new valuation model; stating a forecast as fact
III(B) Fair Dealing	Fairness among clients when disseminating recommendations and allocating trades	Giving favored clients a rating change first; allocating the best fills to the largest accounts
VI(B) Priority of Transactions	Order of trades: clients and employer before accounts the member beneficially owns	Buying for one's own account ahead of a client order or a pending recommendation

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KEY CONCEPT

Deciding fact in the stem	Typical conclusion
Information came from a general newswire	Public, so II(A) is not the issue; look at priority of transactions or fair dealing instead
Member told a relative before acting for clients or employer	VI(B) **Priority of Transactions**
Member receives a benefit tied to employer business	IV(B): obtain written consent
A client's bonus rewards results already achieved and carries no condition on future performance	I(B): a client gift, disclosed to the employer; IV(B) written consent applies when the bonus depends on future performance
Member refers clients or shares fees for referrals	VI(C): disclose to clients and prospects
Member serves on outside boards or owns stock under coverage	VI(A): disclose to the employer, and to clients and prospects when it could affect research or advice
Management revised the analyst's numbers	V(A): the analyst must review them independently
Only general topics of the exam were shared	Still VII(A) if they reveal which topics were tested or not tested

Example. An analyst learns from a radio broadcast that an issuer will buy back shares at a premium, first come, first served. She tenders her sister's shares first and the pension clients' shares an hour later. The information is public, so II(A) is not the issue. Putting a family account ahead of clients violates VI(B) Priority of Transactions.

COMMON EXAM TRAPS

- "Resign" or "refuse the benefit" is usually too extreme when disclosure or written consent solves the conflict.
- The mosaic theory makes many "wait until it is announced" answers wrong when the pieces were nonmaterial or public.

EXAM SHORTCUTS

- If the information in a stem came from a general newswire, it is public, so Standard II(A) is not the issue and the answer usually lies in priority of transactions or fair dealing.

BOTTOM LINE

- Application questions test which Standard a fact pattern touches and whether the conduct crosses the line, and the explanation must name the fact that decides the outcome.

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- Under Standard I(A), a member who cannot stop a violation through supervisors or compliance must dissociate, and a partial fix that leaves some clients affected is not enough.
- Members cannot contract out of their duties to clients, so a client agreement may not excuse the firm from acting in clients' best interests or restrict clients' legal claims.
- Soft dollars may pay only for research and services that benefit clients, never for office furniture or personal expenses.
- A fee-based premium service is allowed if all clients know it exists and it never delivers recommendation changes ahead of other clients, and disclosing the service does not make early delivery acceptable.
- Under Standard IV(B), a benefit that competes with or could create a conflict with the employer's interest needs written consent from all parties involved, and when the outside party has already made its offer in writing, the employer's written consent is what remains.
- Putting a family member's account ahead of clients violates Standard VI(B) even when the information acted on is public.
- Candidates may share general impressions of exam difficulty but not specific questions or which topics were or were not tested, and soliciting such information also violates Standard VII(A).

QUICK CHECK

Question 1

Kwame Mensah, CFA, finds that his manager has breached a national securities regulation. Mensah raises the matter with his firm's compliance department, which does nothing about it. Under the Code and Standards, Mensah is required to:

A report the breach to the securities regulator.

B confront his manager and try to stop the breach.

C dissociate himself from his manager's activity.

Answer: C

Under Standard I(A) Knowledge of the Law, Mensah has already tried to stop the violation by going to compliance. Because that failed, he must dissociate from the activity; one way is to request a move away from the work involved. Reporting to regulators is not something the Code and Standards demand unless applicable law requires it, and a personal confrontation with his manager is not required either.

Why the other options are wrong

- **A.** Reporting to regulators may be advisable, but the Code and Standards do not require it unless the law does.
- **B.** He has already made a reasonable attempt to stop the violation through compliance; a personal confrontation is not required.

Key takeaway Escalate internally first; if that fails, dissociate. Reporting to authorities is required only when the law requires it.

Practice questions

Question 2

Dmitri Volkov, CFA, an investment adviser, uses soft dollars generated by client trades to pay for two things: a subscription to a macroeconomic research service that he uses in managing client portfolios, and new leather chairs for the reception area where he meets clients and prospects. Under Standard III(A) Loyalty, Prudence, and Care:

A only one of the two purchases is a violation.

B both of the two purchases are violations.

C neither of the two purchases is a violation.

Question 3

Imogen Achterberg, CFA, runs portfolios for about 350 private clients and has little spare research capacity. Several of her largest clients ask her to add a 4% to 6% allocation to emerging market local-currency bonds, so she plans to base her bond selections on reports bought from an outside research provider that specializes in that sector. In judging whether the provider's research gives her a reasonable basis, the factor Achterberg should *least likely* rely on is:

A the assumptions behind the provider's analysis and how rigorous that analysis is.

B the high return its model portfolio earned last year.

C how timely the research is and whether its recommendations are objective and independent.

Question 4

Tariq Haddad and Lena Vogt each cleared the Level I exam and have registered to sit for Level II. The headline of Haddad's online professional profile reads "Tariq Haddad, CFA Level I." Vogt's résumé describes her as a "Level II Candidate in the CFA Program." Under Standard VII(B) Reference to CFA Institute, the CFA Designation, and the CFA Program:

A Haddad violated the Standard, but Vogt did not.

B Vogt violated the Standard, but Haddad did not.

C Both Haddad and Vogt violated the Standard.

Answers: 2. A · 3. B · 4. A

Why each answer is right and why each other option is wrong: credguild.com/cfa/level-1/notes/ethics/application-of-the-code-and-standards-level-i/. The full bank has 28 questions on this reading.

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The notes are the start. These are free on the site and do what a PDF cannot:

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